School of Mathematics and Physics, UQ

MATH2001, Assignment 2, Summer Semester, 2024-2025

Submit your assignment on Blackboard by 2:00pm (Brisbane time), January 9, 2025. Each question is marked out of 10 and then homogeneously rescaled up to total marks of 14. **Total marks:** $\frac{14}{60}(Q1+Q2+Q3+Q4+Q5+Q6)$.

Submit your assignment online via the Assignment 2 submission link in Blackboard. Indicate your name and student number at the beginning of the pdf document you will upload.

(1) Let

$$A = \begin{pmatrix} -3 & 0 & -4 \\ -2 & 2 & -1 \\ 2 & 0 & 3 \end{pmatrix}$$

a) Find a matrix P that diagonalizes A.

b) Use part the result of part a) to find the general solution $\mathbf{x}(t) = \big(x_1(t), x_2(t), x_3(t)\big)^T$ to the system of ODEs

$$\begin{aligned} \dot{x}_1 &= -3x_1 - 4x_3, \\ \dot{x}_2 &= -2x_1 + 2x_2 - x_3, \\ \dot{x}_3 &= 2x_1 + 3x_3. \end{aligned}$$

Show all the working.

(2) You are given that the function

$$f(x,y,z) = ax^2 + b\left(y^2+z^2\right) + cxy + dxz, \quad a,b,c,d \in \mathbb{R}$$

has a critical point at $(x_0, y_0, z_0) = (0,0,0)$, meaning that the gradient of the function at the origin is zero. Assuming that the product of the a and b parameters is positive, $a \times b > 0$, and that the parameters c, d are not simultaneously equal to zero, classify the critical point. Show all the working.

(3) Take a general complex 2×2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Impose the following conditions on the previous matrix:

(i) A is Hermitian;
(ii) A is unitary;
(iii) $b, c \neq 0$ are purely imaginary non-zero numbers.

Obtain the one-parameter family of simultaneous solutions of these three conditions. The result will be complex 2×2 matrices that are both Hermitian and unitary and whose entries are not all real numbers. Show all the working.

(4) Compute the following double integral over the rectangle $R = [-1, 0] \times [1, 2]$

$$\iint_R \frac{e^x}{2y} - \frac{4x - 1}{y^2} dA. \tag{1}$$

You may choose any order of integration.

(5) Determine the volume that lies under $f(x, y) = \frac{e^x}{2y} - \frac{4x-1}{y^2}$ within $R = [-1, 0] \times [1, 2]$ in the xy-plane by implementing the definition of the double integral in a numerical algorithm. Write a computer code (Matlab or python) that approximates the volume by breaking up the integration domain R into $N \times N$ squares of equal sizes and within each square approximate the function by its value in the middle of each square.

Use your code to evaluate the volume under $f(x, y)$ in R using $N = 10, 50, 250$ and 1250. Create a table showing the values obtained numerically. Plot the logarithm of the error (i.e. the absolute value of the difference between the exact value obtained at the previous question and the numerical result) as a function of $\log N$. Include your computer code in your response.

(6) Evaluate $\iint_D x(y - 1)dA$ where D is the region bounded by $y = 1 - x^2$ and $y = x^2 - 3$ in two ways using different orders of integration.

Hint: for one of them you may need to break up the integral into two parts.