DISCRETE STRUCTURES

TUTORIAL - 2

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Q1 Given that the student has prepared, the probability of passing a certain entrance exam is 0.99. Given that the student did not prepare, the probability of passing the certain entrance exam is 0.05. Assume that the probability of preparing is 0.7. The student fails in the exam. What is the probability that he/she did not prepare.

ANS 1. let A be the event that a student prepares for the examination;
let B be the event that a student passes the examination.
we are given the following information :

$$\therefore P(B \mid A) = 0.99$$
$$P(B \mid A') = 0.05$$
$$P(A) = 0.7$$

$$\therefore P(B' \mid A) = 1 - P(B \mid A) = 1 - 0.99 = 0.01$$
$$P(B' \mid A') = 1 - P(B \mid A') = 1 - 0.05 = 0.95$$
$$P(A') = 1 - P(A) = 1 - 0.7 = 0.3$$

∴ Probability that a student has not prepared for the examination given that the student failed the examination

$$\therefore P(A' \mid B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(B' \mid A')\,P(A')}{P(B' \mid A)P(A) + P(B' \mid A')P(A')}$$
$$= \frac{(0.95)(0.3)}{(0.01)(0.7) + (0.95)(0.3)}$$
$$= 0.97603$$

Q2 Using Pigeonhole principle, show that
(i) in any room of people who have been doing some handshaking, there will always be atleast two people who have shaken hands the same number of times

ANS (i) let there be N people in the room. A person can shake hands with between 0 to N-1 people since you cannot shake hands with yourself. That is N possibilities. If one person has shaken hands with everyone else, then there is no one who hasn't shook hands with no one. And the other way around. so 0 and N-1 possibilities are mutual exclusive. So we are down to N-1 possibilities of people each person can shake hands with.

So if there are N people and N-1 possibilities to the number of people each person can shake hands with at least 2 people have shook hands with an equal amount of people

(ii) A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. what is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color.

ANS (ii) No of colors $(n) = 3$

No of marbles $(K+1) = 4 \Rightarrow K = 3$

$\therefore$ Minimum no of marbles required $= Kn + 1$

$$Kn + 1 = 3(3) + 1 = 10$$

$$Kn + \frac{1}{n} = 4$$

$$Kn + \frac{1}{3} = 4 \qquad \text{[Considering worst case scenario]}$$

$$Kn + 1 = 10$$

$\therefore$ 3 red + 3 white + 3 blue + 1 (red or white or blue) = 10

Q3 Use mathematical induction to show that

i) $1 + 5 + 9 + \dots + 4n - 3 = n(2n - 1)$

ANS i) Let $n = 1$

$\therefore P(1) = 1(2) - 1 = 1$ which is true

Hence $P(1)$ is true

Consider $n = K$ is true

$\therefore P(K): 1 + 5 + 9 + \dots + (4K - 3) = K(2K - 1) \longrightarrow (1)$

Now $n = K + 1$

$\therefore P(K+1): 1 + 5 + 9 + \dots + (4K - 3) + [4(K+1) - 3] = (K+1)(2K+1)$

$K(2K - 1) + 4K + 1 = 2K^2 + 3K + 1$

$\therefore \quad 2K^2 + 3K + 1 = 2K^2 + 3K + 1$

$\therefore$ LHS = RHS

Hence $P(K+1)$ is true whenever $P(K)$ is true.

$\therefore$ By principle of mathematical induction $P(n)$ is true for any natural number.

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ii) $2 + 5 + 8 + \dots + (3n-1) = n(3n+1)/2$

ANS (ii) Checking for $n = 1$ if $P(n)$ is true.

$P(1) = 2 = 1(4)/2 = 2$

$\therefore 2 = 2$

$\therefore P(n)$ is true for $n = 1$

lets ~~check~~ assume $P(n)$ for $n = K$ is true

$\therefore P(K) = 2 + 5 + 8 + \dots + (3K-1) = K(3K+1) \div 2$

To prove $P(K+1)$ is true

$P(K+1) = 2 + 5 + 8 + \dots + (3K-1) + 3K+2 = (K+1)(3K+4)/2$

$\therefore K(3K+1)/2 + 3K+2 = (K+1)(3K+4)/2$

$3K^2 + K + 6K + 4 = 3K^2 + 7K + 4$

$\therefore 3K^2 + 7K + 4 = 3K^2 + 7K + 4$

$\therefore$ LHS = RHS

$\therefore P(n)$ is true for $n = K+1$

Thus, $P(n)$ is true for all $n \in N$

Q4 (i) Two dice are rolled, find the probability that the sum is

a) Equal to 1

b) Equal to 4

c) Less than 13

ANS (i) Total possible outcomes $= 6^2 = 36$

a] Sum is equal to 1

$\therefore$ Favourable outcome $= 0$ [Not possible]

$\therefore$ Required probability $= \frac{0}{36} = 0$ //

b] Sum is equal to 4

$\therefore$ Favourable outcomes $= 3$ [(1,3), (2,2), (3,1)]

$\therefore$ Required probability $= \frac{3}{36} = \frac{1}{12}$ //

c] Sum is less than 13

$\therefore$ Favourable outcomes $= 36$ [All are less than 13]

$\therefore$ Required probability $= \frac{36}{36} = 1$ //

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(ii) A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, NOT replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen.

ANS (ii) Total Blue pens = 4
Total Red pens = 2
Total Black pens = 3
$\therefore$ Total pens = 4 + 2 + 3 = 9

Probability of drawing 2 blue pens $= \frac{^4C_2}{^9C_2} = \frac{4 \times 3}{9 \times 8} = \frac{1}{6}$ $\rightarrow$ (1)

Since pens are not replaced,
Total pens = 7

$\therefore$ Probability of drawing 1 black pen $= \frac{^3C_1}{^7C_1} = \frac{3}{7}$ $\rightarrow$ (2)

Probability of drawing 2 blue pens and 1 black pen

$$= \frac{1}{6} \times \frac{3}{7}$$

$$= \frac{1}{14}$$

Q5 i) How many four digits can be formed out of digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if no digit is repeated twice? How many of these will be greater than 3000

ii) In how many ways can a committee of three faculty members and 2 students can be formed from 7 faculty members and 8 students.

iii) A box contains 6 white balls and 5 red balls. In how many ways can 4 balls be drawn from the box if :-

a] They are of any color

b] All the balls are of same color.

9 8 7 6

ANS i) Four digit number.

Digits availabe for thousand's place : 9

Digits available of hundred's place : $9 - 1 = 8$ [cant repeat]

Digits available for tenth's place : $8 - 1 = 7$

Digits available for units place : $7 - 1 = 6$

Hence total combinations $= 9 \times 8 \times 7 \times 6 = 3024 \longrightarrow$ 1

if no digit is repeated twice

NUMBERS GREATER THAN 3000 :

We first find numbers lesser than 3000. Such numbers will have 1, 2 in their thousands place.

∴ When 1 is fixed in the thousands place : Number of available combinations are $\underline{8 \times 7 \times 6}$ since 1 is already fixed in thousands place and we cant repeat digits.

∴ Numbers having '1' in thousands place $= 8 \times 7 \times 6 = 336$

∴ Numbers having '2' in thousands place $= 8 \times 7 \times 6 = 336$

∴ Total numbers less than 3000 $= 336 + 336 = 672 \longrightarrow$ 2

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$\therefore$ Total numbers greater than 3000 = Total numbers − Total numbers lesser than 3000

$\therefore$ From (1) and (2)

Total numbers greater than 3000 = 3024 − 672 = 2352.

$\therefore$ 3024 numbers can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and out of these, 2352 numbers are greater than 3000.

ANS ii) 3 faculty members can be selected from 7 faculty members in $^7C_3$ ways = 35 ways $\rightarrow$ (i)

2 students can be selected from 8 students in $^8C_2$ ways = 28 ways $\longrightarrow$ (ii)

$\therefore$ Ways to select 3 faculty members and 2 students from 7 faculty members and 8 students $= {}^7C_3 \times {}^8C_2$

$= 35 \times 28$

$= 980$ ways.

$\therefore$ There are 980 ways to select 3 faculty members and 2 students from 7 faculty members and 8 students.

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ANS iii) Number of white balls $= n(W) = 6$
Number of red balls $= n(R) = 5$

a] Balls are of any color.
Hence to select 4 balls from $[n(W) + n(R) = 6 + 5 = 11]$
11 balls, we have $^{11}C_4$ ways $= 330$ ways.
$\therefore$ If the balls can be of any color, we can draw 4 balls from 6 white and 5 red balls in $\boxed{330}$ ways.

b] If all balls are of same color:
Ways to select white balls $= {}^6C_4 \times {}^5C_0 = 15$ ways.
Ways to select red balls $= {}^5C_4 \times {}^6C_0 = 5$ ways.
Hence total ways to select $= 15 + 5 = 20$ ways.

Hence if all four balls are to be of the same color, there are $\boxed{20}$ ways to draw these 4 balls from 6 white balls and 5 red balls

ANSWERS : a) 330 ways
b) 20 ways.