Heat transfer simulations using discrete random processes

Physics lab: research project

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June 9, 2024

Motivation

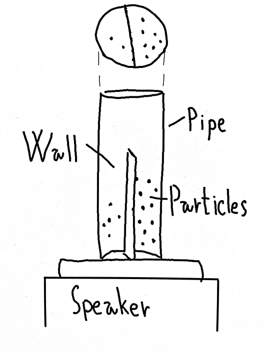


Figure 1: Experiment from International Physics Olympiad 2016 [1].Transparent pipe with two compartments attached to a speaker, particles crossing an energy barrier.

Research questions

1. How well do the discrete dynamics—distribution of oscillating particles into compartments—describe the continuous model of heat flow?
2. What is the relation between the equivalent thermal conductivity and the amplitude of the oscillating table?

The heat equation

Temperature $T(x,t)$ at point x with respect to time t

$$\frac{\partial T}{\partial t}(x,t) = \frac{k}{\sigma\delta}\frac{\partial^2 T}{\partial x^2}(x,t) \tag{1}$$

k—coefficient of thermal conductivity, σ—specific heat, and δ—mass density.

$$D = \frac{k}{\sigma\delta} \tag{2}$$

D—thermal diffusivity.

Predictions

Steady-state heat flux $\dot{q}$ (cf. [2])

$$\dot{q} = -k\frac{T_2 - T_1}{L}. \tag{3}$$

Hot end temperature T_1, cold end temperature T_2, length L.
Coefficient of thermal conductivity k [$\mathrm{W\,m^{-1}\,K^{-1}}$]

Setting $L = 1$, $T_2 = 1$, $T_1 = 0$ arb. units, temperature (0 to 1) in j-th bin at time t (cf. [3, 4])

$$Q_j(t) = \frac{1}{72}(13 - 2j) - \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n^2\pi^2}\left[1 + 11\cos\frac{n\pi}{6}\right]\sin\left[\frac{n\pi}{12}(2j-1)\right]\sin\frac{n\pi}{12}e^{-n^2\pi^2 D t}. \tag{4}$$

Coefficient of thermal diffusivity D [$\mathrm{m^2\,s^{-1}}$]

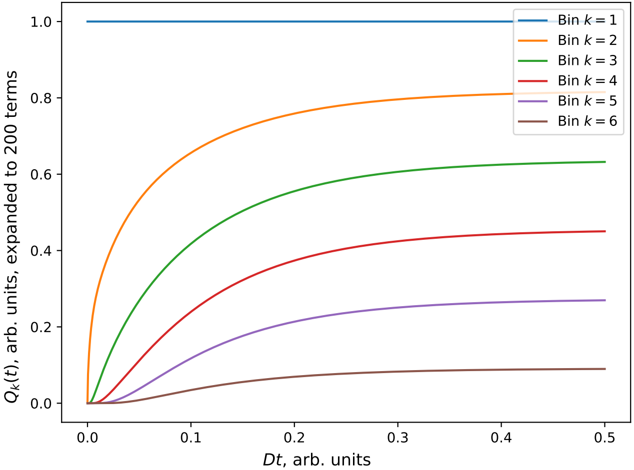
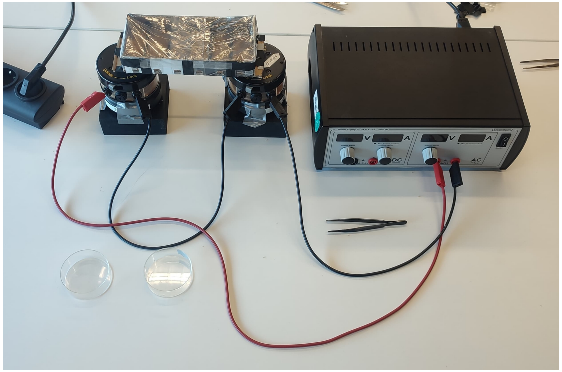


Figure 2: Predicted temperature in each bin.

Setup



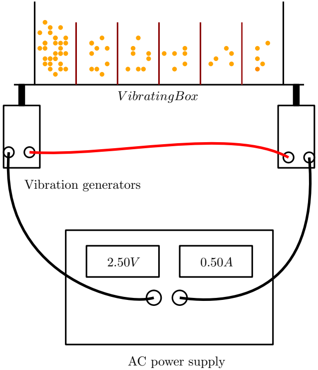


Figure 3: Connections and schematic of the setup

Setup: The box

Figure 4: The filled box

Methods

For seven AC amplitudes (0.5–0.98A):

1. Let the setup vibrate for 5–20s
2. Weigh or count particles in each bin
3. Return particles, refill Hot end, empty Cold end
4. Repeat until steady flux (last bin)

Extensive data analysis and fitting using Python

Steady state

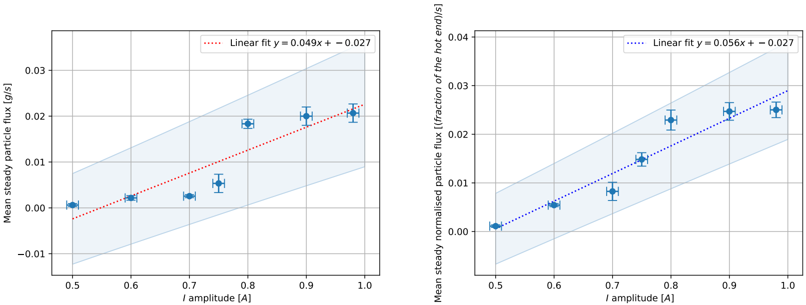


Figure 5: Steady particle flow versus current I. Original (left $a = (0.05 \pm 0.03)\,\text{g/sA}$) Updated, normalised (right $a_{norm} = (0.06 \pm 0.01)\,1/\text{sA}$)

Time evolution

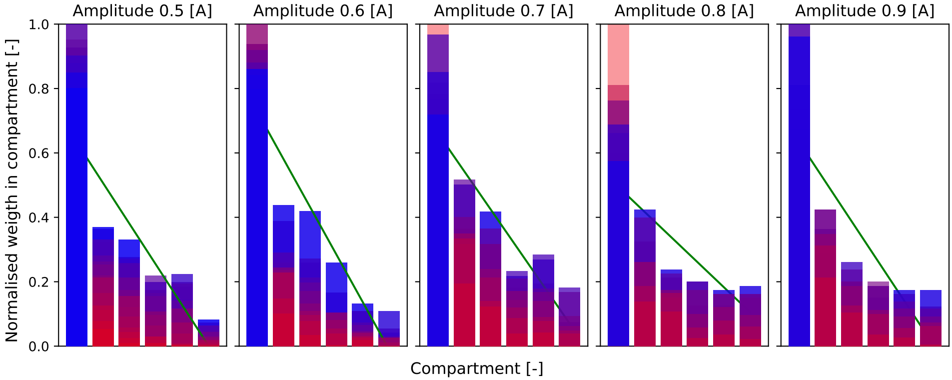


Figure 6: Some normalised weight evolutions. Time increases from red to blue. Green lines are steady-state linear fits.

Linearisation of Distribution

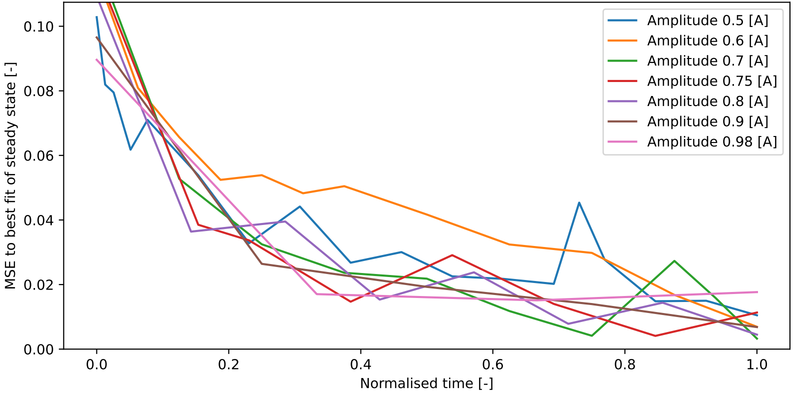


Figure 7: Evolution of Mean square error from linear fit. Normalised stabilisation time.

Stabilisation time

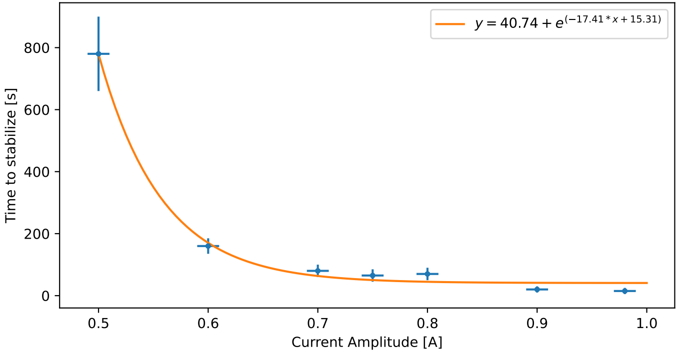


Figure 8: Stabilisation time depending on the current. Best fit in yellow.

Fitting data to the pre-steady state

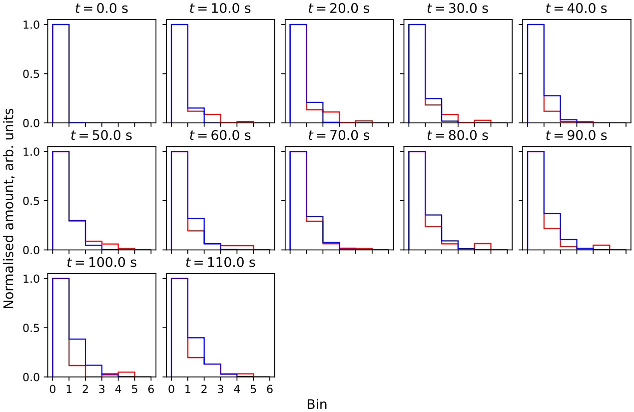


Figure 9: Fitting D. Theoretical, experimental. $I = 0.8\,\mathrm{A}$, $D = (0.08 \pm 0.02)\,\mathrm{Hz\,cm^2}$.

The diffusivity coefficient

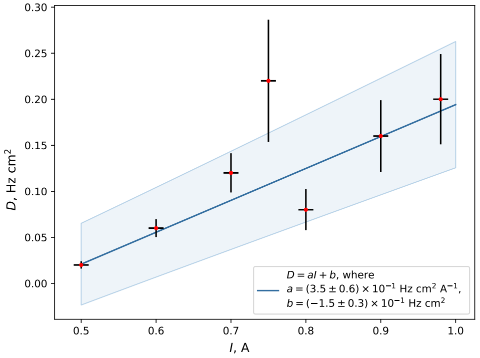


Figure 10: Best-fitted thermal diffusivity D depending on the current I.
$D = 0\,\mathrm{cm}^2/\mathrm{s}$ at $I_{\mathrm{crit}} = (0.4 \pm 0.2)\,\mathrm{A}$.

Discussion

1. Homogeneous heating of the system, $D \propto (I - I_{\text{crit}})$
2. Current amplitude cutoff at $I_{\text{crit}} = (0.4 \pm 0.2)\,\text{A}$
3. Amplitude proportional to current
4. Effective conductivity approximately linear with respect to current $k \propto I$
5. Minimal time to reach steady state $a = (40 \pm 20)\,\text{s}$, $b = (18 \pm 3)\,\text{A}^{-1}$

$$t_{\text{steady}} = Ce^{-bI} + a \tag{5}$$

6. Errors: liquefaction, flimsy box, uneven particle weight, large time between measurements, steady-state time bias, manual fitting

Conclusions

Research questions & hypotheses

1. *Answered & confirmed.* The distribution describes continuous heat conduction with a homogeneous heating term. Thus an analogy exists, confirming the hypothesis.
2. *Answered & confirmed.* The effective coefficient of thermal conductivity is proportional to the amplitude of vibrations. The spread is thus faster, confirming the second hypothesis.

Additional findings

1. Lower bound exists for the time required to reach an approximate steady state for any amplitude
2. Liquefaction observed with too high number of particles

References

[1] C. Aegerter and A. Kish, "47th International Physics Olympiad E-2: Jumping beads—a model for phase transitions and instabilities," 2016.

[2] J. Blijleven and R. Klein-Douwel, "Data and error analysis for physics laboratory: Skills," 2023.

[3] R. Daileda, "Partial differential equations," 2017, lecture notes, Trinity University.

[4] K. R. Hiremath, "Partial differential equations," 2021, lecture notes, Indian Institute of Technology, Jodhpur.