Due **Friday 16 August at 5pm** on blackboard.

Marks will be deducted for sloppy working. Clearly state your assumptions and conclusions, and justify all steps in your work.

Q1 Use a truth table to determine whether the following statement is a contradiction, a tautology or neither. If it is a contradiction or a tautology, verify your answer using logical equivalences.

$$((p \leftrightarrow q) \wedge (p \oplus r)) \rightarrow (q \vee r)$$

(10 marks)

Q2 Using the laws of logical equivalence, show that the following statement is a tautology:

$$((a \rightarrow r) \wedge (b \rightarrow r) \wedge (\sim a \rightarrow b)) \rightarrow r$$

(10 marks)

Q3 Show that the following argument is valid, by adding steps using the rules of inference and/or logical equivalences. Clearly label which rule you used in each step.

1. $a \wedge w \rightarrow p$
2. $\sim a \rightarrow t$
3. $\sim w \rightarrow m$
4. $\sim p$
5. $e \rightarrow \sim (t \vee m)$

$\therefore \sim e$

(5 marks)

Q4 Let $P(x)$, $Q(x)$, $R(x)$ and $S(x)$ denote the following predicates with domain $\mathbb{Z}$:

$$\begin{aligned} P(x)&: \ x^2 = 9, \\ Q(x)&: \ x^2 = 6, \\ R(x)&: \ x \geq 0, \\ S(x)&: \ x \text{ is odd}, \end{aligned}$$

Determine whether each of the following statements is true or false, and give brief reasons.

(a) $\forall x \in \mathbb{Z}, P(x) \rightarrow S(x)$

(b) $\forall x \in \mathbb{Z}, P(x) \rightarrow R(x)$

(c) $\exists x \in \mathbb{Z}$ such that $P(x) \wedge R(x)$

(d) $\forall x \in \mathbb{Z}, Q(x) \rightarrow R(x)$

(e) $\forall x \in \mathbb{Z}, S(x) \rightarrow \sim Q(x)$

(10 marks)

(Continued on the following page...)

Q5 **MATH1061 only:** Find values of a, b and r that show that the following statement is *not* a tautology, and show your working.

$$r \to ((a \to r) \wedge (b \to r) \wedge (\sim a \to b))$$

Note: This is the converse of the statement from Q2.

(10 marks)

Q5 **MATH7861 only:**

- Write down the negation of each of the statements from Q4.
- Consider statements (b), (c) and (d) from Q4. What happens to their true/false values if we change the domain from $\mathbb{Z}$ to $\mathbb{R}$? Do they stay the same, or do they change? Give brief reasons for your answers.

(10 marks)