$x\cos q_1 + y\sin q_1 = a_1$

$y\sin q_1 + x\cos q_1 = a1$

$\sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$

and $t = \tan\frac{x}{2}$

$\Downarrow$

check if $x = \pi + 2k\pi$ is a solution.

q_1

1) $A = \sqrt{x^2+y^2}$

$\tan\varphi = \frac{x}{y} \longrightarrow$ if $x>0 \Rightarrow \varphi = \operatorname{atan}\frac{x}{y}$

elseif $x<0 \Rightarrow \varphi = \operatorname{atan}\frac{x}{y} + \pi$

$A\sin(q_1+\varphi) = a1$

$q_{1,1} = \operatorname{asin}\left(\frac{a1}{A}\right) - \varphi$

$q_{1,2} = \pi - \operatorname{asin}\left(\frac{a1}{A}\right) - \varphi$

2) $y\sin q_1 + x\cos q_1 = a1$

$y\frac{2t}{1+t^2} + \frac{x(1-t^2)}{1+t^2} = a1$

$2ty + x(1-t^2) = a1(1+t^2)$

$2ty + x - xt^2 - a1 - a1t^2 = 0$

$-(x+a1)t^2 + 2yt + x - a1 = 0$

$\Delta = 4y^2 - 4a1(x+a1)$

$t_{1,2} = \frac{-2y \pm \sqrt{\Delta}}{-2(x+a1)} \longrightarrow \tan\left(\frac{q_1}{2}\right) = t_{1,2}$

$q_{1,1} = \operatorname{atan}(t_1)$ $q_{1,3} = \operatorname{atan}(t_2)$

$q_{1,2} = \operatorname{atan}(t_1) + \pi$ $q_{1,4} = \operatorname{atan}(t_2) + \pi$

q_3

$$\frac{y - a_1 \sin q_1}{\cos q_1} \sin\alpha + (z - d_1)\cos\alpha = a_3 \sin q_3$$

$$q_{3,1} = \operatorname{asin}\left(\frac{\frac{y - a_1 \sin q_1}{\cos q_1} \sin\alpha + (z - d_1)\cos\alpha}{a_3} \right)$$

$$q_{3,2} = -\pi - \operatorname{asin}\left(\frac{\frac{y - a_1 \sin q_1}{\cos q_1} \sin\alpha + (z - d_1)\cos\alpha}{a_3} \right)$$

q_2

$$q_2 = \frac{z - d_1 - a_3 \sin(\alpha + q_3)}{\sin\alpha} - d_2$$