Experiment 2: Measurement of g

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Plots

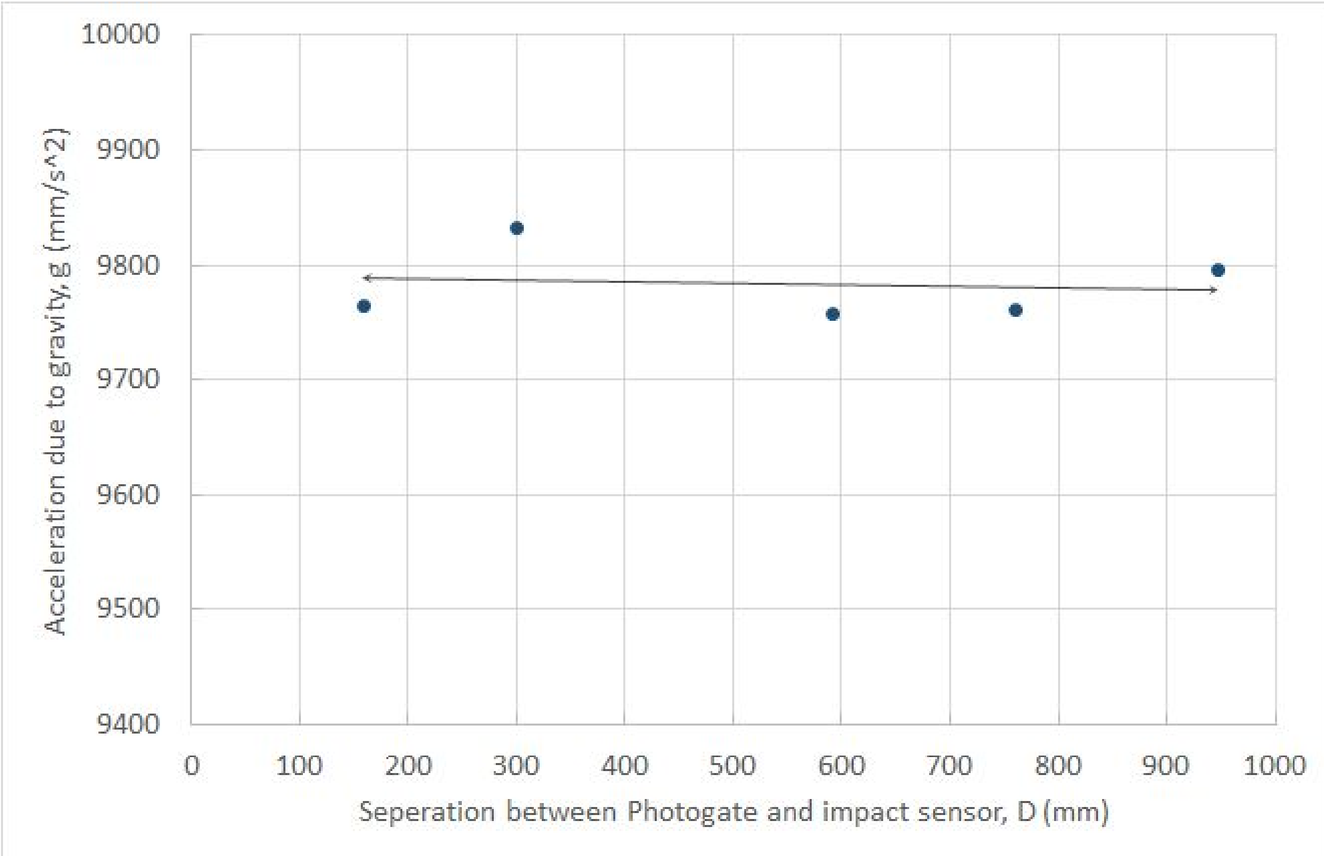

Figure 2.1 Graph of measured distance between lower photogate and impact sensor and calculated acceleration (g_{best}) **to show independence of** g **on** D**.** The dots mark the calculated accelerations for D = 158mm, 300mm, 591mm, 760mm and 947mm. The best fit line has the equation:
$g = -.013D + 9790.7$

Figure 2.1 illustrates that the slope of the best-fit line for the g_{best} vs D graph is consistent with 0 and the intercept is equal to 9790.7mm/s^2. As expected, the almost 0 slope of the best fitting line of g vs D graph indicates that the acceleration due to gravity is independent of the height of release. Thus the dependence of g on D can be qualitatively ruled out by observing the almost horizontal fit-line. The slope is not exactly equal to 0 since there may be some errors in measuring instruments due to which the calculated value may not be exact.

Data Table

Trial	Photogate spacing d (cm)	Gap to impact sensor D (cm)	Measured acceleration g (m/s^2)
1	$8.5 \pm .05$	$15.8 \pm .05$	$9.76 \pm .02$
2	$8.5 \pm .05$	$30.0 \pm .05$	$9.83 \pm .02$
3	$8.5 \pm .05$	$59.1 \pm .05$	$9.75 \pm .01$
4	$8.5 \pm .05$	$76.0 \pm .05$	$9.76 \pm .01$
5	$8.5 \pm .05$	$94.7 \pm .05$	$9.79 \pm .01$

Table 2.1 Experimental results and calculated acceleration values. The spacing, d was measured between the two photogates and the spacing D was measured between the lower photogate and the impact sensor. Equation 2.1 was used to obtain value of acceleration in cm/s^2 and then it was converted to m/s^2. Systematic and statistical uncertainties were added to obtain the value of uncertainty for acceleration. The calculated value of g in this experiment is $g = 9.78 \pm .02\ m/s^2$

Systematic Uncertainty:

Uncertainty in time: .000005s

Uncertainty in distance (for d and D): .05cm=.0005m

The upper and lower limits of d and D along with the best values for T_1 and T_2 were used to calculate g_{max} and g_{min}

Upper limits,

$$d = d_{best} + \delta d \text{ and } D = D_{best} + \delta D$$

Lower limits,

$$d = d_{best} - \delta d \text{ and } D = D_{best} - \delta D$$

Then $(g_{max} - g_{min})/2$ was calculated for each D. This value is the systematic contribution to δg

Gap to impact sensor D (cm)	Uncertainty in measured acceleration g (m/s^2)
$15.8 \pm .05$	$\pm .01$
$30.0 \pm .05$	$\pm .01$
$59.1 \pm .05$	$\pm .01$
$76.0 \pm .05$	$\pm .005$
$94.7 \pm .05$	$\pm .01$

Table 2.2 Systematic uncertainty in measured acceleration

Statistical Uncertainty

For statistical uncertainty, the sample standard deviation is used. The statistical uncertainty for a set of N points is given by:

$$\delta_x = \frac{\sigma_x}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\overline{x})^2} \qquad \text{(Eq ii.13)}$$

Excel's STDEV function was used to calculate the uncertainties:

Gap to impact sensor D (cm)	Uncertainty in measured acceleration g (m/s^2)
$15.8 \pm .05$	$\pm .01$
$30.0 \pm .05$	$\pm .01$
$59.1 \pm .05$	$\pm .006$
$76.0 \pm .05$	$\pm .005$
$94.7 \pm .05$	$\pm .005$

Table 2.3 Statistical uncertainty in measured acceleration

Derivation

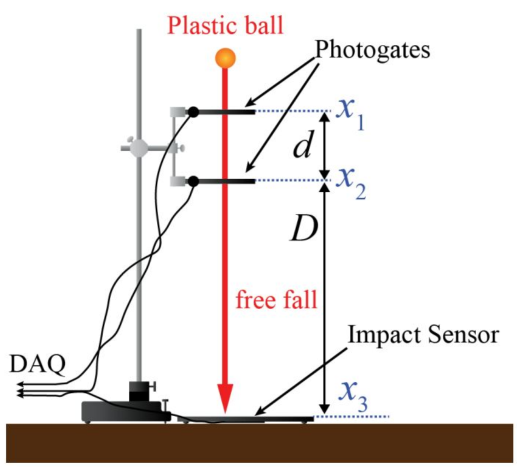

Figure 2.1 Experimental setup for calculating acceleration in free fall. A small ball is released at the top of the upper photogate and it passes through the lower photogate before striking the impact sensor. The DAQ measures the time to cover distance d and D. The D is changed for different trials to check if acceleration depends on height.

We know that acceleration is the rate of change of velocity. Here, the acceleration g is the ratio of difference in average velocities for covering d and D and the time taken for covering the distance between middle of upper region to middle of lower region.

v_d = average velocity in covering distance $d = \frac{d}{T_1}$
v_D = average velocity in covering distance $D = \frac{D}{T_2}$
Time taken for covering the distance between middle of upper region to middle of lower region= $(T_1 + T_2)/2$
Then,

$$g = \frac{v_D - v_d}{(T_1 + T_2)/2}$$

Substituting values of v_d and v_D
Thus,

$$g = \frac{2}{T_1 + T_2}\left(\frac{D}{T_2} - \frac{d}{T_1}\right) \qquad \text{(Eq. 2.1)}$$

Conclusion
The calculated value of g in this experiment is $g = 9.78 \pm .02\ m/s^2$. This value approaches the expected value of $g = 9.7955 \pm .0003\ m/s^2$. It can be seen by Figure 2.1 that the value of g is independent of *D.*

Both systematic and statistical uncertainties were taken into account for calculating the uncertainty in g. Systematic uncertainties may arise due to a various sources of error. Some possible sources of error are air resistance to the falling ball, measurement of distance of interval (d and D), imperfect calibration of instruments etc. Table 2.2 lists the systematic error in each of the trials.
Statistical uncertainties were calculated using Eq. ii.13 and Table 2.3 lists them for each trial.

A conservative method was used for calculating the total uncertainty in measurement of acceleration by adding the systematic and statistical uncertainties. This was used because it produces a larger uncertainty and it provides a better guarantee that the true value of g lies within the range specified by the uncertainty.

Presentation Mini-Report

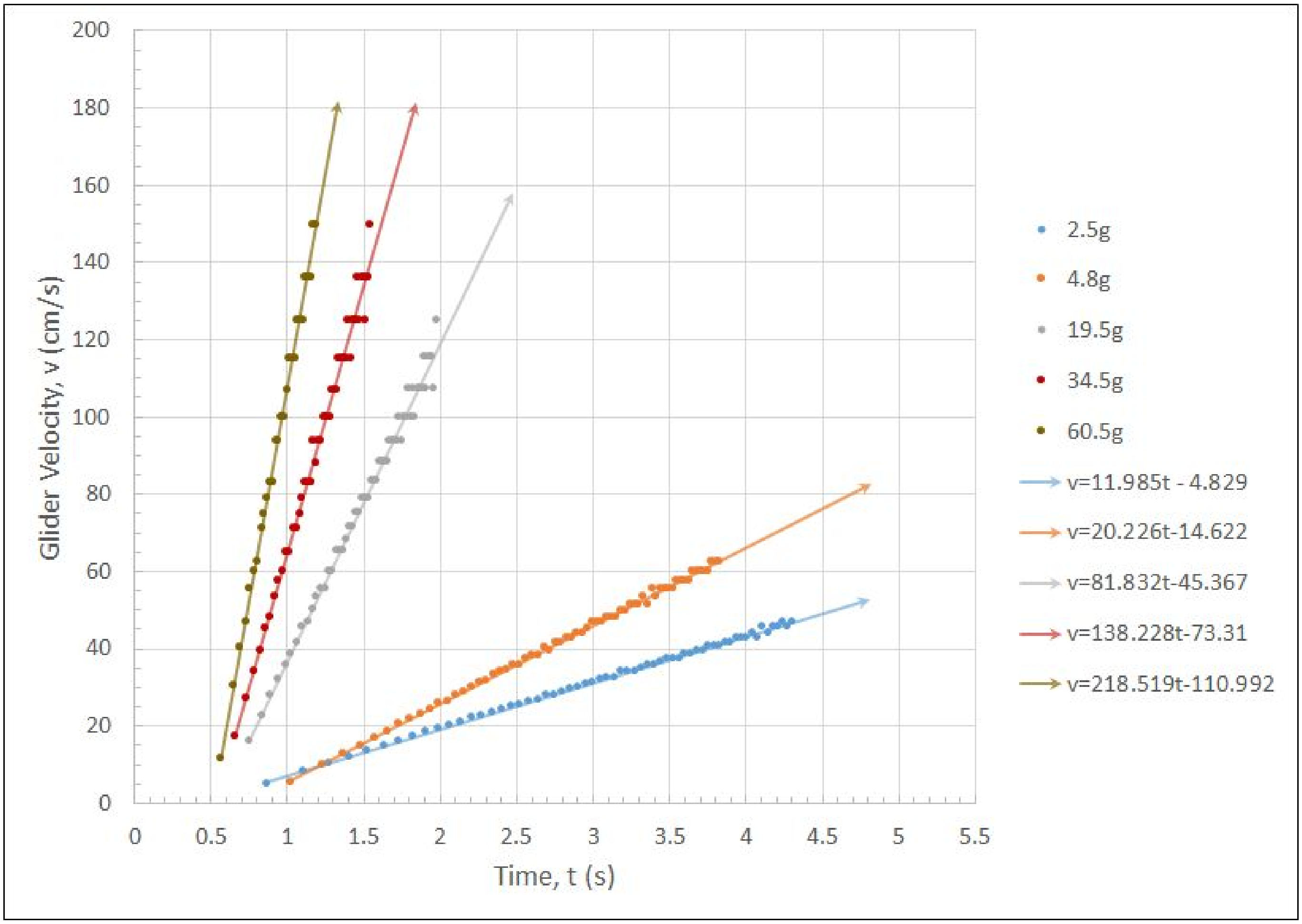

Figure 2.2 Velocity of glider for calculating constant acceleration. The colored dots on this graph represent data points recorded using a photogate for different hanging masses. Velocity as a function of time was calculated by obtaining a derivative of distance covered by the glider. The lines of best fit are represented by the solid linear lines as shown in the graph. The legend shows the corresponding masses and equations. Acceleration for each mass is the slope of the best-fit line.

Figure 2.2 shows the velocity of the glider on a level air-track with different masses attached to it using a string and hung over a smart pulley, at different times. The air-track is used to minimize the friction. The smart pulley measures the time intervals between the spokes blocking and unblocking the sensor of the pulley. These timings are sent to the DAQ and a table is created with block-count and time as the table fields. The linear distance covered by the string for each block count is roughly equal to 1.50 cm. By using this value, the distance covered by the string, and thus the distance covered by the hanging masses is calculated for each time reading that is recorded. For plotting the velocity vs time graph, the velocity for each system (hanging mass + glider mass) was calculated. Velocity as a function of time was calculated by obtaining a derivative of distance covered by the glider with respect to time. For obtaining the derivative, the ratio of change in position to the difference in consecutive time readings was taken. It can be clearly seen that the points plotted in Figure 2.2 are almost linear. The equations on the legend show the slope and intercept of the best-fit lines. The linear best-fit lines illustrate that the acceleration is constant since acceleration is given by the slope of the velocity vs time graph. Figure 2.2 also illustrates that the magnitude of the acceleration i.e the slope of the best-fit line increases as the hanging mass increases. Some data points do not align perfectly with the best-fit line because there might be a some sources of error in experimenting. For eg. the air track does not have exactly 0 friction. Moreover, the time readings were not precise enough. Since the slope of the graphs is constant for each mass and it increases as the hanging mass is increased, the figure agrees with the derived formula $a = \frac{mg}{M+m}$ Thus Figure 2.2 clearly demonstrates that the acceleration for a system of a hanging mass and glider is constant within experimental error.