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DISCRETE STRUCTURES SECTION - A TT2

Q1 $A = \{1, 2, 3, 4, 5\}$

$R = \{(1,1), (1,4), (2,2), (3,4), (3,5), (4,1), (5,2), (5,5)\}$

To find R^{∞} we have to find transitive ~~to~~ ~~warshall's~~ closure for R by Warshall's Algorithm.

$$W_0 = M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore M_R^{\infty} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$R^{\infty} = \{ (1,1), (1,4), (2,2), (3,1), (3,2), (3,4),$
$(3,5), (4,1), (4,4), (5,2), (5,5) \}$

Q2 let R be a binary relation. let $S = \{(a,b) \mid (a,c) \in R$ and $(c,b) \in R$ for some $c\}$ show that R is an equivalence relation then S is an equivalence relation

ANS To Prove : S is reflexive

Since R is reflexive $(a,a) \in R \ \forall a \in A$. Clearly $(a,a) \in S \ \forall a \in A$. This proves that S is reflexive

To Prove : S is symmetric

$(a,b) \in S \rightarrow \exists x \ (a,x) \in R, (x,b) \in R$.

Since R is symmetric $(x,a) \in R, (b,x) \in R$

Therefore by given defination, $(b,a) \in S$

This proves that S is symmetric.

To Prove : S is transitive

If $(a,b) \in S$ and $(b,c) \in S$ we need to prove that $(a,c) \in S$.

$(a,b) \in S \rightarrow \exists d \ (a,d), (d,b) \in R$

R is symmetric $\rightarrow (d,a), (b,d) \in R$

$\Rightarrow (a,b) \in R, (b,a) \in R$

$(b,c) \in S \rightarrow \exists e \ (b,e), (e,c) \in R$

R is symmetric $\Rightarrow (e,b), (c,e) \in R$

$\Rightarrow (b,c) \in R, (c,b) \in R$

Since R is transitive, $(a,c) \in R, (c,a) \in R \longrightarrow (1)$

Since R is reflexive, $(c,c) \in R \longrightarrow (2)$

From (1) and (2) it follows that $(a,c) \in S$

therefore S is transitive and hence an equivalent relation