Universtiy of Queensland
School of Mathematics and Physics

MATH2302 Discrete Mathematics II
Semester 2 2025
Problem Set 4

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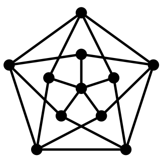
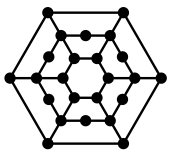
Applied Class #1
Due 3pm Friday 28 October 2025

Question 1: 6 marks

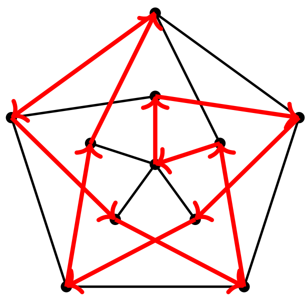
For each of the graphs shown below, determine whether it is Hamiltonian. Justify your answer.

(a) 

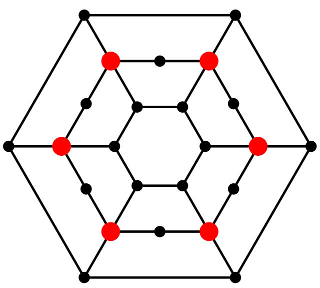
(b) 

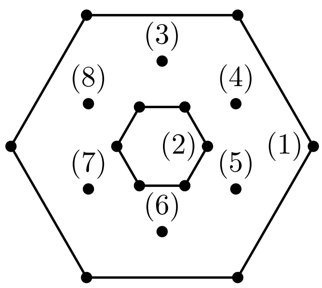
Solution:

Graph (a) is Hamiltonian. I present one such spanning cycle:



Graph (b) is not Hamiltonian. We show this by considering Theorem 26.85. Consider the set S, which contains only the highlighted vertices. Then consider the graph $G-S$, in particular, we count the number of components the graph has.





We highlighted 6 vertices for $S \subseteq G$, hence $|S| = 6$. The graph $G - S$, has componets: the outer ring, the inner ring, and the 6 isolated vertices. Hence, $|\text{Comp}(G-S)| = 8$.

$$|S| = 6 < 8 = |\text{Comp}(G-S)|\,.$$

Therefore, by Theorem 26.85, the graph is not Hamiltonian.

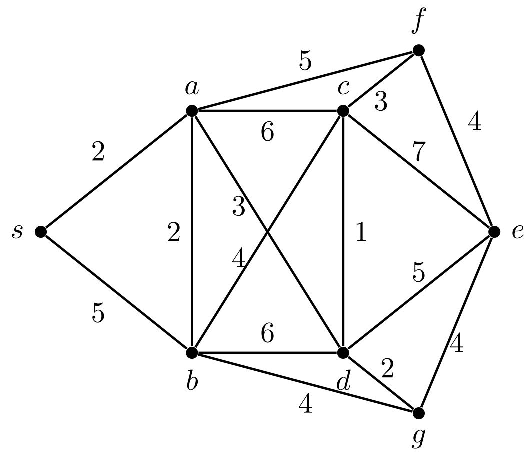
Question 2: 6 marks

For any integer $n \geq 4$, let K_n be the complete graph on vertices $1, 2, \ldots, n$. Let G_n be the graph obtained from K_n by deleting two edges $\{1, 2\}$ and $\{3, 4\}$. What is the vertex chromatic number of G_n? Justify your answer.

Solution:

Question 3: 8 marks

Use Dijkstra's algorithm to determine the distance from s to each other vertex in the weighted graph shown below, and state a shortest path from s to e.



Solution:

s	a	b	c	d	f	g	e	Vertex Added to S
$\boxed{0}$	$(2, s)$	$(5, s)$	∞	∞	∞	∞	∞	s
	$\boxed{(2, s)}$	$(4, a)$	$(8, a)$	$(5, a)$	$(7, a)$	∞	∞	a
		$\boxed{(4, a)}$	$(8, a)$	$(5, a)$	$(7, a)$	$(8, b)$	∞	b
			$(6, d)$	$\boxed{(5, a)}$	$(7, a)$	$(7, d)$	$(10, d)$	d
			$\boxed{(6, d)}$		$(7, a)$	$(7, d)$	$(10, d)$	c
					$\boxed{(7, a)}$	$(7, d)$	$(10, d)$	f
						$\boxed{(7, d)}$	$(10, d)$	g
							$\boxed{(10, d)}$	e

From the final entry in the table, we can see that the shortest path from s to e has distance 10. Starting at the end, e, we can work our way back to d. Back to a. Then back to the starting point s. Therefore, the shortest path is

$$s \to a \to d \to e \quad \text{with} \quad d(s, e) = 10.$$

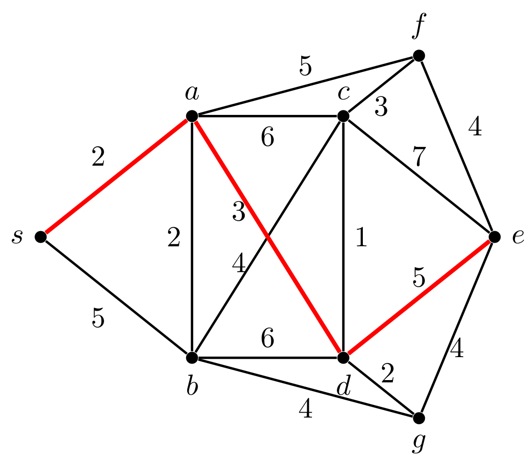


Figure 1: An illustraion of the shortest path, $s \to a \to d \to e$ which has distance $= 10$

Question 4: 10 marks

Use Algorithm 33.113 from the notes to find a maximum weight perfect matching in the weighted complete bipartite graph G with parts $V_1 = \{v_1, v_2, ..., v_6\}$ and $V_2 = \{u_1, u_2, ..., u_6\}$. The weight $w(v_i u_j)$ of the edge $v_i u_j$ is given by $w(v_i u_j) = m_{ij}$, where $M = [m_{ij}]$ is given by

$$\begin{bmatrix} 7 & 5 & 9 & 4 & 7 & 6 \\ 6 & 9 & 9 & 7 & 5 & 5 \\ 5 & 6 & 4 & 8 & 9 & 7 \\ 8 & 7 & 5 & 6 & 8 & 4 \\ 6 & 5 & 7 & 5 & 9 & 8 \\ 7 & 6 & 6 & 9 & 5 & 9 \end{bmatrix}.$$

Solution: