Junaid Girkar
SE COMPS
60004190057

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MATHS TUTORIAL - 1

1 Show that: $L\{\sinh^5 t\} = \dfrac{5!}{(s^2-1)(s^2-9)(s^2-25)}$

2 Find $L\{f(t)\}$ $\quad f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ e^t & 1 \le t \le 4 \\ 0 & t \ge 4 \end{cases}$

3 find $L\{t e^{-2t} \cos 3t . \sin 2t\}$

4 find $L\left[\dfrac{\cos 6t - \cos 4t}{t}\right]$

5 Find $L\left\{\dfrac{e^{-2t} \sin 3t . \sinh t}{t}\right\}$

6 Find $L\left\{e^{-t} \int_0^t \dfrac{e^t \sin t}{t} dt\right\}$

7 Find $L\left\{\int_0^t u \cos^2 u \, du\right\}$

1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

ANS 1

$$L\{\sinh^5 t\} = L(f(t))$$

$$f(t) = \left(\frac{e^t - e^{-t}}{2}\right)^5$$

$$= \frac{1}{32}\left[e^{5t} - 5e^{3t} + 10e^{t} - 10e^{-t} + 5e^{-3t} - e^{-5t}\right]$$

$$= \frac{1}{16}\left\{\frac{e^{5t} - e^{-5t}}{2} - \frac{5e^{3t} - 5e^{-3t}}{2} + 10\frac{(e^{t} - e^{-t})}{2}\right\}$$

$$= \frac{1}{16}\left[\sinh 5t - 5\sinh 3t + 10\sinh t\right]$$

$$L(f(t)) = \frac{1}{16}\left[\frac{5}{s^2 - 25} - \frac{15}{s^2 - 9} + \frac{10}{s^2 - 1}\right] \qquad \left[L(\sinh at) = \frac{a}{s^2 - a^2}\right]$$

$$\cancel{= \frac{1}{16}\left[5s^2 - 45 + 5s^2 - 5\right.}$$

$$= \frac{1}{16}\left[\frac{5(s^2-1)(s^2-9) - 15(s^2-25)(s^2-1) + 10(s^2-25)(s^2-9)}{(s^2-1)(s^2-9)(s^2-25)}\right]$$

$$= \frac{1}{16}\left[\frac{\cancel{5s^4} - \cancel{50s^2} + 45 - \cancel{15s^4} + \cancel{390s^2} - 375 + \cancel{10s^4} - \cancel{340s^2} + 2250}{(s^2-1)(s^2-9)(s^2-25)}\right]$$

$$= \frac{1}{16}\left[\frac{1920}{(s^2-1)(s^2-9)(s^2-25)}\right]$$

$$= \frac{120}{(s^2-1)(s^2-9)(s^2-25)}$$

$$= \frac{5!}{(s^2-1)(s^2-9)(s^2-25)}$$

$= RHS$

Hence Proved.

ANS 2 Find $L(f(t))$ where $f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ e^t & 1 \le t \le 4 \\ 0 & 4 < t \end{cases}$

$$L(f(t)) = \int_0^\infty e^{-st} \cdot f(t)\,dt$$

$$= \int_0^1 e^{-st}(1)\,dt + \int_1^4 e^{-st} e^{t}\,dt + \int_4^\infty e^{-st}(0)\,dt$$

$$= \int_0^1 e^{-st}\,dt + \int_1^4 e^{(-s+1)t}\,dt$$

$$= \frac{-1}{s}\left[e^{-st}\right]_0^1 + \frac{1}{1-s}\left[e^{(-s+1)t}\right]_1^4$$

$$= \frac{-1}{s}\left[e^{-s} - 1\right] + \frac{1}{1-s}\left[e^{4(1-s)} - e^{(1-s)}\right]$$

$$= \frac{1-e^{-s}}{s} + \frac{e^{4-4s} + e^{1-s}}{1-s}$$

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ANS 3

$$L\left\{ t e^{-2t} \cos 3t . \sin 2t \right\}$$

$$\sin 2t . \cos 3t = \frac{1}{2}[\sin 5t - \sin t]$$

$$L(\sin 2t . \cos 3t) = L\left[\frac{1}{2}(\sin 5t - \sin t)\right]$$

$$= \frac{1}{2}\left[\frac{5}{s^2+25} - \frac{1}{s^2+1}\right]$$

$$= \bar{f}(s)$$

$$L(e^{-2t} . \bar{f}(s)) = \bar{f}(s+2) = \frac{1}{2}\left[\frac{5}{(s+2)^2+25} - \frac{1}{(s+2)^2+1}\right]$$

$$= \bar{g}(s)$$

$$L(t\bar{g}(s)) = (-1)^1 \frac{d}{ds}\left[\frac{1}{2}\left(\frac{5}{(s+2)^2+25} - \frac{1}{(s+2)^2+1}\right)\right]$$

$$= -\frac{1}{2}\left[\frac{-5 \times 2(s+2)}{((s+2)^2+25)^2} - \frac{-1 \times 2(s+2)}{((s+2)^2+1)^2}\right]$$

$$= \frac{5(s+2)}{((s+2)^2+25)^2} - \frac{s+2}{((s+2)^2+1)^2}$$

ANS 4 $\quad L\left\{\dfrac{\cos 6t - \cos 4t}{t}\right\}$

$$L(\cos 6t - \cos 4t) = \bar{f}(s) = \frac{s}{s^2+36} - \frac{s}{s^2+16}$$

$$L\left(\frac{f(t)}{t}\right) = \int_s^{\infty} L(f(t))\,ds$$

$$= \frac{1}{2}\int_s^{\infty} \frac{2s}{s^2+36} - \frac{2s}{s^2+16}\,ds$$

$$= \frac{1}{2}\left[\log(s^2+36) - \log(s^2+16)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+36}{s^2+16}\right)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log\left(\frac{1+36/s^2}{1+16/s^2}\right)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log(1) - \log\left(\frac{s^2+36}{s^2+16}\right)\right] + c$$

$$= \frac{1}{2}\left[\log(s^2+16) - \log(s^2+36)\right] + c$$

$$= \log\left(\sqrt{\frac{s^2+16}{s^2+36}}\right) + c$$

ANS 5

$$L\left\{\frac{e^{-2t}\sin 3t.\sinh t}{t}\right\} = L(f(t))$$

$$f(t) = \frac{1}{t}\, e^{-2t}\left(\frac{e^{t}-e^{-t}}{2}\right)\sin 3t$$

$$= \frac{1}{2t}\left(e^{-t}-e^{-3t}\right)\sin 3t$$

$$= \frac{1}{2t}e^{-t}\sin 3t - \frac{1}{2t}e^{-3t}\sin 3t.$$

$$L(f(t)) = \frac{1}{t}\left[\frac{e^{-t}\sin 3t}{2} - \frac{e^{-3t}\sin 3t}{2}\right] = \frac{g(t)}{t}$$

$$L(g(t)) = L\left[\frac{e^{-t}}{2}\frac{3}{s^2+9} - \frac{e^{-3t}}{2}\frac{3}{s^2+9}\right]$$

$$= \frac{3}{2}\frac{1}{(s+1)^2+9} - \frac{3}{2}\frac{1}{(s+3)^2+9}$$

$$L\left(\frac{g(t)}{t}\right) = \int_s^\infty g(t)\,ds$$

$$= \frac{3}{2}\int_s^\infty \frac{1}{(s+1)^2+9}\,ds - \frac{1}{(s+3)^2+9}$$

$$= \frac{3}{2}\left[\frac{1}{3}\tan^{-1}\left(\frac{s+1}{3}\right) - \frac{1}{3}\tan^{-1}\left(\frac{s+3}{3}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\pi/2 - \tan^{-1}\left(\frac{s+1}{3}\right) - \frac{\pi}{2} + \tan^{-1}\left(\frac{s+3}{3}\right)\right]$$

$$= \frac{\tan^{-1}\left(\frac{s+3}{3}\right) - \tan^{-1}\left(\frac{s+1}{3}\right)}{2}$$ //

ANS 6 $L\left[e^{-t}\int_0^t \frac{e^t \sin t}{t}\,dt\right]$

let $g(t) = \int_0^t \frac{e^t \sin t}{t}\,dt$

$L(\sin t) = \frac{1}{s^2+1}$

$L(e^t \sin t) = \frac{1}{(s-1)^2+1}$

$L\left(\frac{e^t \sin t}{t}\right) = \int_s^\infty \frac{1}{(s-1)^2+1}\,ds$

$= \left[\tan^{-1}(s-1)\right]_s^\infty$

$= \frac{\pi}{2} - \tan^{-1}(s-1)$

$= \cot^{-1}(s-1)$

$L\left[\int_0^t \frac{e^t \sin t}{t}\right] = \frac{1}{s}\cot^{-1}(s-1)$

$L\left[e^{-t}\int_0^t \frac{e^t \sin t}{t}\,dt\right] = \frac{1}{s+1}\cot^{-1}(s+1-1)$

$= \frac{\cot^{-1}(s)}{s+1}$ //

ANS 7 $L\left\{\int_0^t u\cos^2 u\, du\right\}$

$$L(\cos^2 u) = L\left(\frac{1+\cos 2u}{2}\right) = \frac{1}{2}\, L(1+\cos 2u)$$

$$= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$$

$$L(u\cos^2 u) = \frac{-1}{2}\frac{d}{ds}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$$

$$= \frac{-1}{2}\left[\frac{-1}{s^2} + \frac{s^2+4-s(2s)}{(s^2+4)^2}\right]$$

$$= \frac{1}{2s^2} - \frac{4-s^2}{2(s^2+4)^2}$$

$$L\left[\int_0^t u\cos^2 u\, du\right] = \frac{1}{s}\left[\frac{1}{2s^2} + \frac{s^2-4}{2(s^2+4)^2}\right]$$

$$= \frac{1}{2s^3} + \frac{s^2-4}{2s(s^2+4)^2} \quad //$$