School of Mathematics and Physics, UQ

MATH2001/MATH7000 practice problems
Sheet 1

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(1) Find the general solution to the equation

$$y' + xy = xy^{-1}$$

using two approaches:

(a) The ODE is separable.

(b) Multiply both sides of the ODE by ye^{x^2} to make it exact.

(2) Find the general solution to the following ODE:

$$(x^2 - 2x)y' = 2(x-1)y.$$

Now consider the initial value problem with $y(x_0) = y_0$ and determine all initial conditions (x_0, y_0) such that the initial value problem has (a) no solutions, (b) more than one solution, and (c) precisely one solution. What do the existence and uniqueness criteria from lectures tell us about the solutions?

(3) Consider the initial value problem

$$y' = y + 1 - x, \quad y(0) = 0.$$

Let $\phi_0(x) = 0$, and define $\{\phi_n(x)\}$ using the method of successive approximations from lectures.

(a) Determine $\phi_n(x)$ for an arbitrary value of n.

(b) Plot $\phi_n(x)$ for $n = 1, 2, 3, 4$. Do they appear to be converging?

(c) Express $\lim_{n\to\infty} \phi_n(x)$ in terms of elementary functions.

(4) Consider the initial value problem

$$y' = x^2 + y^2, \quad y(0) = 0.$$

Let $\phi_0(x) = 0$, and define $\{\phi_n(x)\}$ using the method of successive approximations from lectures.

(a) Determine $\phi_n(x)$ for $n = 1, 2, 3$.

(b) Plot $\phi_n(x)$ for $n = 1, 2, 3$. Do they appear to be converging?

(5) Show that the equation

$$2x^2 + xy^2 + x^2yy' = 0$$

is exact then use this fact to solve the initial value problem with $y(1) = -2$. Note you should be able to write $y(x)$ explicitly.

(6) Let a, b, c, d be constants. Under what condition is the equation

$$ax + by + (cx + dy)\frac{dy}{dx} = 0$$

exact? Impose this condition and solve the exact equation.

(7) Show that $\operatorname{arcosh}(x) = \ln(x + \sqrt{x^2 - 1})$. Note that the domain of $\operatorname{arcosh}(x)$ is $[1, \infty)$ and the range is $[0, \infty)$.

(8) (a) Show $e^{ix} = \cos x + i \sin x$ using Taylor series.

(b) Show that

$$\sinh 2x = 2 \sinh x \cosh x$$

(c) Use the results of parts (a) and (b) to show that

$$\sin 2x = 2 \sin x \cos x.$$