FA

TUTORIAL - 4.

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Q1 Let $\Sigma = \{a, b\}$ show that $L = \{ww^R \mid w \in \Sigma\}$ is not ~~RE~~ regular

ANS STEP 1: Assume L is regular language

STEP 2: Let n be pumping lemma constant

Let $w = a^n b$; $w^R = ba^n$

Let $z = ww^R = a^n bba^n$

Representing z in pumping lemma constant

$\therefore$ let $z = a^n bba^n$

Dividing z into 3 parts such that $z = uvw$ and $|uv| \leq n$ and $1 \leq |v| \leq w$

$\therefore$ Selet values of (u, v, w) satisfying the above two conditions

$\therefore$ Let ~~aa~~ $u = a^{n-2}$

$v = aa$

$w = bba^n$

Then $uv^iw = a^{n-2}(aa)^i bba^n$

For $i = 0$ we get $uv^0w = a^{n-2}bba^n \notin L$

$\because$ Resultant $a^{n-2}b$ cannot be a reverse of ba^n

Hence it contradicts our assumption.

$\therefore$ L is not regular.

Q2 Let $\Sigma = \{0\}$ show that $L = \{0^n \mid n \text{ is a prime}\}$ is not regular

ANS STEP 1: Let us assume L^* is a regular language

$\therefore L = \{00, 000, 00000, 0000000, \ldots\}$

STEP 2: Consider $w = 0000000$ $n = 7$

STEP 3: Divide w in three parts

$x = 000$

$y = 0$

$z = 000$

STEP 4: Here, $|y| > 0$

$|xy| \leq n$

Now we check, xy^iz

$w_0 = xy^0z = 000000 \notin L$

$w_1 = xyz = 0000000 \in L$

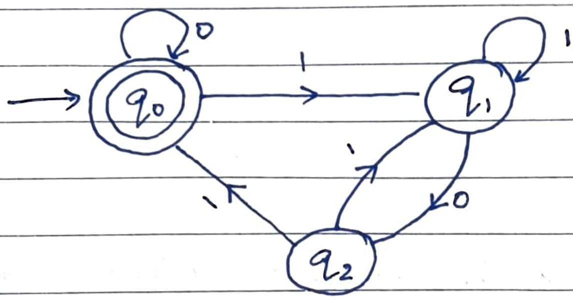
$w_2 = xy^2z = 00000000 \notin L$

$\because \forall\, xy^iz \notin L$ for all $i \geq 0$

$\therefore$ our assumption is wrong

$\therefore$ Given language is NOT regular.

Q3 Convert the following FA to RE.



STEP 1: No dead states

STEP 2: $q_0 = q_0 0 + q_2 1 + \varepsilon$

$q_1 = q_0 1 + q_1 1 + q_2 1$

$q_2 = q_1 0$

STEP 3: substitute q_2 in q_1

$\therefore q_1 = q_0 1 + q_1 1 + q_1 01$

$q_1 = q_0 1 + q_1 (1 + 01)$

$R = Q + RP$ [ARDEN'S THEOREM]

$\therefore R = QP^* = q_0 1 (1+01)^*$

$\therefore q_1 = q_0 1 (1+01)^*$ $\qquad \therefore q_2 = q_0 1 (1+01)^* 0$

Now, substitute q_1 and q_2 in q_0

$\therefore q_0 = q_0 0 + q_0 1 (1+01)^* 01 + \varepsilon$

$\therefore q_0 = \varepsilon + q_0 [0 + 1(1+01)^* 01]$

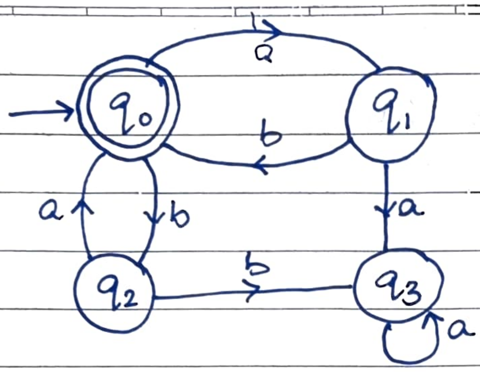
i.e. $R = Q + RP$ [ARDEN'S THEOREM]

$\therefore R = QP^* = \varepsilon [0 + 1(1+01)^* 01]^*$

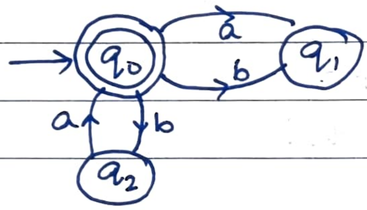
$\therefore q_0 = [0 + 1(1+01)^* 01]^*$ $\qquad [\because \varepsilon R^* = R^*)$

$\therefore$ Regular Expression of given FA is $[0 + 1(1+01)^* 01]^*$

Q4



ANS STEP 1 : q_3 is removed as it has no outgoing transition and thus is a dead state



STEP 2 :
$$q_0 = q_1 b + q_2 a + \epsilon$$
$$q_1 = q_0 a$$
$$q_2 = q_0 b$$

STEP 3 : Substitute values of q_1 and q_2 in q_0

$\therefore \quad q_0 = q_0 ab + q_0 ba + \epsilon$

$\therefore \quad q_0 = \epsilon + q_0 (ab + ba)$

i.e. $R = Q + RP$ [ARDEN'S THEOREM]

$\therefore \quad R = QP^* = \epsilon (ab + ba)^*$

$q_0 = (ab + ba)^*$ $\quad [\because \epsilon R^* = R^*]$

$\therefore$ Regular Expression of given DFA is $(ab + ba)^*$