JUNAID. GIRKAR
60004190057

DM
TUTORIAL - 6

Q1 Let $f(x) = x+2$, $g(x) = x-2$ and $h(x) = 3x$. For $x \in R$ where R = set of real numbers, find $(g \circ f)$, $(f \circ g)$, $(f \circ h \circ g)$

Q2 Test whether the following function is one to one, onto or both : $f : Z \to Z$, $f(x) = x^2 + x + 1$

ANS 1 Given: $f(x) = x + 2$
$g(x) = x - 2$
$h(x) = 3x$

To find : (i) $g \circ f$
(ii) $f \circ g$
(iii) $f \circ h \circ g$

(i) $g \circ f(x) = g[f(x)] = g[x+2]$
$\therefore g \circ f = x + 2 - 2 = x$
$\therefore g \circ f = x$

(ii) $f \circ g(x) = F[g(x)] = F[x-2]$
$\therefore f \circ g = x - 2 + 2 = x$
$\therefore f \circ g = x$

(iii) $f \circ h \circ g(x) = F\{h[g(x)]\}$
$\therefore h[g(x)] = h(x-2) = 3(x-2) = 3x - 6$
$\therefore f\{h[g(x)]\} = f(3x-6) = 3x - 6 + 2 = 3x - 4$
$\therefore f \circ h \circ g = 3x - 4$

ANS 2 Function $f: Z \rightarrow Z$, defined by $f(x) = x^2 + x + 1$

* One to one (Injectivity)

Let x, y be any two elements in the domain (Z) such that $f(x) = f(y)$

$\therefore \; x^2 + x + 1 = y^2 + y + 1$

$\therefore \; x^2 - y^2 + x - y = 0$

$(x+y)(x-y) + (x-y) = 0$

$(x-y)(x+y+1) = 0$

$\therefore \; x = y$ or $x = -y-1$

The second equality gives $x = -y-1 \Rightarrow x \neq y$

Hence f is not injective (one to one)

* ONTO (surjectivity):

Let $y = f(x) = x^2 + x + 1$. We cannot solve the equation for x, i.e. we can't express x in terms of y.

So trying for $y = 0 \Rightarrow x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2} \longrightarrow \text{IMAGINARY}$$

For $y = 6$: $x^2 + x + 1 = 6 \Rightarrow x^2 + x - 5 = 0$

$$\therefore \; x = \frac{-1 \pm \sqrt{1+20}}{2} = \frac{-1 \pm \sqrt{21}}{2} \longrightarrow x \notin Z$$

A function is surjective if for every element in domain, there is some element in range where

f(element in domain) = element in range.

Here, $f(z) \neq z$ for all elements.

Hence f is **NOT** surjective (ONTO)

$\therefore$ f is neither one to one nor onto.