**SCHOOL OF MATHEMATICS AND PHYSICS, UQ**

# MATH1072
# Assignment 2
# Semester Two 2024

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*Submit your answers by 1pm on Monday, 2nd September, using the Blackboard assignment submission system. Assignments must consist of a single PDF.*

You may find some of these problems challenging. Attendance at weekly tutorials is assumed.

Family name:

Given names:

Student number:

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Marker's use only

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Each question marked out of 3.

- Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.

- Mark of 1: Your submission has some relevance, but does not demonstrate deep understanding or sound mathematical technique.

- Mark of 2: You have the right approach, but need to fine-tune some aspects of your calculations.

- Mark of 3: You have demonstrated a good understanding of the topic and techniques involved, with well-executed calculations.

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|---|---|---|---|---|
| Q1(a): | Q2(a): | Q3: | Q4: | Q5(a): |
| Q1(b): | Q2(b): | | | Q5(b): |
| | | | | Q5(c): |

Total (out of 27):

**1.** The one-dimensional heat equation is an example of a *partial differential equation*, so named because of its utility in describing the change in distribution of heat in a rod over time. The equation is given by

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, \quad \alpha \in \mathbb{R}.$$

A function $u(x,t)$ that satisfies this equation is said to be a *solution*. Verify that the following functions are solutions of the heat equation.

(a) $u(x,t) = \dfrac{1}{2\alpha\sqrt{\pi t}} e^{-x^2/4\alpha^2 t}$

(b) $u(x,t) = e^{-\alpha^2 k^2 t} \sin(kx), \quad k \in \mathbb{R}$

**2.** Consider the function

$$f(x,y) = \frac{y-1}{x+1}.$$

(a) Give the equation of the tangent plane to the surface $z = f(x,y)$ at the point $(x,y) = (0,0)$.

(b) Use a linear approximation to estimate $f(0.1, 0.2)$. Give the error in the estimate.

**3.** Let $g$ denote acceleration due to gravity. The period $\tau$ of a pendulum of length $r$ with small oscillations is given by the formula

$$\tau = 2\pi\sqrt{\frac{r}{g}}.$$

Suppose that experimental values of $r$ and $g$ have maximum errors of at most 0.5% and 0.1% respectively. Use differentials to approximate the maximum percentage error in the calculated value of $\tau$.

**4.** Let $f(x,y)$ be differentiable at the point $(x_0, y_0)$. Prove that $f(x,y)$ is continuous at $(x_0, y_0)$. Hint: Consider the function

$$\varepsilon(\Delta x, \Delta y) = \frac{f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0) - \Delta x f_x(x_0, y_0) - \Delta y f_y(x_0, y_0)}{\sqrt{(\Delta x)^2 + (\Delta y)^2}}.$$

**5.** Consider the function

$$f(x,y) = xye^{-x-y}.$$

(a) Find and classify all critical points of $f(x,y)$.

(b) Use the `meshgrid` and `surf` functions in MATLAB to plot $f(x,y)$ over a suitable domain that includes all critical points. Also use the `hold on` and `plot3` functions to plot the critical points on the same figure, making the markers filled, red circles of size 10. Make sure that the plot is oriented so that all critical points are visible.

(c) For any maxima or minima found in your answer to part (a), demonstrate that `fminsearch` can identify these critical points.

Make sure that you submit all MATLAB code used for this question.