

Probabily Homework

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Exercises for Section 5.1

1. A card is randomly selected from a deck of 52 cards. What is the chance that the card is red or a king?
 $|S| = 52$, red = 26, kings = 4, $|E| = 28$
 $P(E) = \frac{28}{52}$
- 5 Toss a dice 5 times in a row. What is the probability that you will get the same number on each roll?
 $|S| = 6x6x6x6x6 = 6^5$, $|E| = \{all1, all2, all3, all4, all5, all6\} = 6$ $p(E) = \frac{6}{6^5}$
- 7 You have a pair of dice, a white one and a black one. Toss them both. What is the probability that they show the same number?
 $|S| = 6x6$, $|E| = \{11, 22, 33, 44, 55, 66\} = 6$
 $p(E) = \frac{6}{6x6}$
- 11 Toss a coin 8 times. Find the probability that the first and last tosses are heads.
 $|S| = 2^8$, $|E| = 1x2x2x2x2x2x2x1 = 1^2x2^6$
 $P(E) = \frac{2^6}{2^8}$
- 13 Five cards are dealt from a shuffled 52-card deck. What is the probability of getting three red cards and two clubs?
 $|S| = C_{52}^5$, $|E| = C_{26}^3xC_{13}^2$
 $P(E) = \frac{C_{26}^3xC_{13}^2}{C_{52}^5}$
- 15 Alice and Bob each randomly pick an integer from 0 to 9. $|S| = 10x10$
 - What is the probability that they pick the same number?
 $|E| = 10$, $P(E) = \frac{10}{10x10}$
 - What is the probability that they pick different numbers?
 $|E^c| = 100 - 10 = 90$, $P(E^c) = \frac{90}{10x10}$

Exercises for Section 5.2

1. A card is taken off the top of a shuffled 52-card deck. What is the probability that it is black or an ace?
 $|S| = 52$, $p(black) = \frac{26}{52}$, $p(ace) = \frac{4}{52}$, $p(A \cap B) = \frac{2}{52}$
 $p(A \cup B) = 4/52 + 26/52 - 2/52 = \frac{28}{52}$
- 3 What is the probability that a 5-card hand dealt off a shuffled 52-card deck contains at least one red card?
 $|S| = C_{52}^5$, $|E| = C_{26}^5$, $p(E^c) = 1 - \frac{C_{26}^5}{C_{52}^5}$
- 7 Two cards are dealt off a shuffled 52-card deck. What is the probability that the cards are both red or both aces?
 $|S| = C_{52}^2$, $|A| = C_{26}^2$, $|B| = C_4^2$, $|A \cap B| = 1$
 $p(A \cup B) = \frac{C_4^2}{C_{52}^2} + \frac{C_{26}^2}{C_{52}^2} - \frac{1}{C_{52}^2}$
- 11 A dice is rolled 5 times. Find the probability that not all of the tosses are even.
 $|S| = 5^5$, $|E| = 3^5$, $|E^c| = 5^5 - 3^5$
 $p(E^c) = \frac{5^5 - 3^5}{5^5}$
- 17 In a shuffled 52-card deck, what is the probability that neither the top nor bottom card is a heart?
 $|S| = 52x51$, $|E| = 39x38$
 $p(E) = \frac{39x38}{52x51}$

Exercises for Section 5.3

- 2 A box contains six tickets: A A B B B E . You remove two tickets, one after the other. What is the probability that both tickets are vowels?
 $|S| = 6x5$, $|E| = 3x2$, $p(E) = \frac{3x2}{6x5}$
- 4 A card is drawn off a 52-card deck. Let A be the event “The card is a heart.” Let B be the event “The card is a queen.” Are these two events independent or dependent?
 $|S| = 52$, $p(A) = 13/52 = 1/4$, $p(B) = 4/52 = 1/13$, $p(A|B) = 1/4$, $p(B|A) = 1/13$
They’re independent
- 7 Say A and B are events with $P(A) = 2/3$, $P(A|B) = 3/4$, and $P(B|A) = 1/2$. Find $p(B)$.
 $p(A) \cdot p(B|A) = p(A \cap B) = p(B) \cdot p(A|B)$
 $2/3 \cdot 1/2 = p(B) \cdot 1/4$
 $p(B) = \frac{4}{9}$
- 9 A box contains 2 red balls, 3 black balls, and 4 white balls. One is removed, and then another is removed. What is the probability that no black balls were drawn?
 $|S| = 9x8$, $|E| = 6x5$, $p(E) = \frac{6x5}{9x8}$
- 11 A coin is flipped 5 times, and there are more tails than heads. What is the probability that the first flip was a tail?
 $|S| = 2^5 = 32$, $A = \text{more tails}$, $B = \text{first tail}$, $p(B|A) = p(A \cap B)/p(A)$
 $|A \cap B| = C_4^2 + C_4^3 + C_4^4 = 11$, $|A| = C_5^3 + C_5^4 + C_5^5 = 16$
 $p(B|A) = \frac{11}{16}$

Exercises for Section 5.5

1. At a certain college, 40% of the students are male, and 60% are female. Also, 20% of the males are smokers, and 10% of the females are smokers. A student is chosen at random. If the student is a smoker, what is the probability that the student is female?

S1 = male, S2 = female, $p(s2|E) = p(s2) \cdot p(e|s2) / p(s2) \cdot p(e|s2) + p(s1) \cdot p(e|s1)$

$$p(s2|E) = \frac{0.6 \cdot 0.10}{0.6 \cdot 0.10 + 0.4 \cdot 0.2}$$

2. At a certain college, 30% of the students are freshmen. Also, 80% of the freshmen live on campus, while only 60% of the non-freshman students live on campus. A student is chosen at random. If the student lives on campus, what is the probability that the student is a freshman?

S1 = non-freshman, S2 = freshmen, $p(s2|E) = p(s2) \cdot p(e|s2) / p(s2) \cdot p(e|s2) + p(s1) \cdot p(e|s1)$

$$p(s2|E) = \frac{0.3 \cdot 0.8}{0.3 \cdot 0.8 + 0.7 \cdot 0.6}$$

3. A jar contains 4 red balls and 5 white balls. A random ball is removed, and then another is removed. If the second ball was red, what is the probability that the first ball was red?

$|S| = 9$, A = 1st is red, B = 2nd is red

$$p(A|B) = \frac{3x4}{8x4} = 3/8$$