* Class of sets :

A set is a collection of objects called elements. But in some situation we may consider set of elements. Such a collection of sets is called a class of sets or a family of sets.

Let $A = \{a, b, c\}$

$S = \{\{a\}, \{b\}, \{a, b\}\}$.

$\boxed{*}$ Let S be a set whose elements are subset of A.

* Power Set :

If S is a given set then set of all subsets of S is called power set of S & denoted by P(S). Clearly ϕ & S are the elements of P(S).

E.g. 1. $S = \{a, b\} \Rightarrow$ $N = 2$

then $P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ $2^2 = \boxed{4}$

2. $S = \{a, b, c\}$ then

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$ $N = 3$

If S has N elements then $|P| = 2^3 = 8$

$P(S) = 2^N$.

N

2^N

$S = \{a, b, c\}$ $A = \{0\}$

$A = \{a, b\}$ $0 \in A \rightarrow T$

$A \subseteq S$ $\boxed{\phi \in A = \text{False}}$

$\{\phi\} \subseteq A$ $\notin$

* Partition of Sets:

A collection $\{A_i\}$ of non-empty subsets of ~~set~~ S is called a partition of S if

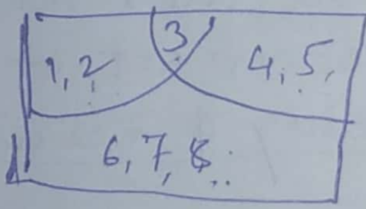
i) Each element of S belongs to one subset A_i i.e. $\cup A_i = S$

ii) the subsets A_i are mutually disjoint i.e. $A_i \cap A_j = \phi$

Let E.g. D. $S = \{1,2,3,4,5,6,7,8\}$

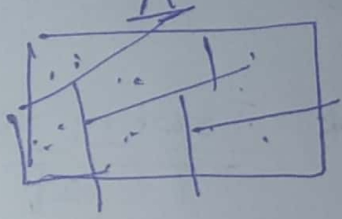
$A_1 = \{\{1,2,3\}, \{3,4,5\}, \{6,7,8\}\}$

X



→ X

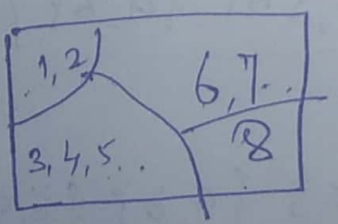
A



i) $A_1, A_2, A_3, A_4, A_5, A_6$

$\cup A_i = A$

$A_2 = \{\{1,2,3\}, \{4,5\}, \{6,7\}, \{8\}\}$



✓

ii) $A_i \cap A_j = \phi$

i) ✓

ii) ✓

A_2 partition of S.

Cardinality of set:

$A = \{1,2,3\}$

$\boxed{|A| = 3}$

* Inclusion - Exclusion Principal :

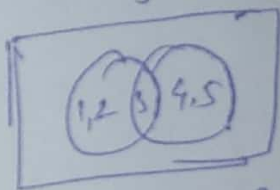
While counting the elements in a situation if some elements are not counted already they are to be included and if some elements are already counted they are to be excluded.

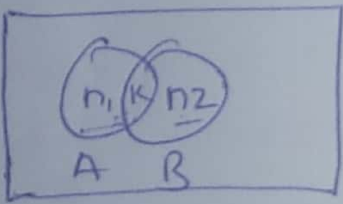
Theorem 1 : If A & B are the two sets & $n(A)$, $n(B)$ denote the number of elements in A and B then

$A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$





$$n(A \cup B) = n_1 + k + n_2$$

$$= (n_1 + k) + (n_2 + k) - k$$

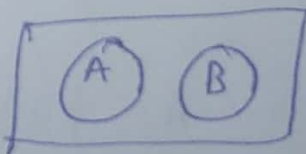
$$= n(A) + n(B) - n(A \cap B)$$

$n(A) = 3$

$n(B) = 3$

If A & B are disjoint sets then

$$\boxed{n(A \cup B) = n(A) + n(B)}$$



Theorem 2 :

$$\boxed{\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - \\ &\quad n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \end{aligned}}$$

B

$n(A \cup B) = 5$ $\qquad$ $n(A \cap B) = 1$

$n(A) + n(B \cup C) - n[A \cap (B \cup C)]$

$- [n(A \cap B) + n(A \cap C)]$

Cartesion Product :

The cartesian product of two sets A & B denoted as A×B is the set of all ordered pair (a, b) where $a \in A$ and $b \in B$ thus

$$A \times B = \{(a, b) \mid a \in A,\ b \in B\}$$

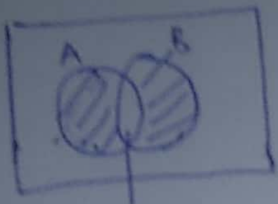
E.g. 1. $A = \{1, 2, 3\}$ & $B = \{a, b, c\}$

$$A \times B = \{(1, a), (1, b), (1, c) \\ (2, a)\ (2, b), (2, c) \\ (3, a)\ (3, b), (3, c)\}$$

2. $$B \times A = \{(1, a), (1, b), (1, c) \\ (2, a)\ (2, b), (2, c) \\ (3, a), (3, b), (3, c)\}$$

3. $A = \{a\}$, $B = \{b, c\}$, $C = \{d, e, f\}$

$$A \times B \times C = \{\{a, b, d\}, \{a, b, e\}, \{a, b, f\} \\ \{a, c, d\}, \{a, c, e\}\ \{a, c, f\}\}$$



$A \cap B$

1. $n(A-B) = n(A) - n(A \cap B)$
2. $n(B-A) = n(B) - n(A \cap B)$
3. $n(A \oplus B) = n(A-B) + n(B-A) = n(A) + n(B) - 2n(A \cap B)$
4. $n(\bar{A}) = n(S) - n(A)$
5. $n(\overline{A \cup B}) = n(S) - n(A \cup B)$

1) Find the no. of integers ≤ 200 which are divisible 2 or 5

$A = \{2, 4, 6, 8 \dots 200\}$

$B = \{5, 10, 15, 20 \dots 200\}$

$n(A \cup B) = ?$ $\quad n(A \cap B) = \frac{200}{2 \times 5} = 20$ $\quad 200/2 = 100$

$200/5 = 40$

$n(A) = 100$ $\quad n(A \cap B) = 20$

$n(B) = 40$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 100 + 40 - 20$

$\boxed{n(A \cup B) = 120}$

$n(S) = 60$

$n(F) = 30$

$n(Se) = 25$

$\boxed{\underline{20} \text{ did not get first class in either}}$

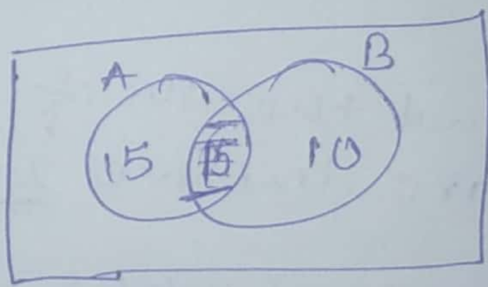
how many students got first class in both exam.

$n(A) = 30$

$n(B) = 25$

$\underline{n(\overline{A \cup B})} = \underline{20}$

$n(A \cap B)$



$\underline{n(A \cup B) = 60 - 20 = 40}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$40 = 30 + 25 - n(A \cap B)$

$\boxed{n(A \cap B) = \underline{15}}$

cartesian product:

$A = \{1,2,3\}$, $B = \{1,2\}$

$|A| = 3$ $\quad$ $|B| = 2$

A, B

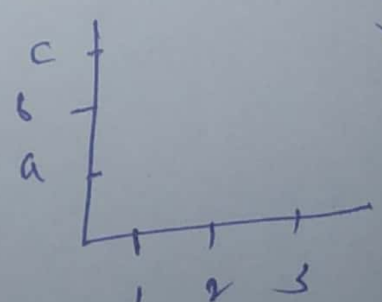
$A \times B = \{ (a,b) \mid a \in A \ \& \ b \in B \}$

$A = \{1,2,3\}$ $\quad$ $B = \{a,b,c\}$

$A \times B = \{ (1,a), (1,b), (1,c)$
$(2,a), (2,b), (2,c)$
$(3,a), (3,b), (3,c) \}$

$= \{(1,a), (2,a),$
$(2,b), (2,c)$
$(1,c), (1,c)$
$(3,a)\ (3,b), (3,c)$
$\}$

$B \times A = \{ (a,1), (a,2), (a,3)$
$(b,1)\ (b,2), (b,3)$
$(c,1)\ (c,2)\ (c,3) \}$



$A \times A =$
$B \times B =$

* $A = \{a\}$ $\quad$ $B = \{b,c\}$, $\quad$ $C = \{d,e,f\}$

$A \times B \times C = \{ (a,b,d)\ (a,b,e)\ (a,b,f)$
$(a,c,d)\ (a,c,e)\ (a,c,f) \}$

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