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American Monte Carlo

Comparison: Tsitsiklis-Van Roy vs. Tilley

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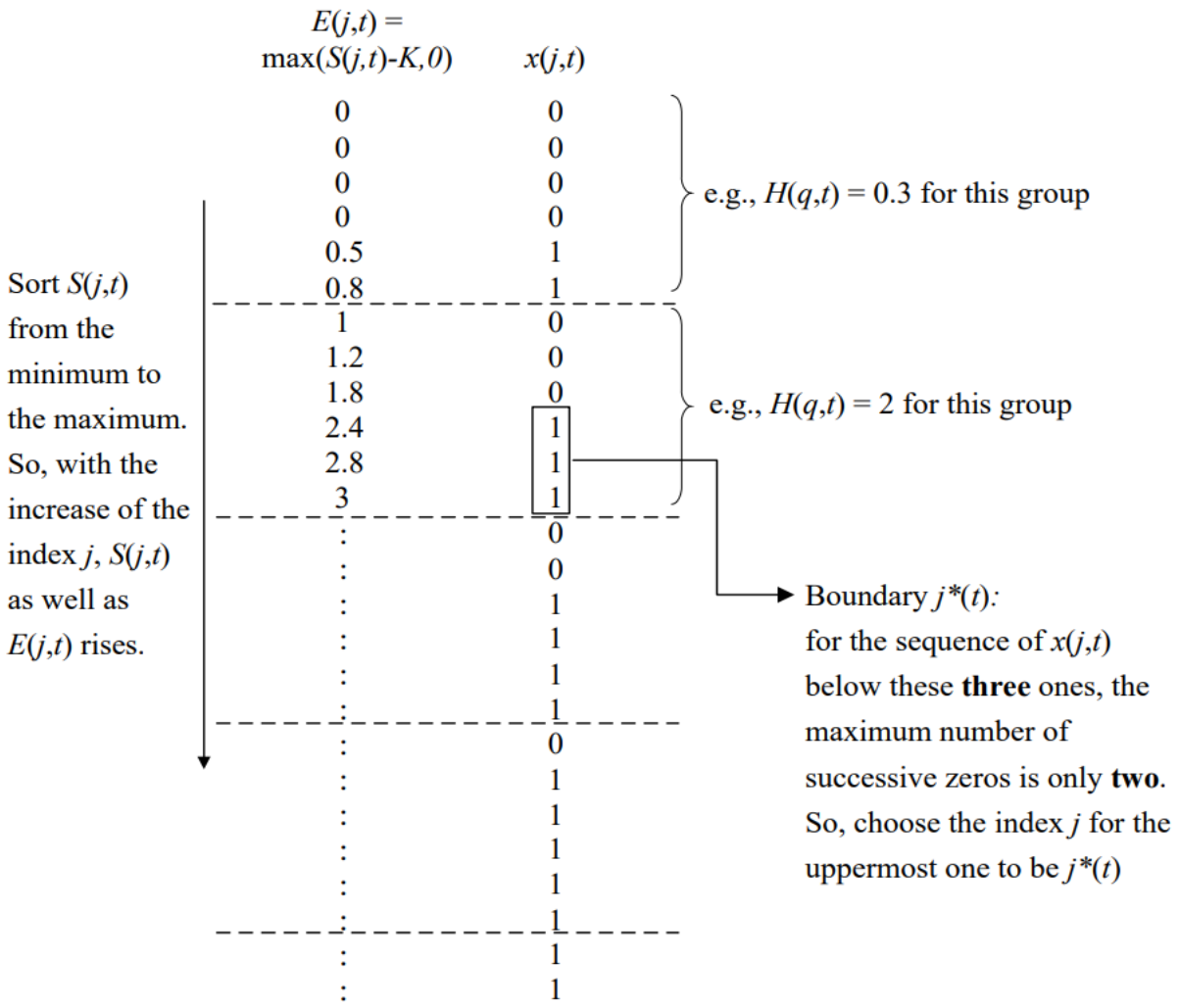
Tilley
Algorithm

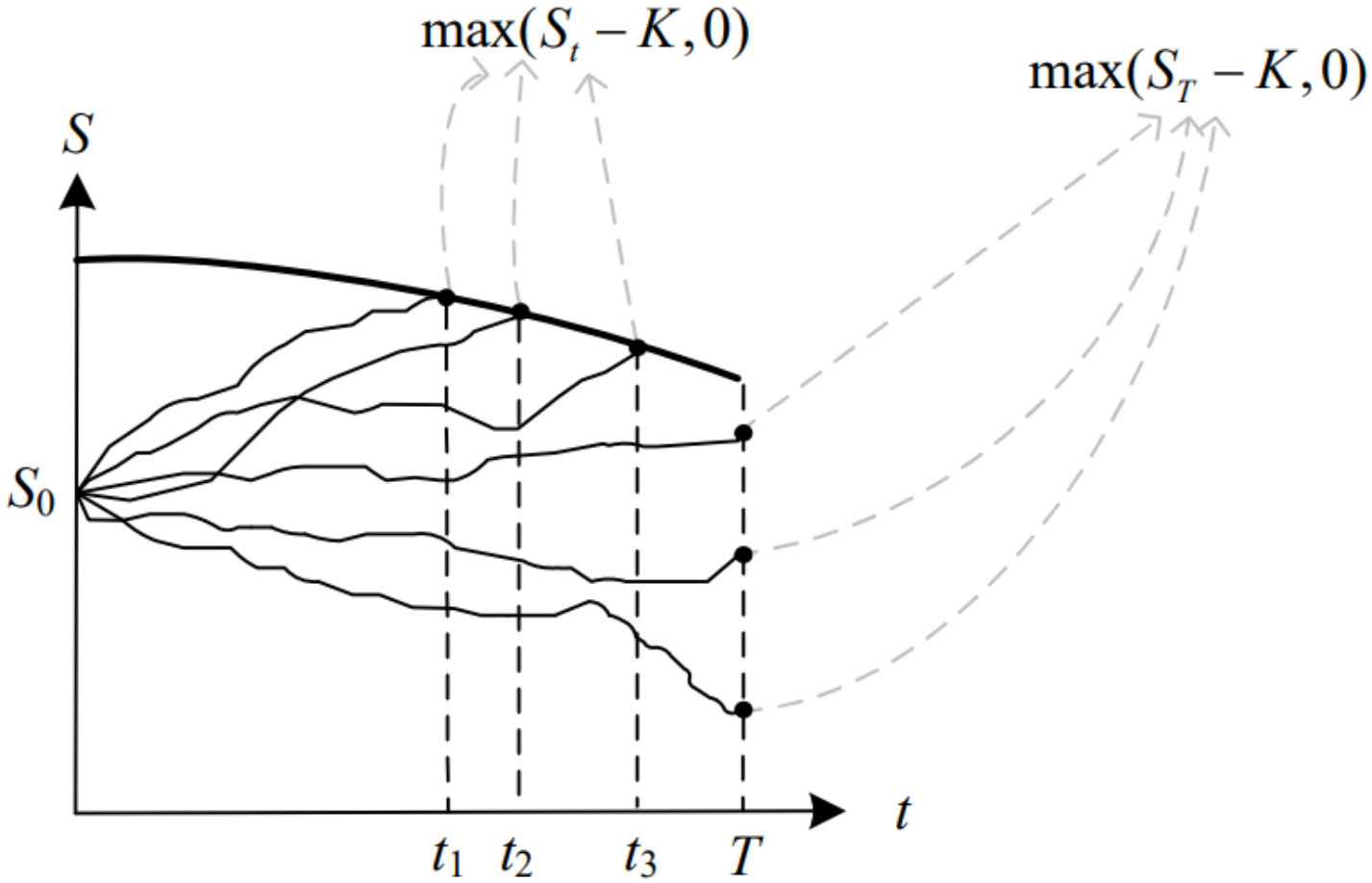
- The main idea is to devise a method based on the Monte Carlo simulation to decide the early exercise boundary.

- Once the early exercise boundary is determined, an American option can be viewed as a knocked-and-exercised option.

Tilley
Algorithm

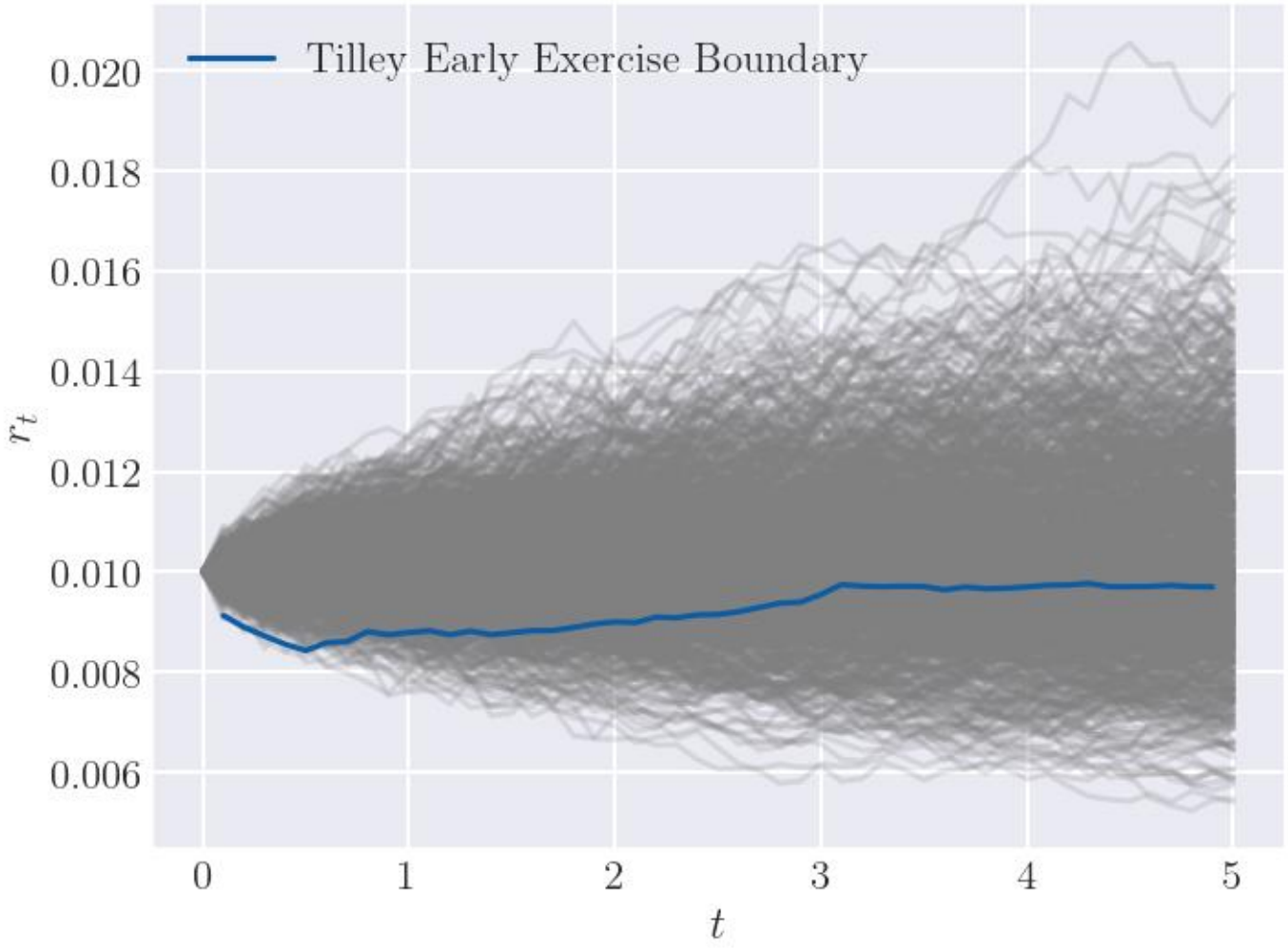


Tilley
Algorithm



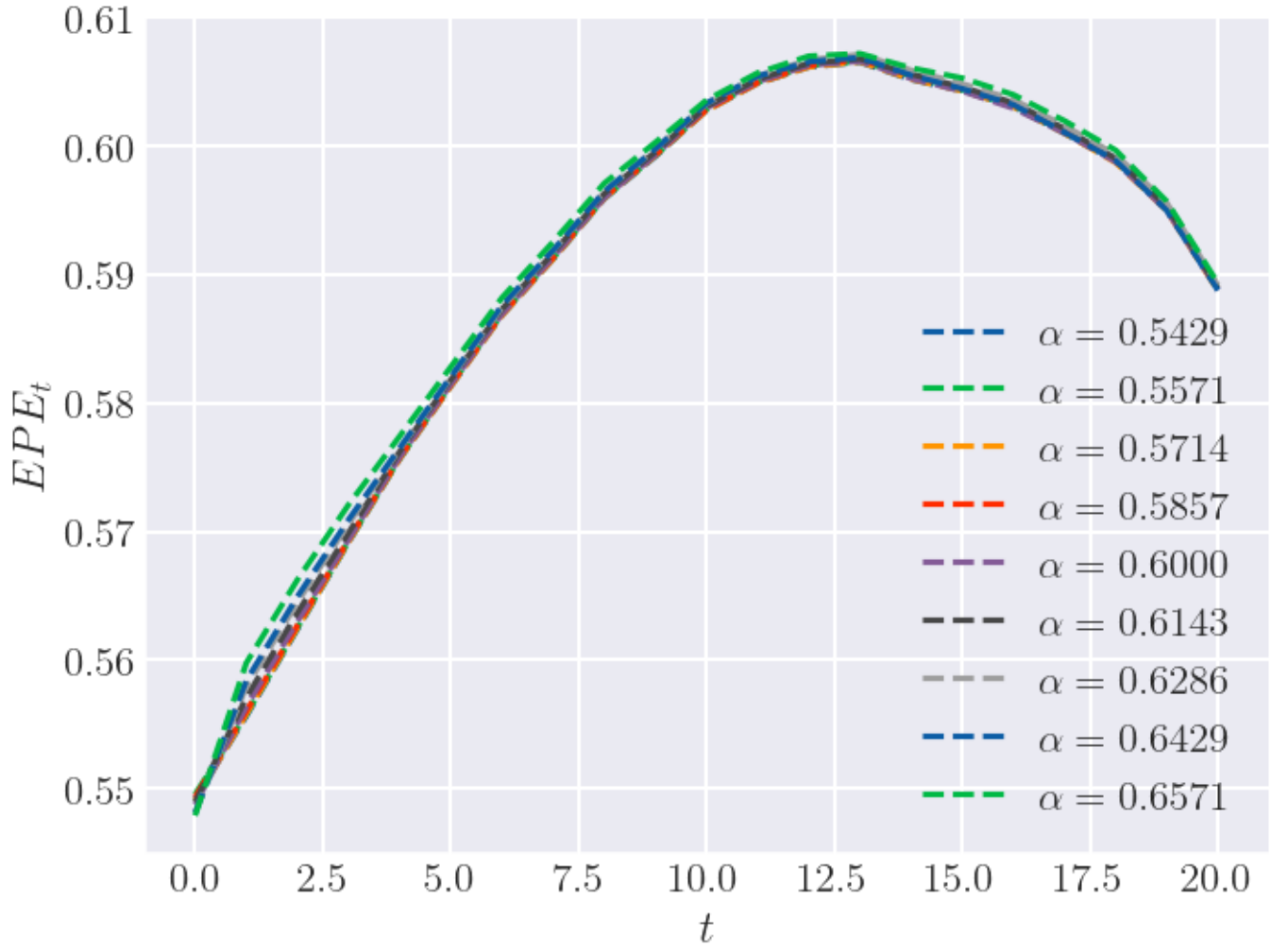
Tilley

Early exercise boundary – American Put



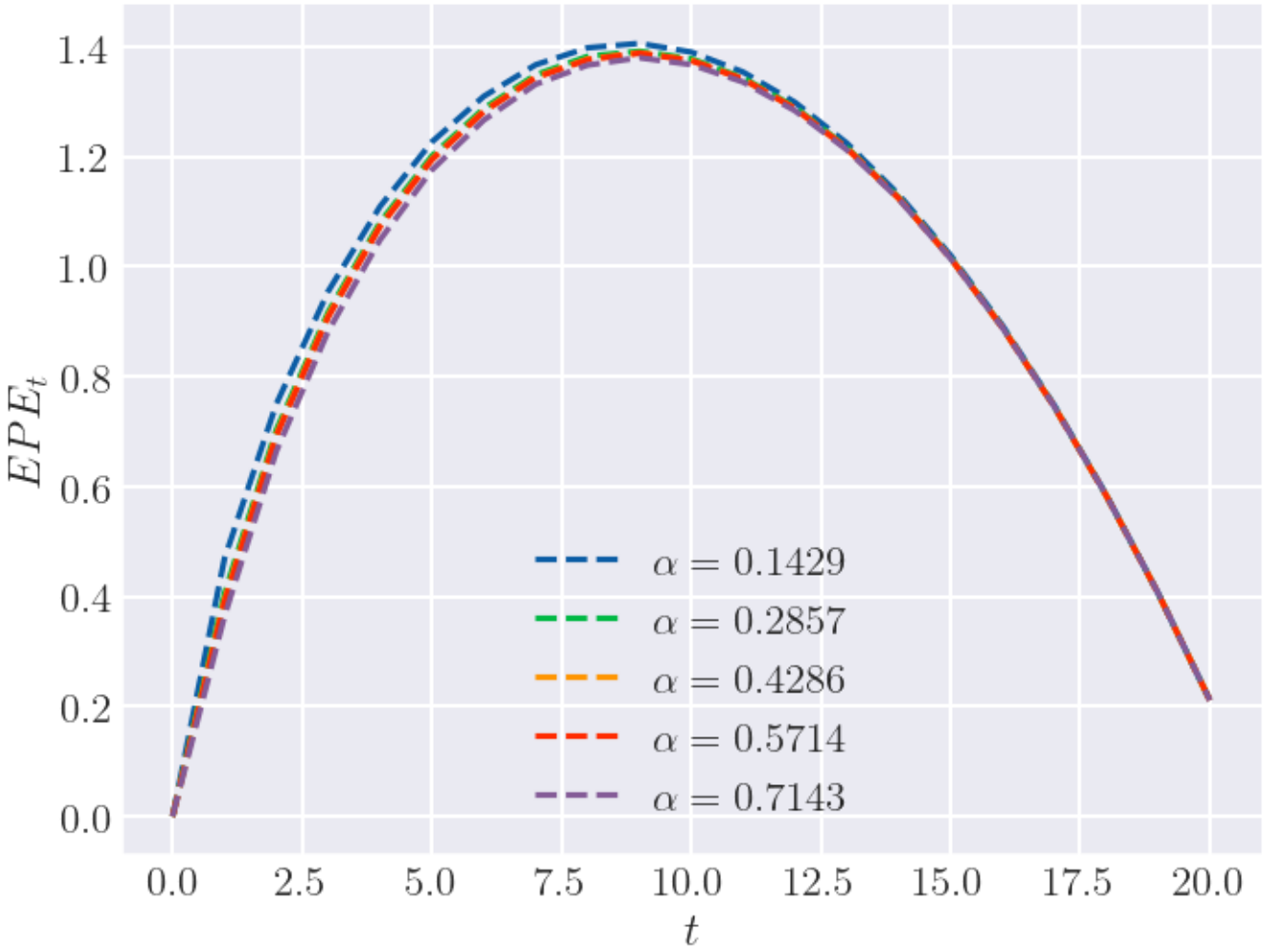
Tilley

Bundling parameter α



Tilley

Bundling parameter α



Tsitsiklis-Van Roy
Algorithm

- We use the regression technique to approximate the conditional expectation function at each exercising date:

$$C_i(x) = \mathbb{E}[V_{i+1}(X_{i+1})|X_i = x]$$

- We are calculating value of an asset by using:

$$\hat{V}_{i,j} = \max\{h_i(X_{i,j}), \widehat{C}_i(X_{i,j})\}$$

- Difference between Longstaff - Schwartz and Tsitsiklis -Van Roy algorithms:

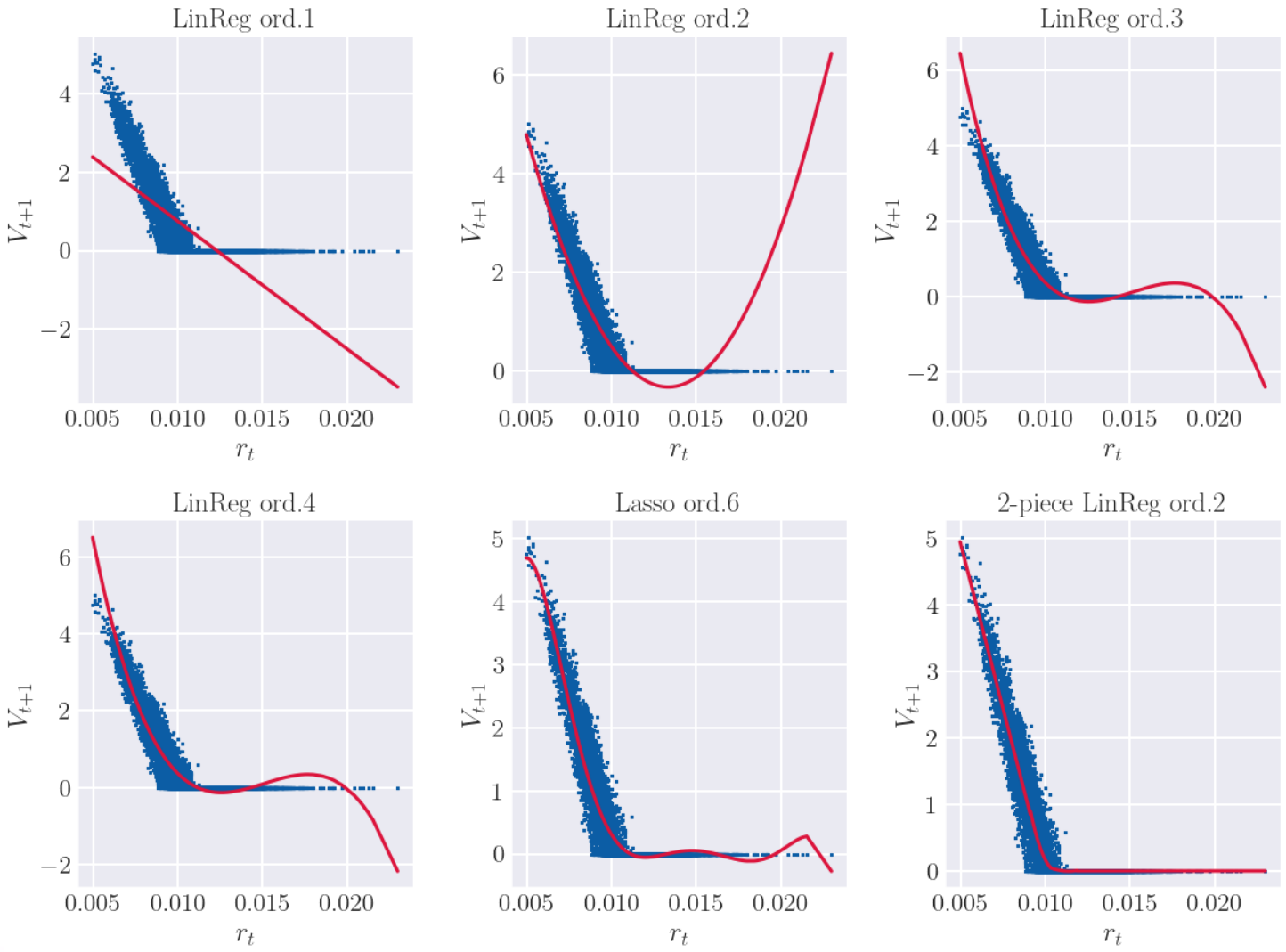
- Tsitsiklis - Van Roy:

$$\hat{V}_{i,j} = \max\{h_i(X_{i,j}), \widehat{C}_i(X_{i,j})\}$$

- Longstaff –Schwartz:

$$\widehat{V}_{ij} = \begin{cases} h_i(X_{i,j}) & h_i(X_{i,j}) \geq \widehat{C}_i(X_{i,j}) \\ \widehat{V}_{i+1,j} & h_i(X_{i,j}) < \widehat{C}_i(X_{i,j}) \end{cases}$$

Tsitsiklis-Van Roy
Choice of regression



Simulation and derivatives setup

Interest rate (GBM):

- Number of paths – 1000
- Number of time steps – 21
- $T = 5$ years
- $\mu = 5\%$
- $\sigma = 20\%$

Receiver Swap:

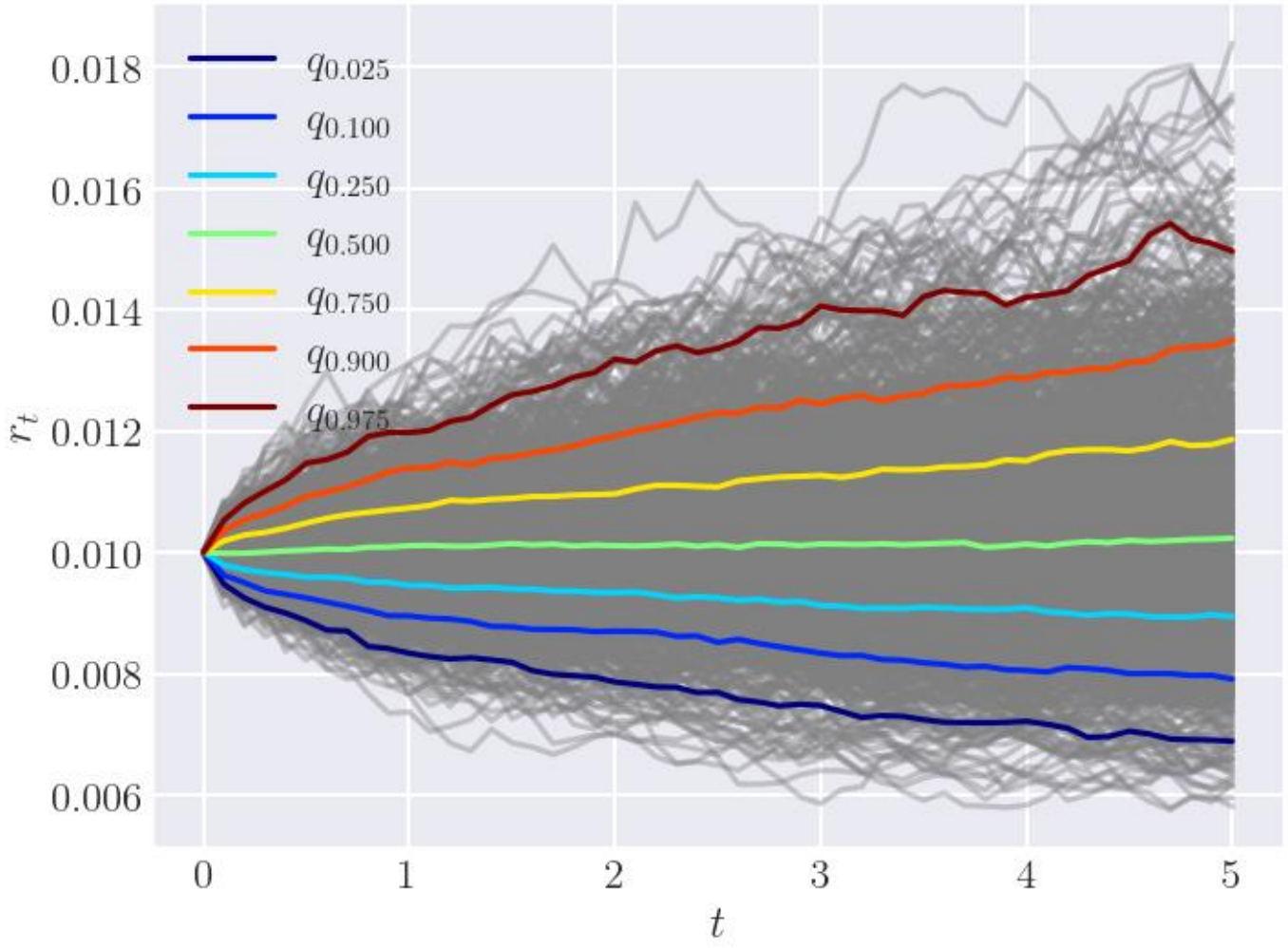
- Swap rate – $1{,}05\%$
- Maturity – 5 years
- Payment frequency – 3 months
- Notional – $\$1000$

American Option:

- Notional – $\$1000$
- Strike - 1%
- Expiry – 5 years
- Possible exercise dates – each interest rate step
- option_type – „put”

Interest rate

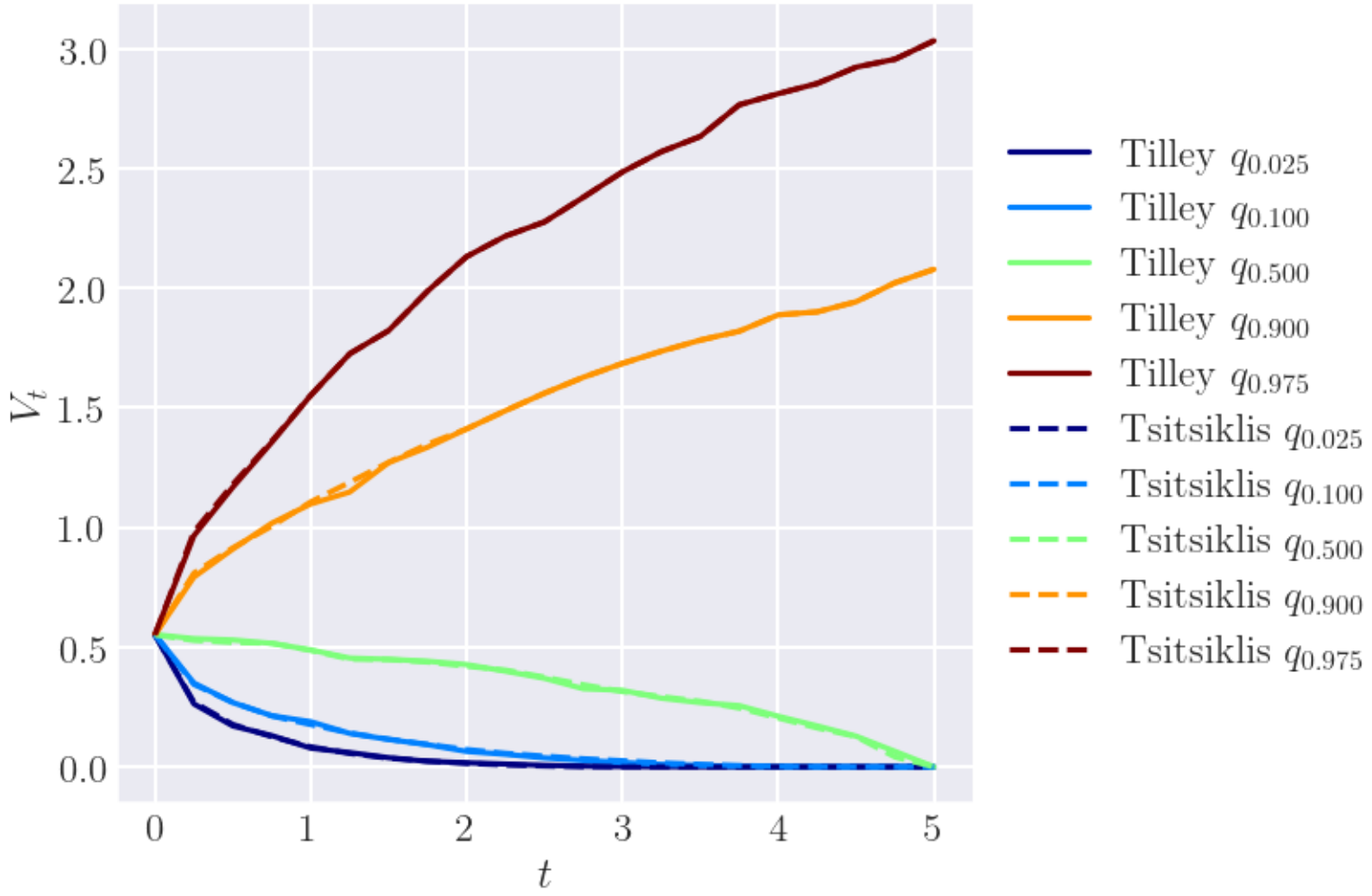
Distribution quantiles



American Option

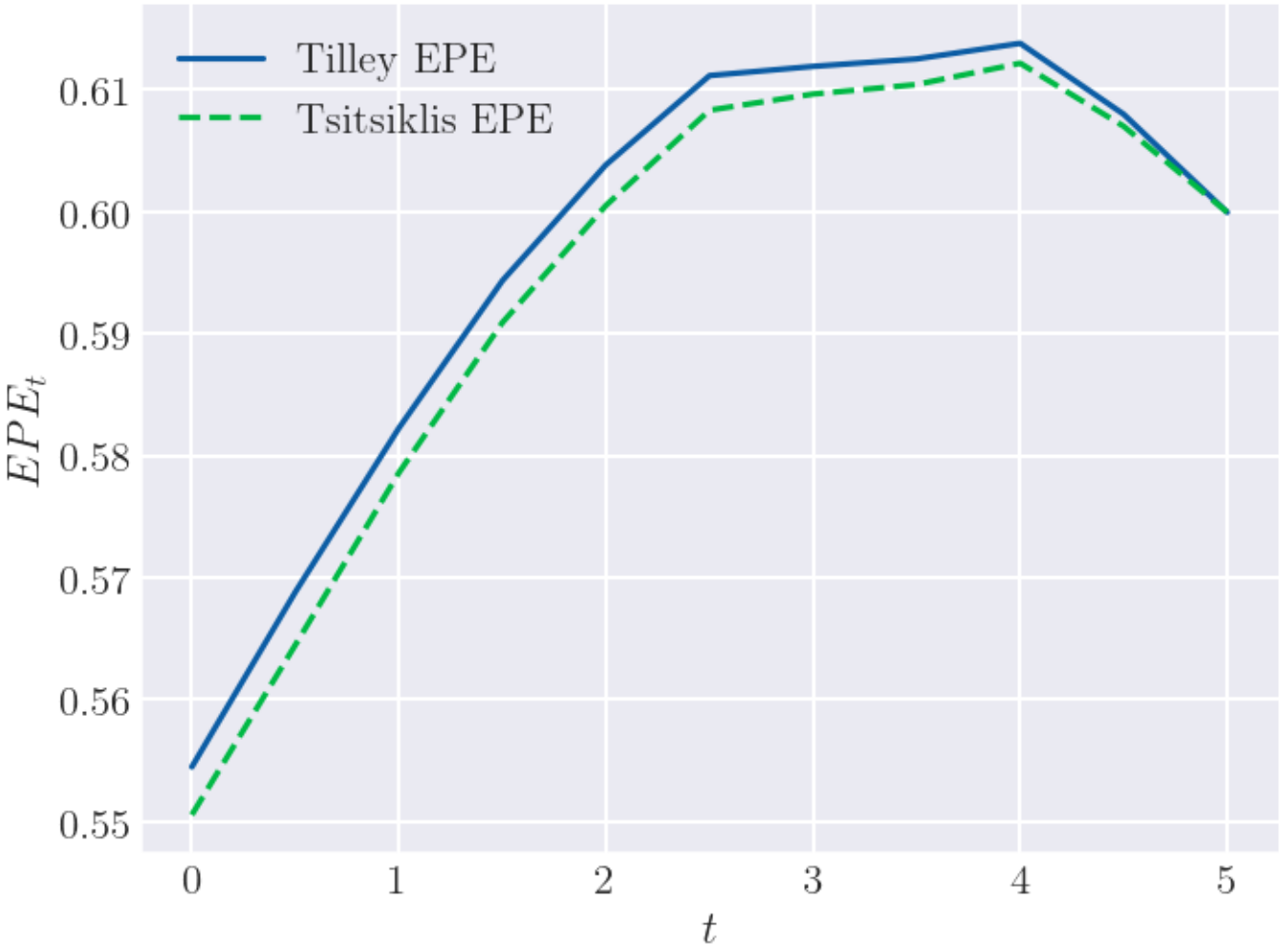
American Option

Tsitsiklis-Van Roy vs Tilley – exposure profiles



American Option

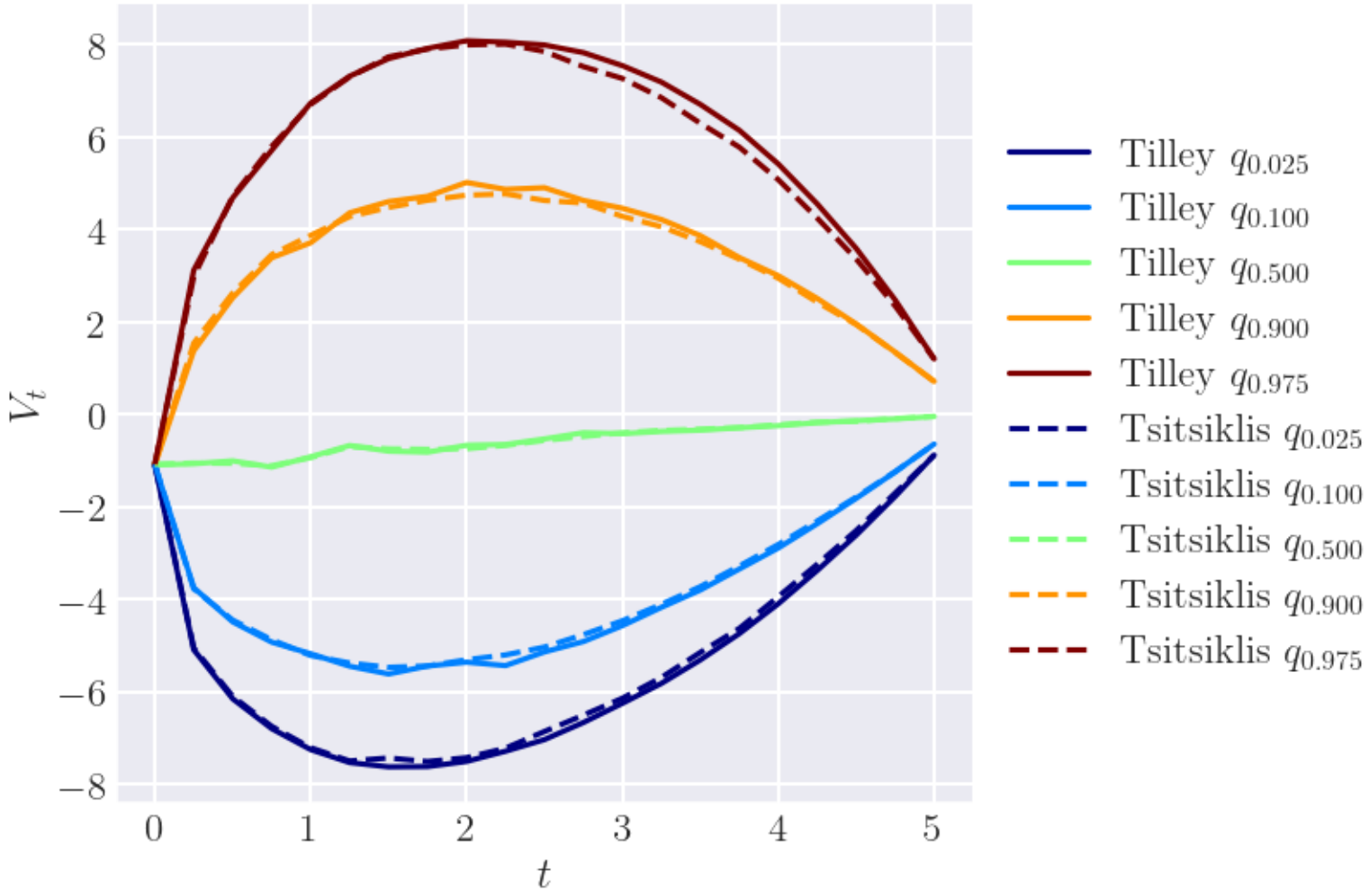
Tsitsiklis vs Tilley – Expected positive exposure



Interest Rate Swap

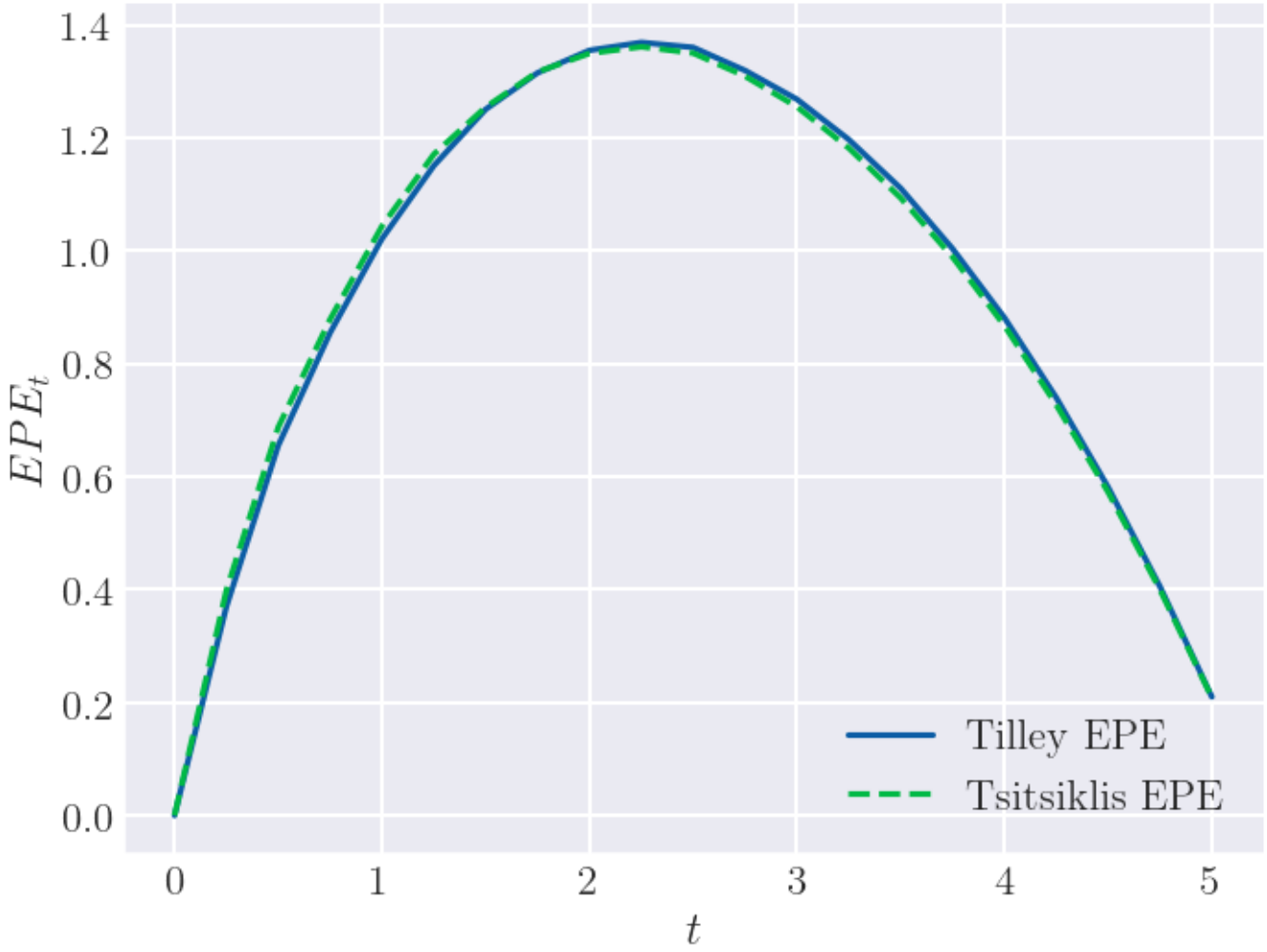
Interest Rate Swap

Tsitsiklis-Van Roy vs Tilley – Exposure profiles



Interest Rate Swap

Tsitsiklis-Van Roy vs Tilley – Expected positive exposure




THANK
YOU !