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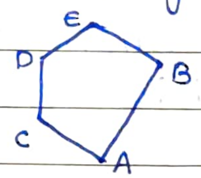
DM
TUTORIAL - 5

Q1 Let $A = \{1, 4, 7, 13\}$ and $R = \{(1,4), (4,7), (7,4), (1,13)\}$
Find transitive closure using Warshall's algoritm.

Q2 Define Lattice. check if the following diagram is a lattice or not :



Q3 Mention all the elements in D_{36}. Also specify R on D_{36} as aR_b if $a|b$. Mention domain and range of R. Explain if the relation is equivalence relation or a partially ordered relation. If it is a partially ordered relation, draw its Hasse diagram.

Q4 $A = \{1, 2, 3\}$. Draw Hasse diagram.

ANS 1. STEP 1: We first write the matrix M_R of the relation R and denote it by W_0.

$$W_0 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

STEP 2: Now, we write a blank matrix of order 4, denote it by W_1 and transfer all 1's from W_0 to W_1

$\therefore \; W_1 = \begin{bmatrix} \cdots & 1 & \cdots & 1 \\ \cdots & \cdots & 1 & \cdots \\ \cdots & 1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$

We now observe the first column and the first row. Since 1 does not appear in the first column ($p_i = 0$) and hence $p_i q_j = 0$ and there is no addition of 1 in W_1, i.e. $W_2 = W_1$

$\therefore \; W_2 = \begin{bmatrix} \cdots & 1 & \cdots & 1 \\ \cdots & \cdots & 1 & \cdots \\ \cdots & 1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$

STEP 3 : Now we observe the second column and the second row. Now in the second column, 1 appears in the 1^{st} and 3^{rd} positions. Hence we add 1 in (1,3) and (3,3) positions.

$\therefore \; W_3 = \begin{bmatrix} \cdots & 1 & 1 & 1 \\ \cdots & \cdots & 1 & \cdots \\ \cdots & 1 & 1 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$

STEP 4 : Now we observe the third column and the third row. In the third column, 1 appears in 2^{nd} and 3^{rd} positions. So we add 1's in (1,2), (1,3), (2,2), (2,3), (3,2) and (3,3) positions if there is no 1.

$\therefore \; W_4 = \begin{bmatrix} \cdots & 1 & 1 & 1 \\ \cdots & 1 & 1 & \cdots \\ \cdots & 1 & 1 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$

2

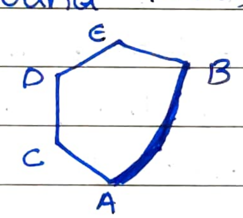
STEP 5: Now we observe the fourth column and the fourth row. In the fourth row, there is no 1. Hence there will be no additions of 1's. Hence $p_i = 0$ and $p_i q_j = 0$. W_4 is our required matrix.

$$W_4 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 11 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 11 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Thus, the transitive column of R, i.e.

$R^{\infty} = \{(1,4), (1,7), (1,11), (4,4), (4,7), (7,4), (7,7)\}$ //

ANS 2 Lattice: A poset $(L, \leq)$ in which every pair $\{a, b\}$ of 2 elements of L has a least upper bound (LUB) and a greatest lower bound (GLB) is called a lattice.



Preparing tables for LUB and GLB, we get:

LUB

V	A	B	C	D	E
A	A	B	C	D	E
B	B	B	E	E	E
C	C	E	C	D	E
D	D	E	D	D	E
E	E	E	E	E	E

Joins exist for all pairs. All pairs have LUB

GLB

$\wedge$	A	B	C	D	E
A	A	A	A	A	A
B	A	B	A	A	B
C	A	A	C	C	C
D	A	A	C	D	D
E	A	B	C	D	E

Meets exist for all pairs. All pairs have GUB. Since every pair of elements has a GLB & LUB, the relation is a lattice.

ANS 3 $D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$R = \{ R \mid a \mid b,\ a \in D_{36}\ \&\ b \in D_{36} \}$

$R = \{ (1,1), (1,2), (1,3), (1,4), (1,6), (1,12), (1,18), (1,36),$
$(2,2), (2,4), (2,6), (2,12), (2,18), (2,36), (3,3), (3,6),$
$(3,9), (3,12), (3,18), (3,36), (4,4), (4,12), (4,36), (6,6),$
$(6,12), (6,18), (6,36), (9,9), (9,18), (9,36), (12,12), (12,36),$
$(18,18), (18,36), (36,36) \}$

Domain of $R = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$
Range of $R = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$\Rightarrow$ R is reflexive since for all $(a,b) \in R$
$aRb \Rightarrow a \mid b$ [a divides b]
$aRa \Rightarrow a \mid a$ [a divides a]

⇒ R is antisymmetric because for all $(a,b) \in R$

$a R b \Rightarrow a | b$ [a divides b]

$a R a \Rightarrow a | a = 1$ [a doesn't divide b until a = b!]

⇒ Relation R is transitive because for all $(a,b) \in R$

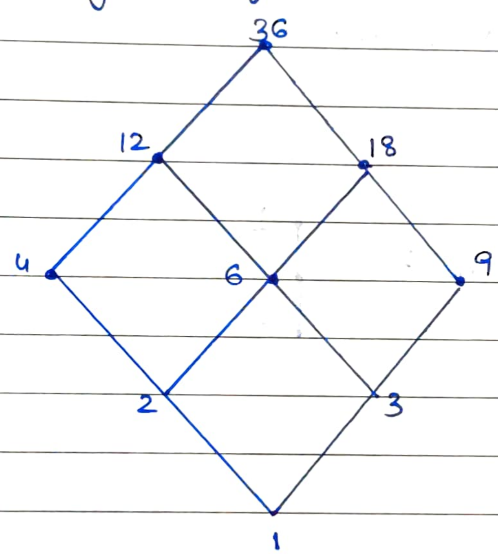
$a R b \Rightarrow a | b$ [a divides b]

$b R c \Rightarrow b | c$ [b divides c]

$a R c \Rightarrow a | c$ [a divides c]

∴ R is a partially ordered relation (poset)

∴ Hasse diagram of R is :-



ANS 4 $A = \{1, 2, 3\}$

Let $R_1 = (A, \leq)$ and $R_2 = (A, \geq)$

For every $(a,b) \in R$,

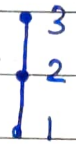
(i) $\because a \leq a$, R_1 is reflexive

(ii) If $a \leq b$ and $b \leq a$ then $a = b$, R_1 is antisymmetric.

(iii) If $a \leq b$, $b \leq c$ then $a \leq c$. R_1 is transitive.

$\therefore R_1$ is a poset.

Hasse diagram of R_1 :



For every $(a,b) \in R_2$

(i) Since $a \geq a$, R_2 is reflexive

(ii) If $a \geq b$ and $b \geq a$ then $a = b$

$\therefore R_2$ is antisymmetric.

(iii) If $a \geq b$, $b \geq c$ and $a \geq c$

$\therefore R_2$ is transitive

$\therefore R_2$ is a poset.

$\therefore$ Hasse diagram of R_2 :