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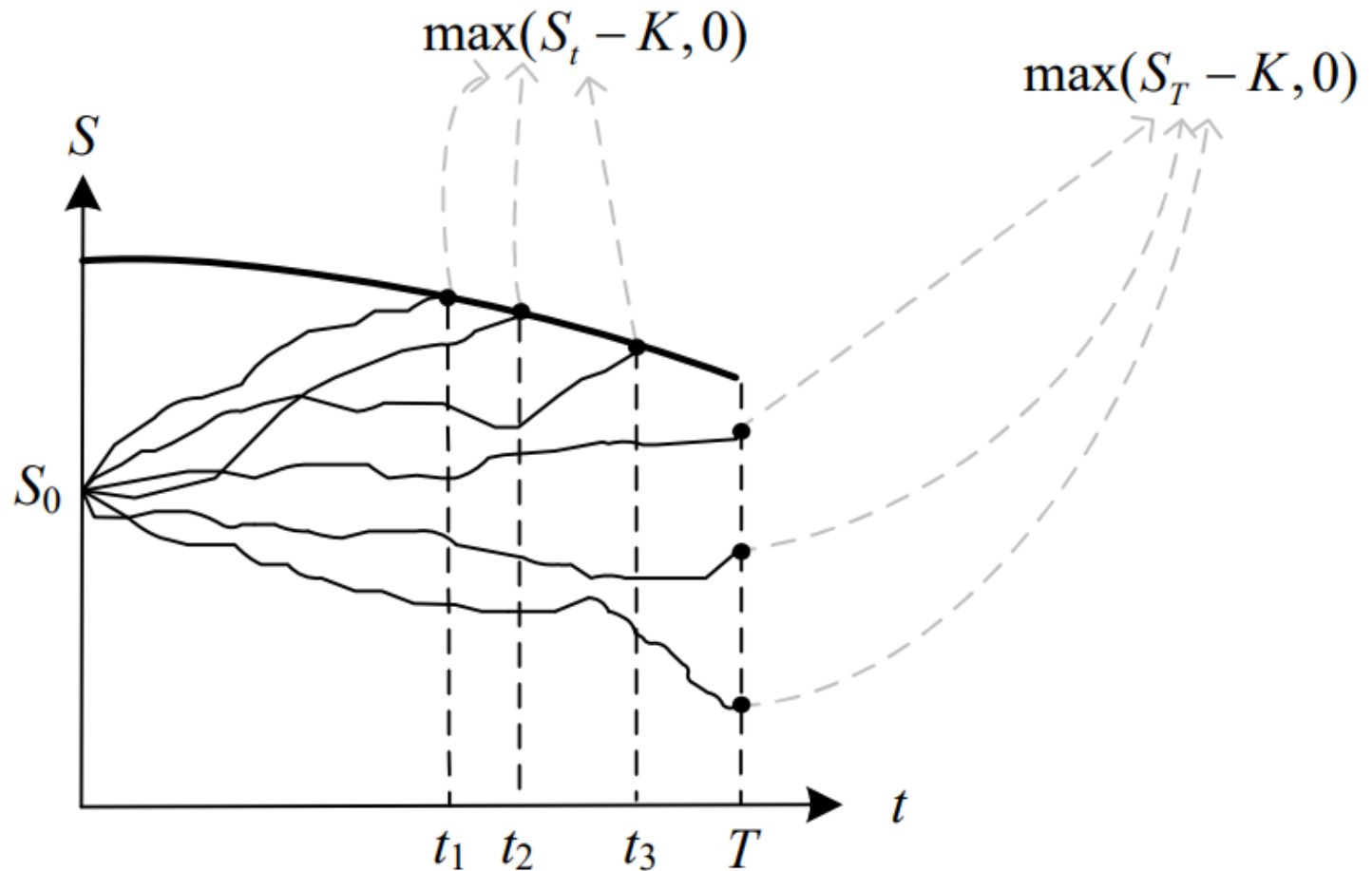
# **Tsitsiklis-Van Roy vs. Tilley**

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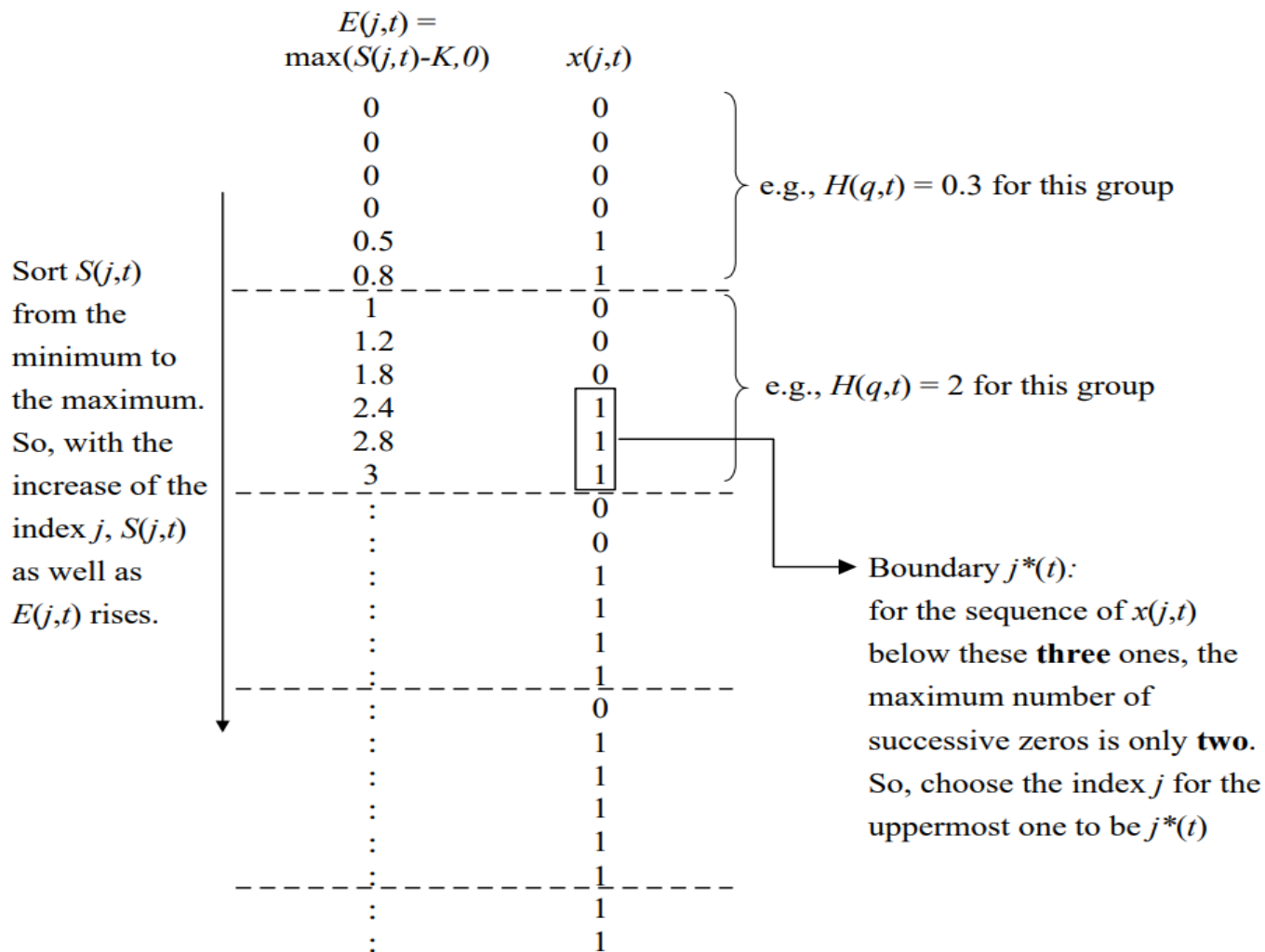
## Tilley algorithm

- The main idea is to devise a method based on the Monte Carlo simulation to decide the early exercise boundary .
- Once the early exercise boundary is determined, an American option can be viewed as a knocked-and-exercised option.

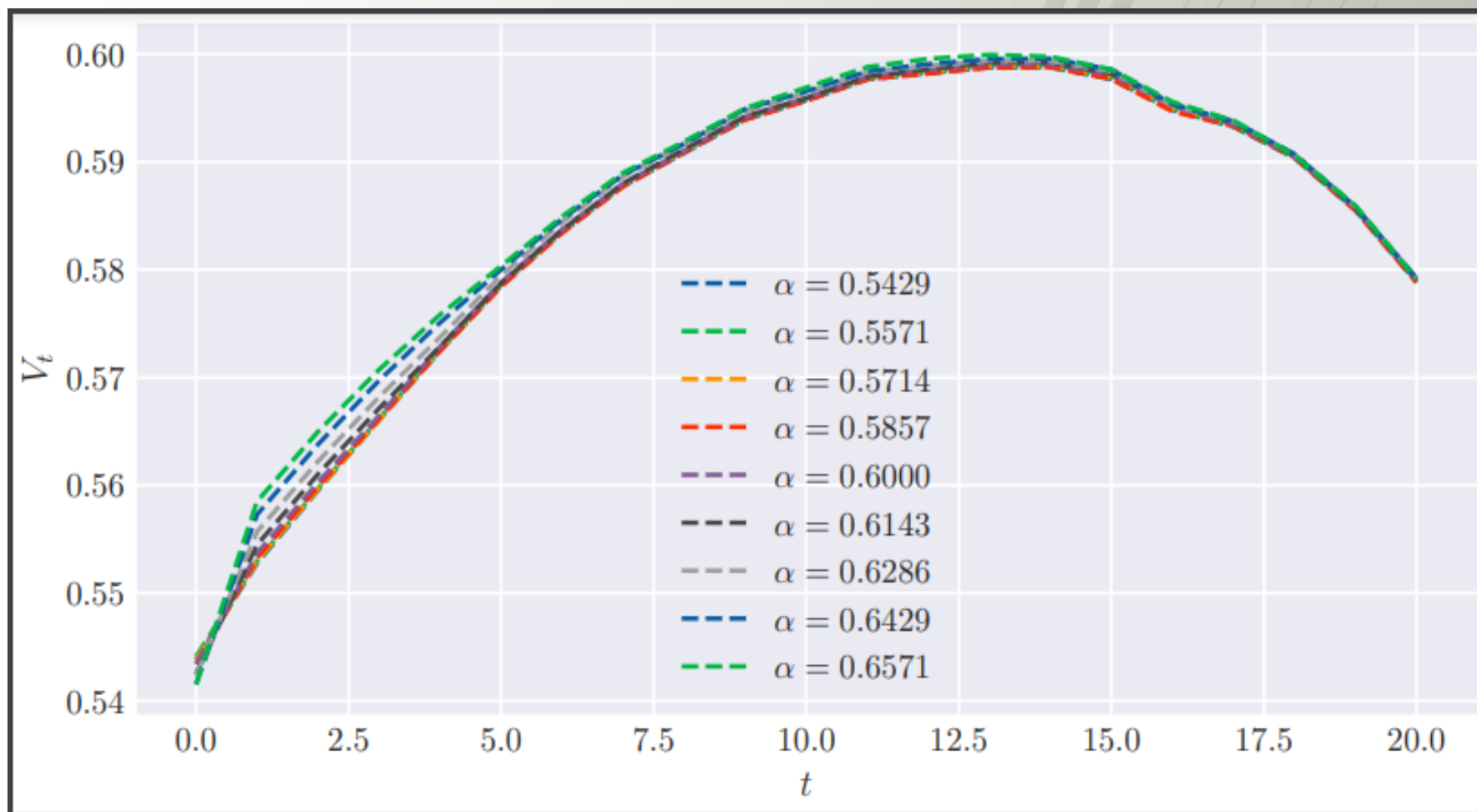
# Tilley algorithm



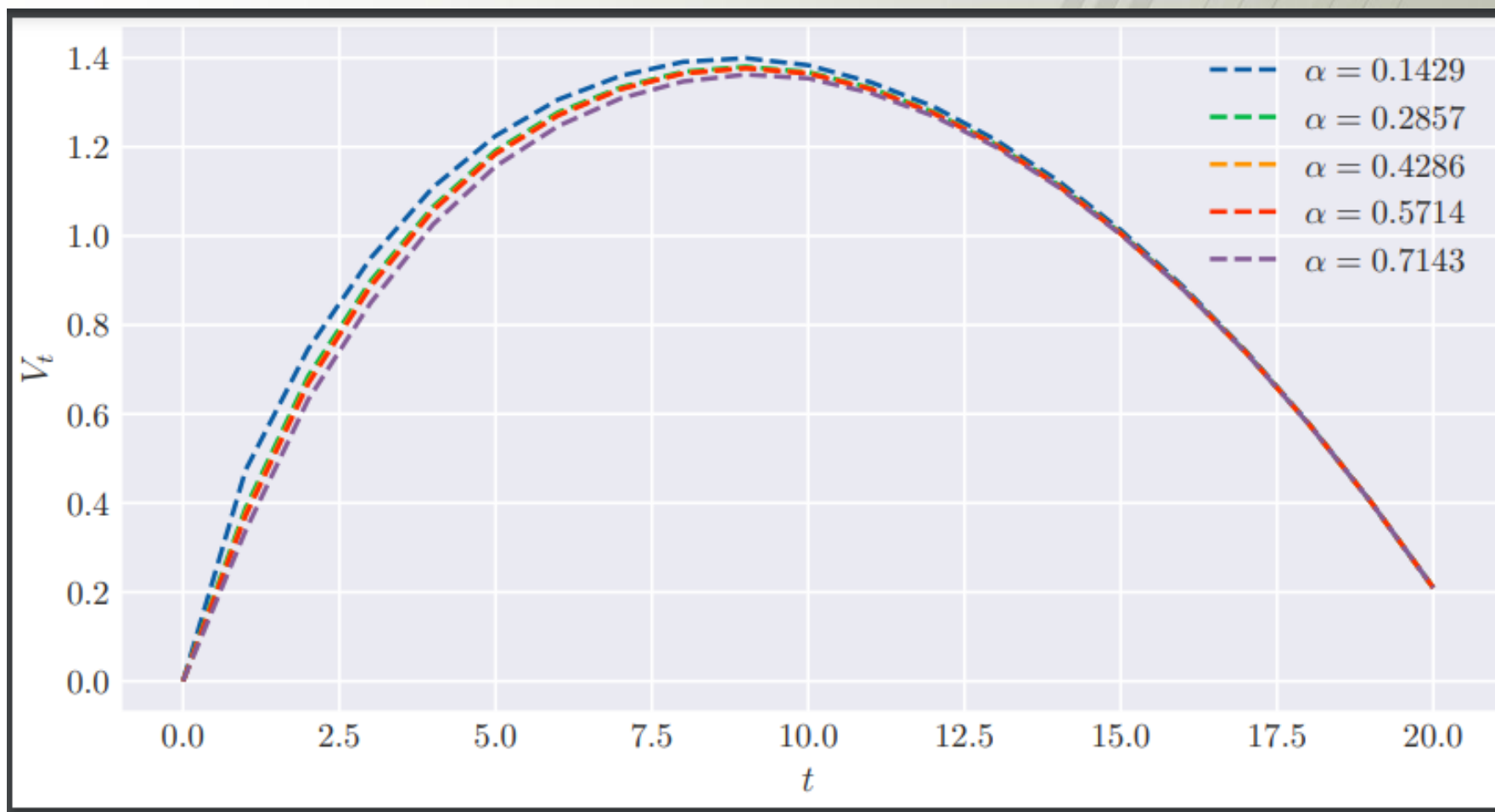
# Tilley algorithm



# American Option Tilley - alpha



# Interest Rate Swap Tilley - alpha



# Tsitsiklis-Van Roy algorithm

- We use the regression technique to approximate the conditional expectation function at each exercising date:

$$C_i(x) = \mathbb{E}[V_{i+1}(X_{i+1})|X_i = x]$$

- We are calculating value of an asset by using:

$$\hat{V}_{i,j} = \max\{h_i(X_{i,j}), \hat{C}_i(X_{i,j})\}$$

Difference between Longstaff - Schwartz and Tsitsiklis -Van Roy algorithms:

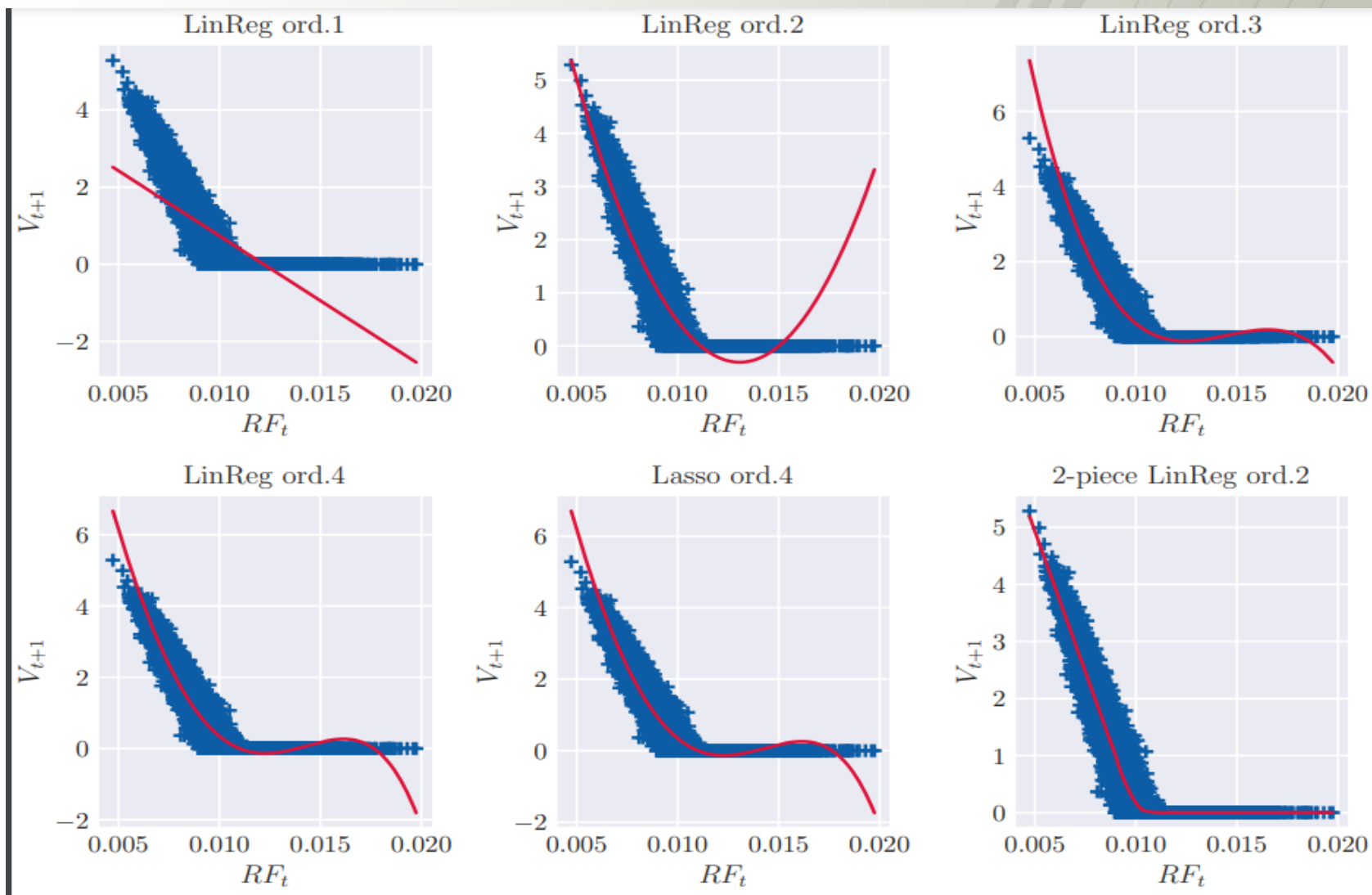
- Tsitsiklis - Van Roy:

$$\hat{V}_{i,j} = \max\{h_i(X_{i,j}), \hat{C}_i(X_{i,j})\}$$

- Longstaff -Schwartz:

$$\hat{V}_{ij} = \begin{cases} h_i(X_{i,j}) & h_i(X_{i,j}) \geq \hat{C}_i(X_{i,j}) \\ \hat{V}_{i+1,j} & h_i(X_{i,j}) < \hat{C}_i(X_{i,j}) \end{cases}$$

# American Option Tsitsiklis - regression





## **Geometric Brownian motion:**

- Number of paths – 10000
- Number of steps - 21
- $T$  – 5 years
- $\mu$  – 5%
- $\sigma$  – 20%

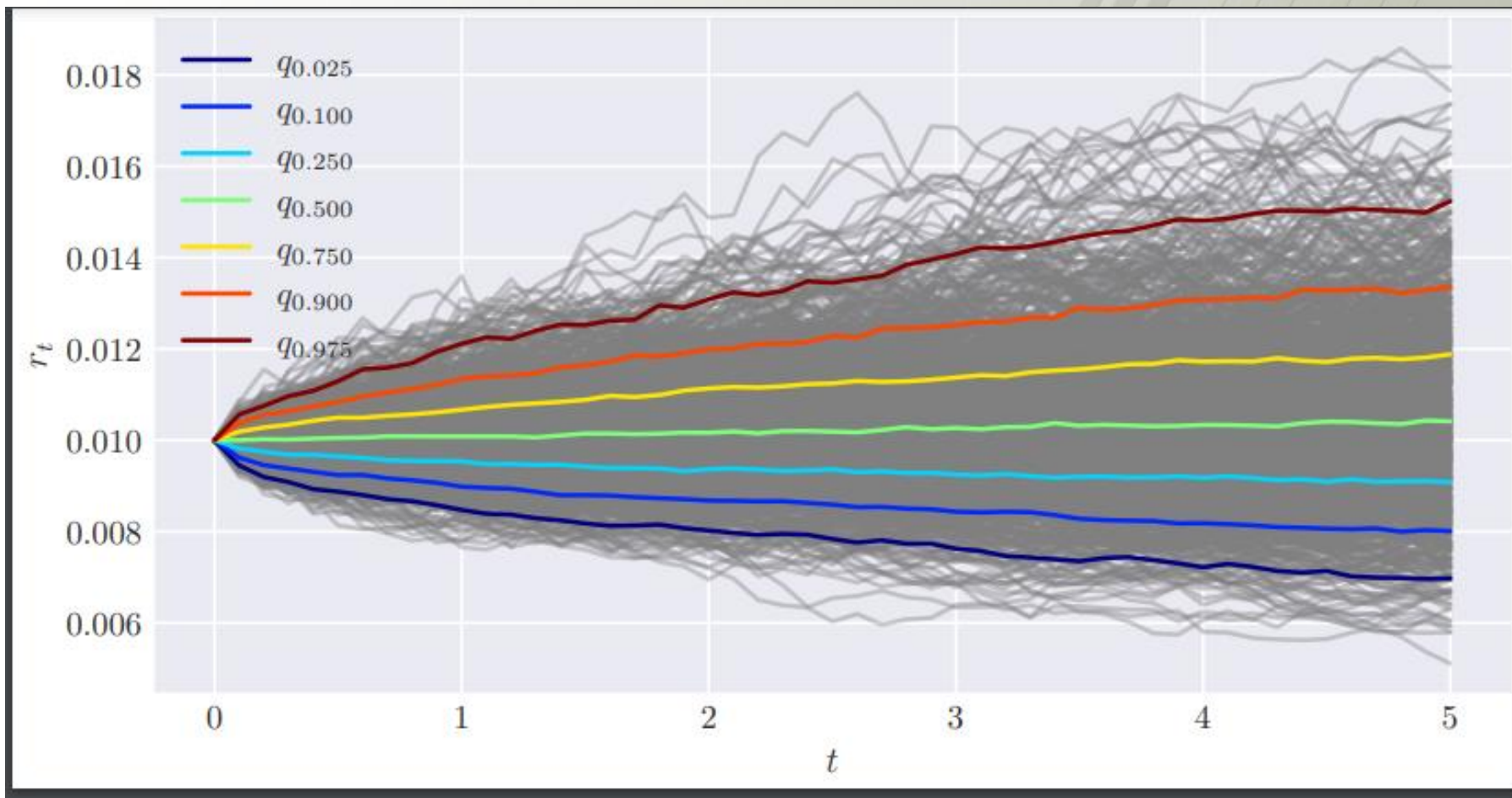
## **American Option:**

- Notional – 1000 USD
- Strike - 1%
- Expiry – 5 years
- Time steps – 21
- option\_type – „put”

## **Receiver Swap:**

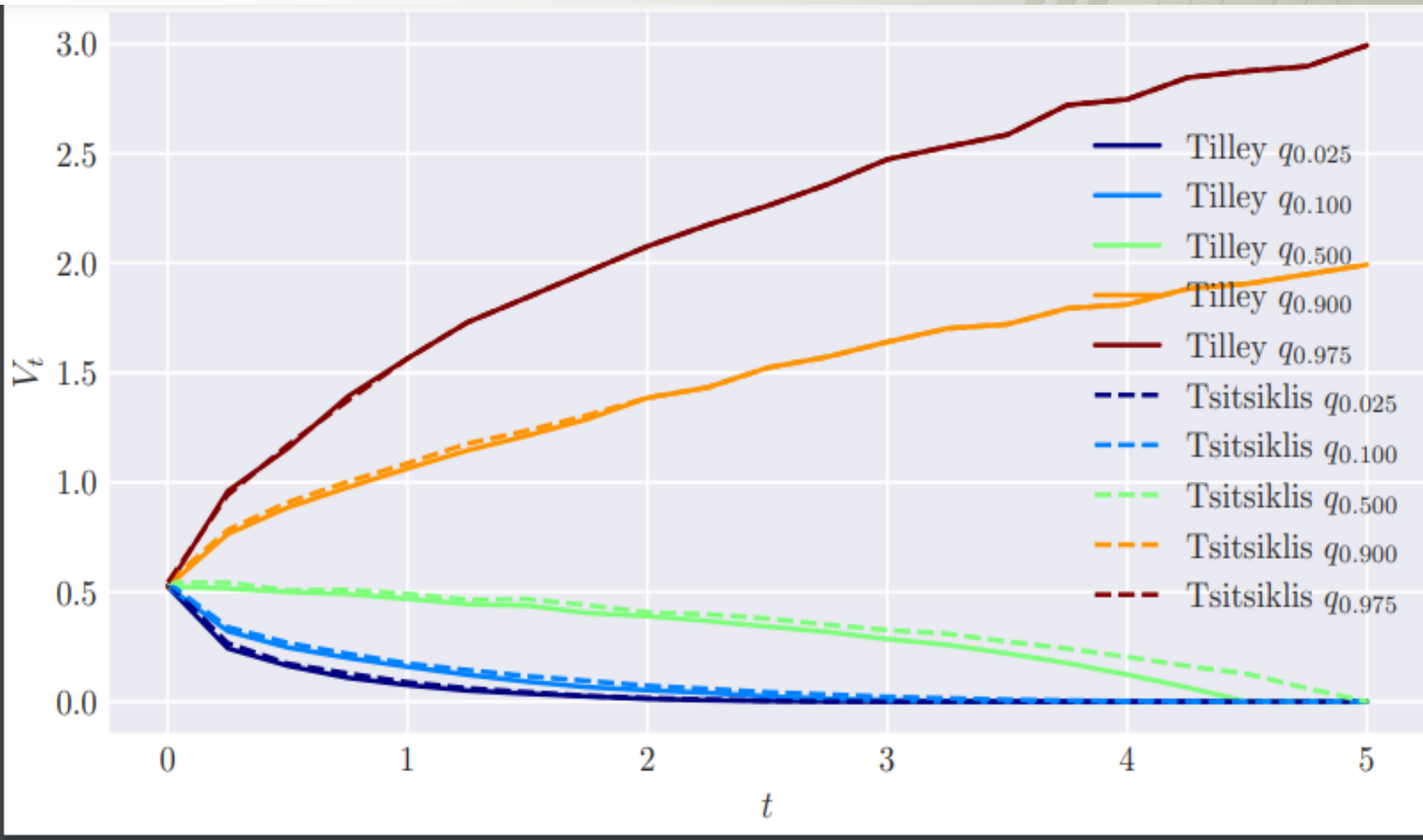
- Fixed rate – 1,5 %
- Maturity – 5 years
- Payment frequency – 3 months
- Notional – 1000 USD

# Interest rate distribution quantiles

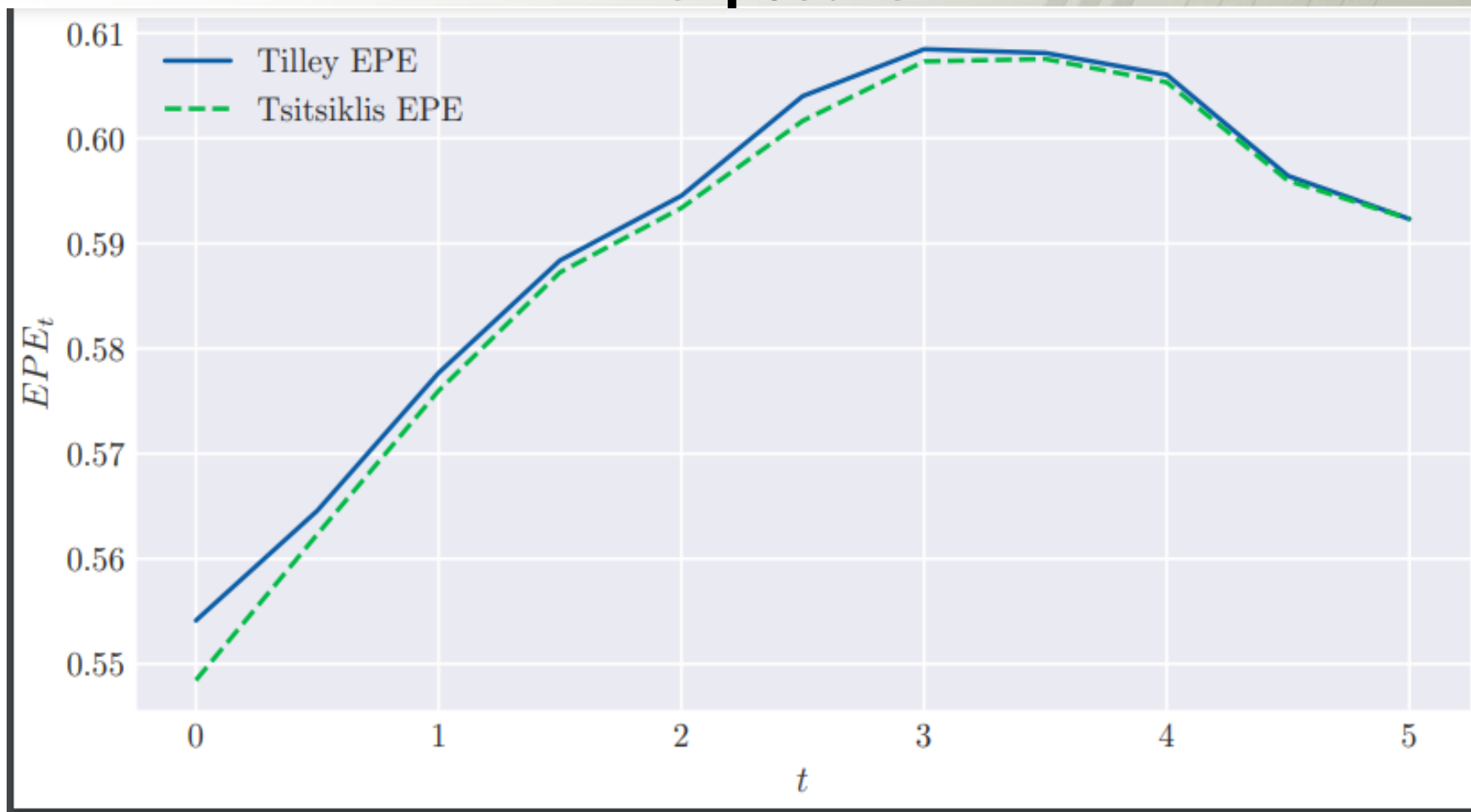


# American Option

# American Option Tsitsiklis vs Tilley – exposure profile



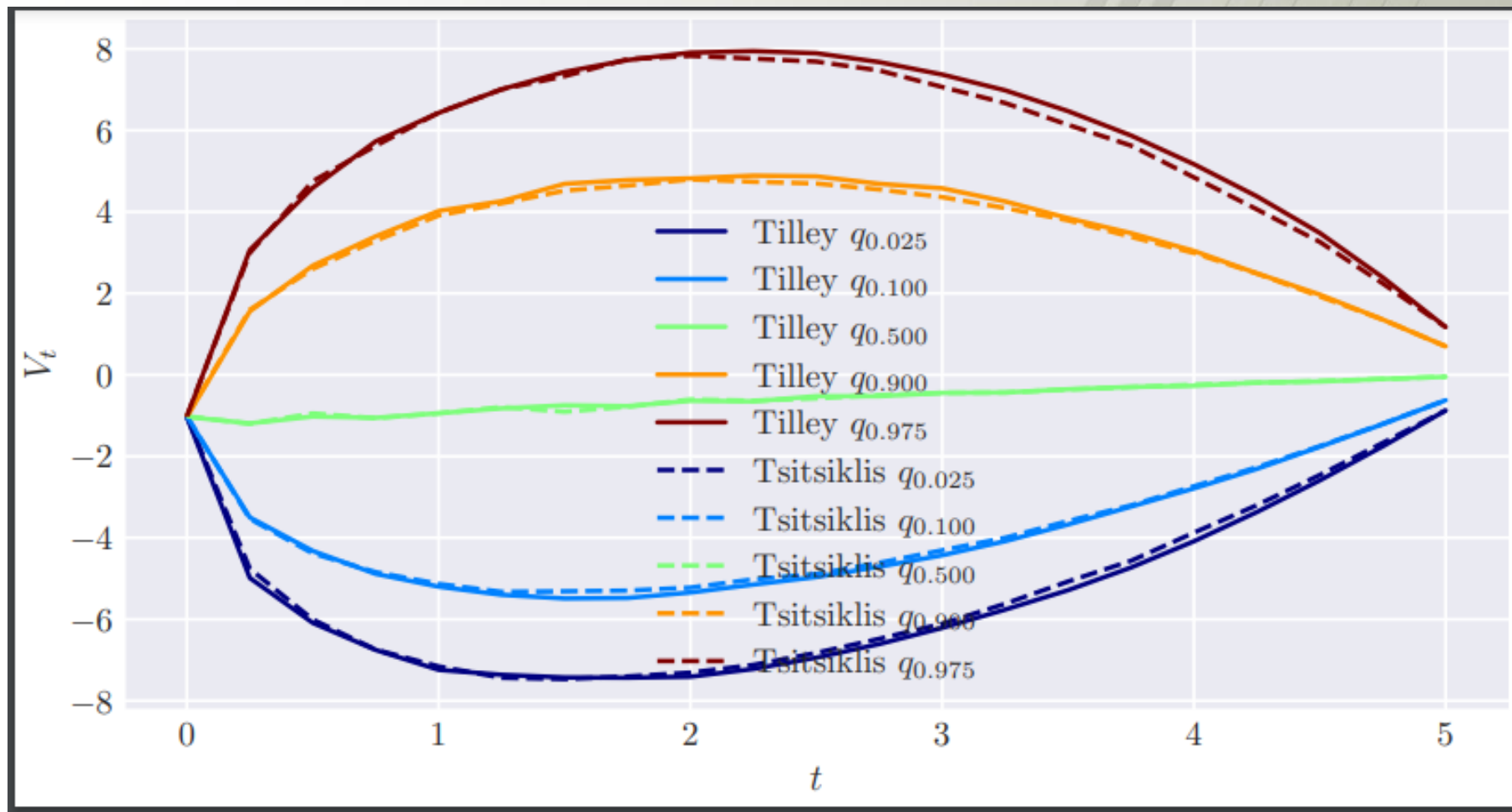
# American Option Tsitsiklis vs Tilley – Expected positive exposure



# Interest Rate Swap

# Interest Rate Swap

## Tsitsiklis vs Tilley – exposure profile



# Interest Rate Swap

## Tsitsiklis vs Tilley – Expected positive exposure

