

6.1. Write down the elements in each of the following sets, and then state the size of the set; that is, find $|A|$, $|B|$, $|C|$ and $|D|$.

(a) $A = \{x \in \mathbb{Z} \mid x = 1 + (-1)^i, \text{ for } i \in \mathbb{Z}\}.$

(b) $B = \{y \in \mathbb{Z} \mid -8 < y \leq -5\}.$

(c) $C = \{s \in \mathbb{Z} \mid s^2 + 1 = 0\}.$

(d) $D = \{t \in \mathbb{Z} \mid t^4 = 1\}.$

6.2. Let $V = \{x \in \mathbb{Z} \mid 2 \leq x \leq 11 \text{ and } x \text{ is odd}\}$, $X = \{x \in \mathbb{Z} \mid -1 \leq x \leq 4\}$, $Y = \{2, 3, 7, 9\}$ and $Z = \{x \in \mathbb{Z} \mid x \neq 0\}$.

Write down the following sets, listing their elements and using brackets as appropriate.

(a) $V \cap X$

(b) $V \cup Y$

(c) $X \cap Z$

(d) $X \cap (Z - Y)$

(e) $(X \cup Y) \cap V$

(f) $X \cup (Y \cap V)$

6.3. Are the following statements true or false?

(a) $x \in \{x\}$

(b) $\{x\} \subseteq \{x\}$

(c) $\{x\} \in \{x\}$

(d) $\{\emptyset\} \in \{\{\emptyset\}\}$

(e) $\emptyset \subseteq \{x\}$

(f) $\emptyset \in \{x\}$

6.4. Let $A = \mathcal{P}(\{\emptyset\})$, $B = \{A, \emptyset\}$, $C = \mathcal{P}(\emptyset)$ and $D = \{A, B\}$.

(a) Write down the following sets, listing their elements and using braces as appropriate.

(i) $C \cup D$

(ii) $D \cap B$

(iii) $\mathcal{P}(D)$

(iv) $D \times B$

(b) State whether the following are true or false.

(i) $A = B$

(ii) $B \subset D$

(iii) $C \subset B$

(iv) $A \cap D = \emptyset$

6.5. (a) Using the element method, prove that for all sets X , Y and Z , $(X - Z) \cap (Y - Z) \cap (X - Y) = \emptyset$.

(b) Using the set identities, prove that for all sets A , B and C , $(A - B) - C = A - (B \cup C)$.

6.6. Prove or disprove the following:

(a) For all sets A and B , $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

(b) Let A and B be any sets. Then $\mathcal{P}(A) = \mathcal{P}(B) \iff A = B$.

(c) There exists some set A such that $\mathcal{P}(A) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$.

(d) Let A, B and C be sets. If $B \subseteq C$, then $A \times B \subseteq A \times C$.

Here are two **puzzles** that you can think about during week 7. Feel free to ask your tutors or lecturer for more hints!

K. After doing some mathematics, Ole and Sara went for a brisk walk. When they returned, Sara wondered how far they had gone, and Ole promptly replied “A bit more than 2.9km—I counted more than 4096 steps, and each step is about 70cm.” He can’t have been counting his steps in his head as they were continuously discussing mathematics during their walk. In hindsight, Sara realised that Ole’s fingers are never still when he walks, especially the ones on his right hand. How did he count his steps whilst maintaining a conversation?

L. Five couples went to a party (so there were 10 people in total). Some of the people shook hands when they met, but no partners shook hands with each other, no pair of people shook hands more than once, and nobody shook hands with themselves.

Afterwards, Simone (one of the people there) asked everybody else how many times they had shaken hands, and she received a different answer from each person. How many times did Simone’s *partner* shake hands?

Extra practice questions from the textbook (Solutions at the back of the book.)

Epp 5th ed.:

Section 6.1, pp. 388–390: Questions 1ace, 2, 3ab, 5, 8a, 9a, 10abcde, 14a, 15a, 16a, 17a, 18ab, 19, 21, 22, 24.

Section 6.2, pp. 404–407: Questions 5, 7, 8, 9, 14, 15, 16, 19, 21, 23, 28, 30, 32, 33, 36, 40, 43.

Section 6.3, pp. 412–414: Questions 1, 3, 5, 6, 9, 11, 12, 14, 15, 17, 18, 19, 22a, 23, 30, 31, 36, 39, 41, 44ab, 46a, 47, 48.

Section 7.1, pp. 435–439: Questions 4a, 15, 38, 41, 43, 45.

Section 7.2, pp. 457–461: Questions 6, 10, 11, 15, 16, 20, 21, 26, 28, 36, 38, 42, 44, 45, 46, 47, 50.

Section 7.3, pp. 471–472: Questions 1, 3, 6, 10, 12, 19, 24, 26.

Epp 4th ed.:

Section 6.1, pp. 349–351: Questions 1ace, 2, 3ab, 5, 8a, 9a, 10abcde, 13bd, 14a, 15a, 16a, 17a, 18ab, 19, 21, 22, 24.

Section 6.2, pp. 364–366: Questions 3, 5, 6, 7, 8, 9, 11, 12, 13, 16, 18, 20, 25, 27, 29, 30, 33, 37, 40.

Section 6.3, pp. 372–373: Questions 1, 3, 5, 6, 9, 11, 12, 14, 15, 17, 18, 19, 22a, 23, 30, 31, 36, 39, 41, 44, 46a, 47, 48.

Section 7.1, pp. 393–396: Questions 4a, 15, 38, 41, 43, 45.

Section 7.2, pp. 413–416: Questions 6, 10, 11, 15, 16, 20, 21, 26, 28, 36, 38, 42, 44, 45, 46, 47, 50.

Section 7.3, pp. 426–427: Questions 1, 3, 6, 9, 16, 21, 23.

Puzzle hints:

Stuck on the puzzles from week 6?

- I. How many numbers in the list start with 1...? How many start with 2...? How many start with 10...? 12...?
- J. Each weighing gives three possible results. How many possible answers can you distinguish between with just one weighing? Two weighings? Three weighings?