MATH1061
Discrete Mathematics I
In-Lecture Questions

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Chapter 1

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Question 1

Which of the following are statements?

(a) ”Is it going to rain tomobbrow?”

(b) ”She is happy.”

(c) ”23 July 2024 is a Tuesday”

(d) $x = 5y + 2$

(e) $65 < 2$

Solution: (a) No, a question. (b) No, ”she” undefined. (c) Yes. (d) No, x, y undefined. (e) Yes.

Question 2

Let p, q, r be statements.

- $p =$ “it is cold.”
- $q =$ “it is snowing.”
- $r =$ “it is sunny.”

Translate these to symbols:

(a) “It is not cold but it is snowing.”

(b) “It is neither snowing nor cold, but it is sunny.”

Translate these to English:

(c) $\sim p \wedge q$

(d) $(p \wedge q) \vee r$

Solution: (a) $\sim p \wedge q$ (b) $\sim p \wedge \sim q \wedge r$ (c) “It is not cold but it is snowing” (d) “It is either snowing and cold, or sunny, or it’s both.”

Question 3

Construct the truth table for $(p \wedge \sim q) \vee (q \wedge r)$

Solution:

p	q	r	$\sim q$	$p \wedge \sim q$	$q \wedge r$	$(p \wedge \sim q) \vee (q \wedge r)$
T	T	T	F	F	T	T
T	T	F	F	F	F	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	T	F	F	F	F	F
F	F	T	T	F	F	F
F	F	F	T	F	F	F

Question 4

Using De Morgan's Law, write down a statement which is logically equivalent to the negation of "5 is even and 6 is even."

Solution: "5 is even and 6 is even." $\equiv p \wedge q$. The solution we want is the negation, $\sim(p \wedge q)$, which, by De Morgan's Law is the same as $\sim p \vee \sim q$ which in English is "5 is odd or 6 is odd."

Question 5

Show that

$$\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \equiv p$$

using a truth table, and by laws of logical equivalence.

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim p \wedge \sim q$	$(\sim p \wedge q) \vee (\sim p \wedge \sim q)$	$\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q))$
T	T	F	F	F	F	F	T
T	F	F	T	F	F	F	T
F	T	T	F	T	F	T	F
F	F	T	T	F	T	T	F

$\therefore \sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \equiv p$ by exhaustion.

$$\begin{aligned}
\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) &\equiv \sim(\sim p \wedge q) \wedge \sim(\sim p \wedge \sim q) && \text{(De Morgan's Law)}\\
&\equiv (p \vee \sim q) \wedge (p \vee q) && \text{(De Morgan's Law)}\\
&\equiv p \vee (q \wedge \sim q) && \text{(Distributivity)}\\
&\equiv p \vee \top && \text{(Negation Law)}\\
&\equiv p && \text{(Identity)}
\end{aligned}$$

$\square$

$\therefore \sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \equiv p$ by logical equivalence.

1.2 Lecture 3

Questions

Question 6

Which of the following sentences have the same meaning as “If I am worried then I did not sleep”?

(a) if I am worried then I do not sleep.

(b) if I am not worried then I do sleep.

(c) If I do not sleep then I am worried.

(d) I am worried and I do sleep.

(e) If I do sleep then I am not worried.

(f) I am worried or I do not sleep

(g) I do not sleep or I am not worried.

Solution:

- Original: $p \to \sim q$
- (a): $p \to \sim q$, equivalent.
- (b): $\sim p \to q$, not equivalent.
- (c): $\sim q \to p$, not equivalent.
- (d): $p \wedge q$, not equivalent.
- (e): $q \to \sim p$, equivalent, the contrapositive.
- (f): $p \vee \sim q$, not equivalent.
- (g): $\sim q \vee \sim p$, equivalent, logically equivalent.

Question 7

Express the operations $\vee$, $\to$, and $\leftrightarrow$ using only $\sim$ and $\wedge$.

Solution:

$$\begin{aligned}
p \vee q &\equiv \sim(\sim(p \vee q)) \\
&\equiv \sim(\sim p \wedge \sim q) \\
p \to q &\equiv \sim p \vee q \\
&\equiv \sim(\sim(\sim p \vee q)) \\
&\equiv \sim(p \wedge \sim q) \\
p \leftrightarrow q &\equiv (p \to q) \wedge (q \to p) \\
&\equiv (\sim p \vee q) \wedge (\sim q \vee p) \\
&\equiv \sim(\sim(\sim p \vee q)) \wedge \sim(\sim(\sim q \vee p)) \\
&\equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)
\end{aligned}$$

Question 8: Challenge

Consider the NAND operation $p \barwedge q \equiv \sim(p \wedge q)$ can you express $\wedge$, $\vee$, $\sim$, and $\to$ using only $\barwedge$ operations? Can you express using only $\sim$ and $\oplus$?

Solution: Generating expressions using only NANDs:

$$
\begin{aligned}
\sim p &\equiv \sim(p \wedge p) \\
&\equiv p \barwedge p \\
p \wedge q &\equiv (p \wedge q) \wedge (p \wedge q) \\
&\equiv \sim(\sim(p \wedge q) \wedge \sim(p \wedge q)) \\
&\equiv \sim((p \barwedge q) \wedge (p \barwedge q)) \\
&\equiv (p \barwedge q) \barwedge (p \barwedge q) \\
p \vee q &\equiv \sim(\sim(p \vee q)) \\
&\equiv \sim(\sim p \wedge \sim q) \\
&\equiv \sim p \barwedge \sim q \\
&\equiv (p \barwedge p) \barwedge (q \barwedge q) \\
p \to q &\equiv \sim p \vee q \\
&\equiv \sim(\sim(\sim p \vee q)) \\
&\equiv \sim(p \wedge \sim q) \\
&\equiv p \barwedge \sim q \\
&\equiv p \barwedge (q \barwedge q)
\end{aligned}
$$

These sick fucks had me testing and observing truth tables for two hours. I was suspicious at times, but I assumed there must be a solution...
There is not. No matter how many XOR and NOT operations you apply, ultimately, you will always have 2 Falses and 2 Trues, or 4 Falses, or 4 Trues. **AHHHHHHHH**.

Question 9

Show that $\sim(p \to q) \not\equiv \sim p \to \sim q$.

Solution: By counterexample. Suppose $p = \text{True}$ & $q = \text{True}$

$$
\begin{aligned}
\sim(p \to q) &\equiv \sim(\text{True} \to \text{True}) \\
&\equiv \sim(\text{True}) \\
&\equiv \text{False} \\
\sim p \to \sim q &\equiv \sim\text{True} \to \sim\text{True} \\
&\equiv \text{False} \to \text{False} \\
&\equiv \text{True} \not\equiv \sim(p \to q)
\end{aligned}
$$

□

Question 10

Which of the following sentences have the opposite truth valuse as "If I am worried then I did not sleep"?

(a) if I am worried then I do not sleep.

(b) if I am not worried then I do sleep.

(c) If I do not sleep then I am worried.

(d) I am worried and I do sleep.

(e) If I do sleep then I am not worried.

(f) I am worried or I do not sleep

(g) I do not sleep or I am not worried.

Solution:

- Original: $p \to \sim q$
- (a): $p \to \sim q$, No, equivalent statement.
- (b): $\sim p \to q$, No, True 3/4 times, same as our statement. Not possible to be opposite.
- (c): $\sim q \to p$, No, True 3/4 times.
- (d): $p \wedge q$, Yes! $p \to \sim q \equiv \sim p \vee \sim q \equiv \sim(p \wedge q)$, exactly the opposite.
- (e): $q \to \sim p$, No, equivalent statement.
- (f): $p \vee \sim q$, No, True 3/4 times.
- (g): $\sim q \vee \sim p$, No, equivalent statement.

Question 11

Show that
$$p \to (q \vee r) \equiv (p \wedge \sim q) \to r.$$

Solution:

$$\begin{aligned} p \to (q \vee r) &\equiv \sim p \vee (q \vee r) \\ &\equiv (\sim p \vee q) \vee r \\ &\equiv \sim(\sim(\sim p \vee q)) \vee r \\ &\equiv \sim(p \wedge \sim q) \vee r \\ &\equiv (p \wedge q) \to r \end{aligned}$$

Question 12

Let n be a positive integer. Find conditions that are:

(a) necessary, but not sufficent for n to be a multiple of 10.

(b) sufficient but not necessary for n to be divisible by 10.

(c) necessary and sufficient for n to be divisible by 10.

Solution:

(a) n is a multiple of 10 $\to$ n is necessarily even.

(b) n is a multiple of 50 $\to$ sufficient to conclude that n is divisible by 10.

(c) n's last digit is a 0 $\leftrightarrow$ n is divisible by 10.

Chapter 2

Week 2

2.1 Lecture 4

Questions

Question 13

Write the following arguments symbolically

- If wages are raised, then buying increases.
- If there is a depression, then buying does not increase.
- Therefore, there is not a depression, or wages are not raised.

Decide whether this argument is valid, using three methods:

(a) a truth table

(b) rules of inference

(c) configuring truth values

Solution:

Let $p =$ "Wages are raised."
Let $q =$ "There is a depression."
Let $r =$ "Buying increases."

$$\begin{array}{rl} 1. & p \to r \\ 2. & q \to \sim r \\ \hline \therefore & \sim p \vee \sim q \end{array}$$

Solution: (a)

Variables			Premises		Conclusion
p	q	r	$p \to r$	$q \to \sim r$	$\sim p \vee \sim q$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Consider rows 3, 6, 7, and 8. Whenever all the premises are true, the conclusion is true. Therefore the argument is valid.

$\square$

Solution: (b)

$$\begin{array}{lll}
1. & p \rightarrow r & \\
2. & q \rightarrow \sim r & \\
3. & r \rightarrow \sim q & \text{(Contrapositive of 2.)} \\
4. & p \rightarrow \sim q & \text{(Transitivity of 1. and 3.)} \\
5. & \underline{\sim p \vee \sim q} & \text{(Expansion of } \rightarrow\text{)} \\
\therefore & \sim p \vee \sim q &
\end{array}$$

Therefore, by using laws of inference, we've proven that the conclusion follows from the premises. Therefore the argument is valid.

$\square$

Solution: (c)

Suppose the argument is invalid
Then, the premises are all true, and the conclusion is false.
So, $\sim p \vee \sim q$ is False.

$$\begin{array}{lll}
1. & p \rightarrow r & \\
2. & q \rightarrow \sim r & \\
3. & \sim(p \wedge q) \text{ is False} & \text{(De Morgan's Law)} \\
4. & p \wedge q \text{ is True} & \text{(Follows from 3.)} \\
5. & p \text{ is True} & \text{(Specalisation)} \\
6. & q \text{ is True} & \text{(Specalisation)} \\
7. & r \text{ is True} & \text{(Follows from 1., given 5.)} \\
8. & \underline{r \text{ is False} \quad ※} & \text{(Follows from 2., given 6.)} \\
\therefore & \sim p \vee \sim q &
\end{array}$$

We've identified a contradiction! If we assume invalidity, we see contradictions arise. Which means our original assumption was incorrect. Which means the argument is valid.

$\square$

Question 14

Is the following argument valid? Again, use all three methods.

$$\begin{array}{ll}
1. & p \rightarrow q \\
2. & p \vee r \\
3. & \underline{p \vee \sim r} \\
\therefore & q
\end{array}$$

Solution: (a)

Variables			Premises			Conclusion	
p	q	r	$p \rightarrow q$	$p \vee r$	$p \vee \sim r$	q	
T	T	T	T	T	T	T	←
T	T	F	T	T	T	T	←
T	F	T	F	T	T	F	
T	F	F	F	T	T	F	
F	T	T	T	T	F	T	
F	T	F	T	F	T	T	
F	F	T	T	T	F	F	
F	F	F	T	F	T	F	

Observing rows 1, and 2, we can see that when all the premises are True, the conclusion is True. Therefore the argument is valid.

□

Solution: (b)

$$\begin{array}{lll}
1. & p \to q & \\
2. & p \vee r & \\
3. & p \vee \sim r & \\
4. & \sim r \vee p & \text{(Commutativity of 3.)} \\
5. & r \to p & \text{(Logical Equivalence of 4.)} \\
6. & r \to q & \text{(Transitivity of 5. and 1.)} \\
7. & \underline{q} & \text{(Division of Cases of 2. by 1., 6.)} \\
\therefore & q &
\end{array}$$

Therefore, we've shown using rules of inference that the Argument is logically valid.

□

Solution: (c)
Suppose the argument is invalid
Then, the premises are all true, and the conclusion is false.
So, q is False.

$$\begin{array}{lll}
1. & p \to q & \\
2. & p \vee r & \\
3. & p \vee \sim r & \\
4. & p \text{ is False} & \text{(We know from 1. given } q\text{)} \\
5. & r \text{ is True} & \text{(We know from 2. given } p\text{)} \\
6. & \underline{r \text{ is False} \quad ※} & \text{(We know from 3. given } p\text{)} \\
\therefore & q &
\end{array}$$

We've identified a contradiction! If we assume invalidity, we see contradictions arise. Which means our original assumption was incorrect. Which means the argument is valid.

Question 15

Is the following argument valid?

$$\begin{array}{ll}
1. & p \to q \\
2. & q \to r \\
3. & \underline{\sim p \vee \sim q} \\
\therefore & r
\end{array}$$

Solution: I'll use rules of inference, since it's my weakest solution method.

$$\begin{array}{lll}
1. & p \to q & \\
2. & q \to r & \\
3. & \sim p \vee \sim q & \\
4. & \underline{p \to \sim q \quad ※} & \text{(Collection of 1. } \vee \text{ to } \to\text{)} \\
\therefore & r &
\end{array}$$

This is a contradiction, because p cannot similateously imply q and $\sim q$, no matter what truth value it takes. Therefore the argument is invalid.

□

Question 16

Determine whether or not the following argument is valid, using all three methods.

- If new messages are queued, then the filesystem is locked.

- The filesystem is not locked if and only if the system is functioning normally.
- New messages will not be sent to the message buffer only if they are queued.
- New messages will not be sent to the message buffer.
- Therefore, the system is functioning normally.

Solution:

Let p be the statement "New messages are queued."
Let q be the statement "The filesystem is locked."
Let r be the statement "New messages are sent to the message buffer."
Let s be the statement "The system is functioning normally."
Then the argument can be written symbolically

$$\begin{array}{ll} 1. & p \rightarrow q \\ 2. & \sim q \leftrightarrow s \\ 3. & p \rightarrow \sim r \\ 4. & \sim r \\ \hline \therefore & s \end{array}$$

Let's check this arguments validity with a truth table.

Variables				Premises				Conclusion	
p	q	r	s	$p \rightarrow q$	$\sim q \leftrightarrow s$	$p \rightarrow \sim r$	$\sim r$	s	
T	T	T	T	T	F	F	F	T	
T	T	T	F	T	T	F	F	F	
T	T	F	T	T	F	T	T	T	
T	T	F	F	T	T	T	T	F	$\leftarrow$
T	F	T	T	F	T	F	F	T	
T	F	T	F	F	F	F	F	F	
T	F	F	T	F	T	T	T	T	
T	F	F	F	F	F	T	T	F	
F	T	T	T	T	F	T	F	T	
F	T	T	F	T	T	T	F	F	
F	T	F	T	T	F	T	T	T	
F	T	F	F	T	T	T	T	F	$\leftarrow$
F	F	T	T	T	T	T	F	T	
F	F	T	F	T	F	T	F	F	
F	F	F	T	T	T	T	T	T	
F	F	F	F	T	F	T	T	F	

Observe rows 4 and 12, where in all the premises are true, but the conclusion is false. Therefore, the argument is invalid.
Now let's prove this invalidity using rules of inference.

$$\begin{array}{lll} 1. & p \rightarrow q & \\ 2. & \sim q \leftrightarrow s & \\ 3. & p \rightarrow \sim r & \\ 4. & \sim r & \\ 5. & s \rightarrow \sim q & \text{(Falls from biconditional 2.)} \\ 6. & q \rightarrow \sim s & \text{(Contrapositve of 5.)} \\ 7. & p \rightarrow \sim s & \text{(Transitivity of 1. to 6.)} \\ 8. & \sim p \vee q & \text{(Expansion of 1. } \rightarrow\text{)} \\ 9. & \sim p \vee \sim r & \text{(Expansion of 3. } \rightarrow\text{)} \\ \hline \therefore & s & \end{array}$$

Therefore, we can see that the argument is invalid.
Finally, lets check in validity by searching for values for the variables.
Let's assume the argument is valid. All the premises are true, and the conclusion is false.
s is false.

1.	$p \to q$	
2.	$\sim q \leftrightarrow s$	
3.	$p \to \sim r$	
4.	$\sim r$	
5.	$s \to \sim q$	(Falls from biconditional 2.)
6.	$\sim q$ is False	(Follows from 5.)
7.	q is True	(Follows from 6.)
8.	p is True	(Follows from 1. given q)
9.	r is False	(Negation of 4.)
$\therefore$	s	

2.2 Lecture 5

Questions

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example

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example

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