Section 2 homework

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Exercises for Section 2.1

A. Write each of the following sets by listing their elements between braces.

1. $\{5x-1 : x \in \mathbb{Z}\} = \{\ldots, -11, -6, -1, 4, 9, \ldots\}$
2. $\{3x+2 : x \in \mathbb{Z}\} = \{\ldots, -4, -1, 2, 5, 8, \ldots\}$
3. $\{x \in \mathbb{Z} : -2 \leq x < 7\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
4. $\{x \in \mathbb{N} : -2 < x \leq 7\} = \{1, 2, 3, 4, 5, 6, 7\}$
5. $\{x \in \mathbb{R} : x^2 = 3\} = \{-\sqrt{3}, \sqrt{3}\}$
6. $\{x \in \mathbb{R} : x^2 = 9\} = \{-3, 3\}$
7. $\{x \in \mathbb{R} : x^2 + 5x = -6\} = \{-3, -2\}$

11 $\{x \in \mathbb{Z} : |x| < 5\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

12 $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2, -1, 0, 1, 2\}$

13 $\{x \in \mathbb{Z} : |6x| < 5\} = \{0\}$

14 $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$

B. Write each of the following sets in set-builder notation.

17 $\{2, 4, 8, 16, 32, 64 \ldots\} = \{2 \cdot 2^x : x \geq 0, x \in \mathbb{Z}\}$

19 $\{\ldots, -6, -3, 0, 3, 6, 9, 12, 15, \ldots\} = \{3x : x \in \mathbb{Z}\}$

24 $\{-4, -3, -2, -1, -0, 1, 2\} = \{x : -4 \leq x \leq 2, x \in \mathbb{Z}\}$

25 $\{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\} = \{2^x : x \in \mathbb{Z}\}$

26 $\{\ldots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \ldots\} = \{3^x : x \in \mathbb{Z}\}$

C. Find the following cardinalities of the following sets.

29 $\{\{1\}, \{2, \{3, 4\}\}, \phi\} = 3$

30 $\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\phi\}\} = 5$

31 $\{\{\{1\}, \{2, \{3, 4\}\}, \phi\}\} = 1$

32 $\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\phi\}\}\} = 1$

33 $\{x \in \mathbb{Z} : |x| < 10\} = 19$

34 $\{x \in \mathbb{N} : |x| < 10\} = 9$

35 $\{x \in \mathbb{Z} : x^2 < 10\} = 7$

36 $\{x \in \mathbb{N} : x^2 < 10\} = 3$

37 $\{x \in \mathbb{N} : x^2 < 0\} = 0$

38 $\{x \in \mathbb{N} : 5x \leq 20\} = 4$

Exercises for Section 2.2

A Write out the indicated sets by listing their elements between braces.

2 Suppose $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$.

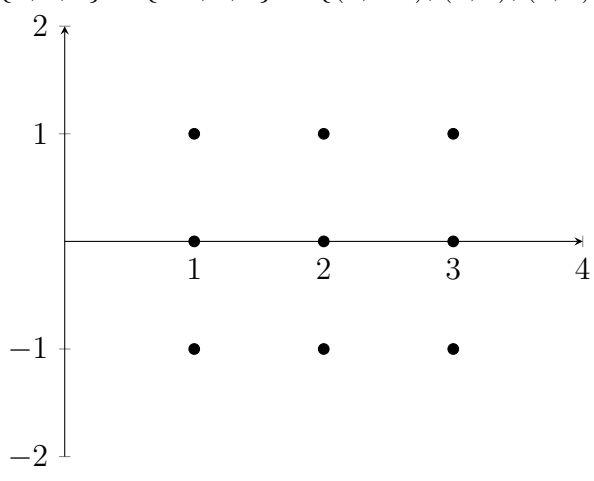
* $A \times B = \{(\pi, 0), (\pi, 1), (e, 0), (e, 1), (0, 0), (0, 1)\}$
* $B \times A = \{(0, \pi), (0, e), (0, 0), (1, \pi), (1, e), (1, 0)\}$
* $A \times A =$
  $\{(\pi, \pi), (\pi, e), (\pi, 0), (e, \pi), (e, e), (e, 0), (0, \pi), (0, e), (0, 0)\}$
* $B \times B = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$
* $A \times \phi = \{(\pi), (e), (0)\}$
* $(A \times B) \times B =$
  $\{((\pi, 0), 0), ((\pi, 0), 1), ((\pi, 1), 0), ((\pi, 1), 1), ((e, 0), 0), ((e, 0), 1),$
  $((e, 1), 0), ((e, 1), 1), ((0, 0), 0), ((0, 0), 1), ((0, 1), 0), ((0, 1), 1)\}$
* $A \times (B \times B) =$
  $\{(\pi, (0, 0)), (\pi, (0, 1)), (\pi, (1, 0)), (\pi, (1, 1)),$
  $(e, (0, 0)), (e, (0, 1)), (e, (1, 0)), (e, (1, 1)),$
  $(0, (0, 0)), (0, (0, 1)), (0, (1, 0)), (0, (1, 1))\}$
* $A \times B \times B =$
  $\{(\pi, 0, 0), (\pi, 0, 1), (\pi, 1, 0), (\pi, 1, 1),$
  $(e, 0, 0), (e, 0, 1), (e, 1, 0), (e, 1, 1),$
  $(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1)\}$

6 $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\} = \{(0, 1), (1, 1)\}$

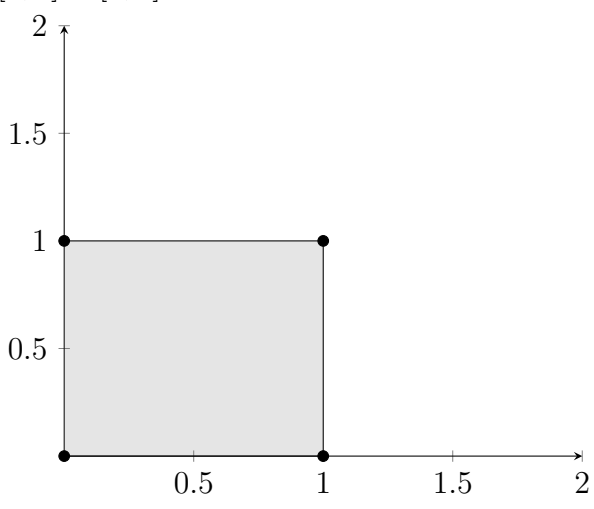
8 $\{0, 1\}^4 =$
$\{(((0, 0), 0), 0), (((0, 0), 0), 1), (((0, 0), 1), 0), (((0, 0), 1), 1), (((0, 1), 0), 0),$
$(((0, 1), 0), 1), (((0, 1), 1), 0), (((0, 1), 1), 1), (((1, 0), 0), 0), (((1, 0), 0), 1),$
$(((1, 0), 1), 0), (((1, 0), 1), 1), (((1, 1), 0), 0), (((1, 1), 0), 1),$
$(((1, 1), 1), 0), (((1, 1), 1), 1)\}$

B Sketch these Cartesian products on the $x - y$ plane $\mathbb{R}^2$ (or $\mathbb{R}^3$for the last two.)

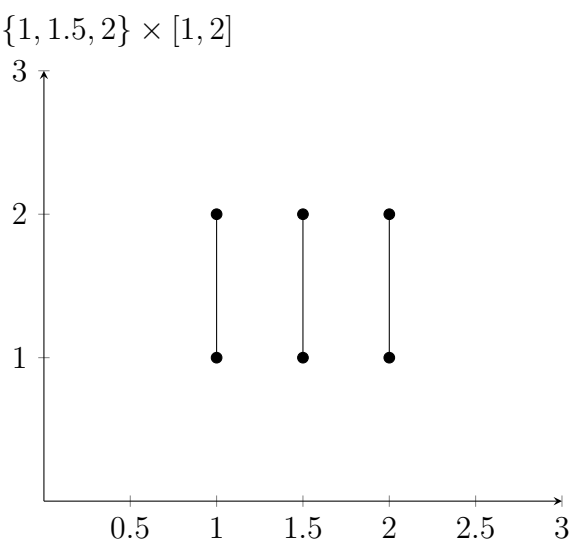
9 $\{1, 2, 3\} \times \{-1, 0, 1\} = \{(1, -1), (1, 0), (1, 1), (2, -1), (2, 0), (2, 1), (3, -1), (3, 0), (3, 1)\}$



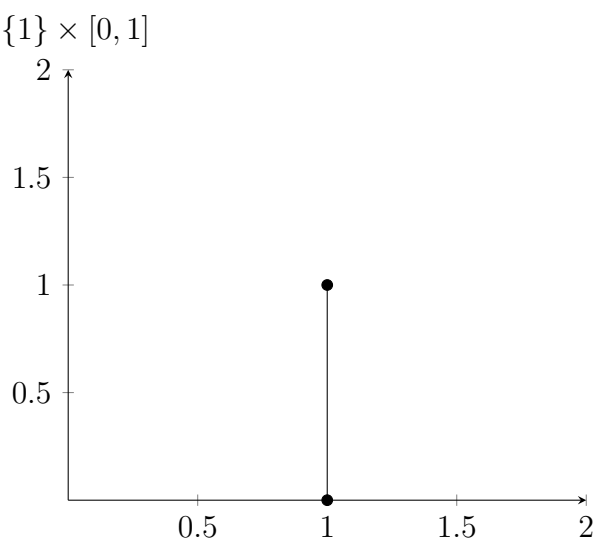
11 $[0, 1] \times [0, 1]$



13 $\{1, 1.5, 2\} \times [1, 2]$



15 $\{1\} \times [0, 1]$



Exercises for Section 2.3

A List all the subsets of the following sets.

1. $\{1, 2, 3, 4\} = \{\}, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\},$
   $\{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}$
2. $\{1, 2, \phi\} = \{\}, \{1\}, \{2\}, \{\phi\}, \{1, 2\}, \{1, \phi\}, \{2, \phi\}, \{1, 2, \phi\}$
3. $\{\{\mathbb{R}\}\} = \{\}, \{\{\mathbb{R}\}\}$
4. $\phi = \{\}$
5. $\phi = \{\}, \{\phi\}$
8. $\{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\} = \{\}, \{\{0, 1\}\}, \{\{0, 1, \{2\}\}\}, \{\{0\}\}, \{\{0, 1\}, \{0, 1, \{2\}\}\},$
   $\{\{0, 1\}, \{0\}\}, \{\{0, 1, \{2\}\}, \{0\}\}, \{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\}$

B Write out the following sets by listing their elements between braces.

10. $\{X \subseteq \mathbb{N} : |X| \leq 1\} = \{\{1\}\}$
11. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 4\} = \{\}$
12. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 1\} = \{\{3\}, \{2\}, \{a\}\}$

C Decide if the following statements are true or false. Explain.

13 $\mathbb{R}^3 \subseteq \mathbb{R}^3$. True, since it's a subset of the same set.

14 $\mathbb{R}^2 \subseteq \mathbb{R}^3$

15 $\{(x, y) : x - 1 = 0\} \subseteq \{(x, y) : x^2 - x = 0\}$

16 $\{(x, y) : x^2 - x = 0\} \subseteq \{(x, y) : x - 1 = 0\}$

Exercises for Section 2.4

A Find the indicated sets.

1. $P(\{\{a, b\}, \{c\}\}) = \{\{\{a, b\}\}, \{\{c\}\}, \{\{a, b\}, \{c\}\}, \phi\}$

5 $P(P(\{2\})) = \{\phi, \{\phi\}, \{\{2\}\}, \{\phi, \{2\}\}\}$

7 $P(\{a, b\}) \times P(\{0, 1\}) =$
$\{(\phi, \phi), (\phi, \{0\}), (\phi, \{1\}), (\phi, \{0, 1\}), (\{a\}, \phi), (\{a\}, \{0\}), (\{a\}, \{1\}), (\{a\}, \{0, 1\}),$
$(\{b\}, \phi), (\{b\}, \{0\}), (\{b\}, \{1\}), (\{b\}, \{0, 1\}),$
$(\{a, b\}, \phi), (\{a, b\}, \{0\}), (\{a, b\}, \{1\}), (\{a, b\}, \{0, 1\})\}$

9 $P(\{a, b\} \times \{0\}) = \{\phi, \{(a, 0)\}, \{(b, 0)\}, \{(a, 0), (b, 0)\}\}$

10 $\{X \in P(\{1, 2, 3\}) : |X| \leq 1\} = \{\phi, \{1\}, \{2\}, \{3\}\}$

11 $\{X \subseteq P(\{1, 2, 3\}) : |X| \leq 1\} = \{\phi, \{\phi\}, \{\{1\}\}, \{\{2\}\}, \{\{3\}\},$
$\{\{1, 2\}\}, \{\{1, 3\}\}, \{\{2, 3\}\}, \{\{1, 2, 3\}\}\}$

12 $\{X \in P(\{1, 2, 3\}) : 2 \in X\} = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

B Suppose that $|A| = m$ and $|B| = n$. Find the following cardinalities.

13 $|P(P(P(A)))| = 2^{2^{2^m}}$

14 $|P(P(A))| = 2^{2^m}$

15 $|P(A \times B)| = 2^{mn}$

16 $|P(A) \times P(B)| = 2^{m+n}$

17 $|X \in P(A) : |X| \leq 1| = m + 1$

18 $|P(A \times P(B))| = 2^{m \cdot 2^n}$

19 $|P(P(P(A \times \phi)))| = 4$

20 $|X \subseteq P(A) : |X| \leq 1| = 1 + 2^m$