

# MATH1061 Tutorial Week 2

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## Question 1: 1.1

- (i) Suppose you know that  $(\sim p \wedge q) \vee p$  is false. What can you conclude about the truth values of each of the two variables?
- (ii) Suppose you know that  $(p \vee q) \wedge \sim r$  is true. What can you conclude about the truth values of each of the three variables?
- (iii) Suppose you know that  $(\sim p \wedge \sim q) \wedge r$  is false. What can you conclude about the truth values of each of the three variables?

**Solution:** (i)

$$\begin{aligned}(\sim p \wedge q) \vee p &\equiv \text{False} \\ \Rightarrow \sim p \wedge q &\equiv \text{False} \\ \wedge p &\equiv \text{False} \\ \Rightarrow \sim p &\equiv \text{True} \\ \Rightarrow q &\equiv \text{False}\end{aligned}$$

$\therefore p \equiv \text{False}$  and  $q \equiv \text{False}$ .

**Solution:** (ii)

$$\begin{aligned}(p \vee q) \wedge \sim r &\equiv \text{True} \\ \Rightarrow \sim r &\equiv \text{True} \\ \Rightarrow r &\equiv \text{False} \\ \Rightarrow p \vee q &\equiv \text{True}\end{aligned}$$

Case 1  $\Rightarrow p \equiv \text{True} \vee q \equiv \text{False}$

Case 2  $\Rightarrow p \equiv \text{False} \vee q \equiv \text{True}$

Case 3  $\Rightarrow p \equiv \text{True} \vee q \equiv \text{True}$

$\therefore (p, q, r) \in \{(\text{True}, \text{False}, \text{False}), (\text{False}, \text{True}, \text{False}), (\text{True}, \text{True}, \text{False})\}$

**Solution:** (iii)

$$\begin{aligned}\text{If } (\sim p \wedge \sim q) \wedge r &\equiv \text{True} \\ \Rightarrow r &\equiv \text{True} \\ \sim p \equiv \text{True} &\Rightarrow p \equiv \text{False} \\ \sim q \equiv \text{True} &\Rightarrow q \equiv \text{False}\end{aligned}$$

$\therefore (\sim p \wedge \sim q) \wedge r \equiv \text{True} \iff (p, q, r) \in \{(\text{False}, \text{False}, \text{True})\}.$

$\therefore (\sim p \wedge \sim q) \wedge r \equiv \text{False} \iff (p, q, r) \notin \{(\text{False}, \text{False}, \text{True})\}.$

$\therefore (\sim p \wedge \sim q) \wedge r \equiv \text{False} \iff (p, q, r) \in U \setminus \{(\text{False}, \text{False}, \text{True})\}.$

### Question 2: 1.2

For each of the following, write down a truth table for the statement, and determine whether the statement is a tautology, a contradiction, or neither.

(i)  $((p \wedge q) \vee (q \wedge r)) \vee \sim q$

(ii)  $(\sim p \vee q) \vee (p \wedge \sim q)$

**Solution:** (i)

$p$	$q$	$r$	$p \wedge q$	$q \wedge r$	$(p \wedge q) \vee (q \wedge r)$	$\sim q$	$((p \wedge q) \vee (q \wedge r)) \vee \sim q$
T	T	T	T	T	T	F	T
T	T	F	T	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	F	T	T	F	T
F	T	F	F	F	F	F	F
F	F	T	F	F	F	T	T
F	F	F	F	F	F	T	T

The statement is neither a contradiction nor a tautology.

**Solution:** (ii)

$p$	$q$	$\sim p$	$\sim p \vee q$	$\sim q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	T	F	F	T
T	F	F	F	T	T	T
F	T	T	T	F	F	T
F	F	T	T	T	F	T

Since the statement is always true, it is a tautology.

### Question 3: 1.3

(i) Use a truth table to show that  $(p \vee q) \wedge \sim p \equiv q \wedge \sim p$ .

(ii) Use a truth table to show that  $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$ .

(iii) Use the laws of logical equivalence and the fact that  $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$ , to show that  $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$ .

**Solution:** (i)

$p$	$q$	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$q \wedge \sim p$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	F

**Solution:** (ii)

$p$	$q$	$r$	$p \oplus q$	$p \wedge r$	$q \wedge r$	$(p \oplus q) \wedge r$	$(p \wedge r) \oplus (q \wedge r)$
T	T	T	F	T	T	F	F
T	T	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

**Solution:** (iii)

$$\begin{aligned}(p \oplus q) \wedge r &\equiv ((p \vee q) \wedge \sim(p \wedge q)) \wedge r \\ &\equiv (p \vee q) \wedge (\sim p \vee \sim q) \wedge r \\ &\equiv ((r \wedge p) \vee (r \wedge q)) \wedge (\sim p \vee \sim q) \\ &\equiv (((r \wedge p) \vee (r \wedge q)) \wedge \sim p) \vee (((r \wedge p) \vee (r \wedge q)) \wedge \sim q) \\ &\equiv (r \wedge p \wedge \sim p) \vee (r \wedge q \wedge \sim p) \vee (r \wedge p \wedge \sim q) \vee (r \wedge q \wedge \sim q) \\ &\equiv \perp \vee (r \wedge q \wedge \sim p) \vee (r \wedge p \wedge \sim q) \vee \perp \\ &\equiv r \wedge ((q \wedge \sim p) \vee (p \wedge \sim q)) \\ &\equiv r \wedge ((q \wedge \sim p) \vee p) \wedge ((q \wedge \sim p) \vee \sim q) \\ \text{GOAL} &\equiv ((p \wedge r) \vee (q \wedge r)) \wedge \sim((p \wedge r) \wedge (q \wedge r)) \\ &\equiv ((p \wedge r) \vee (q \wedge r)) \wedge \sim(p \wedge q \wedge r) \\ &\equiv (r \wedge (p \vee q)) \wedge \sim(p \wedge q \wedge r) \\ &\equiv r \wedge (p \vee q) \wedge \sim(p \wedge q \wedge r) \\ &\equiv r \wedge (p \vee q) \wedge (\sim p \vee \sim q \vee \sim r)\end{aligned}$$

AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA THIS IS HARD.

#### Question 4: 1.4

- (i) Use the laws of logical equivalence to show that  $p \wedge q \equiv \sim(\sim p \vee \sim q)$ .
- (ii) Use the laws of logical equivalence to show that  $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$ .

**Solution:** (i)

$$\begin{aligned}p \wedge q &\equiv \sim(\sim(p \wedge q)) \\ &\equiv \sim(\sim p \vee \sim q)\end{aligned}$$

**Solution:** (ii)

$$\begin{aligned}\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) &\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) \\ &= \sim p \wedge (q \vee \sim q) \\ &= \sim p \wedge \top \\ &= \sim p\end{aligned}$$

#### Question 5: 1.5

Using De Morgan's Law, rewrite these sentences retaining equivalence.

- (i) t is not true that I am studying Computer Science and I am studying Engineering.
- (ii) I am not going to the movies this weekend or I am not going swimming this weekend.

**Solution:** (i) It is not true that I am studying Computer Science and I am studying Engineering.  $\equiv \sim(p \wedge q) \equiv \sim p \vee \sim q \equiv$  I am not studying CS or I am not studying engineering.

**Solution:** (ii) I am not going to the movies this weekend or I am not going swimming this weekend.  $\equiv \sim p \vee \sim q \equiv \sim(p \wedge q) \equiv$  I am not going to the movies and swimming this weekend.

### Question 6: 1.6

For each of the following, write down a truth table for the statement, and determine whether the statement is a tautology, a contradiction, or neither.

(i)  $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$

(ii)  $(p \rightarrow (q \vee r)) \leftrightarrow ((p \wedge \sim q) \rightarrow r)$

**Solution:** (i) For fun and practice I will prove this by logical equivalence.

$$\begin{aligned}
 (\sim p \wedge (p \rightarrow q)) \rightarrow \sim q &\equiv (\sim p \wedge (\sim p \vee q)) \rightarrow \sim q \\
 &\equiv \sim(\sim p \wedge (\sim p \vee q)) \vee \sim q \\
 &\equiv \sim(\sim p \wedge (\sim p \vee q) \wedge q) \\
 &\equiv \sim((\sim p \wedge q) \wedge (\sim p \vee q)) \\
 &\equiv \sim((\sim p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge q)) \\
 &\equiv \sim((\sim p \wedge q) \vee (\sim p \wedge q)) \\
 &\equiv \sim(\sim p \wedge (q \vee q)) \\
 &\equiv \sim(\sim p \wedge q) \\
 &\equiv p \vee \sim q
 \end{aligned}$$

Which is not a contradiction nor a tautology. Now we will answer the question.

$p$	$q$	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \wedge (p \rightarrow q)$	$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

**Solution:** (ii)

$p$	$q$	$r$	$\sim q$	$q \vee r$	$p \wedge \sim q$	$p \rightarrow (q \vee r)$	$\leftrightarrow$	$(p \wedge \sim q) \rightarrow r$
T	T	T	F	T	F	T	T	T
T	T	F	F	T	F	T	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T	F
F	T	T	F	T	F	T	T	T
F	T	F	F	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	F	F	T	T	T

Therefore the statement in question is a tautology.

### Question 7: 1.7

Write each of the following statements in the form “if.. then...”.

(i) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

(ii) Jane gets seasick whenever she is on a boat.

Now negate the following two statements.

(iii) If it rains, then Sue takes her umbrella.

(iv) The cakes burn if the oven temperature is too high.

**Solution:** (i) If you bought the computer less than the warranty is good.

**Solution:** (ii) If Jane is on a boat, then she gets seasick.

**Solution:** (iii) It rains, and Sue does not take her umbrella.

**Solution:** (iv) The oven temperature is too high, and the cake does not burn.