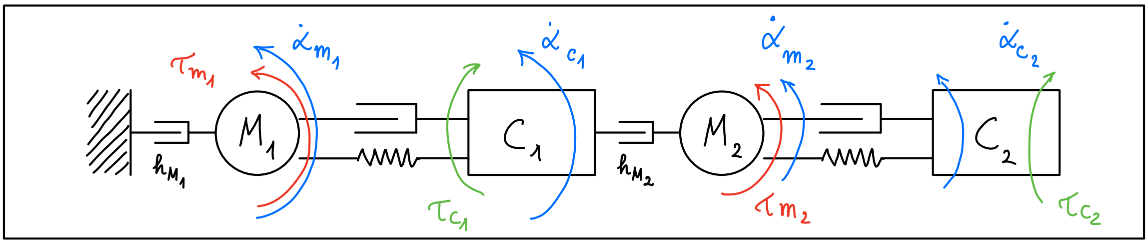
Modello a parametri concentrati: SCARA robot



$$
\begin{cases}
J_{M_1}\ddot{\alpha}_{m_1} = \tau_{m_1} - h_{M_1}\dot{\alpha}_{m_1} + K_1(\alpha_{c_1}-\alpha_{m_1}) + h_1(\dot{\alpha}_{c_1}-\dot{\alpha}_{m_1}) \\
J_{c_1}\ddot{\alpha}_{c_1} = -\tau_{c_1} - K_1(\alpha_{c_1}-\alpha_{m_1}) - h_1(\dot{\alpha}_{c_1}-\dot{\alpha}_{m_1}) + h_{M_2}(\dot{\alpha}_{m_2}-\dot{\alpha}_{c_1}) \\
J_{M_2}\ddot{\alpha}_{m_2} = \tau_{m_2} - h_{M_2}(\dot{\alpha}_{m_2}-\dot{\alpha}_{c_1}) + K_2(\alpha_{c_2}-\alpha_{m_2}) + h_2(\dot{\alpha}_{c_2}-\dot{\alpha}_{m_2}) \\
J_{c_2}\ddot{\alpha}_{c_2} = -\tau_{c_2} - K_2(\alpha_{c_2}-\alpha_{m_2}) - h_2(\dot{\alpha}_{c_2}-\dot{\alpha}_{m_2})
\end{cases}
$$

$$
\underline{x} = \begin{bmatrix} \alpha_{m_1} \\ \alpha_{c_1} \\ \alpha_{m_2} \\ \alpha_{c_2} \\ \dot{\alpha}_{m_1} \\ \dot{\alpha}_{c_1} \\ \dot{\alpha}_{m_2} \\ \dot{\alpha}_{c_2} \end{bmatrix}
\qquad
\dot{\underline{x}} = \begin{bmatrix} \dot{\alpha}_{m_1} \\ \dot{\alpha}_{c_1} \\ \dot{\alpha}_{m_2} \\ \dot{\alpha}_{c_2} \\ \ddot{\alpha}_{m_1} \\ \ddot{\alpha}_{c_1} \\ \ddot{\alpha}_{m_2} \\ \ddot{\alpha}_{c_2} \end{bmatrix}
= \begin{bmatrix} \underline{x}(5) \\ \underline{x}(6) \\ \underline{x}(7) \\ \underline{x}(8) \\ J_{m_1}^{-1}\left(\underline{u}(1) - h_{m_1}\underline{x}(5) + K_1(\underline{x}(2)-\underline{x}(1)) + h_1(\underline{x}(6)-\underline{x}(5))\right) \\ J_{c_1}^{-1}\left(-\underline{u}(2) - K_1(\underline{x}(2)-\underline{x}(1)) - h_1(\underline{x}(6)-\underline{x}(5)) + h_{M_2}(\underline{x}(7)-\underline{x}(6))\right) \\ J_{m_2}^{-1}\left(\underline{u}(3) - h_{m_2}(\underline{x}(7)-\underline{x}(6)) + K_2(\underline{x}(4)-x(3)) + h_2(\underline{x}(8)-x(7))\right) \\ J_{c_2}^{-1}\left(-\underline{u}(4) - K_2(\underline{x}(4)-x(3)) - h_2(\underline{x}(8)-x(7))\right) \end{bmatrix}
$$

$$
\underline{u} = \begin{bmatrix} \tau_{m_1} \\ \tau_{c_1} \\ \tau_{m_2} \\ \tau_{c_2} \end{bmatrix}
\qquad
\underline{y} = \begin{bmatrix} \alpha_{c_1} \\ \alpha_{c_2} \\ \dot{\alpha}_{c_1} \\ \dot{\alpha}_{c_2} \end{bmatrix} = \begin{bmatrix} \underline{x}(2) \\ \underline{x}(4) \\ \underline{x}(6) \\ \underline{x}(8) \end{bmatrix}
$$

$$
\begin{cases}
\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \\
\underline{y} = C\underline{x} + \cancel{D\underline{u}}
\end{cases}
$$

$x: 8\times 1$ $\quad$ $u: 4\times 1$ $\quad$ $y: 4\times 1$

$A: 8\times 8$ $\quad$ $B: 8\times 4$ $\quad$ $C: 4\times 8$ $\quad$ $D: 4\times 4$

$$
A = \begin{array}{r} \\ \\ \\ \\ J_{m_1}^{-1}\cdot \\ J_{c_1}^{-1}\cdot \\ J_{m_2}^{-1}\cdot \\ J_{c_2}^{-1}\cdot \end{array}
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-K_1 & K_1 & 0 & 0 & -(h_{M_1}+h_1) & h_1 & 0 & 0 \\
K_1 & -K_1 & 0 & 0 & h_1 & -(h_1+h_{M_2}) & h_{M_2} & 0 \\
0 & 0 & -K_2 & K_2 & 0 & h_{M_2} & -(h_{M_2}+h_2) & h_2 \\
0 & 0 & K_2 & -K_2 & 0 & 0 & h_2 & -h_2
\end{bmatrix}
$$

$$B = \begin{array}{r} \\ \\ \\ \\ J_{m_1}^{-1}\cdot \\ J_{c_1}^{-1}\cdot \\ J_{m_2}^{-1}\cdot \\ J_{c_2}^{-1}\cdot \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$