MATHS TT 1

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Q1 Find laplace transform of $f(t) = \sin^6 t$

ANS

$$\sin t = \frac{e^{it} - e^{-it}}{2i}$$

$$\sin^6 t = \left(\frac{e^{it} - e^{-it}}{2i}\right)^6$$

$$= \frac{1}{64 i^6}\left[e^{6it} - 6e^{4it} + 15e^{2it} - 20 + 15e^{-2it} - 6e^{-4it} + e^{-6it}\right]$$

$$= \frac{-1}{64}\left[(e^{6it} + e^{-6it}) - 6(e^{4it} + e^{-4it}) + 15(e^{2it} + e^{-2it}) - 20\right]$$

$$= \frac{-1}{64}\left[2\cos 6t - 6\cos 4t \times 2 + 30\cos 2t - 20\right]$$

$$= \frac{-1}{32}\left[\cos 6t - 6\cos 4t + 15\overset{\cos 2t}{\cancel{\sin 2t}} - 10\right]$$

$$L\{\sin^6 t\} = \frac{-1}{32} L\left\{\cos 6t - 6\cos 4t + 15\overset{\cos 2t}{\cancel{\sin 2t}} - 10\right\}$$

$$L\{\cos at\} = \frac{s}{s^2+a^2}$$

$$\therefore L\{\sin^6 t\} = \frac{-1}{32}\left[L\{\cos 6t\} - 6L\{\cos 4t\} + 15L\{\cos 2t\} - 10L\{1\}\right]$$

$$= \frac{-1}{32}\left[\frac{s}{s^2+36} - \frac{6s}{s^2+16} + \frac{15s}{s^2+4} - \frac{10}{s}\right]$$

$$= \frac{1}{32}\left[\frac{10}{s} + \frac{6s}{s^2+16} - \frac{15s}{s^2+4} - \frac{s}{s^2+36}\right]$$ //

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Q2 To find laplace transform of $f(t) = t\sin 2t.\cosh t$

let $f(t) = \sin 2t$

$$L\{f(t)\} = L\{\sin 2t\} = \frac{2}{s^2+4} \longrightarrow (1)$$

$$L\{tf(t)\} = (-1)^1 \frac{d^1}{ds^1}\left(\frac{2}{s^2+4}\right) = \frac{(-2)(-2s)}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2} \longrightarrow (2)$$

$$L\{g(t)\} = \frac{4s}{(s^2+4)^2} = \bar{g}(s)$$

$$\cosh t = \frac{e^t + e^{-t}}{2}$$

$$\therefore L\{g(t)\cosh t\} = \frac{1}{2} L\{g(t)e^t + g(t)e^{-t}\}$$

$$= \frac{1}{2}\left[\bar{g}(s-1) + \bar{g}(s+1)\right]$$

$$= \frac{1}{2}\left[\frac{4(s-1)}{((s-1)^2+4)^2} + \frac{4(s+1)}{((s+1)^2+4)^2}\right]$$

$$= \frac{2(s-1)}{((s-1)^2+4)^2} + \frac{2(s+1)}{((s+1)^2+4)^2}$$

$$\therefore L\{t\sin 2t.\cosh t\} = \frac{2(s-1)}{((s-1)^2+4)^2} + \frac{2(s+1)}{((s+1)^2+4)^2}$$

$$= 2\left[\frac{s-1}{((s-1)^2+4)^2} + \frac{s+1}{((s+1)^2+4)^2}\right]$$ //

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Q3 Find $L\left[\cosh t \int_0^t e^t \cosh t \, dt\right]$

let $f(t) = \cosh t$

$\therefore L\{f(t)\} = \dfrac{s}{s^2 - a^2} = \bar{f}(s)$ $\quad [a = 1]$

$L\{e^t f(t)\} = \bar{f}(s-1) = \dfrac{s-1}{(s-1)^2 - a^2} = \dfrac{s-1}{s^2 - 2s + 1 - 1^2} = \dfrac{s-1}{s^2 - 2s} = \bar{g}(s)$

$L\{g(t)\} = \bar{g}(s)$

$\therefore L\left\{\int_0^t g(t)\,dt\right\} = \dfrac{1}{s}\bar{g}(s) = \dfrac{1}{s}\dfrac{(s-1)}{s(s-2)}$

$h(t) = \dfrac{s-1}{s^3 - 2s^2} = \bar{h}(s)$

$\therefore L\{\cosh t \cdot h(t)\} = L\{h(t)\left[\dfrac{e^t + e^{-t}}{2}\right]$

$= \dfrac{1}{2}\left\{L\{h(t)e^t\} + L\{h(t)e^{-t}\}\right\}$

$= \dfrac{1}{2}\left[\bar{h}(s-1) + \bar{h}(s+1)\right]$

$= \dfrac{1}{2}\left[\dfrac{(s-1)-1}{(s-1)^3 - 2(s-1)^2} + \dfrac{(s+1)-1}{(s+1)^3 - 2(s+1)^2}\right]$

$= \dfrac{1}{2}\left[\dfrac{s-2}{(s-1)^3 - 2(s-1)^2} + \dfrac{s}{(s+1)^3 - 2(s+1)^2}\right]$ //

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Q 4a Find inverse laplace transform using convolution theorem

$$\phi(s) = \frac{(s+2)^2}{(s^2+4s+8)^2}$$

$$L^{-1}\{\phi(s)\} = L^{-1}\left[\frac{(s+2)^2}{((s+2)^2+4)^2}\right]$$

$$= e^{-2t} L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$$

$$= e^{-2t} L^{-1}\left[\frac{s}{s^2+4} \times \frac{s}{s^2+4}\right]$$

$$= e^{-2t} L^{-1}\left[\bar{f}(s).\bar{g}(s)\right]$$

By convolution theorem : $L^{-1}\{\bar{f}(s)\bar{g}(s)\} = \int_0^t f(u).g(t-u)du$

$$= e^{-2t} \int_0^t f(u).g(t-u)du.$$

$$f(u) = L^{-1}\left[\frac{s}{s^2+4}\right] = \cos 2u = g(u)$$

$$\therefore L^{-1}\{\phi(s)\} = e^{-2t} \int_0^t \cos 2u \cdot \cos(2t-2u)du$$

$$= \frac{e^{-2t}}{2} \int_0^t \cos(2u+2t-2u) + \cos(2u-2t+2u)\,du$$

$$= \frac{e^{-2t}}{2} \int_0^t \cos 2t + \cos(4u-2t)\,du$$

$$= \frac{e^{-2t}}{2} \left[u\cos 2t + \frac{\sin(4u-2t)}{4}\right]_0^t$$

$$= \frac{e^{-2t}}{2} \left[t\cos 2t + \frac{\sin 2t}{4} + \frac{\sin 2t}{4}\right]$$

$$= \frac{e^{-2t}}{4} \left[2t\cos 2t + \sin 2t\right]$$ //

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Q5b Find $L^{-1}\left[\frac{s}{s^4+s^2+1}\right]$

let $\bar{f}(s) = \frac{s}{(s^2+1)^2 - s^2} = \frac{s}{(s^2+1+s)(s^2+1-s)}$

Since the difference is 2s in denominator

$$\therefore \bar{f}(s) = \frac{1}{2}\left[\frac{1}{s^2+1-s} - \frac{1}{s^2+1+s}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(s-1/2)^2+3/4} - \frac{1}{(s+1/2)^2+3/4}\right]$$

$$L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a} ; \quad L^{-1}\{\bar{f}(s-a)\} = e^{at} L^{-1}\{\bar{f}(s)\}$$

$$\therefore L^{-1}\{\bar{f}(s)\} = \frac{1}{2}\left[e^{t/2}\frac{\sin \sqrt{3}/2 t}{\sqrt{3}/2} - e^{-t/2}\frac{\sin \sqrt{3}/2 t}{\sqrt{3}/2}\right]$$

$$= \frac{2}{\sqrt{3}} \times \frac{1}{2} \times \sin\left(\frac{\sqrt{3}t}{2}\right)\left(e^{t/2} - e^{-t/2}\right)$$

$$= \frac{2}{\sqrt{3}} \sinh\left(\frac{t}{2}\right) \cdot \sin\left(\frac{\sqrt{3}t}{2}\right)$$

$$\therefore L^{-1}\left[\frac{s}{s^4+s^2+1}\right] = \frac{2}{\sqrt{3}} \sinh\left(\frac{t}{2}\right) \cdot \sin\left(\frac{\sqrt{3}t}{2}\right) \quad //$$