School of Mathematics and Physics, UQ

MATH2001, Assignment 3, Summer Semester, 2024-2025

Submit your assignment on Blackboard by 2pm, January 23, 2025.
Each question is marked out of 10 then homogeneously rescaled up to a total marks of 13.
Total marks: (13/60)(Q1+Q2+Q3+Q4+Q5+Q6). Submit your assignment online via the Assignment 3 submission link in Blackboard.

(1) Evaluate the following integral by first converting to an integral in polar coordinates.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^0 e^{x^2+y^2}\,dydx$$

(2) Use a triple integral to determine the volume of the region below $z = 6 - x$, above $z = -\sqrt{4x^2 + 4y^2}$ inside the cylinder $x^2 + y^2 = 3$ with $x \le 0$.

(3) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (6x - 2y)\mathbf{i} + x^2\mathbf{j}$ for each of the following curves.

- **(i)** C is the line segment from $(6, -3)$ to $(0, 0)$ followed by the line segment from $(0, 0)$ to $(6, 3)$.
- **(ii)** C is the line segment from $(6, -3)$ to $(6, 3)$.

(4) Find the potential function $f(x, y)$ for the following vector field:
$\mathbf{F} = y^2(1 + \cos(x + y))\mathbf{i} + (2xy - 2y + y^2 \cos(x + y) + 2y \sin(x + y))\mathbf{j}$
that satisfies $\nabla f = \mathbf{F}$.

(5) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = y\mathbf{i} + 2x\mathbf{j} + (z - 8)\mathbf{k}$ and S is the surface of the solid bounded by $4x + 2y + z = 8$, $z = 0$, $y = 0$ and $x = 0$ with the positive orientation. Note that all four surfaces of the solid are included in S.

(6) Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = 2xz\mathbf{i} + (1 - 4xy^2)\mathbf{j} + (2z - z^2)\mathbf{k}$ and S is the surface of the solid bounded by $z = 6 - 2x^2 - 2y^2$ and the plane $z = 0$. Note that both of the surfaces of this solid are included in S.