Universtiy of Queensland
School of Mathematics and Physics

MATH2302 Discrete Mathematics II
Semester 2 2025
Problem Set 2

Michael Kasumagic, 44302669
Applied Class #1
Due 3pm Friday 12 September 2025

Question 1: 10 marks

Consider the set X of all positive divisors of 360. We define a partial order $\preceq$ on X by

$$x \preceq y \quad \text{if and only if} \quad x \text{ divides } y.$$

Every divisor d in X can be written uniquely as $d = 2^a3^b5^c$ with $0 \le a \le 3, 0 \le b \le 2$ and $0 \le c \le 1$. We define the *rank* function

$$\rho : X \longrightarrow \mathbb{Z}_{\ge 0}, \qquad \rho(2^a3^b5^c) = a + b + c.$$

(a) List all divisors of rank 2.

(b) Show that two distinct divisors of the same rank are incomparable.

(c) Determine the size of the maximum antichain in the poset X. Hint: use generating functions to determine the number of divisors in each rank, and then use part (b).

(d) Determine the minimum number of chains needed to partition X (you do not need to list the chains explicitly).

Solution: 1(a)

$$\rho(2^a3^b5^c) = 2 \iff a + b + c = 2.$$

Knowing this, I can exhaustively list every 3-tuple, (a, b, c), with $a \in \{0, 1, 2, 3\}$, $b \in \{0, 1, 2\}$, $c \in \{0, 1\}$ such that $a + b + c = 2$.

a	b	c	$2^a3^b5^c$
2	0	0	4
0	2	0	9
1	1	0	6
1	0	1	10
0	1	1	15

I know that this is the exhaustive list because, ignoring the upper bounds, $a+b+c = 2$ is analogous to $a' + b' + c' = 5$ (where $a' = a + 1,\ b' = b + 1,\ c' = c + 1$), i.e. 5 stars & 3 bars, which has $\#\text{solutions} = \binom{2+3-1}{3-1} = \binom{4}{2} = 6.$

Only one of these solutions, $(0, 0, 2)$, breaks the upper bound restriction. Once I've remove that from the list, I'm left with the exhaustive list of divisors of rank 2,

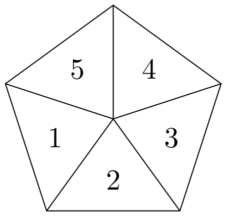
$$\{x \in X \mid \rho(x) = 2\} = \{2, 4, 6, 9, 10, 15\}.$$

Solution: 1(b)

Claim. $\forall x, y \in X : x \neq y, \rho(x) = \rho(y) \implies x \npreceq y,\ y \npreceq x.$

Question 2: 8 marks

A regular pentagon is divided into five congruent triangular regions by drawing lines from its centre to the vertices. Each triangular region is to be coloured using one of three colours. Recall that the symmetry group of the regular pentagon is the dihedral group D_5. Two colourings are considered the same if one can be obtained from another by a symmetry of the pentagon.



(a) Determine the orbit and the stabiliser of the region $\{1, 2\}$ under the action of D_5.

(b) Use the counting theorem to determine the number of distinct colourings of the pentagon.

Solution: 2(a)

Question 3: 8 marks

Using generating functions, determine the number of integer solutions to the inequality

$$x_1 + x_2 + x_3 \leq 15,$$

where $x_1 \geq 1$, $0 \leq x_2 \leq 2$ and $0 \leq x_3 \leq 5$.

Solution:

Question 4: 6 marks

Use a generating function to find a closed form for the recurrence

$$a_n = 3a_{n-1} + 2^n, \qquad a_0 = 0.$$

Solution: