$$x \cos q_1 + y \sin q_1 = a_1$$
$$y \sin q_1 + x \cos q_1 = a1$$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

and $t = \tan \frac{x}{2}$

check if $x = \pi + 2k\pi$ is a solution.

q_1

1) $A = \sqrt{x^2+y^2}$

$$\tan \varphi = \frac{x}{y} \longrightarrow$$

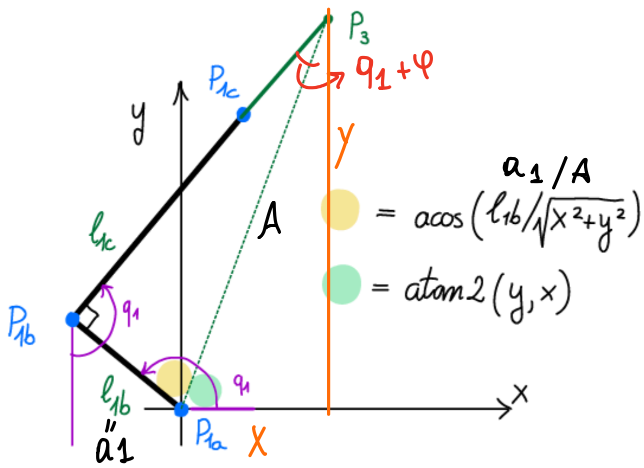
if $y > 0 \Rightarrow \varphi = \operatorname{atan} \frac{x}{y}$

else if $y < 0 \Rightarrow \varphi = \operatorname{atan} \frac{x}{y} + \pi$

$\cos \varphi$

$$A \sin(q_1 + \varphi) = a1$$

$$A \cos\left(\frac{\pi}{2} - (q_1+\varphi)\right) = a_1$$



$$q_{1,1} = \operatorname{asin}\left(\frac{a1}{A}\right) - \varphi$$

$$q_{1,2} = \pi - \operatorname{asin}\left(\frac{a1}{A}\right) - \varphi$$

2) $y \sin q_1 + x \cos q_1 = a1$

$$y \frac{2t}{1+t^2} + \frac{x(1-t^2)}{1+t^2} = a1$$

$$2ty + x(1-t^2) = a1(1+t^2)$$
$$2ty + x - xt^2 - a_1 - a1t^2 = 0$$

$$-(x+a1)t^2 + 2yt + x - a1 = 0$$

$$\Delta = 4y^2 - 4a1(x+a1)$$
$$t_{1,2} = \frac{-2y \pm \sqrt{\Delta}}{-2(x+a1)} \longrightarrow \tan\left(\frac{q_1}{2}\right) = t_{1,2}$$

$q_{1,1} = \operatorname{atan}(t_1) \qquad q_{1,3} = \operatorname{atan}(t_2)$

$q_{1,2} = \operatorname{atan}(t_1) + \pi \qquad q_{1,4} = \operatorname{atan}(t_2) + \pi$

q_3

$$\frac{y - a_1 \sin q_1}{\cos q_1} \sin\alpha + (z - d_1)\cos\alpha = a_3 \sin q_3$$

$$q_{3,1} = \operatorname{asin}\left(\frac{\frac{y - a_1 \sin q_1}{\cos q_1} \sin\alpha + (z - d_1)\cos\alpha}{a_3}\right)$$

$$q_{3,2} = -\pi - \operatorname{asin}\left(\frac{\frac{y - a_1 \sin q_1}{\cos q_1} \sin\alpha + (z - d_1)\cos\alpha}{a_3}\right)$$

q_2

$$q_2 = \frac{z - d_1 - a_3 \sin(\alpha + q_3)}{\sin\alpha} - d_2$$

Caso particolare: $q_1 = \frac{\pi}{2}$

$$\begin{cases} x = +(d_2 + q_2)\cos\alpha + a_3 \cos(\alpha + q_3) & \boxed{\cdot \sin\alpha} \\ y = +a_1 & \ominus \\ z = d_1 + (d_2 + q_2)\sin\alpha + a_3 \sin(\alpha + q_3) & \boxed{\cdot \cos\alpha} \end{cases}$$

$$x \sin\alpha - z\cos\alpha = -d_1 \cos\alpha + a_3\left[\cos(\alpha + q_3)\sin\alpha - \sin(\alpha + q_3)\cos\alpha\right]$$

$$\sin(\cancel{\alpha} - \cancel{\alpha} - q_3) = \frac{x \sin\alpha - z\cos\alpha + d_1\cos\alpha}{a_3}$$

$$\sin q_3 = \boxed{\frac{-x\sin\alpha + (z - d_1)\cos\alpha}{a_3}} \text{ argument}$$

$$q_{3,1} = \operatorname{asin}(\text{argument})$$

$$q_{3,2} = \pi - \operatorname{asin}(\text{argument}) \;\; \text{o} \;\; -\pi - \operatorname{asin}(\text{argument})$$

Caso particolare: $q_2 = \frac{3}{2}\pi$

$$\begin{cases} x = -(d_2 + q_2)\cos\alpha - a_3 \cos(\alpha + q_3) & \boxed{\cdot \sin\alpha} \\ y = -a_1 & + \\ z = d_1 + (d_2 + q_2)\sin\alpha + a_3 \sin(\alpha + q_3) & \boxed{\cdot \cos\alpha} \end{cases}$$

$$x \sin\alpha + z\cos\alpha = +d_1\cos\alpha + a_3\left[-\cos(\alpha + q_3)\sin\alpha + \sin(\alpha + q_3)\cos\alpha\right]$$

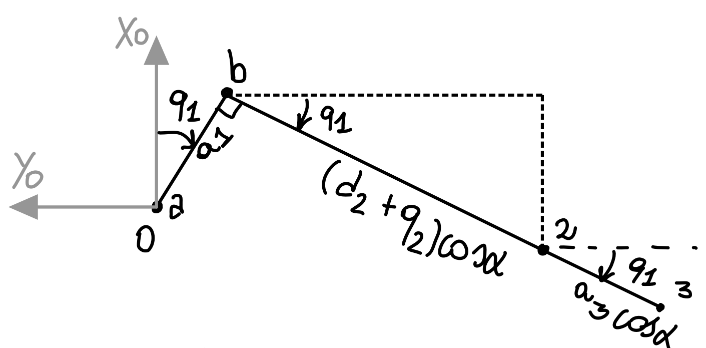
$$\sin(\cancel{\alpha} + q_3 - \cancel{\alpha}) = \frac{x\sin\alpha + z\cos\alpha - d_1\cos\alpha}{a_3} =$$

$$\sin(q_3) = \boxed{\frac{x\sin\alpha + (z - d_1)\cos\alpha}{a_3}} \quad \text{argument}$$

$$q_{3,1} = \text{asin}(\text{argument})$$

$$q_{3,2} = \pi - \text{asin}(\text{argument}) \quad o \quad -\pi - \text{asin}(\text{argument})$$

Plot robot



$$X_{1b} = X_0 + a_1 \cos q_1$$

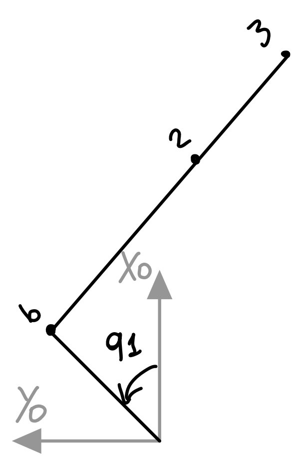
$$y_{1b} = y_0 - a_1 \sin q_1$$

$$z_{1b} = z_{1a}$$

$$x_2 = X_{1b} - (d_2 + q_2)\cos\alpha \sin q_1$$

$$y_2 = y_{1b} - (d_2 + q_2)\cos\alpha \cos q_1$$

$$z_2 = z_{1b} + (d_2 + q_2)\sin\alpha$$



$$x_3 = x_2 - a_3\cos\alpha \sin q_1$$

$$y_3 = y_2 - a_3\cos\alpha\cos q_1$$

$$z_3 = z_2 + a_3 \sin(q_3 + \alpha)$$

$q, \dot{q}, \ddot{q} \longrightarrow M, W, H \longrightarrow x, \dot{x}, \ddot{x}$ Dir. Kin. ✓

$x, \dot{x}, \ddot{x} \longrightarrow q, \dot{q}, \ddot{q}$ Inv. Kin. ?

$$W_{03(0)} = L_{01(0)}\,\dot{q}_1 + L_{12(0)}\,\dot{q}_2 + L_{23(0)}\,\dot{q}_3$$

calcolo $L_{01}(0), L_{12}(0), L_{23}(0) \rightarrow$ funzioni di q_1, q_2, q_3

costruisco $W_{03}(0)$ in funz. di $\underbrace{q_1, q_2, q_3}_{\text{note!}}, \dot{q}_1, \dot{q}_2, \dot{q}_3$

$$\begin{cases} \omega = \dots \\ v_x = \dots \\ v_y = \dots \\ v_z = \dots \end{cases} \Longrightarrow \text{trovo } \dot{q}_1, \dot{q}_2, \dot{q}_3$$

$$L_{01(0)} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad L_{12(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad L_{23(2)} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$L_{12(0)} = M_{01}\, L_{12(1)}\, M_{10} = M_{01}\, L_{12(1)}\, M_{01}^{-1}$$

$$L_{23(0)} = M_{02}\, L_{23(2)}\, M_{02}^{-1}$$

$$W_{03(0)} = L_{01(0)}\,\dot{q}_1 + L_{12(0)}\,\dot{q}_2 + L_{23(0)}\,\dot{q}_3$$

$$W_{O3(0)} = \begin{pmatrix} 0 & -\dot{q}_1 & -\dot{q}_3 \sin q_1 & \frac{\sin q_1}{2}\left(8\dot{q}_3 + \sqrt{3}\,\dot{q}_2 + q_2 \dot{q}_3\right) \\ \dot{q}_1 & 0 & \dot{q}_3 \cos q_1 & -\frac{\cos q_1}{2}\left(8\dot{q}_3 + \sqrt{3}\,\dot{q}_2 + q_2 \dot{q}_3\right) \\ \dot{q}_3 \sin q_1 & -\dot{q}_3 \cos q_1 & 0 & \frac{1}{2}\left(\dot{q}_2 - \sqrt{3}\,\dot{q}_3 (q_2+4)\right) \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} \omega = \dot{q}_1 \\ v_x = \frac{\sin q_1}{2}\left(8\dot{q}_3 + \sqrt{3}\,\dot{q}_2 + q_2 \dot{q}_3\right) \\ v_y = -\frac{\cos q_1}{2}\left(8\dot{q}_3 + \sqrt{3}\,\dot{q}_2 + q_2 \dot{q}_3\right) \\ v_z = \frac{1}{2}\left(\dot{q}_2 - \sqrt{3}\,\dot{q}_3 (q_2+4)\right) \end{cases}$$

$$v_x = \frac{\sin q_1}{2}(8+q_2)\dot{q}_3 + \frac{\sqrt{3}}{2}\sin q_1\, \dot{q}_2$$

$$v_y = -\frac{\cos q_1}{2}(8+q_2)\dot{q}_3 - \frac{\sqrt{3}}{2}\cos q_1\, \dot{q}_2$$

$$v_z = -\frac{\sqrt{3}}{2}(4+q_2)\dot{q}_3 + \frac{\dot{q}_2}{2}$$

$$\dot{q}_2 = 2v_z + \sqrt{3}(4+q_2)\dot{q}_3$$

- $$v_x = \frac{\sin q_1}{2}(8+q_2)\dot{q}_3 + \frac{\sqrt{3}}{\cancel{2}}\sin q_1\, \cancel{2} v_z + \frac{\sqrt{3}}{2}\sin q_1 \cdot \sqrt{3}(4+q_2)\dot{q}_3$$

$$v_x = \sqrt{3}\sin q_1\, v_z + \left[\frac{\sin q_1}{2}(8+q_2) + \frac{3}{2}\sin q_1 (4+q_2)\right]\dot{q}_3$$

$$v_x = \sqrt{3}\sin q_1\, v_z + \sin q_1\left[4 + \frac{q_2}{2} + 6 + \frac{3}{2}q_2\right]\dot{q}_3$$

$$v_x = \sqrt{3}\sin q_1\, v_z + \sin q_1 (10 + 2q_2)\dot{q}_3$$

$$\dot{q}_3 = \frac{v_x - \sqrt{3}\sin q_1\, v_z}{\sin q_1 (10 + 2q_2)}$$

- $$v_y = -\frac{\cos q_1}{2}(8+q_2)\dot{q}_3 - \frac{\sqrt{3}}{2}\cos q_1 \dot{q}_2$$

$$\dot{q}_2 = 2v_z + \sqrt{3}(4+q_2)\dot{q}_3$$

$$v_y = -\frac{\cos q_1}{2}(8+q_2)\dot{q}_3 - \frac{\sqrt{3}}{\cancel{2}}\cos q_1 \cdot \cancel{2}v_z - \frac{\sqrt{3}}{2}\cos q_1 \cdot \sqrt{3}(4+q_2)\dot{q}_3$$

$$v_y = -\sqrt{3}v_z\cos q_1 - \cos q_1\left[4+\frac{q_2}{2}+6+\frac{3}{2}q_2\right]\dot{q}_3$$

$$v_y = -\sqrt{3}v_z\cos q_1 - \cos q_1(10+2q_2)\dot{q}_3$$

$$\dot{q}_3 = \frac{v_y+\sqrt{3}v_z\cos q_1}{-\cos q_1(10+2q_2)}$$

- $$\dot{q}_3 = \frac{v_x - \sqrt{3}\sin q_1 v_z}{\sin q_1(10+2q_2)} \quad \cup \quad \dot{q}_3 = \frac{v_y+\sqrt{3}v_z\cos q_1}{-\cos q_1(10+2q_2)}$$

posso. ⇒ scegliere se imporre v_x o v_y. L'altra risulta di conseguenza.

$$\dot{q}_2 = 2v_z + \sqrt{3}(4+q_2)\dot{q}_3$$

v_z la devo sempre imporre

Inverse kinematics – H

$$W_{01}(0) = L_{01}(0)\,\dot{q}_1 \qquad H_{01}(0) = L_{01}(0)\,\ddot{q}_1 + L_{01}^2(0)\,\dot{q}_1$$

$$W_{12}(0) = L_{12}(0)\,\dot{q}_2 \qquad H_{12}(0) = L_{12}(0)\,\ddot{q}_2$$

$$W_{23}(0) = L_{23}(0)\,\dot{q}_3 \qquad H_{23}(0) = L_{23}\,\ddot{q}_3 + L_{23}^2\,\dot{q}_3$$

$$H_{02}(0) = H_{01}(0) + 2W_{01}(0)\,W_{12}(0) + H_{12}(0)$$

$$H_{03}(0) = H_{02}(0) + 2W_{02}(0)\,W_{23}(0) + H_{23}(0)$$

$$\sigma_1 = \frac{\dot{q}_3 \sin(2q_1)}{2}$$

$$H_{03}(0) = \begin{pmatrix} -\dot{q}_3\sin^2 q_1 - \omega & \sigma_1 - \ddot{q}_1 & -\ddot{q}_3 \sin q_1 - 2\dot{q}_3\omega\cos q_1 & \frac{\sqrt{3}\,\ddot{q}_2 \sin q_1}{2} + \frac{\ddot{q}_3 \sin q_1 (q_2+8)}{2} + \sqrt{3}\,\dot{q}_2\,\omega\cos q_1 + \frac{\sqrt{3}\,\dot{q}_3 \sin q_1 (q_2+4)}{2} + \dot{q}_3\,\omega\cos q_1 (q_2+8) \\ \ddot{q}_1 + \sigma_1 & -\dot{q}_3\cos^2 q_1 - \omega & \ddot{q}_3 \cos q_1 - 2\dot{q}_3\omega\sin q_1 & \sqrt{3}\,\omega\,\dot{q}_2 \sin q_1 - \frac{\ddot{q}_3 \cos q_1 (q_2+8)}{2} - \frac{\sqrt{3}\,\ddot{q}_2\,\omega\cos q_1}{2} - \frac{\sqrt{3}\,\dot{q}_3 \cos q_1 (q_2+4)}{2} + \dot{q}_3\,\omega \sin q_1 (q_2+8) \\ \ddot{q}_3 \sin q_1 & -\ddot{q}_3 \cos q_1 & -\dot{q}_3 & 4\dot{q}_3 + \frac{\ddot{q}_2}{2} - 2\sqrt{3}\,\ddot{q}_3 + \frac{q_2\dot{q}_3}{2} - \frac{\sqrt{3}\,q_2\ddot{q}_3}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Method 2 $L12(0) = M01\, L12(1)\, M10 = M01\, L12(1)\, M01^{-1}$

$$\left.\begin{array}{l} L01\_a \\ L12\_a \\ L23\_a \end{array}\right\} \text{cambiare sdr} \Rightarrow W03(3)$$

$L01\_a = inv(M0a) \cdot L01\_0 \cdot M0a$ — $M0aux$ ✓

$L12\_a = inv(M1a) \cdot L12\_1 \cdot M1a$ — $M10 \cdot M0aux = M1aux$ ✓

$L23\_a = inv(M2a) \cdot L23\_2 \cdot M2a$ — $M20 \cdot M0aux = M2aux$ ✓

$$T = \begin{bmatrix} L01\_a(1:3;4) & L12\_a(1:3,4) & L23\_a(1:3,4) \\ L01\_a(3,2) & \dots & \dots \\ L01\_a(1,3) & \dots & \dots \\ L01\_a(2,1) & \dots & \dots \end{bmatrix}$$

$T\dot{Q} = V$

$\dot{Q} = [q_{1p};\ q_{2p};\ q_{3p}]$

$V = [x_p;\ y_p;\ z_p;\ w_x;\ w_y;\ w_z]$

$$\begin{cases} \dot{x} = (\dot{q}_1 + \dot{q}_2 + \dot{q}_3)\, \sigma_1 \\ \dot{y} = (\dot{q}_1 + \dot{q}_2 + \dot{q}_3)\, \sigma_2 \\ \dot{z} = 0 \\ \omega_x = -\dot{q}_3 \cos q_1 \\ \omega_y = -\dot{q}_3 \sin q_1 \\ \omega_z = \dot{q}_1 \end{cases}$$

$\omega_x \sim \omega_y$

ω_z