Shri Vile Parle Kelavani Mandal's
DWARKADAS J. SANGHVI COLLEGE OF ENGINEERING
(Autonomous College Affiliated to the University of Mumbai)
NAAC Accredited with "A" Grade (CGPA : 3.18)

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TE COMPS A4

# EXPERIMENT - 6

**AIM**: Implement Hidden Markov Model.

**THEORY**:

A Hidden Markov Model (HMM) is a statistical model which is also used in machine learning. It can be used to describe the evolution of observable events that depend on internal factors, which are not directly observable. These are a class of probabilistic graphical models that allow us to predict a sequence of unknown variables from a set of observed variables.

In a Hidden Markov Model (HMM), we have an invisible Markov chain (which we cannot observe), and each state generates in random one out of k observations, which are visible to us.

Let's look at an example. Suppose we have the Markov Chain from above, with three states (snow, rain and sunshine), P - the transition probability matrix and q — the initial probabilities. This is the invisible Markov Chain — suppose we are home and cannot see the weather. We can, however, feel the temperature inside our room, and suppose there are two possible observations: hot and cold, where:

$$\mathbb{P}(\text{Hot}|\text{Snow}) = 0,\ \mathbb{P}(\text{Cold}|\text{Snow}) = 1$$
$$\mathbb{P}(\text{Hot}|\text{Rain}) = 0.2,\ \mathbb{P}(\text{Cold}|\text{Rain}) = 0.8$$
$$\mathbb{P}(\text{Hot}|\text{Sunshine}) = 0.7,\ \mathbb{P}(\text{Cold}|\text{Sunshine}) = 0.3$$

**Basic Example**

As a first example, we apply the HMM to calculate the probability that we feel cold for two consecutive days. In these two days, there are 3*3=9 options for the underlying Markov states. Let us give an example for the probability computation of one of these 9 options:

$$\mathbb{P}((\text{Cold}, \text{Cold}), (\text{Rain}, \text{Snow})) = \mathbb{P}((\text{Cold}, \text{Cold})|(\text{Rain}, \text{Snow})) \cdot \mathbb{P}(\text{Rain}, \text{Snow}) =$$
$$\mathbb{P}(\text{Cold}|\text{Rain}) \cdot \mathbb{P}(\text{Cold}|\text{Snow}) \cdot \mathbb{P}(\text{Snow}|\text{Rain}) \cdot \mathbb{P}(\text{Rain}) = 0.8 \cdot 1 \cdot 0.1 \cdot 0.2 = 0.016$$

**Finding Hidden States**

In some cases we are given a series of observations, and want to find the most probable corresponding hidden states.

A brute force solution would take exponential time (like the calculations above); A more efficient approach is called the Viterbi Algorithm; its main idea is as follows: we are given a sequence of observations $o_1,\ldots,o$ . For each state i and t=1,…,T, we define

$$\eta_t(i) = \max_{i_1,\ldots,i_{t-1}} \mathbb{P}\{x_1 = i_1, \ldots, x_{t-1} = i_{t-1}, x_t = i, o_1, \ldots, o_t\}$$

That is, the maximum probability of a path which ends at time t at the state i, given our observations. The main observation here is that by the Markov property, if the most likely path that ends with i at time t equals to some i* at time t−1, then i* is the value of the last state of the most likely path which ends at time t−1. This gives us the following forward recursion:

$$\eta_t(i) = \max_i p_{i,j} \alpha_j(o_t) \eta_{t-1}(i)$$

here, $\alpha$ (o ) denotes the probability to have o when the hidden Markov state is j

**Application of Hidden Markov Model**

An application, where HMM is used, aims to recover the data sequence where the next sequence of the data can not be observed immediately but the next data depends on the old sequences. Taking the above intuition into account the HMM can be used in the following applications:

- Computational finance
- speed analysis
- Speech recognition
- Speech synthesis
- Part-of-speech tagging
- Document separation in scanning solutions
- Machine translation
- Handwriting recognition
- Time series analysis
- Activity recognition

- Sequence classification
- Transportation forecasting

CODE:

```
import numpy as np
import matplotlib.pyplot as plt

from hmmlearn import hmm

# Now, let's act as the casin and exchange a fair die for a loaded one
# and generate a series of rolls that someone at the casino would
# observe.

# make our generative model with two components, a fair die and a
# loaded die
gen_model = hmm.MultinomialHMM(n_components=2, random_state=99)

# the first state is the fair die so let's start there so no one
# catches on right away
gen_model.startprob_ = np.array([1.0, 0.0])

# now let's say that we sneak the loaded die in:
# here, we have a 95% chance to continue using the fair die and a 5%
# chance to switch to the loaded die
# when we enter the loaded die state, we have a 90% chance of staying
# in that state and a 10% chance of leaving
gen_model.transmat_ = np.array([[0.95, 0.05],
                                [0.1, 0.9]])

# now let's set the emission means:
# the first state is a fair die with equal probabilities and the
# second is loaded by being biased toward rolling a six
gen_model.emissionprob_ = \
    np.array([[1 / 6, 1 / 6, 1 / 6, 1 / 6, 1 / 6, 1 / 6],
              [1 / 10, 1 / 10, 1 / 10, 1 / 10, 1 / 10, 1 / 2]])

# simulate the loaded dice rolls
rolls, gen_states = gen_model.sample(30000)

# plot states over time, let's just look at the first rolls for clarity
fig, ax = plt.subplots()
ax.plot(gen_states[:500])
ax.set_title('States over time')
```

```
ax.set_xlabel('Time (# of rolls)')
ax.set_ylabel('State')
fig.show()

# plot rolls for the fair and loaded states
fig, ax = plt.subplots()
ax.hist(rolls[gen_states == 0], label='fair', alpha=0.5,
        bins=np.arange(7) - 0.5, density=True)
ax.hist(rolls[gen_states == 1], label='loaded', alpha=0.5,
        bins=np.arange(7) - 0.5, density=True)
ax.set_title('Roll probabilities by state')
ax.set_xlabel('Count')
ax.set_ylabel('Roll')
ax.legend()
fig.show()

# split our data into training and validation sets (50/50 split)
X_train = rolls[:rolls.shape[0] // 2]
X_validate = rolls[rolls.shape[0] // 2:]

# check optimal score
gen_score = gen_model.score(X_validate)

best_score = best_model = None
n_fits = 50
np.random.seed(13)
for idx in range(n_fits):
    model = hmm.MultinomialHMM(
        n_components=2, random_state=idx,
        init_params='se')

    model.transmat_ = np.array([np.random.dirichlet([0.9, 0.1]),
                                np.random.dirichlet([0.1, 0.9])])
    model.fit(X_train)
    score = model.score(X_validate)
    print(f'Model #{idx}\tScore: {score}')
    if best_score is None or score > best_score:
        best_model = model
        best_score = score

print(f'Generated score: {gen_score}\nBest score:      {best_score}')

# use the Viterbi algorithm to predict the most likely sequence of
states
```

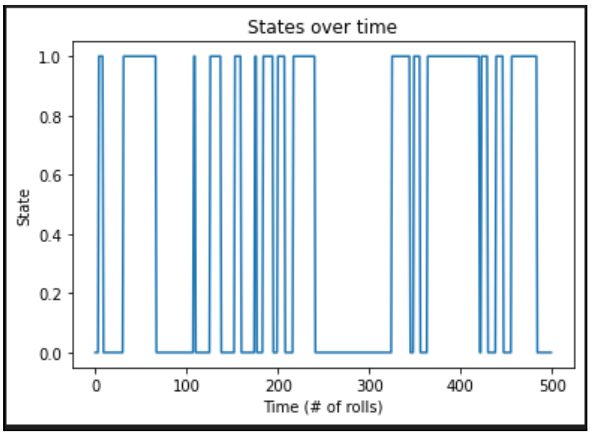
```
# given the model
states = best_model.predict(rolls)

# plot our recovered states compared to generated (aim 1)
fig, ax = plt.subplots()
ax.plot(gen_states[:500], label='generated')
ax.plot(states[:500] + 1.5, label='recovered')
ax.set_yticks([])
ax.set_title('States compared to generated')
ax.set_xlabel('Time (# rolls)')
ax.set_xlabel('State')
ax.legend()
fig.show()


print(f'Transmission Matrix
Generated:\n{gen_model.transmat_.round(3)}\n\n'
      f'Transmission Matrix
Recovered:\n{best_model.transmat_.round(3)}\n\n')


print(f'Emission Matrix
Generated:\n{gen_model.emissionprob_.round(3)}\n\n'
      f'Emission Matrix
Recovered:\n{best_model.emissionprob_.round(3)}\n\n')
```

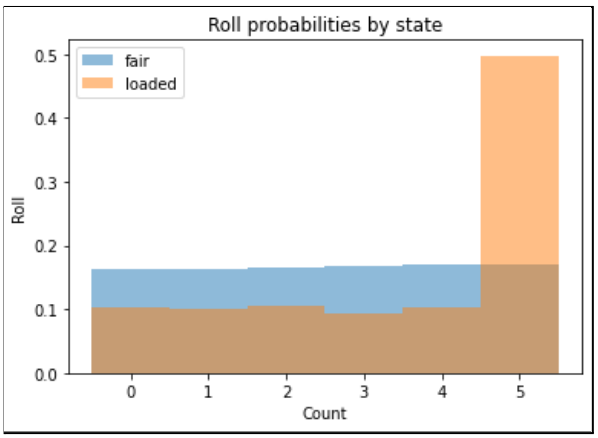
OUTPUT:

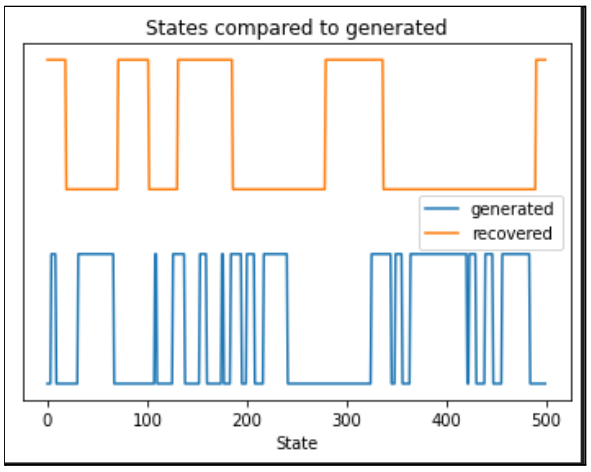
```
Model #0        Score: -26391.3688134072
Model #1        Score: -26395.55036572371
Model #2        Score: -26405.788668012425
Model #3        Score: -26396.290282611586
Model #4        Score: -26395.550365729432
Model #5        Score: -26375.803386308733
Model #6        Score: -26395.484479933773
Model #7        Score: -26300.67439789695
Model #8        Score: -26265.231805566247
Model #9        Score: -26395.550355030126
Model #10       Score: -26317.466352042524
Model #11       Score: -26405.60102609709
Model #12       Score: -26254.60496774158
Model #13       Score: -26395.48205064964
Model #14       Score: -26247.84864999629
```

```
Model #15       Score: -26279.11283863862
Model #16       Score: -26236.968567769647
Model #17       Score: -26320.826969997557
Model #18       Score: -26273.891661064114
Model #19       Score: -26405.955888278735
Model #20       Score: -26405.478747288675
Model #21       Score: -26385.9158985128
Model #22       Score: -26395.485391535083
Model #23       Score: -26395.550366098352
Model #24       Score: -26308.405549046784
Model #25       Score: -26395.550231732344
Model #26       Score: -26296.282172066236
Model #27       Score: -26382.50004338152
Model #28       Score: -26394.036977133965
Model #29       Score: -26396.28994962292
Model #30       Score: -26297.051796993994
Model #31       Score: -26282.17385580235
Model #32       Score: -26315.982257345328
Model #33       Score: -26255.807459691157
Model #34       Score: -26309.4618644052
Model #35       Score: -26395.550365831146
Model #36       Score: -26253.05848049172
Model #37       Score: -26395.550513487622
Model #38       Score: -26395.48290593895
Model #39       Score: -26276.364463630252
Model #40       Score: -26395.55036572371
Model #41       Score: -26395.48180629254
Model #42       Score: -26257.78147660928
Model #43       Score: -26395.550374348983
Model #44       Score: -26282.339321529027
Model #45       Score: -26262.651049578308
Model #46       Score: -26344.068596316894
Model #47       Score: -26229.219361702373
Model #48       Score: -26392.68971014984
Model #49       Score: -26337.637199573255
Generated score: -26230.575868403906
Best score:      -26229.219361702373

Transmission Matrix Generated:
[[0.95 0.05]
 [0.1  0.9 ]]

Transmission Matrix Recovered:
[[0.922 0.078]
```

```
 [0.059 0.941]]


Emission Matrix Generated:
[[0.167 0.167 0.167 0.167 0.167 0.167]
 [0.1   0.1   0.1   0.1   0.1   0.5  ]]

Emission Matrix Recovered:
[[0.106 0.108 0.114 0.105 0.113 0.454]
 [0.168 0.168 0.167 0.171 0.172 0.153]]
```




**CONCLUSION:** We learnt about the Hidden Markov Model and implemented it in python using the library HmmLearn.