Due **Friday 25 October at 5pm** on blackboard.

Marks will be deducted for sloppy working. Clearly state your assumptions and conclusions, and justify all steps in your work.

Q1 Let T and F denote logical "true" and "false".

(a) Is $(\{T, F\}, \wedge)$ a group?

(b) Is $(\{T, F\}, \oplus)$ a group? Here $\oplus$ denotes exclusive OR.

(c) Is $(\{T, F\}, \oplus, \wedge)$ a field?

You should prove your answers correct.

(15 marks)

Q2 (a) Prove that the group $(\mathbb{R}, +)$ is isomorphic to the group $(\mathbb{R}_+, \times)$. Here $\mathbb{R}_+$ denotes the set of all positive reals.

(b) Prove that the group $(\mathbb{Z} \times \mathbb{Z}, +)$ is not isomorphic to the group $(\mathbb{Z}, +)$.

(10 marks)

Q3 Your MATH1061 tutorial group contains 20 people, and together you have all decided to form a party for the coming election.

(a) How many ways could you choose five spokespeople for your party?

(b) How many ways could you choose people for the three leadership roles of leader, deputy leader, and treasurer?

You should give each answer as a single integer (i.e., do not leave it as a formula).
Parts (a) and (b) are intended to be independent questions (i.e., there is not meant to be any link between the choice of spokespeople and the choice of leadership roles).

(5 marks)

Q4 As before, your MATH1061 tutorial group contains 20 people, and together you have all decided to form a party for the coming election. However, now we suppose that these 20 students consist of 8 from the science faculty, 9 from the ITEE faculty, and 3 from the arts faculty (each student belongs to one faculty only).

(a) How many ways could you choose people for the three leadership roles of leader, deputy leader and treasurer, if you insist that these three people come from three different faculties?

(b) How many ways could you choose five spokespeople for your party, if you insist that there must be at least one spokesperson from each faculty?

You should give each answer as a single integer (i.e., do not leave it as a formula).
Again, parts (a) and (b) are intended to be independent questions.

(10 marks)

Q5 **MATH1061 only:** Consider points (x, y) on the 2-dimensional plane $\mathbb{R} \times \mathbb{R}$. Let $S = \{(x, y) \,|\, 0 \leq x, y \leq 1\}$; that is, S represents a unit square, including its boundary. Prove that S has the same cardinality as the entire plane $\mathbb{R} \times \mathbb{R}$.
(Hint: Use the Schröder-Bernstein theorem.)

(10 marks)

Q5 **MATH7861 only:** Prove that the interval $(0, 1)$ and the Cartesian product $(0, 1) \times (0, 1)$ have the same cardinality.
(Hint: Use the Schröder-Bernstein theorem.)

(10 marks)