SCHOOL OF MATHEMATICS AND PHYSICS

MATH1072
Assignment 4
Semester Two 2024

Submit your answers - along with this sheet - by 1pm on the 21st of October, using the blackboard assignment submission system. Assignments must consist of a single PDF.

You may find some of these problems challenging. Attendance at weekly tutorials is assumed.

Family name:

Given names:

Student number:

Marker's use only

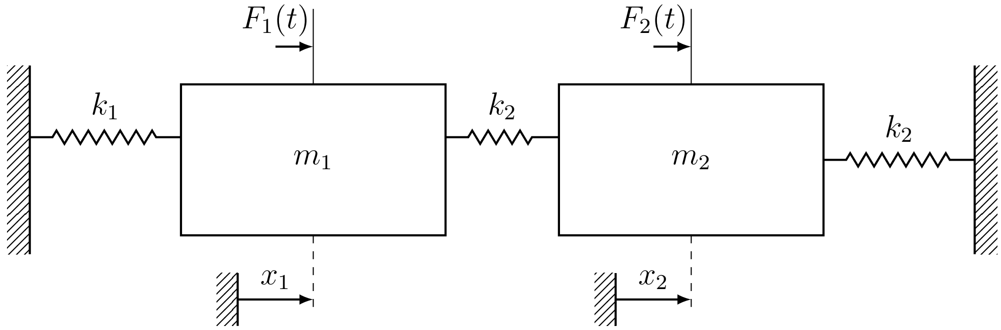
Each question marked out of 3.

- Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.
- Mark of 1: Your submission has some relevance, but does not demonstrate deep understanding or sound mathematical technique.
- Mark of 2: You have the right approach, but need to fine-tune some aspects of your calculations.
- Mark of 3: You have demonstrated a good understanding of the topic and techniques involved, with well-executed calculations.

Q1a	Q1b:	Q1c:	Q2a:
Q2b:	Q2c:	Q2d:	Q2e:
Q3:			

Total (out of):

1 Consider the spring-mass system described by the following image,



a) Derive the system of second order ordinary differential equations that describes the spring-mass system.

b) Write out the reduced system of ordinary differential equations in **vector form** that can be used to solve your system from part a).

c) Use the MATLAB function `ode45.m` to solve your system from part b) over time $[0, 200]$, with the following parameters:

$$F_1(t) = \sin(t),\ F_2(t) = e^{-t}$$
$$k_1 = 2,\ k_2 = 0.5$$
$$m_1 = 3,\ m_2 = 1$$

2 We will explore what it means for the linear combination of solutions to a second-order differential equation, resulting from an application of the *superposition principle*, to be a general solution to the corresponding initial value problem. That is, we will explore what it means for a set of solutions to form a **fundamental set of solutions**.

a) Suppose $y_1(t)$ and $y_2(t)$ are solutions to the second-order differential equation,

$$p(t)y'' + q(t)y' + r(t)y = 0.$$

Use the superposition principle to find a general solution in terms of constant coefficients c_1 and c_2.

b) Consider the initial conditions for the second-order differential equation given by,

$$y(t_0) = y_0, \quad y'(t_0) = y_0'.$$

Apply these initial conditions to your solution from a) and solve for the constants c_1 and c_2 using Cramer's rule.

Cramer's Rule: Given the system of linear equations, $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_0' \end{bmatrix}$, then

$$c_1 = \frac{\begin{vmatrix} y_0 & b_1 \\ y_0' & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad c_2 = \frac{\begin{vmatrix} a_1 & y_0 \\ a_2 & y_0' \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

where $|\cdot|$ denotes the determinant of the matrix.

The quantity in the denominator is called the **Wronskian**.

c) Use the Wronskian for c_1 and c_2 to develop a condition required for this initial value problem to be solvable. The set of solutions that satisfy this condition are called a **fundamental set of solutions**.

d) Consider the second-order differential equation given by,

$$2t^2y'' + ty' - 3y = 0, \ t > 0.$$

Given that $y_1(t) = t^{-1}$ is a solution to the second-order differential equation, find another solution $y_2(t)$ using the reduction of order method by assuming $y_2(t) = u(t)y_1(t)$. For ease of exposition, require that $u(1) = 0$ and $u'(1) = \frac{5}{2}$.

e) Show that the solutions $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions.

3 Solve the following initial value problem for the second-order inhomogeneous differential equation,

$$y'' - 4y' - 12y = 2e^{5t}, \ y(0) = \frac{8}{7}, \ y'(0) = -\frac{1}{7}.$$