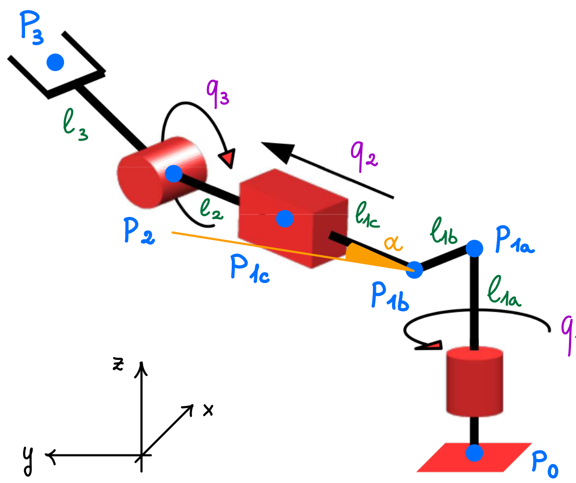
Direct Kinematics



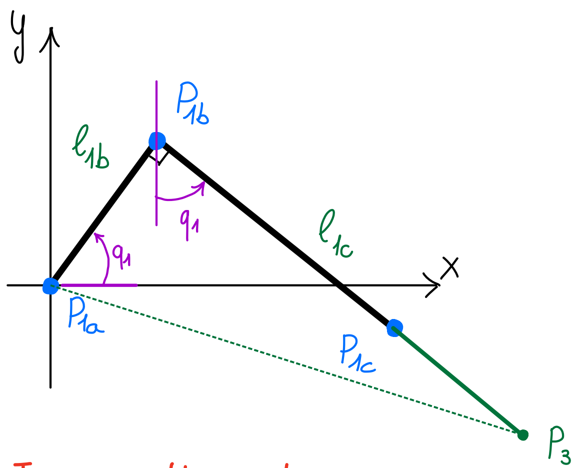
$$P_0 : \begin{cases} x_0 = 0 \\ y_0 = 0 \\ z_0 = 0 \end{cases} \qquad P_{1a} : \begin{cases} x_{1a} = x_0 + 0 \\ y_{1a} = y_0 + 0 \\ z_{1a} = z_0 + \ell_{1a} \end{cases}$$

$$P_{1b} : \begin{cases} x_{1b} = x_{1a} + \ell_{1b} \cdot \cos q_1 \\ y_{1b} = y_{1a} + \ell_{1b} \cdot \sin q_1 \\ z_{1b} = z_{1a} \end{cases}$$

$$P_{1c} : \begin{cases} x_{1c} = x_{1b} + \ell_{1c} \cos(\alpha) \sin q_1 \\ y_{1c} = y_{1b} - \ell_{1c} \cos(\alpha) \cos q_1 \\ z_{1c} = z_{1b} + \ell_{1c} \cdot \sin(\alpha) \end{cases}$$

$$P_2 : \begin{cases} x_2 = x_{1c} + (\ell_2 + q_2) \cos(\alpha) \sin q_1 \\ y_2 = y_{1c} - (\ell_2 + q_2) \cos(\alpha) \cos q_1 \\ z_2 = z_{1c} + (\ell_2 + q_2) \sin(\alpha) \end{cases}$$

$$P_3 : \begin{cases} x_3 = x_2 + \ell_3 \cos(\alpha + q_3) \sin q_1 \\ y_3 = y_2 - \ell_3 \cos(\alpha + q_3) \cos q_1 \\ z_3 = z_2 + \ell_3 \sin(\alpha + q_3) \end{cases}$$



Limits : $q_1 \in [-\pi ; \pi]$
$q_2 \in [0 ; max(q_2)]$
$q_3 \in [-\frac{\pi}{2} ; \frac{\pi}{2}]$

Inverse Kinematics

$$S = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$\begin{cases} x = \ell_{1b} \cos q_1 + \ell_{1c} \cos(\alpha) \sin q_1 + (\ell_2 + q_2) \cos(\alpha) \sin q_1 + \ell_3 \cos(\alpha + q_3) \sin q_1 \\ y = \ell_{1b} \sin q_1 - \ell_{1c} \cos(\alpha) \cos q_1 - (\ell_2 + q_2) \cos(\alpha) \cos q_1 - \ell_3 \cos(\alpha + q_3) \cos q_1 \\ z = \ell_{1a} + \ell_{1c} \sin(\alpha) + (\ell_2 + q_2) \sin(\alpha) + \ell_3 \sin(\alpha + q_3) \end{cases}$$

$$\begin{cases} x = \ell_{1b} \cos q_1 + \left[(\ell_{1c} + \ell_2 + q_2) \cos(\alpha) + \ell_3 \cos(\alpha + q_3) \right] \sin q_1 \\ y = \ell_{1b} \sin q_1 - \left[(\ell_{1c} + \ell_2 + q_2) \cos(\alpha) + \ell_3 \cos(\alpha + q_3) \right] \cos q_1 \\ z = \ell_{1a} + (\ell_{1c} + \ell_2 + q_2) \sin(\alpha) + \ell_3 \sin(\alpha + q_3) \end{cases}$$

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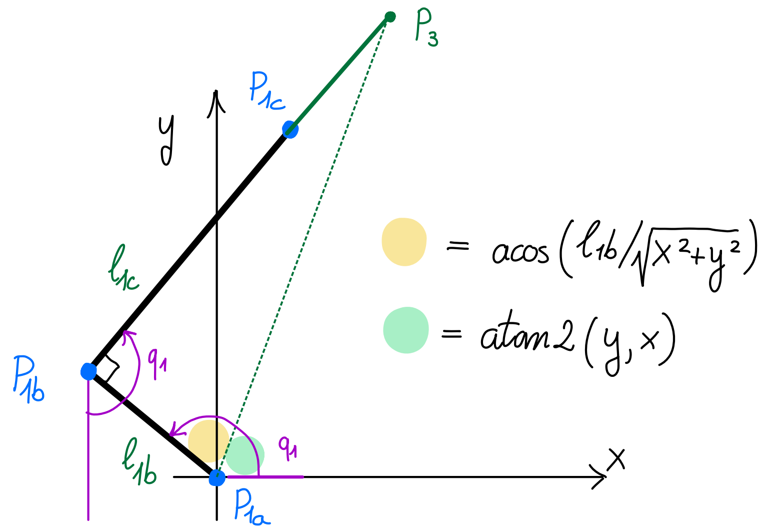
$$\bullet\quad\begin{cases} x\cos q_1 = \ell_{1b}\cos^2 q_1 + \left[(\ell_{1c}+\ell_2+q_2)\cos(\alpha) + \ell_3\cos(\alpha+q_3)\right]\sin q_1 \cos q_1 & \boxed{\cdot \cos(q_1)} \\ y\sin q_1 = \ell_{1b}\sin^2 q_1 - \left[(\ell_{1c}+\ell_2+q_2)\cos(\alpha) + \ell_3\cos(\alpha+q_3)\right]\cos q_1 \sin q_1 & \boxed{\cdot \sin(q_1)} \end{cases}$$

$$\begin{cases} x\cos q_1 + y\sin q_1 = \ell_{1b} & \boxed{1+2} \end{cases} \quad ⊛$$

$$\bullet\quad\begin{cases} x\sin q_1 = \ell_{1b}\cos q_1 \sin q_1 + \left[(\ell_{1c}+\ell_2+q_2)\cos(\alpha) + \ell_3\cos(\alpha+q_3)\right]\sin^2 q_1 & \boxed{\cdot \sin(q_1)} \\ y\cos q_1 = \ell_{1b}\sin q_1 \cos q_1 - \left[(\ell_{1c}+\ell_2+q_2)\cos(\alpha) + \ell_3\cos(\alpha+q_3)\right]\cos^2 q_1 & \boxed{\cdot \cos(q_1)} \end{cases}$$

$$\begin{cases} x\sin q_1 - y\cos q_1 = (\ell_{1c}+\ell_2+q_2)\cos(\alpha) + \ell_3\cos(\alpha+q_3) & \boxed{1-2} \end{cases}$$



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$$x\cos q_1 + y\sin q_1 = \ell_{1b}$$

$$A = \sqrt{x^2+y^2}$$

$$\tan(\varphi) = -\frac{y}{x}$$

$$\longrightarrow A\cos(q_1+\varphi) = \ell_{1b}$$

$$q_1 = acos\left(\frac{\ell_{1b}}{A}\right) - \varphi \longrightarrow q_1 = acos\left(\frac{\ell_{1b}}{A}\right) - atan2\left(\frac{-y}{x}\right)$$

$$\longrightarrow q_1 = acos\left(\frac{\ell_{1b}}{A}\right) + atan2\left(\frac{y}{x}\right)$$

△

$$\begin{cases} x = \ell_{1b}\cos q_1 + \left[(\ell_{1c}+\ell_2+q_2)\cos(\alpha) + \ell_3\cos(\alpha+q_3)\right]\sin q_1 \\ y = \ell_{1b}\sin q_1 - \left[(\ell_{1c}+\ell_2+q_2)\cos(\alpha) + \ell_3\cos(\alpha+q_3)\right]\cos q_1 \\ z = \ell_{1a} + (\ell_{1c}+\ell_2+q_2)\sin(\alpha) + \ell_3\sin(\alpha+q_3) \end{cases}$$

$$\begin{cases} \dfrac{y - l_{1b}\sin q_1}{\cos q_1} = -\left[(l_{1c}+l_2+q_2)\cos(\alpha) + l_3\cos(\alpha+q_3)\right] \\ z - l_{1a} = (l_{1c}+l_2+q_2)\sin(\alpha) + l_3\sin(\alpha+q_3) \end{cases}$$

q_1 is known from above !!

(C) $$\begin{cases} \dfrac{y - l_{1b}\sin q_1}{\cos q_1} = -\left[(l_{1c}+l_2+q_2)\cos(\alpha) + l_3\cos(\alpha+q_3)\right] & \boxed{\cdot \sin(\alpha)} \\ z - l_{1a} = (l_{1c}+l_2+q_2)\sin(\alpha) + l_3\sin(\alpha+q_3) & \boxed{\cdot \cos(\alpha)} \end{cases} \quad \oplus$$

$$\frac{y - l_{1b}\sin q_1}{\cos q_1}\sin\alpha + (z - l_{1a})\cos\alpha = -l_3\sin(\alpha)\cos(\alpha+q_3) + l_3\sin(\alpha+q_3)\cos(\alpha)$$

$$\frac{y - l_{1b}\sin q_1}{\cos q_1}\sin\alpha + (z - l_{1a})\cos\alpha = l_3\sin(\cancel{\alpha}+q_3-\cancel{\alpha})$$

$$q_{3,1} = \arcsin\frac{\frac{y - l_{1b}\sin q_1}{\cos q_1}\sin\alpha + (z - l_{1a})\cos\alpha}{l_3}$$

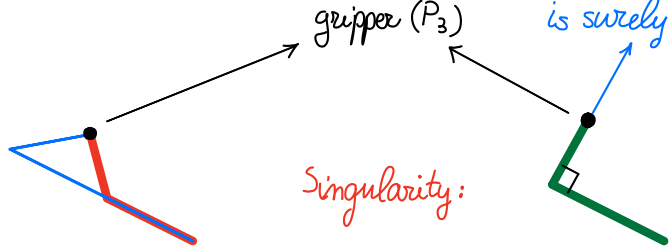
(see function "inv_kin" for correct implementation)

$$q_{3,2} = -\pi - \arcsin\frac{\frac{y - l_{1b}\sin q_1}{\cos q_1}\sin\alpha + (z - l_{1a})\cos\alpha}{l_3}$$

"-", since we want to define q_3 half in the first turn anticlockwise and half in the first turn clockwise

(C) $$q_2 = \frac{z - l_{1a} - l_3\sin(\alpha+q_3)}{\sin\alpha} - l_{1c} - l_2$$

radial velocity of link 3 is surely 0!



There are 2 solutions : $\begin{cases} q_{3,1} \longrightarrow q_{2,1} \\ q_{3,2} \longrightarrow q_{2,2} \end{cases}$

Singularity:

SIDE VIEW

$$x \sin q_1 - y \cos q_1 = (\ell_{1c} + \ell_2 + q_2)\cos(\alpha) + \ell_3 \cos(\alpha + q_3)$$

$$\longrightarrow (\ell_{1c} + \ell_2 + q_2) = \frac{x \sin q_1 - y \cos q_1 - \ell_3 \cos(\alpha + q_3)}{\cos\alpha}$$

$$z = \ell_{1a} + (\ell_{1c} + \ell_2 + q_2)\sin(\alpha) + \ell_3 \sin(\alpha + q_3)$$

$$\longrightarrow z - \ell_{1a} = \frac{x \sin q_1 - y \cos q_1 - \ell_3 \cos(\alpha + q_3)}{\cos(\alpha)} \sin(\alpha) + \ell_3 \sin(\alpha + q_3)$$

$$\underbrace{\frac{z - \ell_{1a} - \tan(\alpha)(x \sin q_1 - y \cos q_1)}{\ell_3}}_{A} = \sin(\alpha + q_3) - \tan(\alpha)\cos(\alpha + q_3)$$

$$B = \sqrt{1 + \tan^2\alpha}$$

$$\tan(\varphi) = \frac{1}{\tan(\alpha)}$$

$$B \cos(\alpha + q_3 + \varphi) = A$$

$$q_{3,1} = \operatorname{acos}\left(\frac{A}{B}\right) - \operatorname{atan2} \frac{1}{\tan(\alpha)} - \alpha$$

$$q_{3,2} = -\operatorname{acos}\left(\frac{A}{B}\right) + \operatorname{atan2} \frac{1}{\tan(\alpha)} + \alpha$$