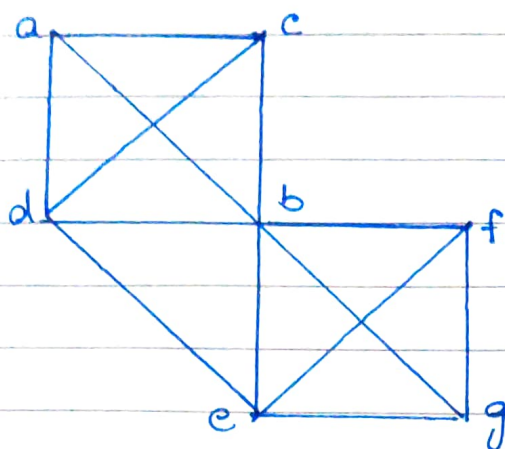


DISCRETE STRUCTURES SEC-B TT2

Q1



there are 4 vertices  $a, c, f, g$  with odd degree 3.

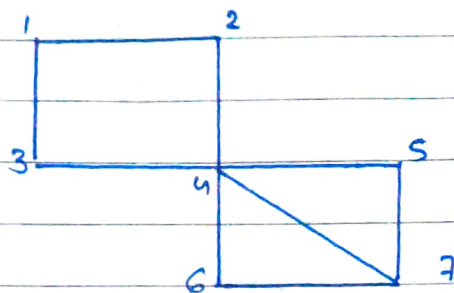
By theorem 1, since there are four vertices of odd degree, there can be no Eulerian circuit.

By theorem 2, since there are more than 2 vertices of odd degree there can be no Eulerian path.

THEOREM 1: A connected graph  $G$  is an Eulerian graph if and only if all vertices of  $G$  are of even degree.

THEOREM 2: If  $G$  is a connected graph having exactly two vertices  $u$  and  $v$  of odd degree then there is a Eulerian path from  $u$  to  $v$  which includes all the edges and all the vertices of  $G$ .

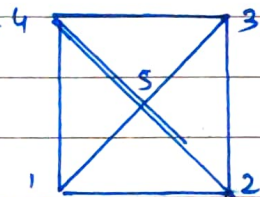
Q2 a)



- It is a simple graph with  $n=7$  vertices. The degree of vertex 1 is 2 and of vertex 7 is 3.
- The sum of these degrees is 5 and is not greater than number of vertices 7.
- Hence by theorem 1, the graph is not Hamiltonian. There is no Hamiltonian circuit.
- But there is an Hamiltonian path

$$\pi : 3, 1, 2, 4, 6, 7, 5$$

b)



- The graph is simply connected. There are  $n=5$  vertices. But the degree of each of the vertices 1, 2, 3, 4 is 3 and the degree of the vertex 5 is 4.
- Since the degree of each vertex is greater than  $n/2 = 2$ , there is by theorem 2, a Hamiltonian circuit.
- The Hamiltonian circuit is  $\pi : 1, 2, 5, 3, 4, 1$
- The graph is Hamiltonian

- $\therefore$  There is a Hamiltonian path

$$\pi : 1, 2, 5, 3, 4$$