MATH1061 Tutorial Week 2

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Question 1: 1.1

(i) Suppose you know that $(\sim p \wedge q) \vee p$ is false. What can you conclude about the truth values of each of the two variables?

(ii) Suppose you know that $(p \vee q) \wedge \sim r$ is true. What can you conclude about the truth values of each of the three variables?

(iii) Suppose you know that $(\sim p \wedge \sim q) \wedge r$ is false. What can you conclude about the truth values of each of the three variables?

Solution: (i)

$$\begin{aligned} (\sim p \wedge q) \vee p &\equiv \text{False} \\ \Rightarrow \sim p \wedge q &\equiv \text{False} \\ \wedge p &\equiv \text{False} \\ \Rightarrow \sim p &\equiv \text{True} \\ \Rightarrow q &\equiv \text{False} \end{aligned}$$

$\therefore p \equiv \text{False}$ and $q \equiv \text{False}$.

Solution: (ii)

$$\begin{aligned} (p \vee q) \wedge \sim r &\equiv \text{True} \\ \Rightarrow \sim r &\equiv \text{True} \\ \Rightarrow r &\equiv \text{False} \\ \Rightarrow p \vee q &\equiv \text{True} \\ \text{Case 1} \Rightarrow p &\equiv \text{True} \vee q \equiv \text{False} \\ \text{Case 2} \Rightarrow p &\equiv \text{False} \vee q \equiv \text{True} \\ \text{Case 3} \Rightarrow p &\equiv \text{True} \vee q \equiv \text{True} \end{aligned}$$

$\therefore (p, q, r) \in \{(\text{True}, \text{False}, \text{False}), (\text{False}, \text{True}, \text{False}), (\text{True}, \text{True}, \text{False})\}$

Solution: (iii)

$$\begin{aligned} \text{If } (\sim p \wedge \sim q) \wedge r &\equiv \text{True} \\ \Rightarrow r &\equiv \text{True} \\ \sim p \equiv \text{True} \Rightarrow p &\equiv \text{False} \\ \sim q \equiv \text{True} \Rightarrow q &\equiv \text{False} \end{aligned}$$

$\therefore (\sim p \wedge \sim q) \wedge r \equiv \text{True} \iff (p, q, r) \in \{(\text{False}, \text{False}, \text{True})\}$.
$\therefore (\sim p \wedge \sim q) \wedge r \equiv \text{False} \iff (p, q, r) \notin \{(\text{False}, \text{False}, \text{True})\}$.
$\therefore (\sim p \wedge \sim q) \wedge r \equiv \text{False} \iff (p, q, r) \in U \setminus \{(\text{False}, \text{False}, \text{True})\}$.

Question 2: 1.2

For each of the following, write down a truth table for the statement, and determine whether the statement is a tautology, a contradiction, or neither.

(i) $((p \wedge q) \vee (q \wedge r)) \vee \sim q$

(ii) $(\sim p \vee q) \vee (p \wedge \sim q)$

Solution: (i)

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \vee (q \wedge r)$	$\sim q$	$((p \wedge q) \vee (q \wedge r)) \vee \sim q$
T	T	T	T	T	T	F	T
T	T	F	T	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	F	T	T	F	T
F	T	F	F	F	F	F	F
F	F	T	F	F	F	T	T
F	F	F	F	F	F	T	T

The statement is neither a contradiction nor a tautology.

Solution: (ii)

p	q	$\sim p$	$\sim p \vee q$	$\sim q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	T	F	F	T
T	F	F	F	T	T	T
F	T	T	T	F	F	T
F	F	T	T	T	F	T

Since the statement is always true, it is a tautology.

Question 3: 1.3

(i) Use a truth table to show that $(p \vee q) \wedge \sim p \equiv q \wedge \sim p$.

(ii) Use a truth table to show that $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$.

(iii) Use the laws of logical equivalence and the fact that $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$, to show that $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$.

Solution: (i)

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$q \wedge \sim p$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	F

Solution: (ii)

p	q	r	$p \oplus q$	$p \wedge r$	$q \wedge r$	$(p \oplus q) \wedge r$	$(p \wedge r) \oplus (q \wedge r)$
T	T	T	F	T	T	F	F
T	T	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Solution: (iii)

$$
\begin{aligned}
(p \oplus q) \wedge r &\equiv ((p \vee q) \wedge \sim(p \wedge q)) \wedge r \\
&\equiv (p \vee q) \wedge (\sim p \vee \sim q) \wedge r \\
&\equiv ((r \wedge p) \vee (r \wedge q)) \wedge (\sim p \vee \sim q) \\
&\equiv (((r \wedge p) \vee (r \wedge q)) \wedge \sim p) \vee (((r \wedge p) \vee (r \wedge q)) \wedge \sim q) \\
&\equiv (r \wedge p \wedge \sim p) \vee (r \wedge q \wedge \sim p) \vee (r \wedge p \wedge \sim q) \vee (r \wedge q \wedge \sim q) \\
&\equiv \bot \vee (r \wedge q \wedge \sim p) \vee (r \wedge p \wedge \sim q) \vee \bot \\
&\equiv r \wedge ((q \wedge \sim p) \vee (p \wedge \sim q)) \\
&\equiv r \wedge ((q \wedge \sim p) \vee p) \wedge ((q \wedge \sim p) \vee \sim q) \\
GOAL &\equiv ((p \wedge r) \vee (q \wedge r)) \wedge \sim((p \wedge r) \wedge (q \wedge r)) \\
&\equiv ((p \wedge r) \vee (q \wedge r)) \wedge \sim(p \wedge q \wedge r) \\
&\equiv (r \wedge (p \vee q)) \wedge \sim(p \wedge q \wedge r) \\
&\equiv r \wedge (p \vee q) \wedge \sim(p \wedge q \wedge r) \\
&\equiv r \wedge (p \vee q) \wedge (\sim p \vee \sim q \vee \sim r)
\end{aligned}
$$

AHHHHHHHHHHHHHHHHHHHHHHH THIS IS HARD.

Question 4: 1.4

(i) Use the laws of logical equivalence to show that $p \wedge q \equiv \sim(\sim p \vee \sim q)$.

(ii) Use the laws of logical equivalence to show that $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$.

Solution: (i)

$$
\begin{aligned}
p \wedge q &\equiv \sim(\sim(p \wedge q)) \\
&\equiv \sim(\sim p \vee \sim q)
\end{aligned}
$$

Solution: (ii)

$$
\begin{aligned}
\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) &\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) \\
&= \sim p \wedge (q \vee \sim q) \\
&= \sim p \wedge \top \\
&= \sim p
\end{aligned}
$$

Question 5: 1.5

Using De Morgan's Law, rewrite these sentences retaining equivalence.

(i) t is not true that I am studying Computer Science and I am studying Engineering.

(ii) I am not going to the movies this weekend or I am not going swimming this weekend.

Solution: (i) It is not true that I am studying Computer Science and I am studying Engineering. $\equiv \sim(p \wedge q) \equiv \sim p \vee \sim q \equiv$ I am not studying CS or I am not studying engineering.

Solution: (ii) I am not going to the movies this weekend or I am not going swimming this weekend. $\equiv \sim p \vee \sim q \equiv \sim(p \wedge q) \equiv$ I am not going to the movies and swimming this weekend.

Question 6: 1.6

For each of the following, write down a truth table for the statement, and determine whether the statement is a tautology, a contradiction, or neither.

(i) $(\sim p \wedge (p \to q)) \to \sim q$

(ii) $(p \to (q \vee r)) \leftrightarrow ((p \wedge \sim q) \to r)$

Solution: (i) For fun and practice I will prove this by logical equivalence.

$$\begin{aligned}
(\sim p \wedge (p \to q)) \to \sim q &\equiv (\sim p \wedge (\sim p \vee q)) \to \sim q \\
&\equiv \sim(\sim p \wedge (\sim p \vee q)) \vee \sim q \\
&\equiv \sim(\sim p \wedge (\sim p \vee q) \wedge q) \\
&\equiv \sim((\sim p \wedge q) \wedge (\sim p \vee q)) \\
&\equiv \sim((\sim p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge q)) \\
&\equiv \sim((\sim p \wedge q) \vee (\sim p \wedge q)) \\
&\equiv \sim(\sim p \wedge (q \vee q)) \\
&\equiv \sim(\sim p \wedge q) \\
&\equiv p \vee \sim q
\end{aligned}$$

Which is not a contradiction nor a tautology. Now we will answer the question.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \wedge (p \to q)$	$(\sim p \wedge (p \to q)) \to \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Solution: (ii)

p	q	r	$\sim q$	$q \vee r$	$p \wedge \sim q$	$p \to (q \vee r)$	$\leftrightarrow$	$(p \wedge \sim q) \to r$
T	T	T	F	T	F	T	T	T
T	T	F	F	T	F	T	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T	F
F	T	T	F	T	F	T	T	T
F	T	F	F	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	F	F	T	T	T

Therefore the statement in question is a tautology.

Question 7: 1.7

Write each of the following statements in the form "if.. then...".

(i) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

(ii) Jane gets seasick whenever she is on a boat.

Now negate the following two statements.

(iii) If it rains, then Sue takes her umbrella.

(iv) The cakes burn if the oven temperature is too high.

Solution: (i) If you bought the computer less then the warranty is good.

Solution: (ii) If Jane is on a boat, then she gets seasick.

Solution: (iii) It rains, and Sue does not take her umbrella.

Solution: (iv) The oven temperture is too high, and the cake does not burn.