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MATHS

TUTORIAL-1

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Q1 Use dual simplex method to solve the following LPP :

Maximize $Z = -4x_1 - 3x_2$

$x_1 + x_3 \leq 2$

$x_2 \geq 1$

$-x_1 + 2x_2 \leq 1$

$x_1, x_2 \geq 0$

ANS MAXIMIZE $Z = -4x_1 - 3x_2$

$x_1 + x_2 \leq 2$

$-x_2 \leq -1$

$-x_1 + 2x_2 \leq 1$

$x_1, x_2 \geq 0$

Standard form :

Maximize $Z = -4x_1 - 3x_2 + 0s_1 + 0s_2 + 0s_3$

$-x_1 + x_2 + s_1 = 2$

$-x_2 + s_2 = -1$

$-x_1 + 2x_2 + s_3 = 1$

$x_1, x_2 \geq 0$ (Basis variable)

$s_1, s_2, s_3 \geq 0$ (slack variable)

ITERATION 1

	c_j		-4	-3	0	0	0	
C_B	x_B	Solution	x_1	x_2	S_1	S_2	S_3	
0	S_1	2	1	1	1	0	0	
0	S_2	-1	0	-1	0	1	0	$\rightarrow$
0	S_3	1	-1	2	0	0	1	
$Z_j - C_j$			4	3	0	0	0	
Max ratio				-3	—	—	—	
				$\uparrow$				

$\because \forall\ Z_j - C_j \geq 0$ but $\forall\ x_{Bi} \ngtr 0$

$\therefore$ For next iteration

S_2 is departing variable

x_2 is entering variable

-1 is key element

ITERATION 2:

C_B	C_j / x_B	Solution	-4	-3	0	0	0	
0	S_1	1	1	0	1	1	0	
-3	x_2	1	0	1	0	-1	0	
0	S_3	-1	-1	0	0	2	1	$\rightarrow$
$Z_j - C_j$			4	0	0	3	0	
Max ratio			-4	–	–	–	–	
			$\uparrow$					

$\because \forall\ Z_j - C_j \geq 0$ but $\forall\ x_{Bi} \ngtr 0$

$\therefore$ For next iteration

S_3 is departing variable

x_1 is entering variable

-1 is key element

ITERATION 3:

C_B	C_j / x_B	Solution	-4 x_1	-3 x_2	0 S_1	0 S_2	0 S_3
0	S_1	0	0	0	1	3	1
-3	x_2	1	0	1	0	-1	0
-4	x_1	1	1	0	0	-2	-1
$Z_j - C_j$		[redacted]	0	0	0	11	4
Max ratio							

$\because \forall Z_j - C_j \geq 0$ and $\forall x_{Bi} \geq 0$

$\therefore$ Current solution is optimal solution

$\therefore x_1 = 1$

$x_2 = 1$

$\therefore Z_{max} = -4(1) - 3(1)$

$= -7$

$\therefore \boxed{Z_{max} = -7}$

Q2 Use dual simplex method to solve the following LPP.

Maximize $Z = -4x_1 - 3x_2$

$x_1 + x_2 \leq 1$

$x_2 \geq 1$

$-x_1 + 2x_2 \leq 1$

$x_1, x_2 \geq 0$

ANS Maximize $Z = -4x_1 - 3x_2$

$x_1 + x_2 \leq 1$

$-x_2 \leq -1$

$-x_1 + 2x_2 \leq 1$

$x_1, x_2 \geq 0$

standard form:

Maximize $Z = -4x_1 - 3x_2 + 0s_1 + 0s_2 + 0s_3$

$x_1 + x_2 + s_1 = 01$

$-x_2 + s_2 = -1$

$-x_1 + 2x_2 + s_3 = 1$

$x_1, x_2 \geq 0$ [Basis variables]

$s_1, s_2, s_3 \geq 0$ [Slack variables]

ITERATION 1:

	C_j		-4	-3	0	0	0	
C_B	x_B	Solution	x_1	x_2	S_1	S_2	S_3	
0	S_1	1	1	1	1	0	0	
0	S_2	-1	0	-1	0	1	0	$\longrightarrow$
0	S_3	1	-1	2	0	0	1	
$Z_j - C_j$			4	3	0	0	0	
Max ratio			–	-3	–	–	–	
				$\uparrow$				

$\because \forall\ Z_j - C_j \geq 0$ but $\forall\ x_{Bi} \ngeq 0$

$\therefore$ For next iteration,

S_2 is departing variable

x_2 is entering variable

-1 is key element

ITERATION 2:

	C_j		-4	-3	0	0	0	
C_B	x_B	Solution	x_1	x_2	S_1	S_2	S_3	
0	S_1	0	1	0	1	1	0	
-3	S_2	1	0	1	0	-1	0	
0	S_3	-1	-1	0	0	2	1	$\longrightarrow$
$Z_j - C_j$			4	0	0	3	0	
Max ratio			-4	–	–	–	–	
			$\uparrow$					

$\therefore \forall\ Z_j - C_j > 0$ but $\forall\ x_{Bi} \ngeq 0$

$\therefore$ For next iteration,

S_3 is departing variable

x_1 is entering variable

-1 is key element

ITERATION 3:

	C_j		-4	-3	0	0	0
C_B	x_B	Solution	x_1	x_2	S_1	S_2	S_3
0	S_1	-1	0	0	1	3	1
-3	x_2	1	0	1	0	-1	0
4	x_1	1	1	0	0	-2	-1
	$Z_j - C_j$		0	0	0	11	4
Max Ratio			-	-	-	-	-

$\because \forall\ Z_j - C_j \geq 0$ but $\forall\ x_{Bi} \not\geq 0$

and Max ratio cannot be found.

$\because$ Key row has all +ve elements

$\therefore$ Given LPP is unbounded

i.e. No feasable solution.

Q3 Use duality to solve the LPP

Minimize $Z = x_1 - x_2$

$x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

ANS Maximize $T = 2y_1 + y_2$

s/t $y_1 - y_2 \leq 1$ [Due to x_1]

$y_1 - y_2 \leq -1$ [Due to x_2]

Standard Form:

Maximize $T = 2y_1 + y_2$

s/t $y_1 - y_2 \leq 1$

$-y_1 + y_2 \geq 1$

$y_1, y_2 \geq 0$

$$\text{Max } T = 2y_1 + y_2 + 0x_1 + 0x_2 - ma_1$$

S/t

$$y_1 - y_2 + x_1 = 1$$

$$-y_1 + y_2 - x_2 + a_1 = 1$$

$y_1, y_2 \geq 0$ [Basic variable]

$x_1 \geq 0$ [slack variable]

$x_2 \geq 0$ [slack variable]

$a_1 \geq 0$ [Artificial variable]

	c_j		2	1	0	0	$-m$	
C_B	x_B	Sol^n	y_1	y_2	x_1	x_2	a_1	Min Ratio
0	x_1	1	1	-1	1	0	0	—
-1	a_1	1	-1	1	0	-1	1	1 →
	$Z_j - C_j$		m-2	m-1 ↑	0	m	0	

	c_j		2	1	0	0	
C_B	x_B	Sol^n	y_1	y_2	x_1	x_2	Min ratio
0	x_1	2	0	0	1	-1	—
1	y_2	1	-1	1	0	-1	—
	$Z_j - C_j$		-3 ↑	0	0	-1	

$\therefore \forall Z_j - C_j \neq 0$ & Minimum ratio can't be found as element of key column are not positive

$\therefore$ Given LPP has unbounded collection

i.e. No feasable solution.