

Model Orbita Sateliti

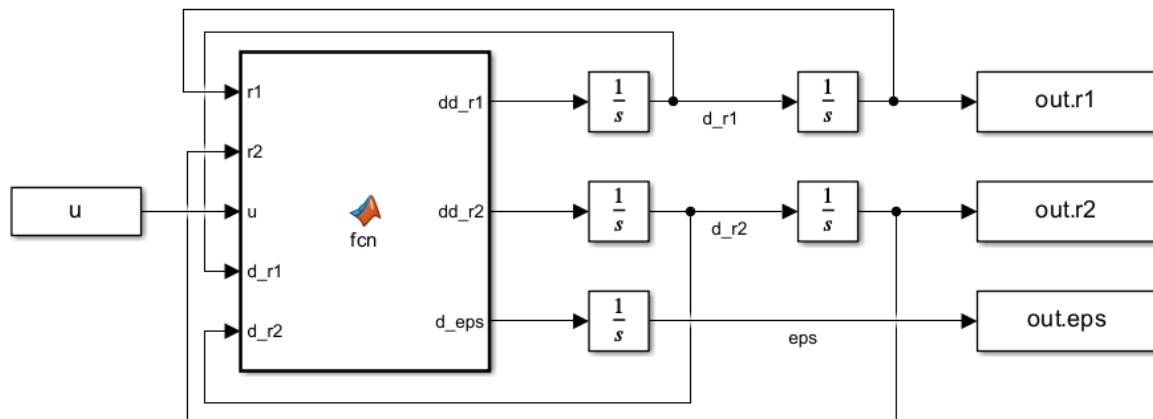
Cerinta 1

```

3 %% Cerinta 1
4 r10=1e6*[-3.111566746661099;2.420733547442338;-5.626803092559423];
5 r20=1e6*[-3.422723421327209;2.662806902186572;-6.189483401815366];
6 d_r10=1e3*[4.953572247000772;-3.787243278806948;-4.362500902062312];
7 d_r20=1e3*[5.448929471700850;-4.165967606687643;-4.798750992268544];
8 eps0=1;
9 x0=[r10;r20;d_r10;d_r20;eps0];

```

Cerinta 2



```

1 function [dd_r1, dd_r2, d_eps] = fcn(r1, r2, u, d_r1, d_r2)
2 dd_r1=g(r1,d_r1)+u;
3 dd_r2=g(r2,d_r2);
4 d_eps=norm(r1-r2)/norm(r2);

```

```

1 function dd_r = g(r, d_r)
2 G=6.674e-11;
3 M=5.972e24;
4 R=6371000;
5 J2=1.08262668e-3;
6 w=7.2921e-5;
7
8
9 x=r(1); y=r(2); z=r(3);
10 norm_r=norm(r);
11 dd_r=-G*M*r/norm_r^3 ...
12 -3/2*J2*G*M*R^2*[x-5*x*z^2; y-5*y*z^2; 3*z-5*z^3]/norm_r^7 ...
13 +w^2*[x;y;0] ...
14 +2*w*[d_r(2);-d_r(1);0];
15 end

```

Cerinta 3

$$x(t) = \begin{bmatrix} r_1 \\ r_2 \\ \dot{r}_1 \\ \dot{r}_2 \\ \varepsilon \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \rightarrow \dot{x}(t) = \begin{bmatrix} \dot{r}_1 \\ \dot{r}_2 \\ \ddot{r}_1 \\ \ddot{r}_2 \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ g(t, x_1, x_3) + u(t) \\ g(t, x_2, x_4) \\ \frac{\|x_1 - x_2\|}{\|x_2\|} \end{bmatrix} = f(t, x)$$

Cerinta 4

```

11      %% Cerinta 4
12      h=1;
13      tmax=10000;
14      t=0:h:tmax;
15      u=0;
16      x=[x0 zeros(13,length(t)-1)];
17
18      for i=1:length(t)-1
19          k1=f(u,x(:,i));
20          k2=f(u,x(:,i)+h*k1/2);
21          k3=f(u,x(:,i)+h*k2/2);
22          k4=f(u,x(:,i)+h*k3);
23          x(:,i+1)=x(:,i)+h/6*(k1+2*k2+2*k3+k4);
24      end
25      [r1_rk,r2_rk,eps_rk]=deal(x(1:3,:),x(4:6,:),x(13,:));

1 function d_x = f(u, x)
2     x1=x(1:3); x2=x(4:6); x3=x(7:9); x4=x(10:12);
3     d_x=[x3; x4; g(x1,x3)+u; g(x2,x4); norm(x1-x2)/norm(x2)];
4 end

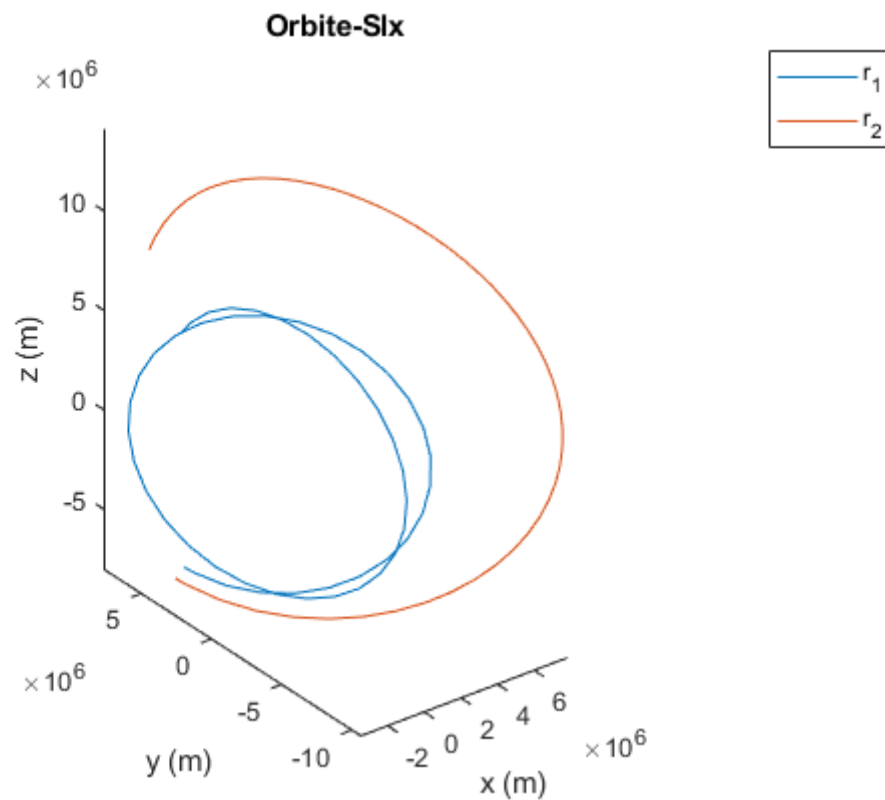
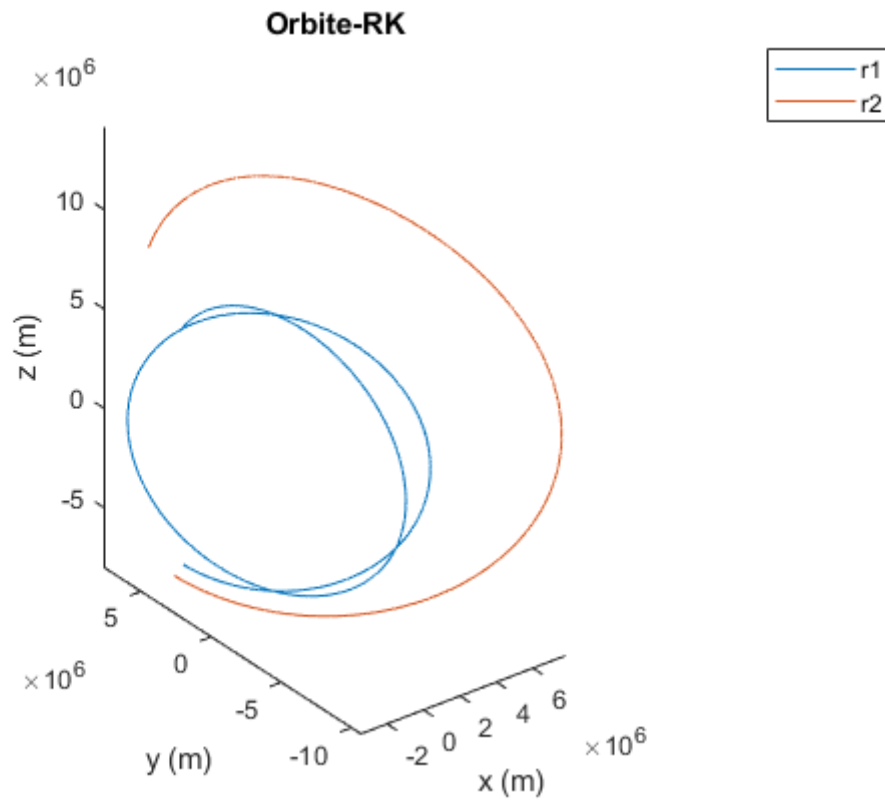
```

Cerinta 5

```

27      %% Cerinta 5
28      u=timeseries(zeros(size(t)),t);
29      load_system('Model_sim')
30      set_param('Model_sim','StopTime',num2str(tmax))
31      out=sim('Model_sim');
32      [r1_slx,r2_slx,eps_slx]=deal(squeeze(out.r1.Data),squeeze(out.r2.Data),squeeze(out.eps.Data));
33
34      figure
35      plot3(r1_rk(1,:),r1_rk(2,:),r1_rk(3,:))
36      hold on
37      plot3(r2_rk(1,:),r2_rk(2,:),r2_rk(3,:))
38      axis equal
39      xlabel("x (m)")
40      ylabel("y (m)")
41      zlabel("z (m)")
42      title("Orbite-RK")
43      legend("r1","r2")
44
45      figure
46      plot3(r1_slx(1,:),r1_slx(2,:),r1_slx(3,:))
47      hold on
48      plot3(r2_slx(1,:),r2_slx(2,:),r2_slx(3,:))
49      axis equal
50      xlabel("x (m)")
51      ylabel("y (m)")
52      zlabel("z (m)")
53      title("Orbite-Slx")
54      legend("r_1","r_2")

```

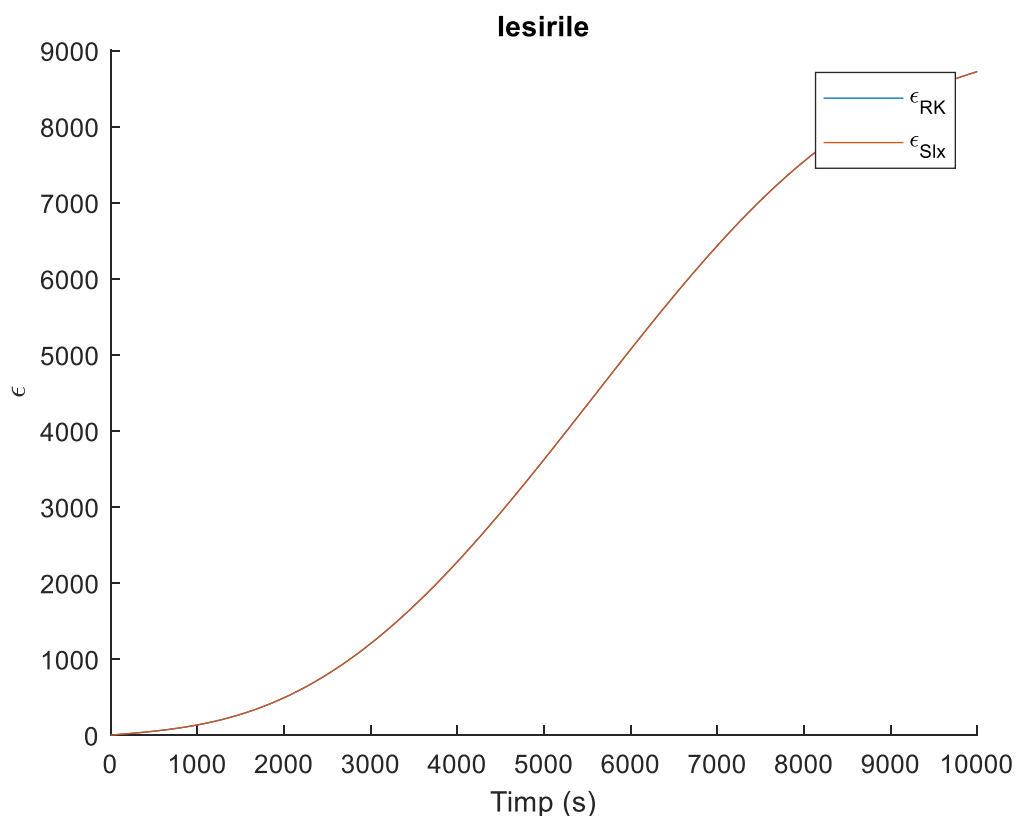


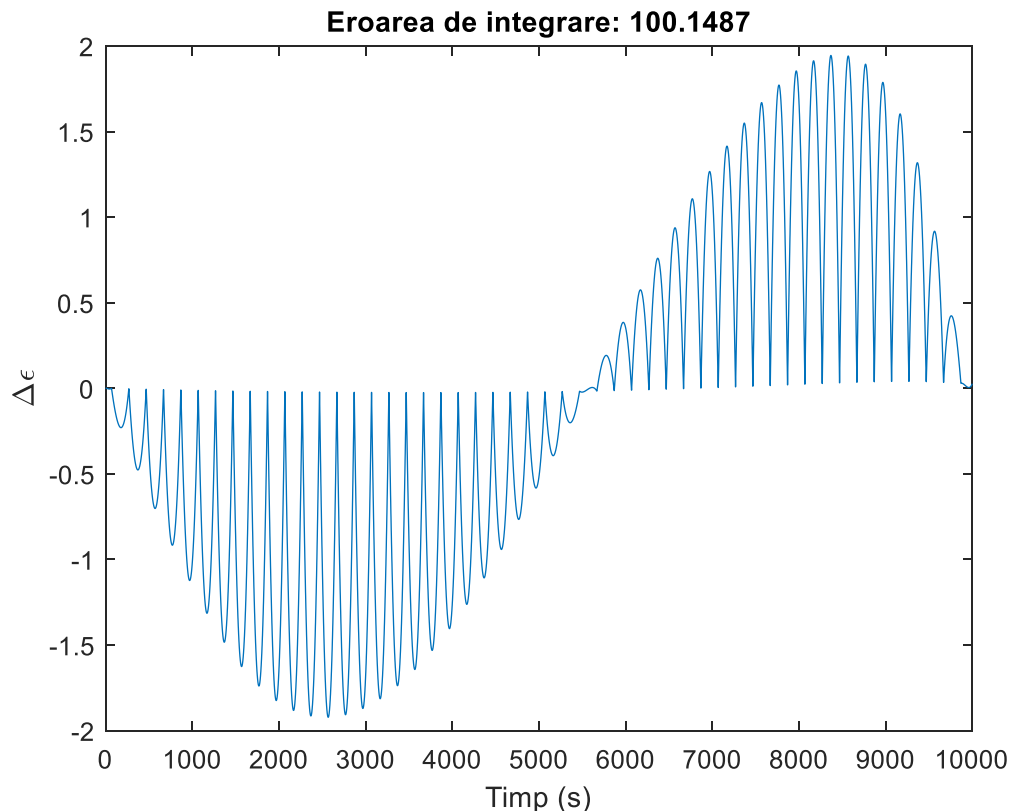
Comparand grafic rezultatele metodei Runge-Kutta cu cele din Simulink, observam ca sunt asemanatoare, insa orbita rezultata din Simulink nu este

la fel de smooth deoarece foloseste un pas adaptiv, fata de RK care foloseste un pas constant destul de mic.

Cerinta 6

```
56 %% Cerinta 6
57 eps_slx_int=interp1(out.tout,eps_slx,t);
58
59 figure
60 hold on
61 plot(t,eps_rk)
62 plot(t,eps_slx_int)
63 xlabel("Timp (s)")
64 ylabel("\epsilon")
65 title("Iesirile")
66 legend("\epsilon_{RK}", "\epsilon_{Slx}")
67
68 figure
69 plot(t,eps_rk-eps_slx_int)
70 xlabel("Timp (s)")
71 ylabel("\Delta\epsilon")
72 title("Eroarea de integrare: "+string(norm(eps_rk-eps_slx_int)))
```





Observam din nou ca cele doua metode dau rezultate asemanatoare, eroarea de integrare la fiecare moment de timp (si norma acesteia) fiind destul de mici, relativ la valorile iesirilor. In plus, eroarea oscileaza periodic cu amplitudine mica, indicand o concordanta buna intre ele.

Cerinta 7/8

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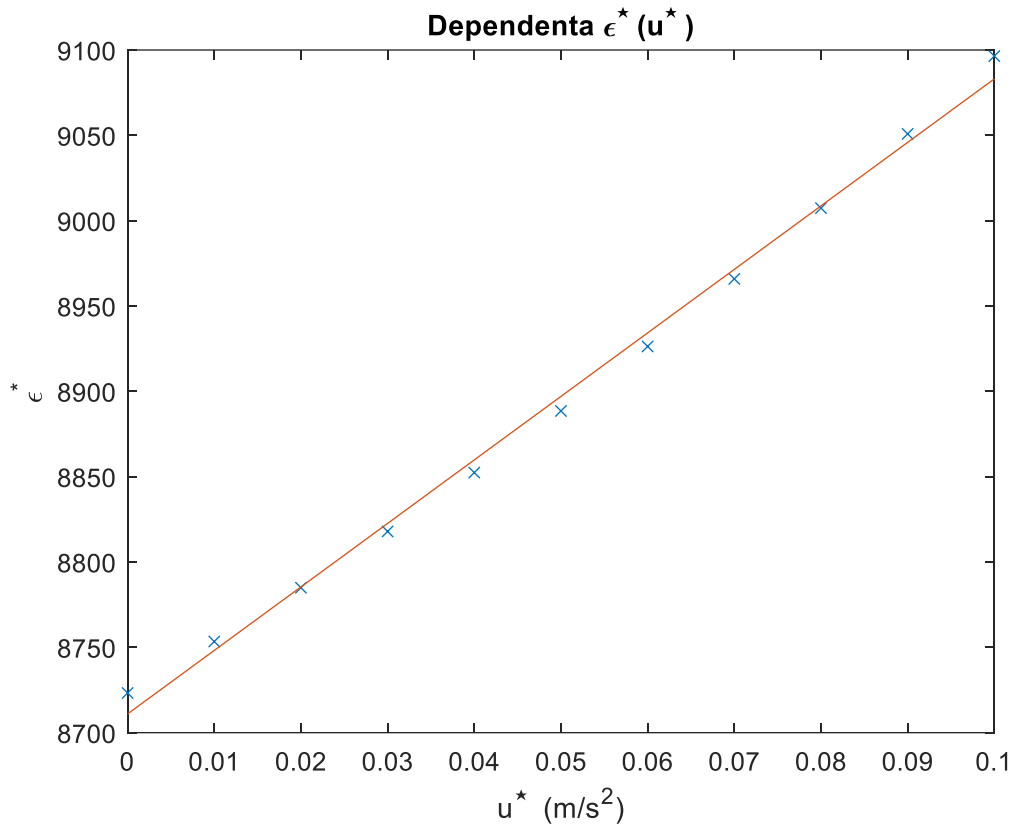
74 %% Cerinta 7
75 i=1;
76 for k=0:10:100
77     u=timeseries(k*1e-3.*double(t>=0),t);
78     out=sim('Model_sim');
79     eps_star(i)=out.eps.Data(end);
80     ustar(i)=k*1e-3;
81     i=i+1;
82 end
83
84 figure
85 plot(ustar,eps_star,'x');
86 xlabel("u^* (m/s^2)");
87 ylabel("\epsilon^*")
88 title("Dependenta \epsilon^*(u^*)")

```

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90 %% Cerinta 8
91 p=polyfit(ustar,eps_star,1);
92 ustar_grid=ustar(1):0.01:ustar(end);
93 eps_int=polyval(p,ustar_grid);
94
95 hold on;
96 plot(ustar_grid,eps_int);

```



Dependenta intre iesirea stationara (eroarea relativa totala intre cei doi sateliti) si semnalul exogen constant este aproximativ liniara, indicand o crestere proportionala cu semnalul.

Cerinta 9/10

```

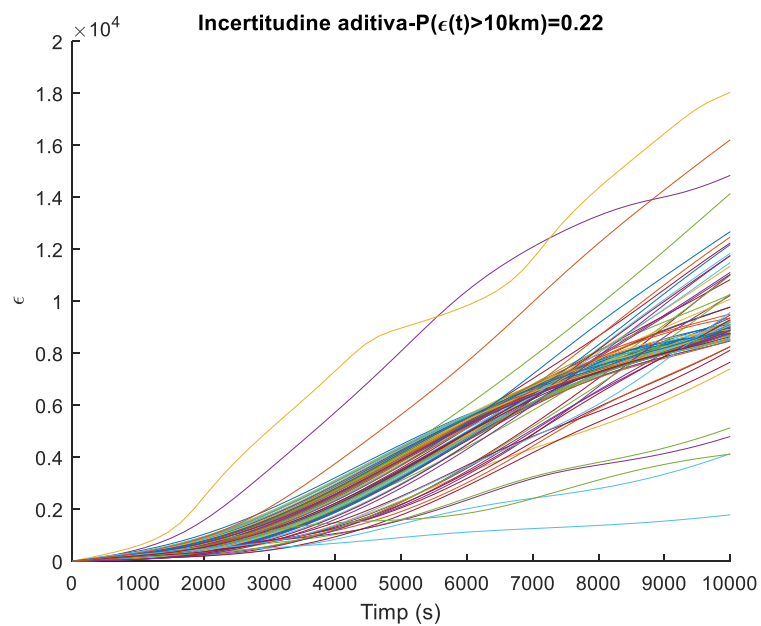
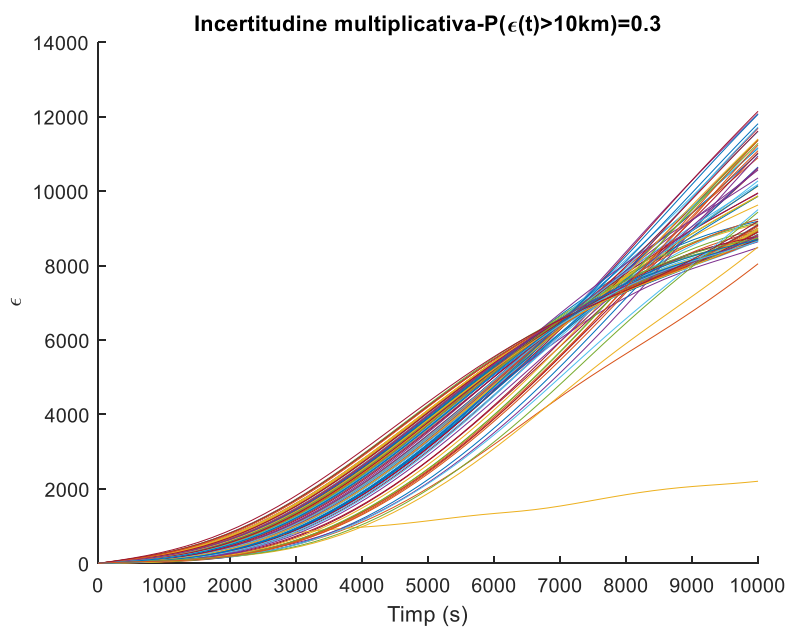
98 %% Cerinta 9
99 alfa=randn(3,100)/10;
100 n=0;
101
102 figure
103 hold on;
104 for i=1:size(alfa,2)
105     x0=[r10;(1+alfa(:,i)).*r20;d_r10;d_r20;eps0];
106     out=sim("Model_sim");
107     plot(out.tout,out.eps.Data)
108     n=n+any(out.eps.Data>1e4);
109 end
110 xlabel("Timp (s)")
111 ylabel("\epsilon")
112 title("Incertitudine multiplicativa-P(\epsilon(t)>10km)="+string(n/size(alfa,2)))

```

```

114 %% Cerinta 10
115 alfa=1e6*randn(3,100);
116 n=0;
117
118 figure
119 hold on;
120 for i=1:size(alfa,2)
121     x0=[r10;alfa(:,i)+r20;d_r10;d_r20;eps0];
122     out=sim("Model_sim");
123     plot(out.tout,out.eps.Data)
124     n=n+any(out.eps.Data>1e4);
125 end
126 xlabel("Timp (s)")
127 ylabel("\epsilon")
128 title("Incertitudine aditiva-P(\epsilon(t)>10km)="+string(n/size(alfa,2)))

```



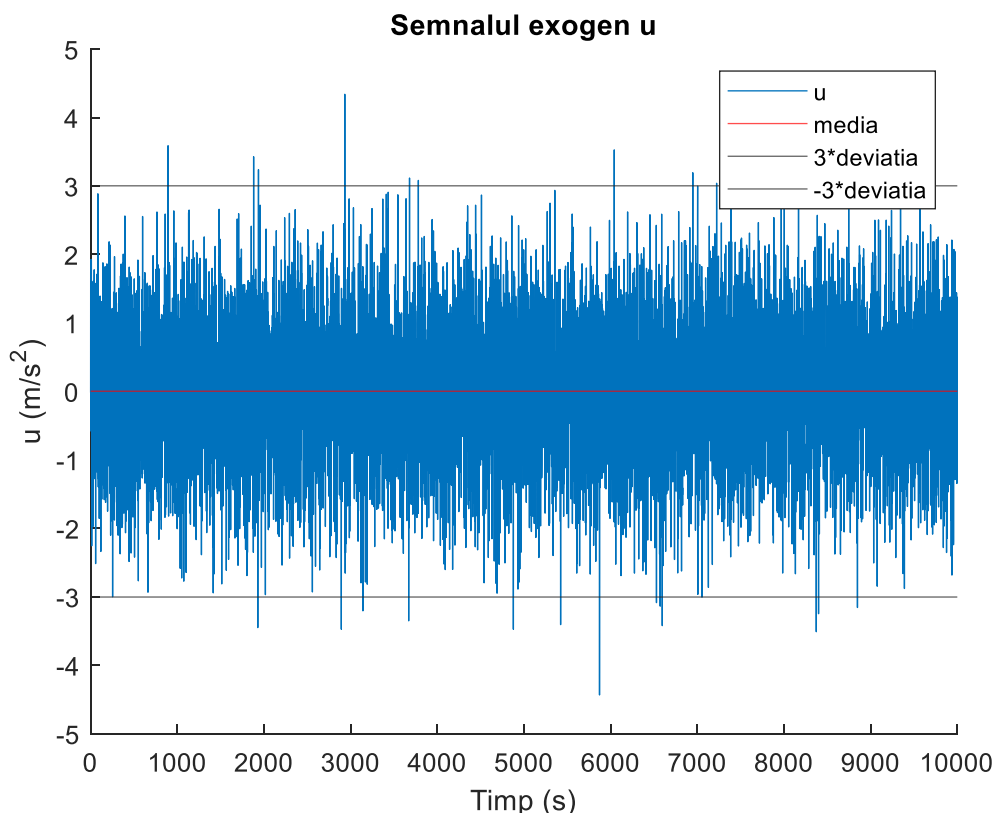
O incertitudine multiplicativa (proportionala) mentine iesirea intr-o anumita zona, pe cand o incertitudine aditiva de aceeaasi scala imprastie mai mult iesirea. Acest lucru se observa si prin calculul probabilitatii ca distanta relativa totala intre cei doi sateliti (iesirea) sa fie mai mare de 10km: in cazul celei multiplicative avem o certitudine mai mare decat in cazul celei aditive.

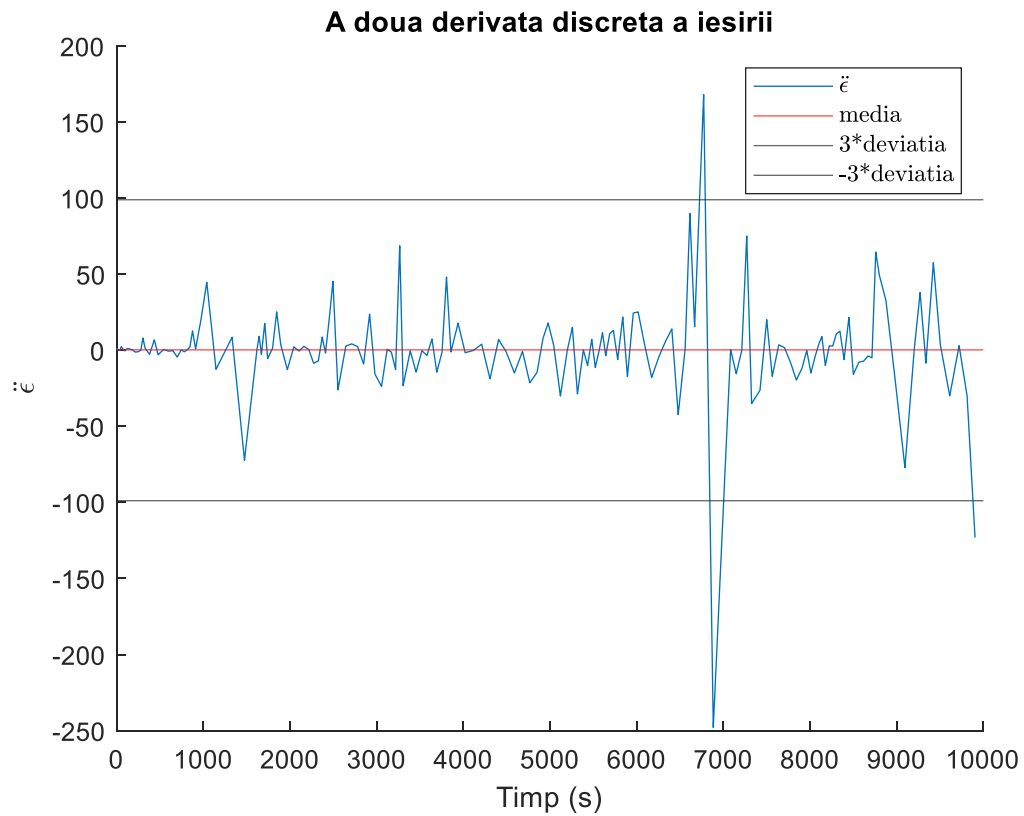
Cerinta 11

```

130 %% Cerinta 11
131 u=timeseries(randn(1,length(t)),t);
132 out=sim("Model_sim");
133
134 figure
135 hold on
136 plot(t,squeeze(u.Data))
137 xlabel("Timp (s)")
138 ylabel("u (m/s^2)")
139 yline(mean(u),'r')
140 yline(3*std(u))
141 yline(-3*std(u))
142 title("Semnalul exogen u")
143 legend("u","media","3*deviatia","-3*deviatia")
144
145 figure
146 hold on
147 dd_eps=diff(diff(out.eps.Data));
148 plot(out.tout(1:end-2),dd_eps)
149 xlabel("Timp (s)")
150 ylabel("$\ddot{\epsilon}$",'Interpreter','latex')
151 yline(mean(dd_eps),'r')
152 yline(3*std(dd_eps))
153 yline(-3*std(dd_eps))
154 title("A doua derivata discreta a iesirii")
155 legend("$\ddot{\epsilon}$","media","3*deviatia","-3*deviatia",'Interpreter','latex')

```





Observam ca a doua derivata discreta a iesirii are tot un comportament aleator de distributie normala ca semnalul exogen, avand tot media 0, dar deviatia mai mare.