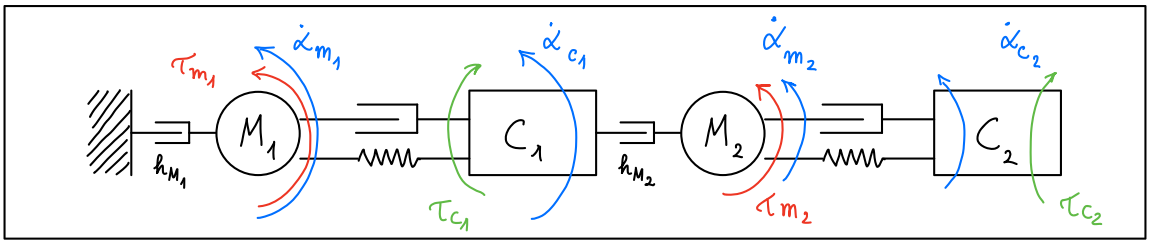
Modello a parametri concentrati SCARA robot



$$
\begin{cases}
J_{M_1}\ddot\alpha_{m_1} = \tau_{m_1} - h_{M_1}\dot\alpha_{m_1} + K_1(\alpha_{c_1} - \alpha_{m_1}) + h_1(\dot\alpha_{c_1} - \dot\alpha_{m_1}) \\
J_{c_1}\ddot\alpha_{c_1} = -\tau_{c_1} - K_1(\alpha_{c_1} - \alpha_{m_1}) - h_1(\dot\alpha_{c_1} - \dot\alpha_{m_1}) + h_{M_2}(\dot\alpha_{m_2} - \dot\alpha_{c_1}) - \tau_{m_2} \\
J_{M_2}\ddot\alpha_{m_2} = \tau_{m_2} - h_{M_2}(\dot\alpha_{m_2} - \dot\alpha_{c_1}) + K_2(\alpha_{c_2} - \alpha_{m_2}) + h_2(\dot\alpha_{c_2} - \dot\alpha_{m_2}) \\
J_{c_2}\ddot\alpha_{c_2} = -\tau_{c_2} - K_2(\alpha_{c_2} - \alpha_{m_2}) - h_2(\dot\alpha_{c_2} - \dot\alpha_{m_2})
\end{cases}
$$

$$
\underline{x} = \begin{bmatrix} \alpha_{m_1} \\ \alpha_{c_1} \\ \alpha_{m_2} \\ \alpha_{c_2} \\ \dot\alpha_{m_1} \\ \dot\alpha_{c_1} \\ \dot\alpha_{m_2} \\ \dot\alpha_{c_2} \end{bmatrix}
\qquad
\dot{\underline{x}} = \begin{bmatrix} \dot\alpha_{m_1} \\ \dot\alpha_{c_1} \\ \dot\alpha_{m_2} \\ \dot\alpha_{c_2} \\ \ddot\alpha_{m_1} \\ \ddot\alpha_{c_1} \\ \ddot\alpha_{m_2} \\ \ddot\alpha_{c_2} \end{bmatrix}
= \begin{bmatrix} \underline{x}(5) \\ \underline{x}(6) \\ \underline{x}(7) \\ \underline{x}(8) \\ J_{m_1}^{-1}\left(\underline{u}(1) - h_{m_1}\underline{x}(5) + K_1(\underline{x}(2) - \underline{x}(1)) + h_1(\underline{x}(6) - \underline{x}(5))\right) \\ J_{c_1}^{-1}\left(-\underline{u}(2) - \underline{u}(3) - K_1(\underline{x}(2) - \underline{x}(1)) - h_1(\underline{x}(6) - \underline{x}(5)) + h_{M_2}(\underline{x}(7) - \underline{x}(6))\right) \\ J_{m_2}^{-1}\left(\underline{u}(3) - h_{m_2}(\underline{x}(7) - \underline{x}(6)) + K_2(\underline{x}(4) - \underline{x}(3)) + h_2(\underline{x}(8) - \underline{x}(7))\right) \\ J_{c_2}^{-1}\left(-\underline{u}(4) - K_2(\underline{x}(4) - \underline{x}(3)) - h_2(\underline{x}(8) - \underline{x}(7))\right) \end{bmatrix}
$$

$$
\underline{u} = \begin{bmatrix} \tau_{m_1} \\ \tau_{c_1} \\ \tau_{m_2} \\ \tau_{c_2} \end{bmatrix}
\qquad
\underline{y} = \begin{bmatrix} \alpha_{m_1} \\ \alpha_{m_2} \\ \dot\alpha_{m_1} \\ \dot\alpha_{m_2} \end{bmatrix}
= \begin{bmatrix} \underline{x}(1) \\ \underline{x}(3) \\ \underline{x}(5) \\ \underline{x}(7) \end{bmatrix}
$$

$$
\begin{cases} \dot{\underline{x}} = A\underline{x} + B\underline{u} \\ \underline{y} = C\underline{x} + \cancel{D\underline{u}} \end{cases}
$$

$x: 8\times 1$ $\quad u: 4\times 1$ $\quad y: 4\times 1$

$A: 8\times 8$ $\quad B: 8\times 4$ $\quad C: 4\times 8$ $\quad D: 4\times 4$

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
J_{m_1}^{-1}\cdot -K_1 & K_1 & 0 & 0 & -(h_{M_1}+h_1) & h_1 & 0 & 0 \\
J_{c_1}^{-1}\cdot K_1 & -K_1 & 0 & 0 & h_1 & -(h_1+h_{M_2}) & h_{M_2} & 0 \\
J_{m_2}^{-1}\cdot 0 & 0 & -K_2 & K_2 & 0 & h_{M_2} & -(h_{M_2}+h_2) & h_2 \\
J_{c_2}^{-1}\cdot 0 & 0 & K_2 & -K_2 & 0 & 0 & h_2 & -h_2
\end{bmatrix}
$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{matrix} \\ \\ \\ \\ J_{m_1}^{-1} \cdot \\ J_{c_1}^{-1} \cdot \\ J_{m_2}^{-1} \cdot \\ J_{c_2}^{-1} \cdot \end{matrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$