Zadanie 1

Znaleźć rozwiązania poniższego równania w zbiorze liczb zespolonych

$z \neq 0$

$$\left(\frac{z-i}{z}\right)^4 = (1-i)^4.$$

$\sqrt[4]{(1-i)^4} = \{1-i,\ (1-i)\cdot i,\ (1+i)i,\ -1-i\}$ $\quad 1+i$

$\Downarrow$

① $\frac{z-i}{z} = 1-i$, ② $\frac{z-i}{z} = 1+i$, ③ $\frac{z-i}{z} = -1+i$, (5 pkt)

Zadanie 2

④ $\frac{z-i}{z} = -1-i$

a) Znaleźć część rzeczywistą i urojoną liczby zespolonej $\frac{-\sqrt{2}-\sqrt{2}i}{1-\sqrt{3}i}$. $z = \ \cdot \frac{1+\sqrt{3}i}{1+\sqrt{3}i}$

$= \frac{-\sqrt{2}-\sqrt{6}i-\sqrt{2}i+\sqrt{6}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4} - \frac{\sqrt{6}+\sqrt{2}}{4}i \Rightarrow \operatorname{Re} z = \frac{\sqrt{6}-\sqrt{2}}{4}$ ✓ (2 pkt)

$\operatorname{Im} z = \frac{\sqrt{6}+\sqrt{2}}{4}$ ✓

b) Obliczyć $\left|\frac{-\sqrt{2}-\sqrt{2}i}{1-\sqrt{3}i}\right|$. $=$

$= \frac{|-\sqrt{2}-\sqrt{2}i|}{|1-\sqrt{3}i|} = \frac{2}{2} = \underline{1}$ ✓ (1.5 pkt)

c) Obliczyć $\operatorname{Arg}\left(\frac{-\sqrt{2}-\sqrt{2}i}{1-\sqrt{3}i}\right) = \underbrace{\operatorname{Arg}(-\sqrt{2}-\sqrt{2}i)}_{5/4\,\pi} - \underbrace{\operatorname{Arg}(1-\sqrt{3}i)}_{\frac{5\pi}{3}} + 2k\pi$ (1.5 pkt)

$\cos\varphi = \frac{1}{2}$, $\sin\varphi = \frac{-\sqrt{3}}{2}$ IV $\Rightarrow \varphi = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$k=1$ $\Downarrow$ $= \frac{15\pi}{12} - \frac{20\pi}{12} + 2\pi = \frac{-5\pi}{12} + \frac{24\pi}{12} = \underline{\underline{\frac{19\pi}{12}}}$

Zadanie 3

Przedstawić na płaszczyźnie zespolonej zbiory spełniające poniższe warunki

a) $|\overline{z} - 2 + i| \geqslant |z - 2 + i|$.

(2.5 pkt)

b) $\frac{\pi}{2} \leqslant \operatorname{Arg}(2iz) < \frac{3\pi}{2}$.

cd. zad 1

(2.5 pkt)

①	②	③
$z - i = z(1-i)$	$z-i = z(1+i)$	$z - i = z(-1+i)$
$z(\cancel{1}-\cancel{1}+i) = i$	$z(1-1-i) = i$	$z(1+1-i) = i$
$\underline{z_1 = 1}$	$\underline{z_2 = -1}$	$z_3 = \frac{i}{2-i}\cdot\frac{2+i}{2+i} = \underline{\frac{-1}{5} + \frac{2}{5}i}$

oraz ④ $z - i = z(-i-1)$

$z(1+1+i) = i$

$z_4 = \frac{i}{2+i}\cdot\frac{2-i}{2-i} = \underline{\frac{1}{5} + \frac{2}{5}i}$

zad 3

$$\overline{z_1} - \overline{z_2} = \overline{z_1 - z_2}$$

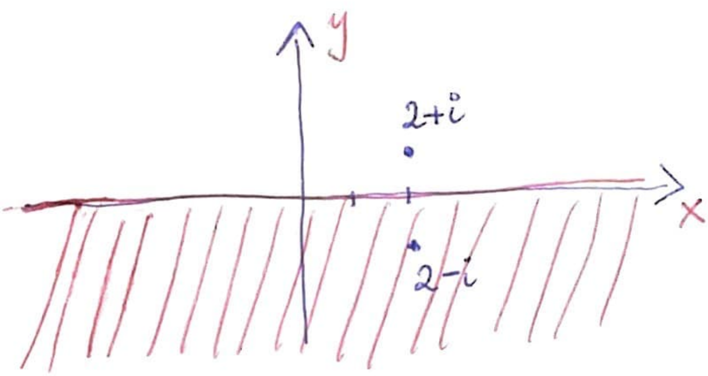
a)

$$|\overline{z_3}| = |z_3|$$

$$|\overline{z} - 2 + i| = |\overline{z} - (2-i)| = |\overline{z} - \overline{(2+i)}| =$$

$$|\overline{z - 2 - i}| = |z-2-i| = |z - (2+i)|$$

$$|z-(2+i)| \geq |z-(2-i)$$



półpłaszczyzna

b) $\frac{\pi}{2} \leq Arg(2iz) < \frac{3\pi}{2}$

$$\frac{\pi}{2} \leq \underbrace{Arg(2i)}_{\frac{\pi}{2}} + Arg(z) \overset{+2k\pi}{} < \frac{3\pi}{2}$$

$$\frac{\pi}{2} \leq \frac{\pi}{2} + Arg(z) + 2k\pi < \frac{3\pi}{2} \quad \Big| - \frac{\pi}{2} - 2k\pi$$

$$-2k\pi \leq Arg(z) < \pi - 2k\pi \quad \leftarrow k = 0$$

$$0 \leq Arg(z) < \pi$$