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MATHS 4

TT-2

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Q1

Maximize $Z = 4x_1 - 2x_2 - x_3$

subject to

$$x_1 + x_2 + x_3 \le 3$$
$$2x_1 + 2x_2 + x_3 \le 4$$
$$x_1 - x_2 \le 0$$

and $x_1, x_2, x_3 \ge 0$

Introducing slack variables (s_1, s_2, s_3)

Maximize $Z = 4x_1 - 2x_2 - x_3 + 0s_1 + 0s_2 + 0s_3$

subject to

$$x_1 + x_2 + x_3 + s_1 = 3$$
$$2x_1 + 2x_2 + x_3 + s_2 = 4$$
$$x_1 - x_2 + s_3 = 0$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$$

ITERATION 1		C_j	4	-2	-1	0	0	0	MIN RATIO
B	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	X_B/x_1
s_1	0	3	1	1	1	1	0	0	3
s_2	0	4	2	2	1	0	1	0	2
s_3	0	0	$\boxed{1}$	-1	0	0	0	1	0 →
$Z=0$	Z_j	0	0	0	0	0	0	0	
		$Z_j - C_j$	-4	2	01	0	0	0	
			↑						

Negetive minimum $Z_j - C_j = -4$ and its column index is 1

∴ Entering variable $= x_1$

Minimum ratio is 0 and its row index is 3

∴ leaving basis variable is s_3

∴ Pivot element = 1

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ITERATION - 2		c_j	4	-2	-1	0	0	0	MIN RATIO
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	X_B/x_2
S_1	0	3	0	2	1	1	0	-1	1.5
S_2	0	4	0	$\boxed{4}$	1	0	1	-2	1 →
x_1	4	0	1	-1	0	0	0	1	-
Z = 0		Z_j	4	-4	0	0	0	4	
		$Z_j - C_j$	0	-2 ↑	1	0	0	4	

Negetive minimum $Z_j - C_j = 2$ and its column index = 2

∴ Entering variable = x_2

Minimum ratio = 1, row index = 2

∴ leaving basis variable = S_2

∴ Pivot element = 4

ITERATION - 3		c_j	4	-2	-1	0	0	0	MIN RATIO
B	C_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	
S_1	0	1	0	0	0.5	1	-0.5	0	
x_2	-2	1	0	1	0.25	0	0.25	-0.5	
x_1	4	1	01	0	0.25	0	0.25	0.5	
Z = 2		Z_j	4	-2	0.5	0	0.5	3	
		$Z_j - C_j$	0	0	1.5	0	0.5	3	

Since all $Z_j - C_j \geq 0$

Hence optimal solution has $x_1 = 1$, $x_2 = 1$, $x_3 = 0$

Maximum $Z = 2$

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Q3

Given $z = 4x_1^2 + 3x_2^2$

Subject to : $x_1 + 2x_2 = 9$

$x_1, x_2 \geq 0$

We have the lagrangian function :

$L(x_1, x_2, \lambda) = 4x_1^2 + 3x_2^2 - \lambda(x_1 + 2x_2 - 9)$

obtaining partial derivatives

$\therefore \frac{\partial L}{\partial x_1} = 0 \Rightarrow 8x_1 - \lambda = 0 \Rightarrow 8x_1 = \lambda \longrightarrow (1)$

$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 6x_2 - 2\lambda = 0 \Rightarrow 6x_2 = 2\lambda \Rightarrow 3x_2 = \lambda \longrightarrow (2)$

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow x_1 + 2x_2 - 9 = 0 \Rightarrow x_1 + 2x_2 = 9 \longrightarrow (3)$

$\therefore 8x_1 = \lambda \quad \& \quad 3x_2 = \lambda$

$8x_1 = 3x_2 \Rightarrow x_1 = \frac{3x_2}{8}$

Substituting value in (3)

$x_1 + 2x_2 = 9$

$3x_2 + 16x_2 = 72$

$\boxed{\therefore x_2 = 3.79}$

$x_1 = \frac{3x_2}{8} =$

$\boxed{x_1 = 1.42}$

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From (1), $\lambda = 8x_1 = 8(1.42)$

$\therefore \quad \lambda = 11.36$

x_0 is $(1.42, 3.79)$

$h(x_1, x_2) = x_1 + 2x_2 - 9 = 0$

$\therefore \quad \frac{\partial h}{\partial x_1} = 1, \quad \frac{\partial h}{\partial x_2} = 2$

$f(x_1, x_2) = 4x_1^2 + 3x_2^2$

$\frac{\partial f}{\partial x_1} = 8x_1 \qquad \frac{\partial f}{\partial x_2} = 6x_2$

$\frac{\partial^2 f}{\partial x_1^2} = 8 \qquad \frac{\partial^2 f}{\partial x_2^2} = 6$

$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0 \qquad \frac{\partial^2 f}{\partial x_2 \partial x_1} = 0$

$$\Delta = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda \frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda \frac{\partial^2 h}{\partial x_1 \partial x_2} \\ \frac{\partial h}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda \frac{\partial^2 h}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda \frac{\partial^2 h}{\partial x_2^2} \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 8 & 0 \\ 2 & 0 & 6 \end{vmatrix}$$

$= 0 - 1(6 - 0 + 2(1 - 16))$

$= -6 - 30$

$= -36$

$\because$ The value of determinent ≤ 0, x_0 is a minima

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$$Z_{min} = 4(1.42)^2 + 3(3.79)^2$$
$$= 4(2.016) + 3(14.364)$$
$$\boxed{= 51.156}$$

Q2 Minimize $z = 5x_1 + 2x_2$

such that
$x_1 - x_2 \leq 1$ — 1
$x_1 + x_2 \geq 4$ — 2
$x_1 - 3x_2 \leq 3$ — 3
$x_1, x_2 \geq 0$

converting to dual form

Number of dual constraints = No of prime variables = 2

~~Number of dual~~

let y_1, y_2, y_3 be dual variables corresponding to 1st, 2nd and 3rd constraints

Minimize $T = y_1 - 4y_2 + 3y_3$

such that
$y_1 - y_2 + y_3 \geq 5$
$-y_1 - y_2 - 3y_3 \geq 2$
$y_1, y_2, y_3 \geq 0$

The standard form is:

Minimize $T = -[\text{Max}(-T = -y_1 + 4y_2 - 3y_3 + 0s_1 + 0s_2 - Ma_1 - Ma_2]$

such that
$y_1 - y_2 + y_3 - s_1 + a_1 = 5$
$-y_1 - y_2 - 3y_3 - s_2 + a_2 = 2$
$y_1, y_2, y_3, s_1, s_2 \geq 0$
$a_1, a_2 \geq 0$

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Iteration 1 :

	c_j		-1	4	-3	0	0	-m	-m	
C_B	X_B	sol	y_1	y_2	y_3	S_1	S_2	a_1	a_2	Min Ratio
-m	a_1	5	1	-1	1	-1	0	1	0	
-m	a_2	2	-1	-1	-3	0	-1	0	1	
$z_j - c_j$			1	2m-4	2m+3	m	m	0	0	

since all $z_j - c_j \geq 0$

But the basis column has artificial variable i.e. artificial variable is not departed from basis column

$\therefore$ The dual of LPP has infeasable solution

$\therefore$ The primal of LPP will have unbounded solution