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DISCRETE STRUCTURES — Junaid. Girkar

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Q1 If A, B, C are the sets of the letters in the words 'college', 'marriage' and 'luggage'. verify $[A - (B \cup C)] = [(A-B) \cap (A-C)]$

ANS Set A contains $\{c, o, l, e, g\}$
Set B contains $\{m, a, r, i, g, e\}$
Set C contains $\{L, u, a, g, e\}$

$\therefore (B \cup C) = \{m, a, r, i, g, e, l, u\}$
$[A - (B \cup C)] = \{c, o\} \longrightarrow (1)$

$(A - B) = \{c, o, l\}$
$(A - C) = \{c, o\}$
$\therefore [(A-B) \cap (A-C)] = \{c, o\} \longrightarrow (2)$

From (1) and (2)
$A - (B \cup C) = (A-B) \cap (A-C)$
Hence verified.

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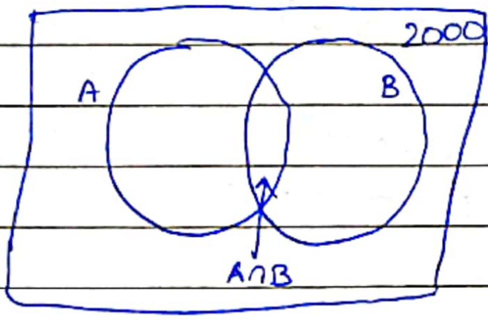
Q2 From amongst 2000 literate individuals of a town. 70% read marathi, 50% read English and 32.5% read Marathi & English. Find:-
(i) At least one of the newspapers
(ii) Neither marathi nor english.
(ii) only one.

ANS literate people count = 2000
People who read marathi = 70% of 2000 = 1400
People who read English = 50% of 2000 = 1000
People who read both = 32.5% of 2000 = 650



$n(A) = 1400$; $n(B) = 1000$; $n(A \cap B) = 650$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 1400 + 1000 - 650$

(i) $\boxed{= 1750}$

People reading neither $= U - n(A \cup B)$
$= 2000 - 1750$

(ii) $\boxed{= 250}$

People reading only one $= n(A \cup B) - n(A \cap B)$
$= 1750 - 650$

(iii) $\boxed{= 1100}$

ANS (i) 1750 people read atleast one newspaper.
(ii) 250 people read neither Marathi nor English newspaper
(iii) 1100 people read only one newspaper.

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Q3 What is power set ?
let $S = \{1, 2, 3\}$ then find power set of S.

ANS For a given set S, Power set P(S) represents the set containing all possible subsets of S as its elements. Power set contains 2^n elements

$S = \{1, 2, 3\}$
$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Q4 Verify the statement using laws of logic: $\overline{A \cap B} \cup (\bar{A} \cap B \cap C) = U$

ANS LHS = $\overline{A \cap B} \cup (\bar{\bar{A}} \cap \bar{B} \cap C)$

$= (\bar{A} \cup \bar{B}) \cup (A \cup B \cup \bar{C})$ [De Morgan's Law]

$= \bar{B} \cup \bar{A} \cup (A \cup B \cup \bar{C})$ [Commutative law]

$= \bar{B} \cup (\bar{A} \cup A) \cup B \cup \bar{C}$ [Associative law]

$= \bar{B} \cup (U) \cup B \cup \bar{C}$ [complement law]

$= (U \cup B) \cup \bar{C}$ [Domination law]

$= U \cup \bar{C}$ [Domination law]

$= U$ [Domination law]

$\therefore$ LHS = RHS
Hence Proved

Q5 Prove that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.

ANS

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow q \wedge q \rightarrow r$	$p \rightarrow r$	Answer.
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore [(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.

Q6 Explain quantifiers with example

ANS In predicate logic, predicates are used alongside quantifiers to express the extent to which a predicate is true over a range of elements. Using quantifiers to create such propositions is called quantification. There are 2 types of quantification:-

• UNIVERSAL QUANTIFICATION ($\forall$): Mathematical statements sometimes assert that a property is true for all the values of a variable in a particular domain, called domain of discourse. Such a statement is expressed using universal quantification

EXAMPLE: Let $P(x)$ be the statement "$x + 2 > x$".

As $x+2$ is always greater than x, $P(x) \equiv T$ for all x

$\therefore \forall x \; P(x) \equiv T$

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• EXISTENTIAL QUANTIFICATION ($\exists$): Some mathematical statements assert that there is an element with a property. Such statements are expressed by existential quantifiers.
Existential quantification can be used to form a proposition that is true if and only if $P(x)$ is true for atleast one value of x in the domain.

EXAMPLE: let $P(x)$ be the statement '$x > 5$'
$P(x)$ is true for all real numbers greater than 5 and false for all real numbers less than 5.

$\therefore \exists x\, P(x) \equiv T$ where $x > 5$.

Statement	when True	when False.
$\forall P(x)$	$P(x)$ is true for all x	There is an x for which $P(x)$ is false.
$\exists P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for all x.

Q7 Prove for $n >= 1$, $1.2 + 2.3 + \ldots n(n+1) = n(n+1)(n+2)/3$

ANS STEP 1: For $n = 1$

$LHS = 1.2 = 2$

$RHS = 1(2)(3)/3 = 2$

$\therefore LHS = RHS$

$\therefore P(1)$ is true.

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STEP 2: let us assume that P(n) is true for n = K

$\therefore 1.2 + 2.3 + \dots + K(K+1) = K(K+1)(K+2)/3 \longrightarrow (1)$

STEP 3: For n = K + 1

we have to prove that, $1.2 + 2.3 + \dots + K(K+1) + (K+1)(K+2) = (K+1)(K+2)(K+3)/3$

$LHS = 1.2 + 2.3 + \dots + K(K+1) + (K+1)(K+2)$

From (1)

$LHS = \frac{K(K+1)(K+2)}{3} + (K+1)(K+2) = (K+1)(K+2)\left(\frac{K}{3} + 1\right)$

$= (K+1)(K+2)(K+3)/3$

$RHS = (K+1)(K+2)(K+3)/3$

$\therefore$ LHS = RHS

$\therefore$ P(K+1) is true

$\because$ P(K+1) is true only when P(K) is true.

Hence our assumption is correct.

$\therefore$ By principle of mathematical induction P(n) is true for all n >= 1

$\therefore 1.2 + 2.3 + \dots + n(n+1) = [(n)(n+1)(n+2)]/3$

Hence Proved.