

Assignment 1:

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In [59]:

```
import sympy as sp
import numpy as np
import scipy
import matplotlib.pyplot as plt
import scipy.misc
import imageio
```

Models of a neuron

1.1 An example of the logistic function is defined by

$$\varphi(\nu) = \frac{1}{1+e^{(-a\nu)}}$$

whose limiting values are 0 and 1. Show that the derivative of $\varphi(\nu)$ with respect to ν is given by

$$\frac{d\varphi}{d\nu} = a\varphi(\nu)[1 - \varphi(\nu)]$$

What is the value of this derivative at the origin?

In [35]:

```
sp.init_printing(use_latex=True)
v = sp.Symbol('v')
a = sp.Symbol('a')

phi = 1/(1+sp.exp(-a*v))

derivative_wrt_v = sp.diff(phi,v)

print "Derivative of phi(v):"
print "dphi/dv = {}".format(derivative_wrt_v)
print "Value of the derivative at the origin:"
print "dphi/dv(0) = {}".format(derivative_wrt_v.subs(v,0))
```

```
Derivative of phi(v):
dphi/dv = a*exp(-a*v)/(1 + exp(-a*v))**2
Value of the derivative at the origin:
dphi/dv(0) = a/4
```

1.2 An odd sigmoid function is defined by

$$\varphi(\nu) = \frac{1-e^{-a\nu}}{1+e^{-a\nu}} = \tanh\left(\frac{a\nu}{2}\right)$$

where \tanh denotes a hyperbolic tangent. The limiting values of this second sigmoid function are -1 and +1.

Show that the derivative of $\varphi(\nu)$ w.r.t. ν is given by

$$\frac{d\varphi}{d\nu} = \frac{a}{2}[1 - \varphi^2(\nu)]$$

What is the value of this derivative at the origin? Suppose that the slope parameter a is made infinitely large. What is the resulting form of $\varphi(\nu)$

In [37]:

```
v = sp.Symbol('v')
a = sp.Symbol('a')

phi = sp.tanh(a*v/2)

derivative_wrt_v = sp.diff(phi, v)

print "Derivative of phi(v):"
print "phi = ", derivative_wrt_v
print "Derivative of the infinitively large slope:"
print "dphi/dv(inf) = ", phi.subs(a,float('Inf'))
```

```
Derivative of phi(v):
phi = a*(-tanh(a*v/2)**2 + 1)/2
Derivative of the infinitively large slope:
dphi/dv(inf) = tanh(inf*v)
```

Network architectures

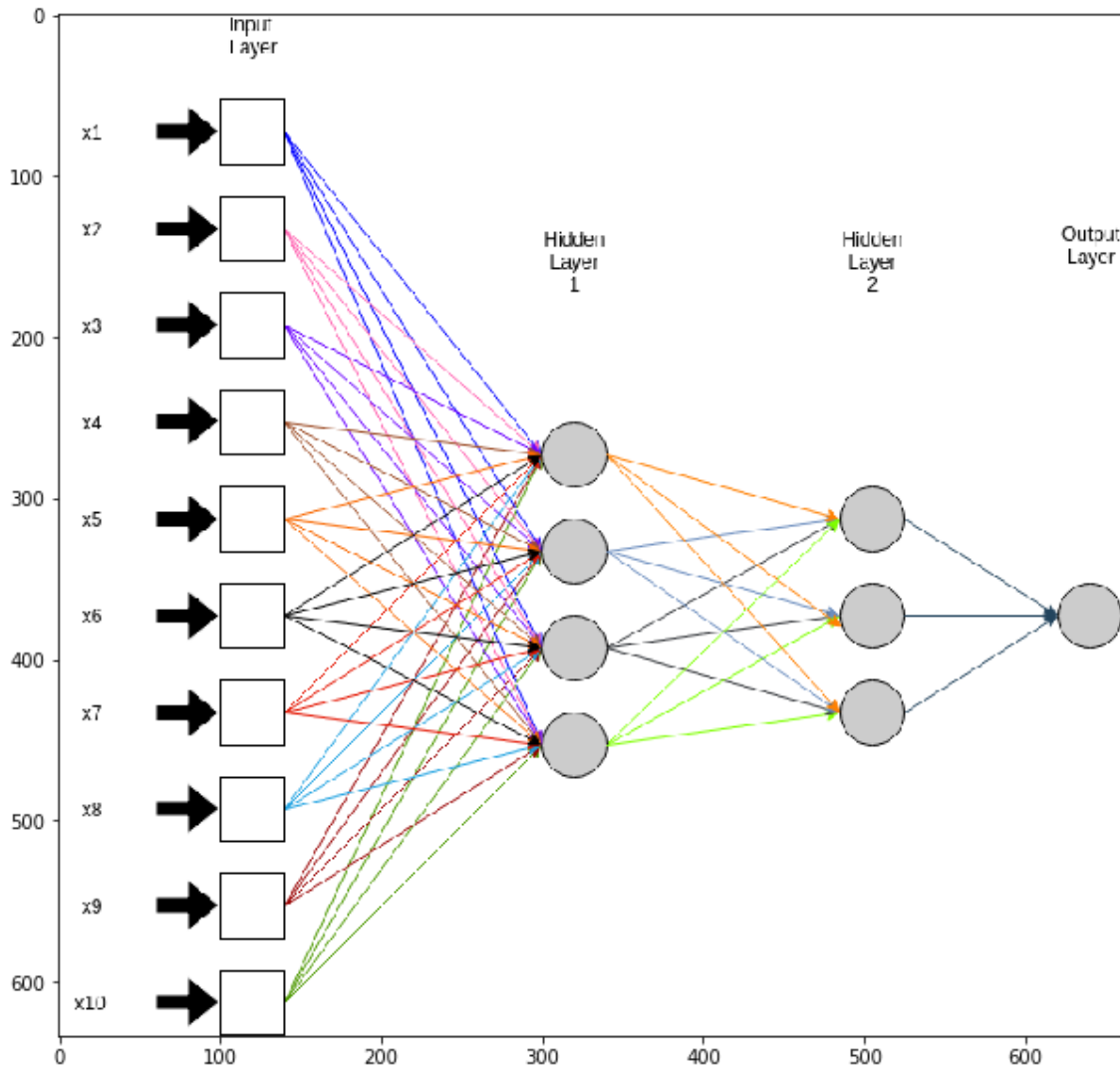
1.12 A fully connected feedforward network has 10 source nodes, 2 hidden layers, one with 4 neurons and the other with 3 neurons, and a single output neuron. Construct an architectural graph of this network.

In [63]:

```
im = imageio.imread('1_12.png')
plt.figure(figsize=(20,10))
plt.imshow(im)
```

Out[63]:

<matplotlib.image.AxesImage at 0x7fc83451ca90>



1.13 Based on the figure P1.13:

a) Write the input-output mapping defined by this network

b) If the output neuron operates in the linear region. Write the input-output mapping defined by this new network.

$$A) \psi[-2\psi[3\psi(5a + b) - 1\psi(-3b + 2a)] + 1\psi[6\psi(-3b + 2a) + 4\psi(5a + b)]]$$

$$B) [-2\psi[3\psi(5a + b) - 1\psi(-3b + 2a)] + 1\psi[6\psi(-3b + 2a) + 4\psi(5a + b)]] \approx 0$$

thus $\psi[0] = 0$, hence it lies in the linear region