MATH1072

Practical 4: Vector Fields and Line Integrals
(Exploring Symbolics in MATLAB)

Motivation: We've explored many aspects of numerics in terms of discretization and point approximations. We will continue to do this throughout the semester, especially when considering numerical solutions to differential equations. At present, we will take a side road to explore some of the various analytical solving tools currently available in MATLAB. These tools actually allow us to get an analytic solution to a problem, for example differentiation or integration, provided a closed-form solution *exists*.
The set of tools used to apply analytics to problems are often referred to as `symbolics` which comes from the fact that you are providing the platform with a representation of a problem in a symbolic form. We will use a few of these tools in the context of vector fields and line integrals. These will be useful for you to check your work on assignment, tutorial, or lecture problems as well.

1 Conservative Fields, Non-conservative Fields, and Line Integrals

(a) Symbolically define variables x and y using the `syms` function and the vector field $F(x,y) = (y^2 - 2, 2xy)$.

```
syms %type the variables you would like to define
   symbolically
F(x,y) = %type out the vector field as a vector in matlab
```

(b) Determine whether the vector field $F(x,y) = (y^2, 2xy)$ is conservative. Hint: From lecture we know, if $\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$ then the field F is conservative. Use the `diff(func,var)` function to evaluate the differential of a symbolic function with respect to a variable.

```
F1(x,y) = ; %define F1
```

```
F2(x,y) = ; %define F2
dF1y = diff() %complete the argument
dF2x = diff() %complete the argument
```

(c) Evaluate the work done by $F(x, y)$ over $(0, 0)$ to $(2, 1)$. Recall that, for conservative fields, $\int_A^B F(r)\, dr = f(B) - f(A)$ where $A = (x_A, y_A)$, $B = (x_B, y_B)$. Use the `int(func,var)` function to evaluate the integral of a symbolic function with respect to a variable.

```
%we will find the potential function
fx = int(); %complete the argument, find the integral of
   the symbolic function F1
fxdy = diff(); %complete the argument, find the derivative
    of fx with respect to y
c = ; %find "c'(y)" by subtracting F2 from fxdy
c = int(); %integrate "c'(y)" to find c
f(x,y) = ; %add c to fx to find the potential function
work = ; %calculate the work done.
```

(d) Evaluate the work done by $F(x, y)$ over the path $x = 2t$, $y = t^2$ for $t \in (0, 1)$. Use the relation,

$$\int_A^B F(r) \cdot dr = \int_a^b (F_1(x(t), y(t))\frac{dx}{dt} + F_2(x(t), y(t))\frac{dy}{dt})\ dt.$$

```
syms %type the variable you would like to define
   symbolically
rx(t) = ; %type out part of the path x = 2t
ry(t) = ; %type out part of the path y = t^2
F_int = int(); %complete the argument
```

(e) Create a figure representing the vector field $F(x, y)$. Use the `quiver` function.

(f) Create a figure representing the vector field $(y, -x)$. Use the `quiver` function. Can you find a *closed* path through this vector field that has nonzero work? Is this a conservative or non-conservative field? How does this relate to the work along a closed path?