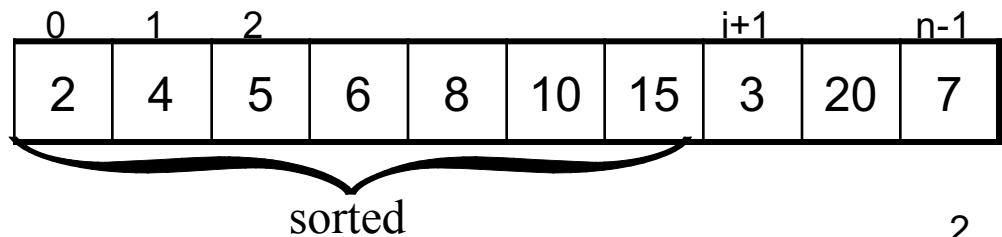
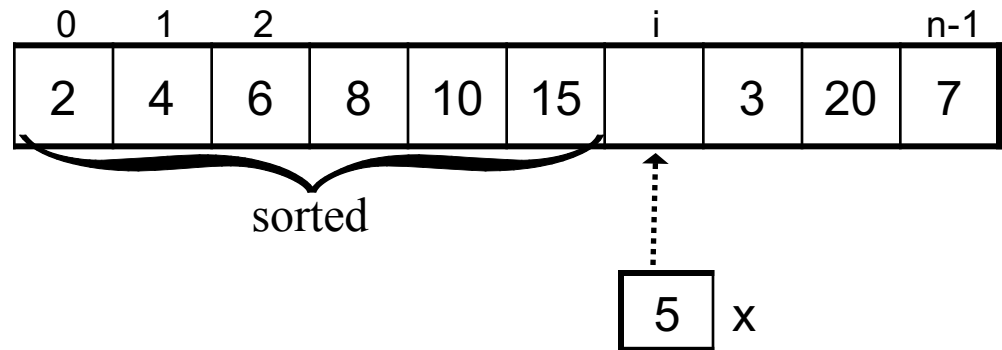
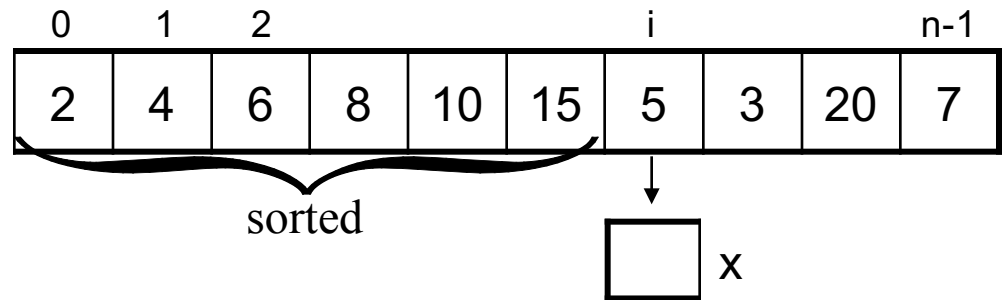


# Quadratic Sorting

## Review

- Insertion Sort with an array
- Selection Sort with an array
- Bubblesort with an array

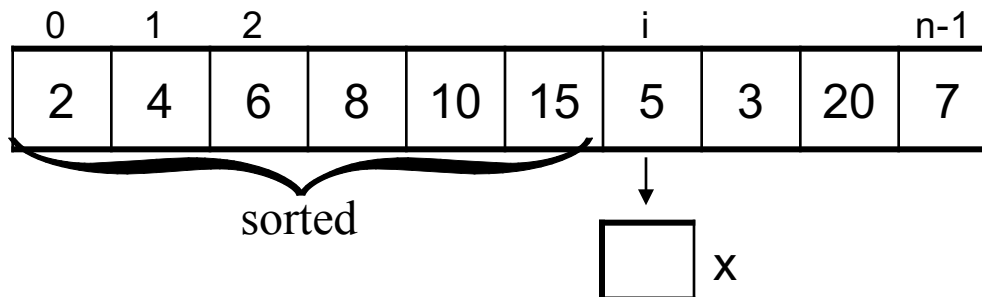
# Insertion Sort (array)



```

for i=1 to n-1
  x ← A[i]
  j ← i-1
  while x.key < A[j].key
    and j >= 0
    A[j+1] ← A[j]
    j ← j-1
  A[j+1] ← x

```



## Complexity

Insertion sort for  
arrays

Number of comparisons:

MIN

$(C_i)_{\min} = 1$   
 $i=1..n-1$   
(already in order)



$$C_{\min} = n-1 = O(n)$$

MAX

$(C_i)_{\max} = i$   
 $i=1..n-1$   
(in reverse  
order)



$$C_{\max} = \sum_{i=1}^{n-1} i$$
$$= n(n-1)/2$$
$$= O(n^2)$$

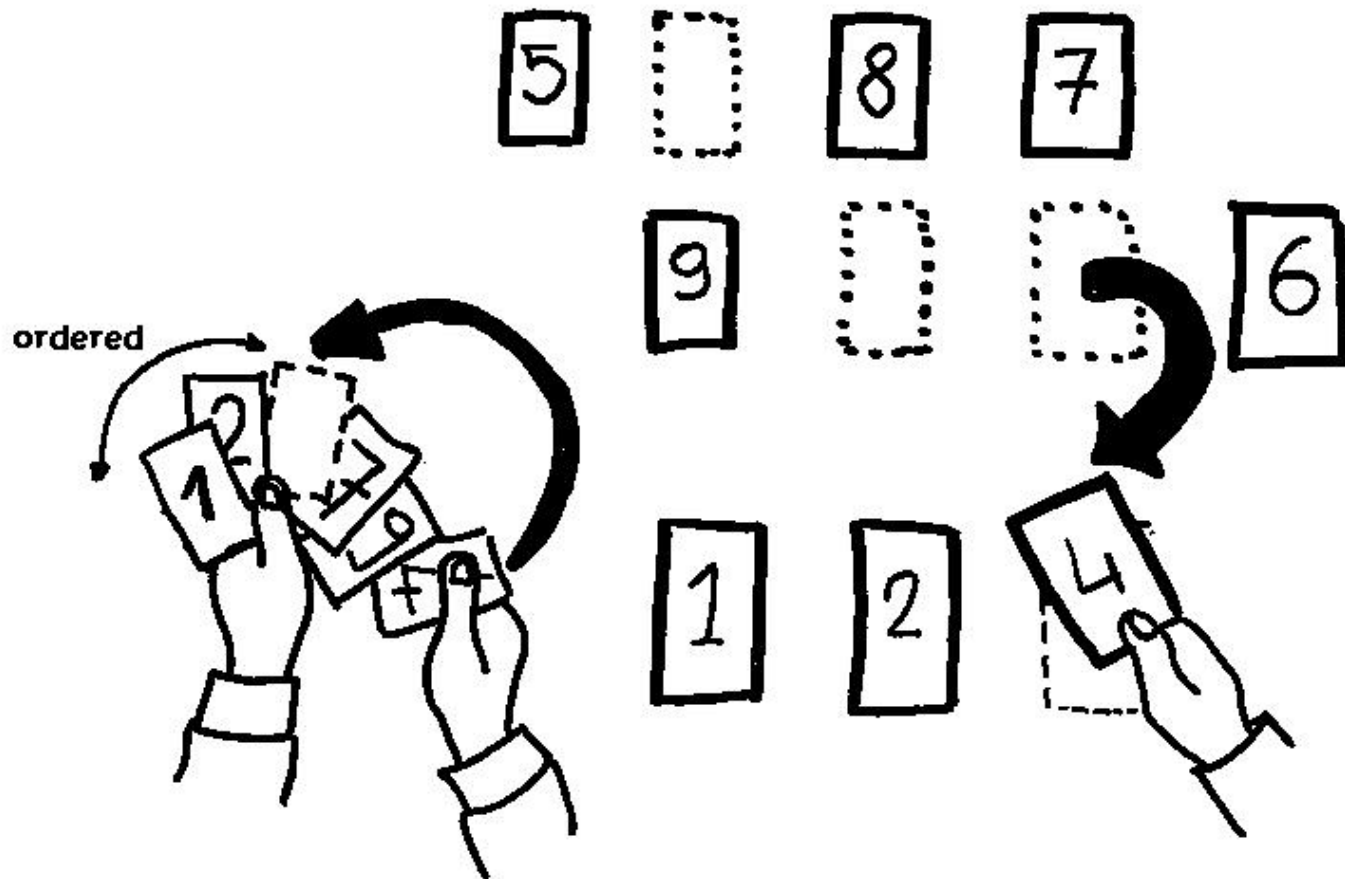
And with Sequence implemented with linked list ?

\_\_\_\_\_ Complexity \_\_\_\_\_ Insertion sort for  
Number of movements: arrays

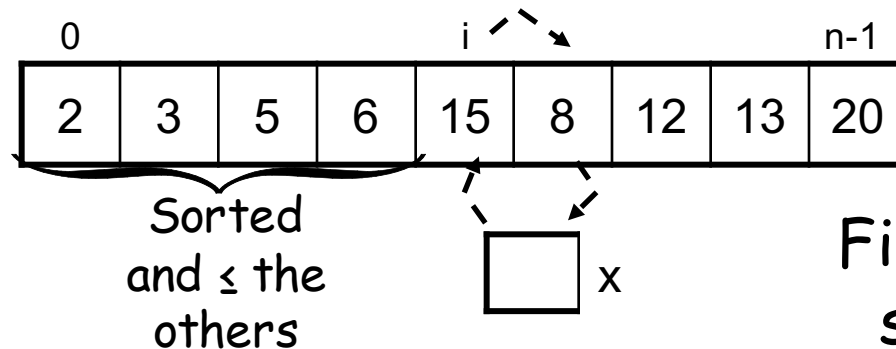
Same as comparisons

And with Sequence implemented with linked list ?

# Selection Sort (array)



# Selection Sort (array)



Finding the  
smallest  
element

```
for i=0 to n-2
  k ← i
  x ← A[i]
  for j=i+1 to n-1
    if A[j].key < x.key
      k ← j
      x ← A[j]
  A[k] ← A[i]
  A[i] ← x
```

# Complexity of Selection Sort (array)

**Comparisons** (Does not depend on the initial order of the elements)

$$\begin{aligned}c &= \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i) = \sum_{i=1}^{n-1} (n-i) = \sum_{i=1}^{n-1} n - \sum_{i=1}^{n-1} i \\&= n(n-1) - n(n-1) / 2 \\&= n(n-1) / 2 \\&= O(n^2)\end{aligned}$$



# Complexity of Selection Sort (array)

## Movements

MIN (already in order) =  $O(n)$

MAX (in reverse order)  
 $(D_i)_{\max} = O(n^2)$

And with Sequence implemented with linked list ?

# Bubble Sort

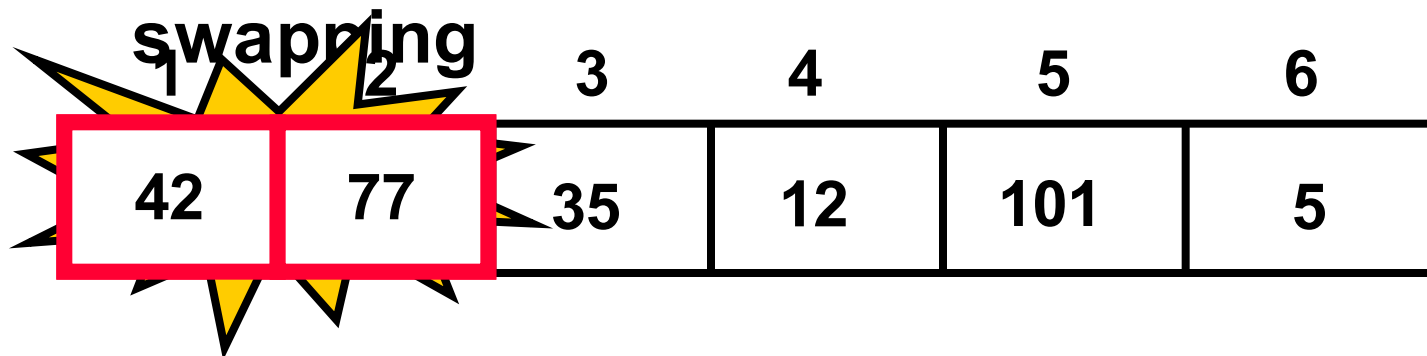
# "Bubbling Up" the Largest Element

- **Traverse a collection of elements**
  - Move from the front to the end
  - “Bubble” the **largest value** to the end using **pair-wise comparisons and swapping**

1	2	3	4	5	6
77	42	35	12	101	5

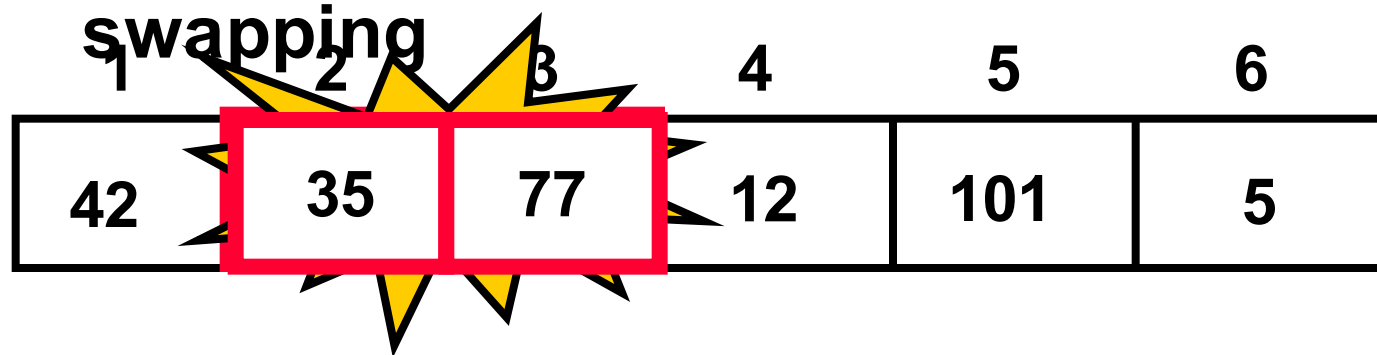
# "Bubbling Up" the Largest Element

- **Traverse a collection of elements**
  - Move from the front to the end
  - “Bubble” the largest value to the end using pair-wise comparisons and



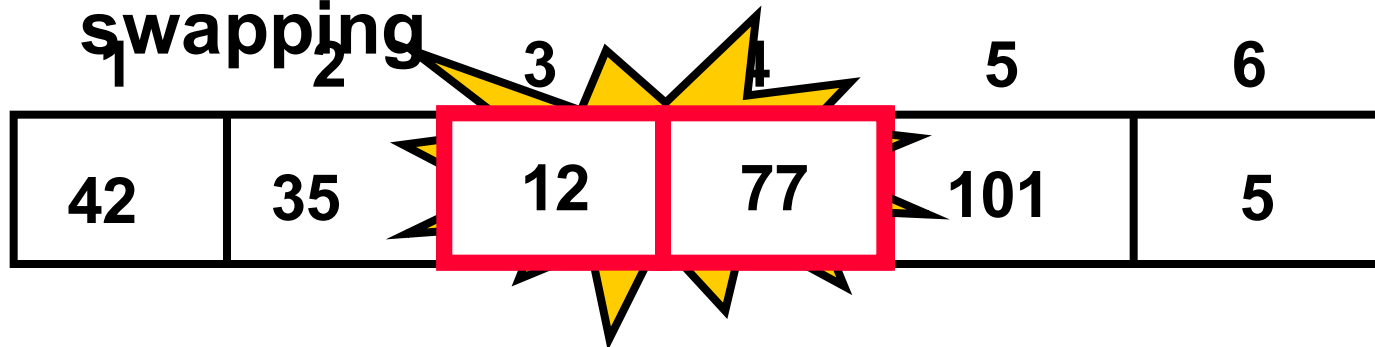
# "Bubbling Up" the Largest Element

- **Traverse a collection of elements**
  - Move from the front to the end
  - “Bubble” the largest value to the end using pair-wise comparisons and



# "Bubbling Up" the Largest Element

- **Traverse a collection of elements**
  - Move from the front to the end
  - “Bubble” the largest value to the end using pair-wise comparisons and swapping



# "Bubbling Up" the Largest Element

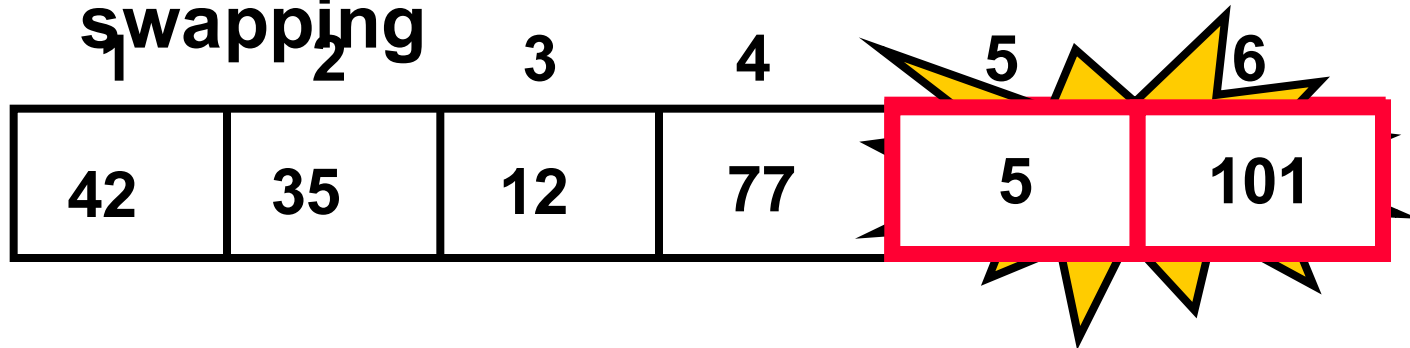
- **Traverse a collection of elements**
  - Move from the front to the end
  - “Bubble” the largest value to the end using pair-wise comparisons and swapping

1	2	3	4	5	6
42	35	12	77	101	5

No need to swap

# "Bubbling Up" the Largest Element

- **Traverse a collection of elements**
  - Move from the front to the end
  - “Bubble” the largest value to the end using pair-wise comparisons and swapping





# "Bubbling Up" the Largest Element

- **Traverse a collection of elements**
  - Move from the front to the end
  - “Bubble” the largest value to the end using pair-wise comparisons and swapping

0	1	2	3	4	5
42	35	12	77	5	101

**Largest value correctly placed**

# Items of Interest

- Notice that only the largest value is correctly placed
- All other values are still out of order
- So we need to repeat this process

0	1	2	3	4	5
42	35	12	77	5	101

Largest value correctly placed

# Repeat “Bubble Up” How Many Times?

- If we have  $N$  elements...
- And if each time we bubble an element, we place it in its correct location...
- Then we repeat the “bubble up” process  $N - 1$  times.
- This guarantees we’ll correctly place all  $N$  elements.

# “Bubbling” All the Elements

1st

0	1	2	3	4	5
42	35	12	77	5	101
35	12	42	5	77	101
12	35	5	42	77	101
12	5	35	42	77	101
5	12	35	42	77	101

# Complexity of Bubblesort (arrays)

And with Sequence implemented with linked list ?

Comparisons

$$c = \sum_{i=1}^{n-1} (n - i) = \frac{n}{2} (n - 1) = O(n^2)$$

Movements

$$D_{\min} = 0 \quad (\text{already in order})$$

$$D_{\max} = 3 \cdot C = O(n^2) \quad (\text{in reverse order})$$

# Already Sorted Collections?

- What if the collection was already sorted?
- What if only a few elements were out of place and after a couple of “bubble ups,” the collection was sorted?
- We want to be able to **detect this and “stop early”!**

0	1	2	3	4	5
5	12	35	42	77	101

# Using a Boolean “Flag”

- We can use a boolean variable to determine if any swapping occurred during the “bubble up.”
- If no swapping occurred, then we know that the collection is already sorted!
- This boolean “flag” needs to be reset after each “bubble up.”

# Bubblesort algorithm

```
j ← 0
swapped ← true
while (swapped)
    swapped ← false
    j ← j+1
    for i=0 to n-j-1
        if A[i].key > A[i+1].key
            tmp= A[i]
            A[i]= A[i+1]
            A[i+1]= tmp
            swapped ← true
```