MATH2504
Programming of Simulation, Analysis, and Learning (Data) Systems

Michael Kasumagic, s4430266

Semester 2, 2024

Contents

Chapter 1

Unit 1: Hello World Bootcamp

1.1 Hello World!

We will learn Julia over Python in this course, because Julia is more mathematical by nature. It is also a natural step forward from MATLAB which was previously taught in the School of Math and Physics.

Let us now print "Hello World!" using Julia...

```
println("Hello World!")
```

Output: `Hello World!`

Note:-
Julia doesn't play nice with GitBash, consider using JuliaUP, a Julia installer and version multiplexer.

1.2 `test.pl`

Let us now create a little test file, which we will approriately name `test.pl`.

```
~ $ mkdir ./MATH2504
~ $ cd ./MATH2504
~/MATH2504 $ vim ./test.pl
```

```
print("Hi what is your name?")
my_name = readline()
println("ahh welcome, so your name is $(my_name). Great to meet you!")
```

Note:-
`readline()` is an interative function, and isn't in the course notes. We'll use it here though.

```
~/MATH2504 $ julia test.jl
Hi what is your name?Michael
ahh welcome, so your name is Michael. Great to meet you!
```

1.3 Jupyter Notebooks

Changing gears now, lets play around with Jupyter notebooks:

```
--- Cell 1 ---
 for i in 1:5
  println("My favourite number is: $i")
 end
```

```
------
 My favourite number is: 1
 My favourite number is: 2
 My favourite number is: 3
 My favourite number is: 4
 My favourite number is: 5

--- Cell 2 ---
 # Top Heading
 ## Smaller Heading
 ### Even smaller Heading!
 This is markdown text!

 In-text latex!

 $\int_0^\infty \frac{1}{x} \textrm{d}x$
------

--- Cell 3 ---
 <html>
 <h1>
   We can do in-text HTML too!
 </h1>
   HTML can do some things better then markdown like links.
   It’s uglier but more versitile
 </html>
------

--- Cell 4 ---
 x = 1
------
1

--- Cell 5 ---
 y = 2
------
2

--- Cell 6 ---
 1 + 1 * 3    # Order of Operations matters!
 (1 + 1) * 3   # Parens give us control of OoO
------
4
6

--- Cell 7 ---
 my_tup = ("Hello", 35, \pi, "world", ’!’)
 typeof(my_tup)
------
Tuple{String, Int64, Irrational{:\pi}, String, Char}

--- Cell 8 ---
 (1984)
 typeof(ans)
 (1984,)
 typeof(ans)
```

```
------
Int64
Tuple{Int64}
```

Note:-

Consider giving Pluto.jl a go! It's like Jupyter, but conserves state continuously, in a way which Jupyter doesn't.

1.4 Some Builtins and Language Features

`print()` prints what is given to it. `println()` will print what is given, then prints a newline; it also lets us specify the an IOBuffer to print to.

```julia
io = IOBuffer()
println(io, "I love", " ", "math", " ", 2+5)
my_str = String(take!(io))
print(my_str)
```

Output: `I love math 7`

Note:-

My lecturer insists that a kilobyte is 1024 bytes! Didn't understand what we were talking about when we pointed out that kibibytes exist smh...

```julia
\eta = 1 / \pi
typeof(\eta)
```

Output: `Float64`

Doing any arthimatic on a type Irational will cast it to a float.

```julia
x = "Hello"
y = "World!"
x * " " * y
```

Output: `Hello World!`

Julia uses the '`*`' operator for string concatonation.

```julia
import Base: +
+(a::String, b::String) = a * b
x + y
```

Output: `HelloWorld!`

We've overloaded the '`+`' operator to concatonate strings!

Chapter 2

Template.tex - Delete me Later!

2.1 Random Examples

Definition 2.1.1: Limit of Sequence in $\mathbb{R}$

Let $\{s_n\}$ be a sequence in $\mathbb{R}$. We say

$$\lim_{n\to\infty} s_n = s$$

where $s \in \mathbb{R}$ if $\forall$ real numbers $\epsilon > 0$ $\exists$ natural number N such that for $n > N$

$$s - \epsilon < s_n < s + \epsilon \text{ i.e. } |s - s_n| < \epsilon$$

Question 1

Is the set $x-$axis$\setminus\{$Origin$\}$ a closed set

Solution: We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls

Note:-

We will do topology in Normed Linear Space (Mainly $\mathbb{R}^n$ and occasionally $\mathbb{C}^n$)using the language of Metric Space

Claim 2.1.1 Topology

Topology is cool

Example 2.1.1 (Open Set and Close Set)

Open Set:
- ϕ
- $\bigcup\limits_{x\in X} B_r(x)$ (Any $r > 0$ will do)
- $B_r(x)$ is open

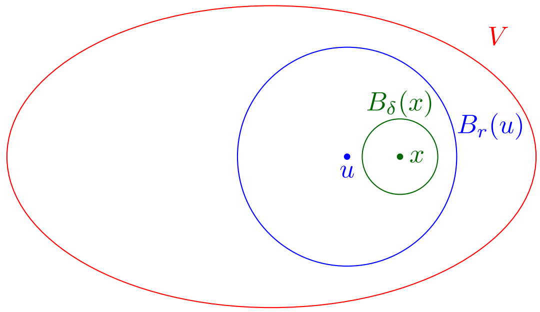
Closed Set:
- $X,\ \phi$
- $\overline{B_r(x)}$

$x-$axis $\cup$ $y-$axis

Theorem 2.1.1

If $x \in$ open set V then $\exists$ $\delta > 0$ such that $B_\delta(x) \subset V$

Proof: By openness of V, $x \in B_r(u) \subset V$



Given $x \in B_r(u) \subset V$, we want $\delta > 0$ such that $x \in B_\delta(x) \subset B_r(u) \subset V$. Let $d = d(u,x)$. Choose δ such that $d + \delta < r$ (e.g. $\delta < \frac{r-d}{2}$)

If $y \in B_\delta(x)$ we will be done by showing that $d(u,y) < r$ but

$$d(u,y) \leq d(u,x) + d(x,y) < d + \delta < r$$

😆

Corollary 2.1.1

By the result of the proof, we can then show...

Lenma 2.1.1

Suppose $\vec{v_1}, \ldots, \vec{v_n} \in \mathbb{R}^n$ is subspace of $\mathbb{R}^n$.

Proposition 2.1.1

$1 + 1 = 2$.

2.2 Random

Definition 2.2.1: Normed Linear Space and Norm $\|\cdot\|$

Let V be a vector space over $\mathbb{R}$ (or $\mathbb{C}$). A norm on V is function $\|\cdot\|$ $V \to \mathbb{R}_{\geq 0}$ satisfying

1. $\|x\| = 0 \iff x = 0 \;\forall\; x \in V$
2. $\|\lambda x\| = |\lambda|\|x\| \;\forall\; \lambda \in \mathbb{R}(\text{or } \mathbb{C}),\, x \in V$
3. $\|x + y\| \leq \|x\| + \|y\| \;\forall\; x, y \in V$ (Triangle Inequality/Subadditivity)

And V is called a normed linear space.

• Same definition works with V a vector space over $\mathbb{C}$ (again $\|\cdot\| \to \mathbb{R}_{\geq 0}$) where 2 becomes $\|\lambda x\| = |\lambda|\|x\|$ $\forall\; \lambda \in \mathbb{C},\, x \in V$, where for $\lambda = a + ib$, $|\lambda| = \sqrt{a^2 + b^2}$

Example 2.2.1 ($p-$Norm)

$V = \mathbb{R}^m$, $p \in \mathbb{R}_{\geq 0}$. Define for $x = (x_1, x_2, \cdots, x_m) \in \mathbb{R}^m$

$$\|x\|_p = \Big(|x_1|^p + |x_2|^p + \cdots + |x_m|^p\Big)^{\frac{1}{p}}$$

(In school $p = 2$)

Special Case $p = 1$: $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_m|$ is clearly a norm by usual triangle inequality.

Special Case $p \to \infty$ ($\mathbb{R}^m$ with $\|\cdot\|_\infty$): $\|x\|_\infty = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$

For $m = 1$ these $p-$norms are nothing but $|x|$. Now exercise

Question 2

Prove that triangle inequality is true if $p \geq 1$ for $p-$norms. (What goes wrong for $p < 1$?)

Solution: **For Property 3 for norm-2**

When field is $\mathbb{R}$:

We have to show

$$\begin{aligned}
&\sum_i (x_i + y_i)^2 \leq \left(\sqrt{\sum_i x_i^2} + \sqrt{\sum_i y_i^2}\right)^2 \\
\implies &\sum_i (x_i^2 + 2x_iy_i + y_i^2) \leq \sum_i x_i^2 + 2\sqrt{\left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]} + \sum_i y_i^2 \\
\implies &\left[\sum_i x_iy_i\right]^2 \leq \left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]
\end{aligned}$$

So in other words prove $\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$ where

$$\langle x, y\rangle = \sum_i x_iy_i$$

Note:-

- $\|x\|^2 = \langle x, x\rangle$
- $\langle x, y\rangle = \langle y, x\rangle$
- $\langle \cdot, \cdot\rangle$ is $\mathbb{R}-$linear in each slot i.e.

  $$\langle rx + x', y\rangle = r\langle x, y\rangle + \langle x', y\rangle \text{ and similarly for second slot}$$

  Here in $\langle x, y\rangle$ x is in first slot and y is in second slot.

Now the statement is just the Cauchy-Schwartz Inequality. For proof

$$\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$$

expand everything of $\langle x - \lambda y, x - \lambda y\rangle$ which is going to give a quadratic equation in variable λ

$$\begin{aligned}
\langle x - \lambda y, x - \lambda y\rangle &= \langle x, x - \lambda y\rangle - \lambda\langle y, x - \lambda y\rangle \\
&= \langle x, x\rangle - \lambda\langle x, y\rangle - \lambda\langle y, x\rangle + \lambda^2\langle y, y\rangle \\
&= \langle x, x\rangle - 2\lambda\langle x, y\rangle + \lambda^2\langle y, y\rangle
\end{aligned}$$

Now unless $x = \lambda y$ we have $\langle x - \lambda y, x - \lambda y\rangle > 0$ Hence the quadratic equation has no root therefore the discriminant is greater than zero.

When field is $\mathbb{C}$:

Modify the definition by

$$\langle x, y\rangle = \sum_i \overline{x_i}y_i$$

Then we still have $\langle x, x\rangle \geq 0$

2.3 Algorithms

Algorithm 1: what

```
Input: This is some input
Output: This is some output
/* This is a comment */

some code here;
x ← 0;
y ← 0;
if x > 5 then
    x is greater than 5 ;                    // This is also a comment
else
    x is less than or equal to 5;
end
foreach y in 0..5 do
    y ← y + 1;
end
for y in 0..5 do
    y ← y − 1;
end
while x > 5 do
    x ← x − 1;
end
return Return something here;
```