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MATHS
TUTORIAL-3

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Q1 Following data gives age and blood pressure of 8 women. Find co-efficient of correlation.

ANS Let age be represented by x and blood pressure be represented by y.

x (Age)	y (B.P)	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
56	147	5.5	7	30.25	49	38.5
42	125	-8.5	-15	72.25	225	127.5
36	118	-14.5	-22	210.25	484	319
47	128	-3.5	-12	12.25	144	42
49	145	-1.5	5	2.25	25	-7.5
42	140	-8.5	0	72.25	0	0
60	155	9.5	15	90.25	225	142.5
72	162	21.5	22	462.25	484	473
$\Sigma x_i = 404$	1120			952	1636	1135

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{404}{8} = 50.5 \quad ; \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{1120}{8} = 140$$

$$r = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2 \, \Sigma (y_i - \bar{y})^2}}$$

$$= \frac{1135}{\sqrt{952 \times 1636}}$$

$$\boxed{r = 0.9095}$$

$\therefore$ Coefficient of correlation is 0.9095.

Q2 Find Rank correlation coefficient between height and weight from following data.

ANS Let height be represented by x and weight be represented by y.

x	y	r_1	r_2	$r_1 - r_2$	d_i^2
48	11	4	1	3	9
40	13	1.5	3	-1.5	2.25
45	14	3	5.5	-2.5	6.25
50	16	6	9.5	-3.5	12.25
55	16	9	9.5	-0.5	0.25
55	15	9	7.5	2.5	2.25
55	15	9	7.5	1.5	2.25
50	14	6	5.5	0.5	0.25
50	13	6	3	3	9
40	13	1.5	3	-1.5	2.25
					$\Sigma d_i^2 = 46$

40 repeated twice $\rightarrow m_1 = 2$

50 repeated thrice $\rightarrow m_2 = 3$

55 repeated thrice $\rightarrow m_3 = 3$

13 repeated thrice $\rightarrow m_4 = 3$

14 repeated twice $\rightarrow m_5 = 2$

16 repeated twice $\rightarrow m_6 = 2$

15 repeated twice $\rightarrow m_7 = 2$

Now,
$$r = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m) + \frac{1}{12}(m_2^3 - m) + \frac{1}{12}(m_3^3 - m) + \frac{1}{12}(m_4^3 - m) + \frac{1}{12}(m_5^3 - m) + \frac{1}{12}(m_6^3 - m) + \frac{1}{12}(m_7^3 - m)\right]}{n^3 - n}$$

$$\therefore r = 1 - \frac{6\left[46 + \frac{6+24+24+24+6+6+6}{12}\right]}{10^3 - 10}$$

$$= 1 - \frac{6[46+8]}{990}$$

$$= 1 - \frac{324}{990}$$

$$= \frac{666}{990}$$

$$= \frac{37}{55}$$

$\therefore r = 0.6727.$

$\therefore$ Rank correlation coefficient is 0.6727

Q3 Find the equations of two regression lines for the following data.

ANS

x_i	y_i	$x_i y_i$	x_i^2	y_i^2
1	2	2	1	4
2	5	10	4	25
3	3	9	9	9
4	8	32	16	64
5	7	35	25	49
$\Sigma x_i =$ 15	25	88	55	151

let equation of line of regression of y on x be,

$$y = a + bx \quad \text{———} (1)$$

Normal equations are,

$\Sigma y = na + b\Sigma x$, $\Sigma xy = a\Sigma x + b\Sigma x^2$

$25 = 5a + 15b$ —(2) , $88 = 15a + 55b$ —(3)

Solving equation (2) and (3) we get,

$a = \frac{11}{10}$, $b = \frac{13}{10}$

Substitute values of a and b in equation (1)

$\therefore$ Equation of regression line y on x, $\boxed{y = \frac{11}{10} + \frac{13}{10}x}$ —(I)

let equation of regression of line of x on y be,

$x = a_1 + b_1 y$ —(4)

Normal equations are,

$\Sigma x = a_1 n + b_1 \Sigma y$, $\Sigma xy = a_1 \Sigma y + b_1 \Sigma y^2$

$15 = 5a_1 + 25b_1$ —(5) $88 = 25a_1 + 151b_1$ —(6)

Solving equation (5) and (6)

$a_1 = \frac{1}{2}$, $b_1 = \frac{1}{2}$

Subvalue of a_1 and b_1 in equation (4)

$\therefore$ Equation of regression line x on y, $\boxed{x = \frac{1}{2} + \frac{1}{2}y}$ —(II)

(1) Most probable value of y when $x = 10$

From (I)

$$y = \frac{11}{10} + \frac{13}{10}x = \frac{11}{10} + \frac{13}{10}(10) = \frac{141}{10}$$

$\therefore y = 14.1$

(2) Most probable value of x when $y = 12$

From (II)

$$x = \frac{1}{2} + \frac{1}{2}(12) = \frac{13}{2}$$

$\therefore x = 6.5$

(3) We know that $r = \sqrt{b_{xy}\, b_{yx}}$

$$\bar{x} = \frac{\Sigma x_i}{n} \qquad \bar{y} = \frac{\Sigma y_i}{n}$$

$$\bar{x} = 3 \qquad \bar{y} = 5$$

Now $b_{yx} = \dfrac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \dfrac{88 - 5(3)(5)}{55 - 5(9)} = \dfrac{88 - 75}{55 - 45}$

$b_{yx} = \dfrac{13}{10}$

Now $b_{xy} = \dfrac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma y^2 - n\bar{y}^2} = \dfrac{88 - 5(3)(5)}{151 - (5)(25)} = \dfrac{88 - 75}{151 - 25} = \dfrac{13}{26}$

$\therefore b_{xy} = \dfrac{1}{2}$

Now, $r = \sqrt{\dfrac{13}{10} \times \dfrac{1}{2}} = 0.8062.$

$\therefore$ Coefficient of correlation is 0.8062.

Q4 If two regression equation are $4x - 5y - 33 = 0$ and $20x - 9y - 107 = 0$. Find correlation coefficient between x & y. Also find value of x when $y = 15$. Find standard deviation of x if variance of $y = 16$

ANS let us assume equation of regression of line of y on x be,

$$4x - 5y - 33 = 0$$
$$\therefore 5y = 4x - 33$$
$$\therefore y = \frac{4}{5}x - \frac{33}{5} \quad \text{——} (1)$$

let us assume equation of regression of line of x on y be,

$$20x - 9y - 107 = 0$$
$$20x = 9y + 107$$
$$\therefore x = \frac{9}{20}y + \frac{107}{20} \quad \text{——} (2)$$

Equation of regression line y on x is given by,

$$(y - \bar{y}) = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$
$$\therefore y - \bar{y} = b_{yx}x - b_{yx}\bar{x} \quad \left[\because r\frac{\sigma_y}{\sigma_x} = b_{yx}\right]$$
$$\therefore y = b_{yx}x - b_{yx}\bar{x} + \bar{y}$$

$\because b_{yx}$, $\bar{x}$ and $\bar{y}$ are constants.

$$\therefore y = b_{yx}x + c \quad \text{——} (3) \quad [c = \bar{y} - b_{yx}\bar{x}]$$

Comparing (1) and (3)

$$\therefore b_{yx} = \frac{4}{5} \quad \text{——} (4) \quad [b_{yx} > 0]$$

Equation of regression line x on y is given by

$$(x-\bar{x}) = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$x-\bar{x} = b_{xy}y - b_{xy}\bar{y} \qquad \left[\because r\frac{\sigma_x}{\sigma_y} = b_{xy}\right]$$

$$x = b_{xy}y - b_{xy}\bar{y} + \bar{x}$$

$\because$ $b_{xy}, \bar{x}, \bar{y}$ are constants.

$$x = b_{xy}y + c \quad \text{—(5)} \qquad [c = \bar{x} - b_{xy}\bar{y}]$$

comparing (2) and (5)

$$\therefore b_{xy} = \frac{9}{20} \quad \text{—(6)} \qquad [b_{xy} > 0]$$

$$r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\frac{4}{5} \times \frac{9}{20}} = \frac{3}{5} = 0.6 \quad \text{—(7)} \qquad [\because b_{xy}, b_{yx} > 0]$$

$\therefore$ Our assumption is correct and coefficient of correlation between x and y is 0.6.

(2) Value of x when $y = 15$

From (2), $x = \frac{9}{20}y + \frac{107}{20}$

$$\therefore x = \frac{9}{20} \times 15 + \frac{107}{20}$$

$$\therefore x = \frac{242}{20}$$

$$x = 12.1$$

(3) Standard deviation of x if variance $y = 16$

$\sigma_y^2 = 16$ $\therefore \sigma_y = 4$

we know that, $b_{yx} = r \frac{\sigma_y}{\sigma_x}$

$\frac{4}{5} = \frac{3}{5} \times \frac{4}{\sigma_x}$ [From (4) and (7)]

$\therefore \sigma_x = 3$

$\therefore$ Standard deviation of x is 3.