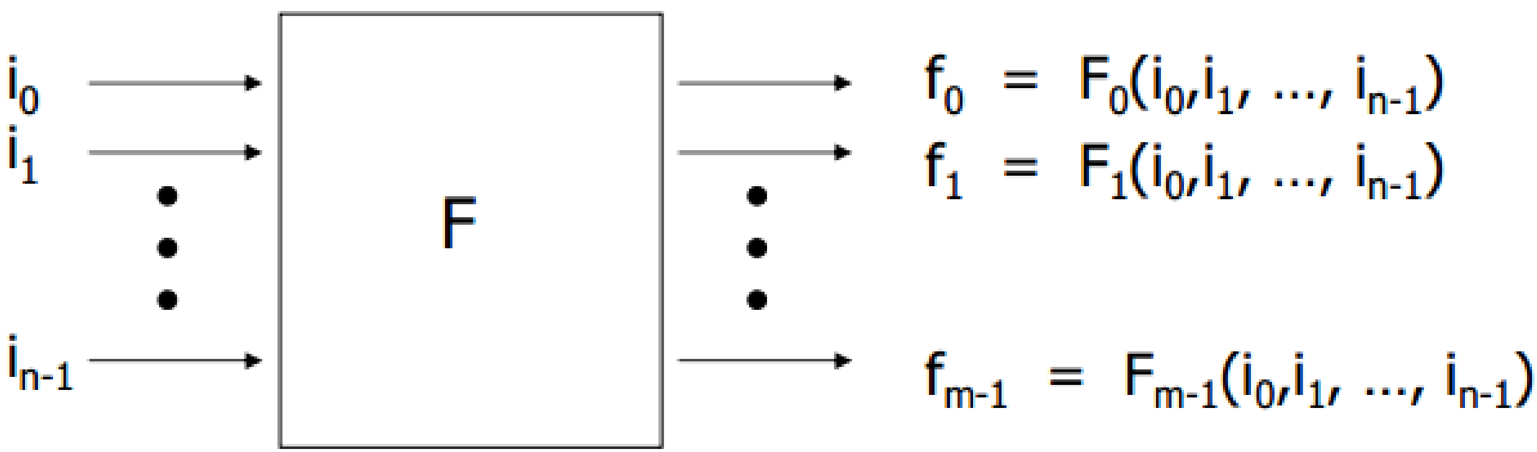
Digital Logic Gates and Boolean Algebra Part 1

Introduction

- **Boolean Algebra: named after mathematician George Boole (1815-1864).**
  - **Base 2 algebra**
    - **All variables hold the literal value True (1) or False (0)**
- **A (5v) digital circuit can have one of two values:**
  - **Signal between 0 and 0.8 volt: 0 value**
  - **Signal between 2 and 5 volts: 1 value**
    - **Signal between 0.8 and 2 volts:**
      - **undefined, gate has no deterministic output.**
- **Gates calculate various functions of one or more input values to generate an output.**
  - **NOT, AND, OR, XOR, etc.**
- **Computers are made up of gates.**
  - **Performs all logical operations**
  - **Composed of connected transistors, resistors, capacitors, etc.**
    - **Digital circuits**

Combinational Logic

- **Translates a set of N input variables (0 or 1) by a mapping function.**
  - **Uses Boolean operations to produce a set of M output variables (0 or 1).**

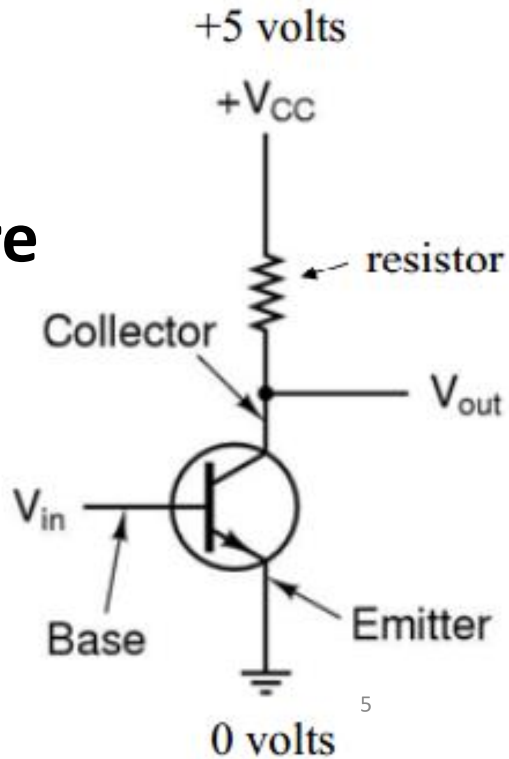


Boolean Operators

- **Basic Operators**
  - AND
  - OR
  - NOT
- **Other Useful Operators**
  - NAND
  - NOR
  - XOR
  - XNOR

NOT Operator (Inverter)

- 1 goes in, 0 comes out and vice versa
- Circuit:
  - When input (V_{in}) is low, transistor turns off (infinite resistance) which means output is V_{cc}
    - (0 in = 1 out)
  - When V_{in} is high, transistor acts like a wire to ground making V_{cc} 0 volts.
    - (1 in = 0 out)
- Instant switching of states?
  - Takes a few nanoseconds.

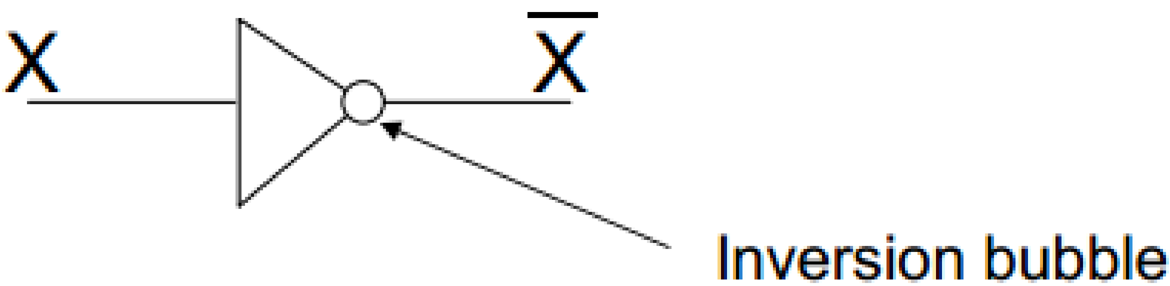


NOT Operator (Inverter)

- **NOT truth table:**

X	$\overline{X}$
0	1
1	0

- **NOT gate symbol**

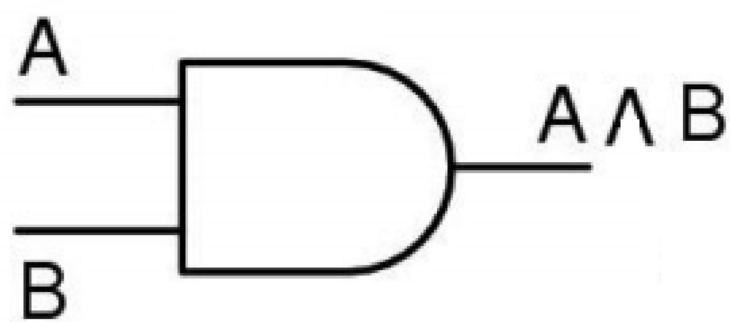


AND Operator

- **AND truth table:**
  - Output is High if all inputs are high.
  - Otherwise, output is low.

A	B	A ∧ B
0	0	0
0	1	0
1	0	0
1	1	1

- **AND gate symbol**

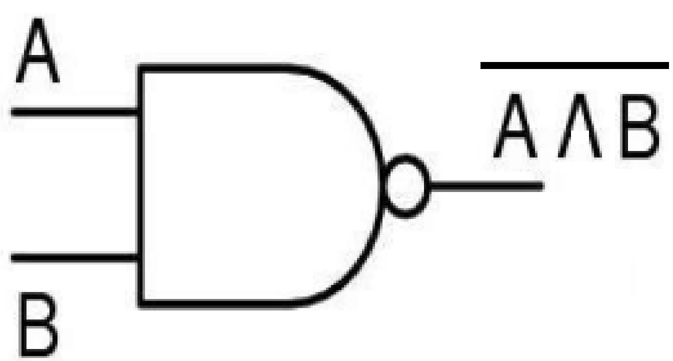


NAND Operator

- **NAND truth table:**
  - **Output is Low if all inputs are high.**
  - **Otherwise, output is high.**

A	B	$\overline{A \wedge B}$
0	0	1
0	1	1
1	0	1
1	1	0

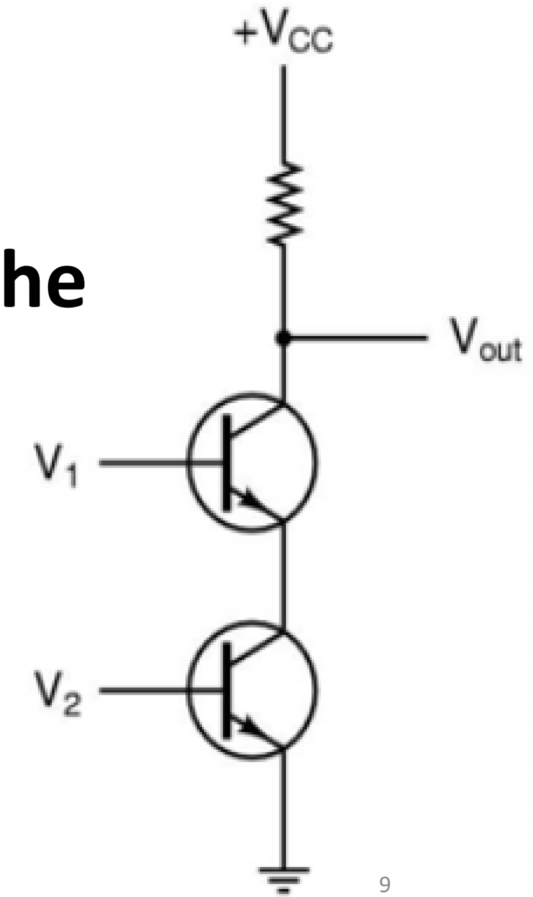
- **NAND gate symbol**



NAND Operator

- **Simplified Circuit:**
  - **If V_1 and V_2 are both high, both transistors will conduct and produce a path to ground. V_{out} will be low (0).**

  - **If either V_1 or V_2, or both, are low, the corresponding transistor resists, not allowing electricity to ground. V_{out} will be high (1).**

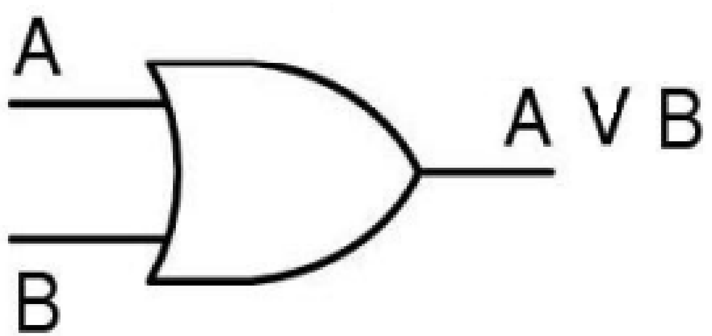


OR Operator

- **OR truth table:**
  - **Output is low if all inputs are low.**
  - **Otherwise, output is high.**

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

- **OR gate symbol**

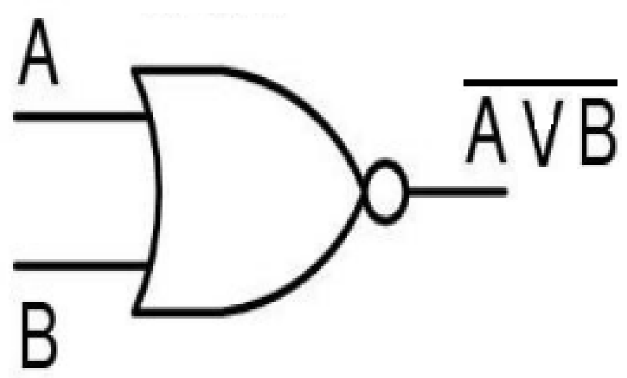


NOR Operator

- **NOR truth table:**
  - **Output is High if all inputs are low.**
  - **Otherwise, output is low.**

A	B	$\overline{A \vee B}$
0	0	1
0	1	0
1	0	0
1	1	0

- **NOR gate symbol**



NOR Operator

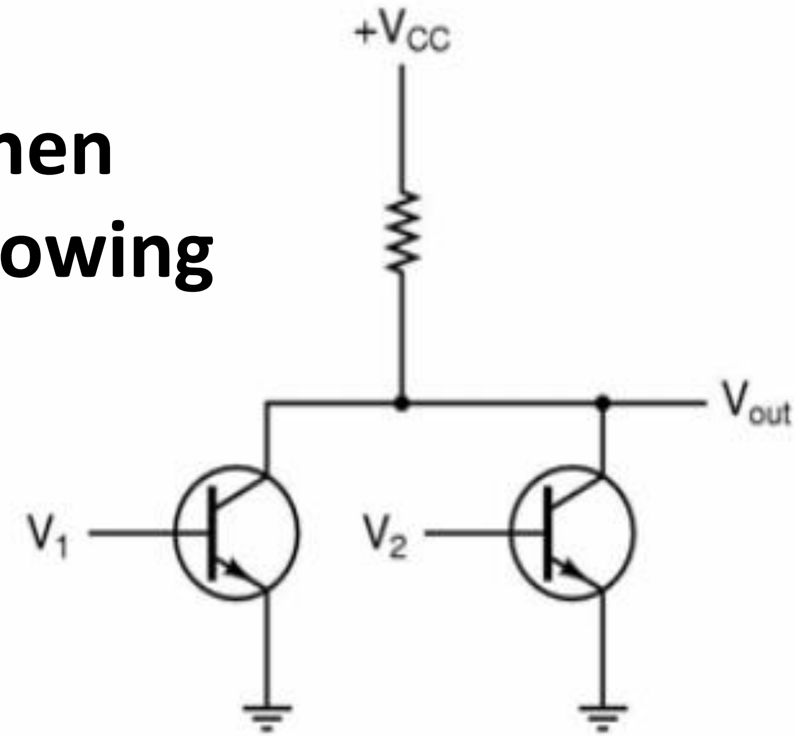
- **Simplified Circuit:**
  - **If either V_1 or V_2, or both, are high, then transistor will conduct and produce a path to ground.**

V_{out} will be low (0).

  - **If both V_1 and V_2, are low, then**

both transistors resist, not allowing electricity to ground.

V_{out} will be high (1).



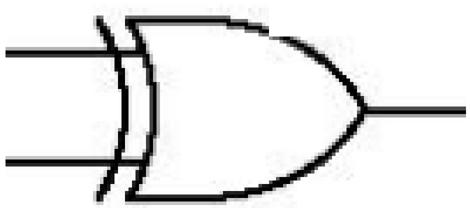
XOR Operator

- **Exclusive-OR truth table:**
  - Output is High if all inputs are mismatched.
  - Otherwise, output is low.

  - XOR = $A \oplus B = \overline{A}B + A\overline{B}$
    - Build this using AND, OR Gates?

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

- **XOR gate symbol**



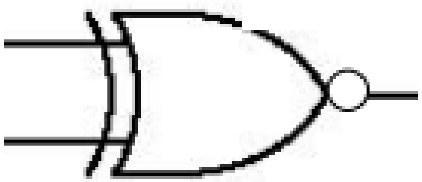
XNOR Operator

- **Exclusive-NOR truth table:**
  - **Output is low if all inputs are mismatched.**
  - **Otherwise, output is high.**

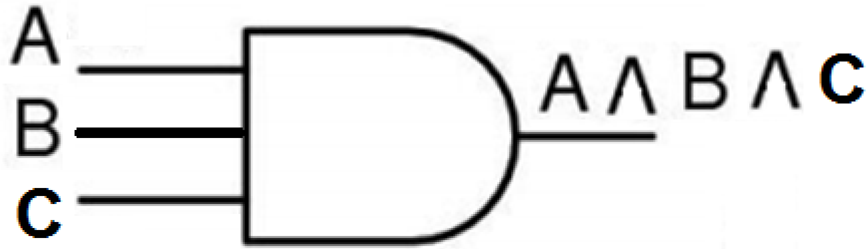
A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

- **XNOR = $A \overline{\oplus} B = AB + \overline{AB}$**
  - **Build this using AND, OR Gates?**

- **XNOR gate symbol**

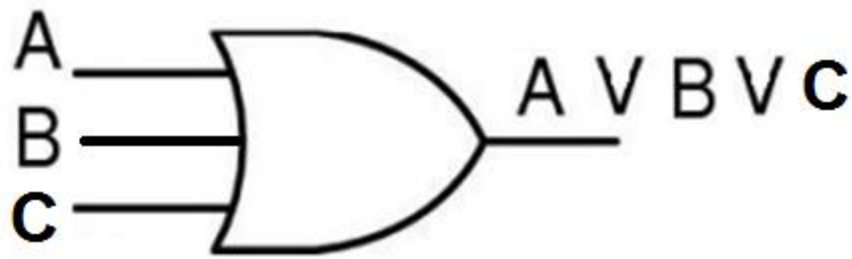


Three Input AND Gate



A	B	C	A ∧ B∧ C
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**0**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

Three Input Or Gate



A	B	C	$A \vee B \vee C$
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**