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END SEM - 3 EXAM
DISCRETE STRUCTURES

Q3

$A = \{2, 3, 6, 12, 24, 36, 72\}$

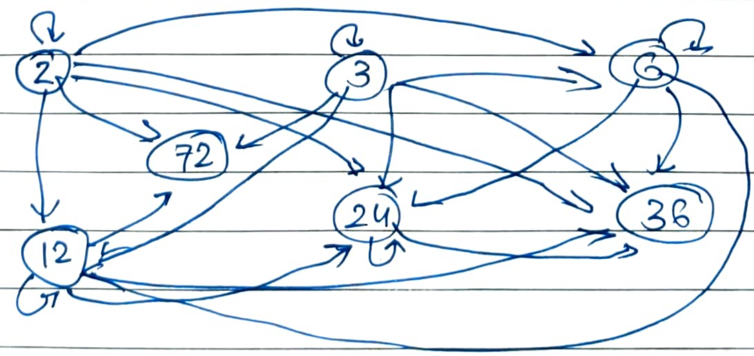
The relation "is divisible by" is given by the following matrix

$$R = \begin{array}{c} \\ 2 \\ 3 \\ 6 \\ 12 \\ 24 \\ 36 \\ 72 \end{array} \begin{array}{c} \begin{array}{ccccccc} 2 & 3 & 6 & 12 & 24 & 36 & 72 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Relation of divisibility : R.

$R = \{(2,2), (2,6), (2,12), (2,24), (2,36), (2,72),$
$(3,3), (3,6), (3,12), (3,24), (3,36), (3,72),$
$(6,6), (6,12), (6,24), (6,36), (6,72), (12,12),$
$(12,24), (12,36), (12,72), (24,24), (24,72),$
$(36,36), (36,72), (72,72)\}$

DIGRAPH :



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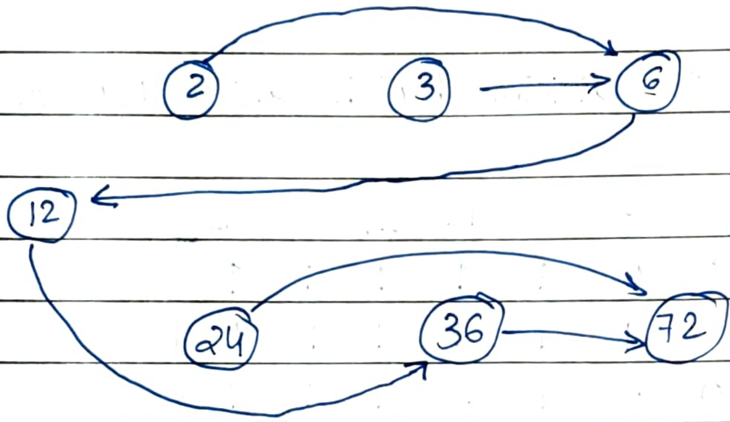
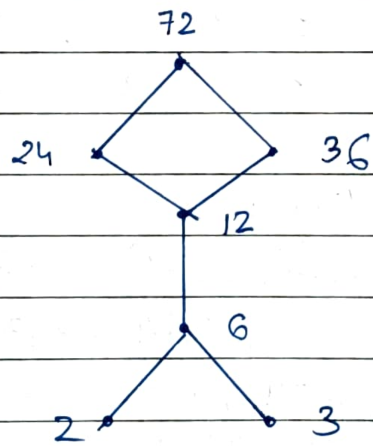
Removing all self loops & transitivity



HASSE DIAGRAM.



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Q3.2 Performing join ($a \vee b$) [least upper bound]

$\vee$	2	3	6	12	24	36	72
2	2	6	6	12	24	36	72
3	6	3	6	12	24	36	72
6	6	6	6	12	24	36	72
12	12	12	12	12	24	36	72
24	24	24	24	24	24	72	72
36	36	36	36	36	72	36	72
72	72	72	72	72	72	72	72

Performing meet ($a \wedge b$) [Greatest lower bound]

The greatest lower bound of pair (2,3) is not possible

Hence its NOT a lattice.

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Q4 Let f be a function from A to B. Let f be defined everywhere i.e. domain f = A

* INJECTIVE

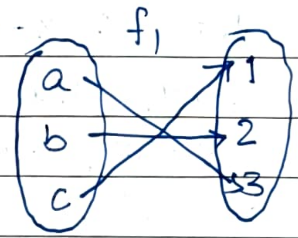
⇒ A function f is said to be injective [one-to-one] if two distinct elements of A correspond to two distinct elements of B

i.e. If a function $f: X \to Y$ is one-to-one if $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$ or $f(x_1) = f(x_2)$ implies $x_1 = x_2$

e.g: let $A = \{a, b, c\}$ $B = \{1, 2, 3\}$

$\therefore f_1 = \{(a,3), (b,2), (c,1)\}$

f_1 is one-to-one.



* SURJECTIVE

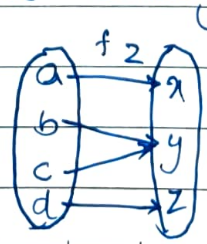
⇒ A function $f: A \to B$ is called surjective or onto if every element of $b \in B$ is an image of atleast one element a of A.

i.e. In other words, the range of f = B

e.g. let $A = \{a, b, c, d\}$ $B = \{x, y, z\}$

$f_2 = \{(a,x), (b,y), (c,y), (d,z)\}$

f_2 is onto or surjective because every element x, y, z of B is an image of atleast one element (pre-image) of A



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★ BIJECTIVE

⇒ If $f : A \to B$ is both one-to-one and onto the f is bijective. Such a function is also called one-to-one correspondence between A and B

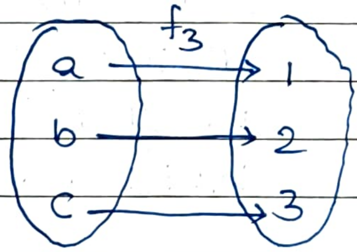
⇒ If f is injective and surjective, it is bijective.

e.g: $A = \{a, b, c\}$ $B = \{1, 2, 3\}$

$f_3 = \{(a,1), (b,2), (c,3)\}$

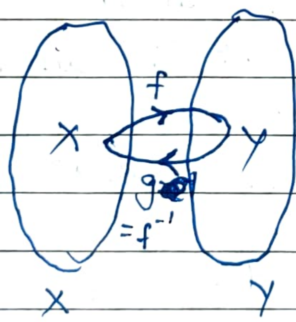
f_3 is one-to-one & onto

f_3 is bijective



★ INVERSE FUNCTION

⇒ let $f : X \to Y$. Suppose g is a function $g : Y \to X$ such that $(g \circ f)_x = x$ for every $x \in X$ and $(f \circ g)_y = y$ for every $y \in Y$, then g is called the inverse of f and is denoted by f^{-1}. Thus $g = f^{-1}$ and $\text{dom}(f) = \text{codom}(f^{-1})$ and $\text{codom}(f) = \text{dom}(f^{-1})$



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Q4.2 $f(x) = \frac{4x+4}{4x-2}$

To find f^{-1}

let $f(x) = y$

$\therefore y = \frac{4x+4}{4x-2}$

$y = \frac{24(x+)}{2(2x-1)}$

$2xy - y = 2x + 2$

$2x(y-1) = y+2$

$\therefore 2x = \frac{y+2}{y-1}$

$\therefore x = \frac{y+2}{2y-2} \Rightarrow f^{-1} = \frac{x+2}{2x-2}$

$f(x^{\circ}) = \frac{4x+4}{4x-2}$

$f^{-1} = \frac{x+2}{2x-2}$

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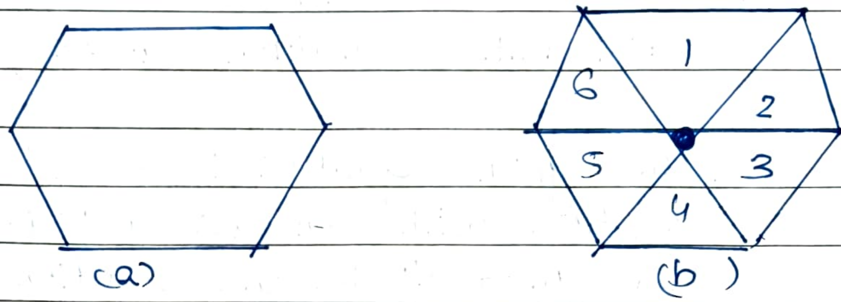
Q6

a)



Divide the region into 6 equilateral triangles.

If seven points are chosen in the region, we can assign each of them to a triangle that contains it. If the point belongs to several triangles arbitrarily assign to one at them. Then the seven points are assigned to six triangular regions. So by the pigeonhole principle atleast two points must belong to the same region. These two cannot be more than 1 unit apart

(b) let $P(n) : 5^n - 1$

(i) BASIS OF INDUCTION

For $n = 1$, $5^1 - 1 = 4$, divisible by 4

(ii) INDUCTION STEP

Assume that $5^k - 1$ is divisible by 4

we have $5^{k+1} - 1 = (5^k \cdot 5 - 5) + 4$

$= 5(5^k - 1) + 4$

By induction hypothesis $5^k - 1$ is divisible by 4.

$\therefore$ Each term on the RHS is divisible by 4.

$\therefore 5^{k+1} - 1$ is divisible by 4.

Hence $5^n - 1$ is divisible by 4 for $n \geq 1$.

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Q7. Solve the following relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ with initial condition $a_0 = 1$, $a_1 = 6$

The given equation is second order linear homogeneous relation with constant coefficient

Let $a_n = x^n$ be the solution

$\therefore x^n - 7x^{n-1} + 10x^{n-2} = 0$

$\therefore x^{n-2}[x^2 - 7x + 10] = 0$

$(x-5)(x-2) = 0$

$\therefore x = 5, 2$

$\therefore$ The roots are real, rational and distinct.

Hence let the general solution be $a_n = A(5)^n + B(2)^n$

We now use the initial condition to find the value of A & B

putting $n = 0$

$\therefore a_0 = A + B = 1$ —— (1)

putting $n = 1$

$\therefore a_1 = 5A + 2B = 6$ —— (2)

Solving (1) and (2)

we get $A = \frac{4}{3}$ and $B = \frac{-1}{3}$

Hence the desired relation solution is

$$\boxed{a_n = \frac{4}{5}(5)^n - \frac{1}{3}(2)^n}$$