Analiza matematyczna

Biovacet 50mg 2x1
rano-wieczór
Acc optima 600mg
rano 30 min
przed śniadaniem

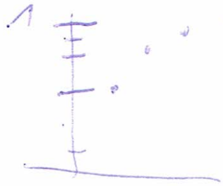
3.1.b) $a_n = (-1)^n n$ nie jest monotoniczna

3.2 a) $a_1 = \frac{1}{2}$

$a_2 = \frac{2}{3}$

$a_3 = \frac{3}{4}$

$a_4 = \frac{4}{5}$



rosnąca

$= a_n = \frac{n}{n+1} = 1 - \frac{1}{n+1}$

rośnie

maleje

rosnąca od 1 wyrazu

3.5) $\lim_{n\to\infty} \frac{a_n}{b_n}$ jest 0

wtedy $\lim_{n\to\infty} a_n = 0$

więc 0

3.6) h) $\lim_{n\to\pm\infty} \frac{2n+(-1)^n}{n} = \lim_{n\to\infty} 2 + (-1)^{n-1} = 1$

j) $\lim_{n\to\frac{2}{3}} \frac{\sqrt{n^2+4}}{3n-2} = \lim_{n\to\frac{2}{3}} \frac{\sqrt{\frac{n^2}{n^2}+\frac{4}{n^2}}}{3-\frac{2}{n}} = \frac{\sqrt{\frac{4}{4}\cdot\frac{9}{4}}}{3-3}$

$3n-2=0$

$3n=2$

$n=\frac{2}{3}$

j) $\lim_{n\to\frac{2}{3}^-} \frac{\sqrt{n^2+4}}{3n-2} = -\infty$

nie ma granicy

$\lim_{n\to\frac{2}{3}^+} \frac{\sqrt{n^2+4}}{3n-2} = \infty$

n) $\cancel{\dfrac{2n+\sqrt{n+n^2}}{3n+\sqrt[3]{n-n^3}}}$

3.6 h) $\cancel{\lim_{n\to\infty} \dfrac{2n+(-1)^n}{n} =}$

3.6 j) $\cancel{\lim_{n\to\infty} \dfrac{\sqrt{n^2+4}}{3n-2} = \infty}$

3.7) c) $\lim_{n\to\infty} \sqrt[n]{\dfrac{2^n+3^n}{3^n+4^n}} = \ldots \dfrac{(2^n+3^n)(3^n-4^n)}{3^{2n}-4^{2n}} = \ldots \cancel{9^n - 8^n}$

$\cancel{\dfrac{\lim_{n\to\infty} \sqrt[n]{2^n+3^n}}{\lim_{n\to\infty} \sqrt[n]{3^n+4^n}}}$

3.8) 6) $\cancel{\lim_{n\to\infty} \left(1-\dfrac{c}{n^2}\right)^n = \lim_{n\to\infty} \sum_{k=0}^{n} 1^{n-k} \left(-\dfrac{1}{n^2}\right)^k \binom{n}{k}}$

~~3.8) 6)~~

3.6 j) $\cancel{\lim_{n\to\infty} \dfrac{\sqrt{n^2+4}}{3n-2} = \dfrac{\sqrt{\lim_{n\to\infty}(n^2+4)}}{\lim_{n\to\infty}(3n-2)} = \dfrac{\infty}{\infty}}$

3.7) c) $\lim_{n\to\infty} \sqrt[n]{\dfrac{2^n+3^n}{3^n+4^n}} = \dfrac{\lim_{n\to\infty} \sqrt[n]{2^n+3^n}}{\lim_{n\to\infty} \sqrt[n]{3^n+4^n}} = \dfrac{a}{b} = \dfrac{3}{4}$

a) $\sqrt[n]{3^n} \le \sqrt[n]{2^n+3^n} \le \sqrt[n]{3^n+3^n}$

$3 \le a \le 3$

b) $\sqrt[n]{4^n} \le b \le \sqrt[n]{4^n+4^n}$

$4 \le b \le 4$

8) c) $\lim_{n\to\infty}\left(\frac{n+5}{n}\right)^n = \lim_{n\to\infty}\left(\frac{n}{n}+\frac{5}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{5}{n}\right)^n = \{(1+0)\}^{\infty} = 1$ ⌊6

b) $\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^n = \{(1-0)^{\infty}\} = 1$

8, 9 będą na kolokwium tego typu

3.9 g) $\left(\frac{2n-1}{n+2}\right)^n = \left(\frac{2-\frac{1}{n}}{1+\frac{2}{n}}\right)^n = \left\{\frac{2-0}{1+0}\right\}^{\infty} = \infty$

3.10 b) $\lim_{n\to\infty}(n^2-2^n) = \lim_{n\to\infty}\left(1-\frac{2^n}{n^2}\right) = \lim_{n\to\infty}\left(1-\frac{\text{natowiche}}{n\cdot n}\right) = -\infty$

rozbieżne

3.11 b) $a_1 = 2$, $a_{n+1} = \frac{a_n}{1+a_n}$

$a_n = \frac{a_{n-1}}{1+a_{n-1}}$

$a_2 = \frac{2}{3}$

$a_3 = \frac{\frac{2}{3}}{\frac{5}{3}} = \frac{2}{5}$ $\qquad a_n = \frac{2}{1+2n}$ $\qquad$ ciąg malejący stale

$a_4 = \frac{\frac{2}{5}}{\frac{7}{5}} = \frac{2}{7}$

$\vdots$

$g = \lim_{n\to\infty}\frac{2}{1+2n} = 0$ $\qquad$ ciąg ograniczony od dołu 0

$g = \frac{g}{1+g}$

$g(1+g) = g$

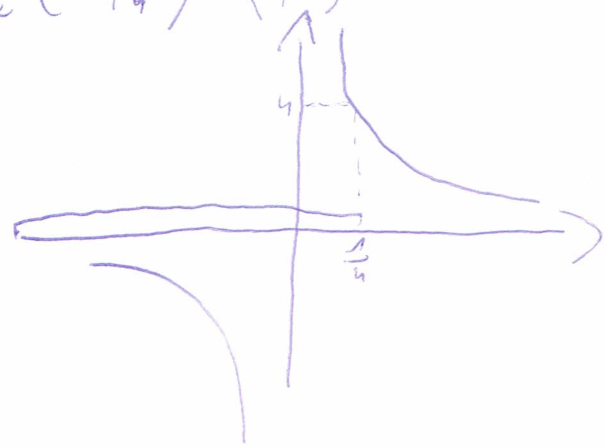
$g + g^2 = g$

$g^2 = 0$

A.12a)

c) $D_f = \mathbb{R}\setminus\{0,1\}$

$x - x^2 \in (-\infty, \frac{1}{4}\rangle \setminus \{0\}$



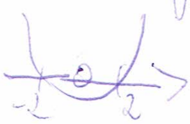
$Zw_f = \cancel{-\infty,}\ \mathbb{R}\setminus\langle 0, 4\rangle$

d) $f(x) = \frac{x^2+1}{x} = y$ Zw. $(-\infty,-2] \cup [2,\infty)$.

$x^2+1 = yx$

$x^2 - yx + 1 = 0$

$\Delta = y^2 - 4 = (y-2)(y+2) \geq 0$



$y \in (-\infty,-2\rangle \cup \langle 2,\infty)$

$$\boxed{TW: ZW_f = D_{f^{-1}}}$$

odwracalna

$$f(x) = \frac{x^2+1}{x} \qquad x \neq 0$$

$$x = \frac{y \pm \sqrt{y^2-4}}{2}$$

$$f^{-1}(x_+) = \frac{x + \sqrt{x^2-4}}{2}$$

$$f^{-1}(x_-) = \frac{x - \sqrt{x^2-4}}{2}$$

$$ZW_f = (-\infty, -2] \cup [2, +\infty)$$

$$|f(x)| \geq 2$$

$$\frac{x^2+1}{|x|} \geq 2$$

$$|x|^2 - 2|x| + 1 \geq 0$$

$$\frac{(|x|-1)^2}{|x|} \geq 0$$

3. b) $g(x) = x^3$

$$g \nearrow \overset{df}{\equiv} \forall_{x,y \in D_g} (x < y \longrightarrow g(x) < g(y))$$

$$D: \forall_{x,y \in R} (x < y \Rightarrow x^3 < y^3)$$

$x^3 - y^3 < 0$

$(x-y)(x^2 + xy + y^2) < 0$

$x - y < 0 \quad \wedge \quad x^2 + xy + y^2 \geq 0$

Prawda

$(x+y)^2 - xy > 0$

$(x+y)^2 > xy$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^2 - 2xy = x^2$

$(x+y)^2 - 2xy = x^2 + y^2$

$x^2 + xy + y^2 \geq 0$

$(x^2 + \frac{1}{2}y)^2 + \frac{3}{4}y^2 \geq 0$

$(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2 \neq 0$ dla $x = 0 = y$

Coś to na wykresie $\Rightarrow$ $f(x) = \sin(2x - \frac{\pi}{3})$

f, g są wz. odw.

$\underset{=}{\text{df}}$

$f \circ g = id_Y \quad , g \circ f = id_X \qquad \forall_{y \in Y} f(g(y)) = y \quad , \forall_{x \in X} g(f(x)) = x$

$id_A(x) = x$ dla $x \in A$

$f: X \to Y$

$f: Y \to X$

$f(x) = -x^2$, $g = \sqrt{-x}$

$f(g(x)) = -(\sqrt{-x})^2 = --x = x$

$g(f(x)) = \sqrt{-(x)^2} = |x|$

$f_1 = f \upharpoonright [0, \infty)$ ↑ drayum

$f_1(x) = f(x) = -x^2$ dla $x \geq 0$