School of Mathematics and Physics, UQ

MATH2001/MATH7000 practice problems
Sheet 2

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(1) Determine the longest interval in which the initial value problem

$$x(x-4)y'' + 3xy' + 4y = 2, \quad y(3) = 0, \ y'(3) = -1$$

is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

(2) If the Wronskian of f and g is $3e^{4x}$, and if $f(x) = e^{2x}$, find $g(x)$.

(3) Can $y = \sin(x^2)$ be a solution on an interval containing $x = 0$ of an equation $y'' + p(x)y' + q(x)y = 0$ with continuous coefficients? Explain.

(4) Solve the initial value problem

$$y'' + y = \sin x, \quad y(0) = 1, \ y'(0) = 0.$$

(5) Solve the initial value problem

$$y'' - 6y' + 9y = 4e^{3x} + 14\cos x, \quad y(0) = 4, \quad y'(0) = 5.$$

(6) Find the general solution to the nonhomogeneous equation

$$x^2y'' + xy' - n^2y = x^m, \quad x > 0,$$

where m and $n \neq 0$ are any real numbers such that $m^2 \neq n^2$.

Hint: to find the general solution to the corresponding homogeneous equation (y_H), assume the solution is of the form $y = x^\lambda$. You should end up with a characteristic equation which you can solve for λ, after which you should be able to write down y_H. Then use variation of parameters to find y_P. Don't forget to rewrite the equation in standard form (ie. with 1 as the coefficient of y'').

(7) Find the general solution to

$$y'' - 2y' + y = e^x/x^3,$$

(8) Determine whether

$$\left\{ \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ -4 & 4 \end{pmatrix} \right\}$$

is a basis for $M_{2\times2}(\mathbb{F})$.

(9) Suppose $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a basis for the vector space V. Show that

$$\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_3 + \mathbf{v}_4, \mathbf{v}_4\}$$

is a basis for V.

(10) Show that

$$\{x^2 + 1, x^2 - 1, 2x - 1\}$$

is a basis for $P_2(\mathbb{F})$.

(11) Find the coordinate vector of $2 - x + x^2$ relative to the ordered $P_2(\mathbb{F})$ basis $\{1 + x, 1 + x^2, x + x^2\}$.

(12) Let $\beta_1, \beta_2, \beta_3$ be ordered bases for the two-dimensional vector space V, and let

$$P_{\beta_1 \to \beta_2} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}, \qquad P_{\beta_2 \to \beta_3} = \begin{pmatrix} 7 & 2 \\ -4 & -1 \end{pmatrix}$$

be transition matrices. Find $P_{\beta_3 \to \beta_1}$.

(13) Let β' be an ordered basis for $\mathbb{R}^2$, and let

$$P_{\beta \to \beta'} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

be the transition matrix from the ordered basis $\beta = \{(1,0), (0,1)\}$ to β'. Find β'.