Relation Homework

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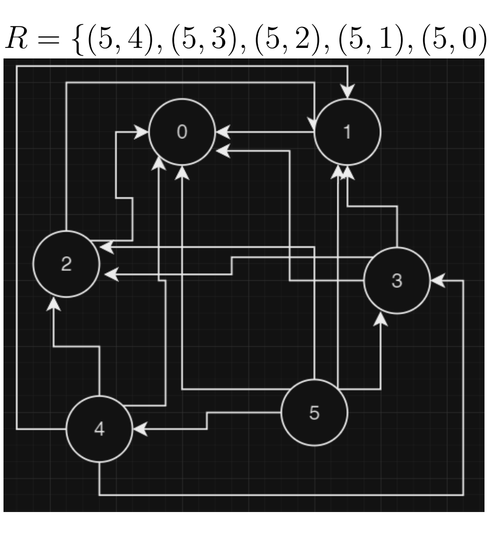
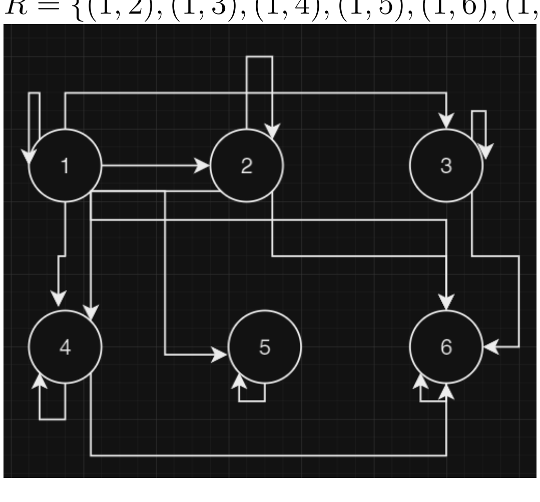
Exercises for Section 16.1

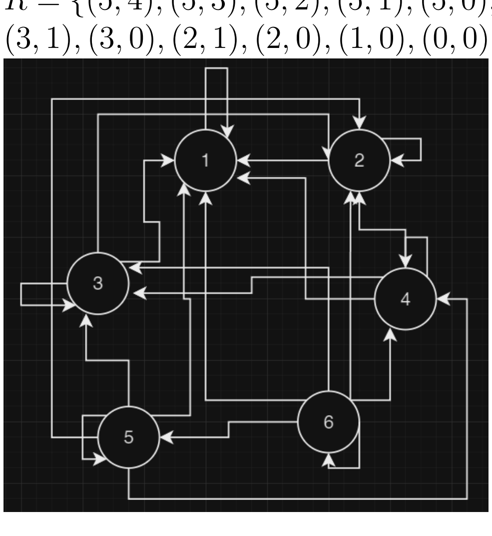
1. $R = \{(5,4),(5,3),(5,2),(5,1),(5,0),(4,3),(4,2),(4,1),(4,0),(3,2),(3,1),(3,0),(2,1),(2,0),(1,0)\}$



2. $R = \{(1,2),(1,3),(1,4),(1,5),(1,6),(1,1),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(4,6),(5,5),(6,6)\}$



3. $R = \{(5,4),(5,3),(5,2),(5,1),(5,0),(4,3),(4,2),(4,1),(4,0),(3,2),$
$(3,1),(3,0),(2,1),(2,0),(1,0),(0,0),(1,1),(2,2),(3,3),(4,4),(5,5)\}$



4. $A = \{0,1,2,3,4,5\}$,
$B = \{(0,0),(0,4),(1,1),(1,3),(2,2),(2,4),(3,3),(3,1),(4,4),(4,0),(4,2),(5,5),(5,1)\}$

5. $A = \{0,1,2,3,4,5\}$, $B = \{(1,2),(2,5),(3,3),(4,3),(4,2),(5,0)\}$

9 $= 2^{36}$

11 $= 2^{|A|x|A|}$

Exercises for Section 16.2

1. it is reflexive, symmetric and transitive.

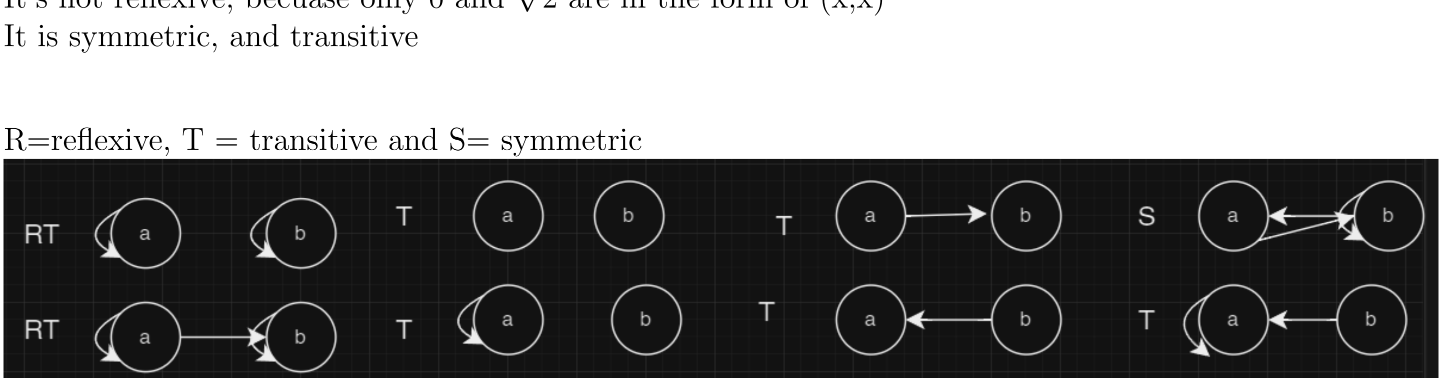
2. It's not reflexive, becuase (a,a) is missing.
It's not symmetric, becuase $(b,a),(c,a)$ is missing.
It is transitive

3. It's not reflexive, becuase $(a,a),(b,b),(c,c)$ is missing.
It's not symmetric, becuase $(b,a),(c,a)$ is missing.
It's not transitive, becuase $(b,b),(c,c)$ is missing.

4. it is reflexive, symmetric and transitive.

5. It's not reflexive, becuase only 0 and $\sqrt{2}$ are in the form of (x,x)
It is symmetric, and transitive

7 R=reflexive, T = transitive and S= symmetric



11 It's reflexive, symmetric and transitive

12

$$
\begin{aligned}
&\text{Reflexive}\\
&x|x, x = xn, n \in \mathbb{Z}\\
&x/x = n, n = 1\\
&x = x(1)\\
&\text{Therefore, it's reflexive}\\
&\text{Transitive, if x|y and y|z, then x|z}\\
&x|y, y = xn, n\mathbb{Z}\\
&y|z, z = ym, m\mathbb{Z}\\
&z = xn(m)\\
&z = x(nm)\\
&\text{Therefore, it's transitive}
\end{aligned}
$$

13

$$
\begin{aligned}
&\text{Reflexive}\\
&xRx, x - x = 0 \in \mathbb{Z}\\
&\text{Therefore, it's reflexive}\\
&\text{Symmetric}\\
&xRy, x - y \in \mathbb{Z}\\
&= -(x - y) = y - x\\
&\text{Therefore, it's symmetric}\\
&\text{Transitive}\\
&xRy \wedge yRz \Rightarrow xRz\\
&x - y \in \mathbb{Z}, y - z \in \mathbb{Z}\\
&(x - y) + (y - z) = x - z \in \mathbb{Z}\\
&\text{Therefore, it's transitive}
\end{aligned}
$$

15 By counterexample, A = 1,2,3, R = (1,1), (1,2), (2,1), (2,2), it's not reflexive.

16

$$
\begin{aligned}
&R = \{(x,y) \in \mathbb{Z}x\mathbb{Z} : xRy \Leftrightarrow x^2 \equiv y^2 (mod\ 4)\}\\
&\text{Reflexive}\\
&xRx, x^2 \equiv x^2 (mod\ 4), 4|(x^2 - x^2)\\
&4|0, true\\
&4|0, x^2 - x^2 = (0)\\
&\text{Therefore, it's reflexive}\\
&\text{Symmetric}\\
&xRy, x^2 \equiv y^2 (mod\ 4), 4|(x^2 - y^2)\\
&4|(y^2 - x^2), true, yRx\\
&\text{Therefore, it's symmetric}\\
&\text{Transitive}\\
&xRy, x^2 \equiv y^2 (mod\ 4), 4|(x^2 - y^2), x^2 = y^2 + 4a, a \in \mathbb{Z}\\
&yRz, y^2 \equiv z^2 (mod\ 4), 4|(y^2 - z^2), y^2 = z^2 + 4b, b \in \mathbb{Z}\\
&x^2 = z^2 + 4a + 4b\\
&x^2 = z^2 + 4(a + b)\\
&a + b = c, c \in \mathbb{Z}, \text{c is a multiple of 4}\\
&4|x^2 - z^2\\
&\text{Therefore, it's transitive}
\end{aligned}
$$

Exercises for Section 16.3

1.

$$
\begin{aligned}
[1] &= \{1\}\\
[2] &= \{2,3\}\\
[3] &= \{3,2\}\\
[4] &= \{4,5,6\}\\
[5] &= \{4,5,6\}\\
[6] &= \{4,5,6\}
\end{aligned}
$$

3 R = (a, d),(b, c),(a, a),(c, c),(b, b),(e, e),(d, d),(d, a),(c, b)

5 R1 = (a, a), (b, b), (a, b), (b, a), R2 = (a, a), (b, b)

6 R1 = (a,a),(b,b),(c,c)
R2 = (a,a),(b,b),(c,c),(a,b),(b,a),(c,a),(a,c),(c,b),(b,c)
R3 = (a,a),(b,b),(c,c), (a,b), (b, a)
R4 = (a,a),(b,b),(c,c), (c, a), (a, c)
R5 = (a,a),(b,b),(c,c), (b, c), (c, b)

7

$$
\begin{aligned}
&\text{Reflexive}\\
&xRx, 3x - 5x = -2x, even\\
&\text{Therefore, it's reflexive}\\
&\text{Symmetric}\\
&xRy, 3x - 5y = 2a\\
&3x - 5y + 8y - 8x = 2a + 8y - 8x\\
&3y - 5x = 2(a + 4y - 4x)\\
&\text{Therefore, it's symmetric}\\
&\text{Transitive}\\
&xRy, 3x - 5y = 2a\\
&yRz, 3y - 5z = 2b\\
&(3x - 5y) + (3y - 5z) = 2a + 2b\\
&3x - 5z = 2a + 2b + 2y\\
&3x - 5z = 2(a + b + y)\\
&\text{Therefore, it's transitive}\\
&\text{Equivalence classes}\\
&[0] = \{x \in \mathbb{Z} : xR0\} = \{x \in \mathbb{Z} : 3x - 0\ even\}\\
&= \{x \in \mathbb{Z} : x\ even\}\\
&[0] = \text{All even integers}\\
&[1] = \{x \in \mathbb{Z} : xR1\} = \{x \in \mathbb{Z} : 3x - 5\ even\}\\
&= \{x \in \mathbb{Z} : x\ odd\}\\
&[1] = \text{All odd integers}
\end{aligned}
$$

9

$$
\begin{aligned}
&\text{Reflexive}\\
&xRx, 4|(4x) = 4|4,\ true\\
&\text{Therefore, it's reflexive}\\
&\text{Symmetric}\\
&xRy, 4|(x + 3y), x + 3y = 4n, n \in \mathbb{Z}\\
&3x + 9y = 12n\\
&y + 3x = 12n - 8y\\
&y + 3x = 4(3n - 2y)\\
&3n - 2y \in \mathbb{Z}, 4|(y + 3x)\\
&\text{Therefore, it's symmetric}\\
&xRy, 4|(x + 3y), x + 3y = 4n, n \in \mathbb{Z}\\
&yRz, 4|(y + 3z), y + 3z = 4m, m \in \mathbb{Z}\\
&x + 3y + y + 3z = 4n + 4m\\
&x + 3z = 4(n + m - y)\\
&n + m - y \in \mathbb{Z}, 4|(x + 3z)\\
&\text{Transitive}\\
&\text{Therefore, it's transitive}\\
&\text{Equivalence classes}\\
&[0] = \{x \in \mathbb{Z} : 4|x\} = \{\cdots, -4, 0, 4, 8, 12, \cdots\}\\
&[1] = \{x \in \mathbb{Z} : 4|x + 3\} = \{\cdots, -3, 1, 5, 9, 13, \cdots\}\\
&[2] = \{x \in \mathbb{Z} : 4|x + 6\} = \{\cdots, -2, 2, 6, 10, 14, \cdots\}\\
&[3] = \{x \in \mathbb{Z} : 4|x + 9\} = \{\cdots, -1, 3, 7, 11, 15, \cdots\}
\end{aligned}
$$

11 This is not true. By the counterexample on the Relation based on Z, where xRy such that 3x-5y is even. It has 2 equivalence classes.

Exercises for Section 16.4

1. a, b, a, b

2. a, b, c, a, b, c, a,b,c, a, b,c, a, c, b

3.

$$
\begin{aligned}
4|(x - 0) &= \{..., -4, 0, 4, 8, ...\}\\
4|(x - 1) &= \{..., -3, 1, 5, 9, ...\}\\
4|(x - 2) &= \{..., -2, 2, 6, 10, ...\}\\
4|(x - 3) &= \{..., -1, 3, 7, 11, ...\}\\
4|(x - 4) &= \{..., 0, 4, 8, ...\} = [0]\\
&[0], [1], [2], [3]
\end{aligned}
$$

5

$$
\begin{aligned}
&xRy, \_ \equiv \_(mod\ n)\\
&2|(x - 0) = \{..., -4, -2, 0, 2, 4, ...\}\\
&2|(x - 1) = \{..., -5, -3, -1, 1, 3...\}
\end{aligned}
$$

Exercises for Section 16.5

1.

+	[0]	[1]
[0]	[0]	[1]
[1]	[1]	[0]

·	[0]	[1]
[0]	[0]	[0]
[1]	[0]	[1]

3

+	[0]	[1]	[2]	[3]
[0]	[0]	[1]	[2]	[3]
[1]	[1]	[2]	[3]	[0]
[2]	[2]	[3]	[0]	[1]
[3]	[3]	[0]	[1]	[2]

·	[0]	[1]	[2]	[3]
[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]
[2]	[0]	[2]	[0]	[2]
[3]	[0]	[3]	[2]	[1]

4

+	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[2]	[3]	[4]	[5]	[0]
[2]	[2]	[2]	[4]	[5]	[0]	[1]
[3]	[3]	[4]	[5]	[0]	[1]	[2]
[4]	[4]	[5]	[0]	[1]	[2]	[3]
[5]	[5]	[0]	[1]	[2]	[3]	[4]

·	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]	[5]
[2]	[0]	[2]	[4]	[0]	[2]	[4]
[3]	[0]	[3]	[0]	[3]	[0]	[3]
[4]	[0]	[4]	[2]	[0]	[1]	[5]
[5]	[0]	[5]	[4]	[3]	[5]	[4]

5

·	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]
[2]	[0]	[2]	[4]	[1]	[3]
[3]	[0]	[3]	[1]	[4]	[2]
[4]	[0]	[4]	[3]	[2]	[1]

Therefore, it's true, either a or b has to be 0

6 No, when [a] = [2] and [b] = [3], it's [0]