

# Planning, Learning and Decision Making: Homework 1.

## Markov chains

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### 1 Exercise 1) a

State Space: {Station A, Stop 1-3, Stop 4, Stop 5-6, Station B}

Transition Probabilities:

	Station A	Stop 1-3	Stop 4	Stop 5-6	Station B
Station A	0	0.5	0.15	0.35	0
Stop 1-3	0	0	0	0	1
Stop 4	0	0	0	0	1
Stop 5-6	0	0	0	0	1
Station B	1	0	0	0	0

### 2 Exercise 1) b

We assume that a 'time step' is defined as a full cycle from Station A back to Station A.

$$\begin{aligned} & P_{\{EachStop\}} \\ &= P_{\{1-3,4,5-6\}} + P_{\{1-3,5-6,4\}} + P_{\{4,1-3,5-6\}} + P_{\{4,5-6,1-3\}} + P_{\{5-6,4,1-3\}} + P_{\{5-6,1-3,4\}} \\ &= 6 \times (0.35 \times 0.15 \times 0.5) = 0.1575 = 15.75\% \end{aligned}$$

The probability that the train stops in each stop at  $t = 3$  is 15.75%.

### 3 Exercise 1) c

We assume that the train does not pass the same stop twice and the train leaves Station A at the described moment.

There are 5 possibilities:

1) Stop 1 - Stop 5 - Stop 4 : Total time is 118 minutes.

*Probability* =  $P_{\{Stop1\}} \times P_{\{Stop5|Stop1\}} = 0.5 \times 0.7 = 0.35$

2) Stop 5 - Stop 1 - Stop 4 : Total time is 118 minutes.

*Probability* =  $P_{\{Stop5\}} \times P_{\{Stop1|Stop5\}} = 0.35 \times 0.77 = 0.27$

3) Stop 1 - Stop 4 : Total time is 70 minutes.

*Probability* =  $P_{\{Stop1\}} \times P_{\{Stop4|Stop1\}} = 0.5 \times 0.3 = 0.15$

4) Stop 5 - Stop 4 : Total time is 58 minutes.

*Probability* =  $P_{\{Stop5\}} \times P_{\{Stop4|Stop5\}} = 0.35 \times 0.23 = 0.08$

5) Stop 4 : Total time is 10 minutes.

*Probability* =  $P_{\{Stop4\}} = 0.15$

*Total* :  $118 \times 0.35 + 118 \times 0.27 + 70 \times 0.15 + 58 \times 0.08 + 10 \times 0.15 = 89.8$  minutes

We calculated the dependent Probabilities by the following formula:

$$P_{\{A|B\}} = \frac{1}{1 - P_{\{B\}}} * P_{\{A\}} \quad (1)$$