Counting Homework

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Exercises for Section 4.2

1. Consider lists made from the letters T, H, E, O, R, Y, with repetition allowed.

   - Length 4 lists: **6x6x6x6**
   - Length 4 lists that begin with T: **1x6x6x6**
   - Length 4 lists that do not begin with T: **5x6x6x6**

3 How many lists of length 3 can be made from the symbols A, B, C, D, E, F if...

   - repetition is allowed: **6x6x6**
   - repetition is not allowed: **6x5x4**
   - repetition is not allowed and the list must contain the letter A:
     (A,,,,) = 1x5x4
     (,,A,,) = 5x1x4
     (,,,,A) = 5x4x1
     = 3(5x4)
   - repetition is allowed and the list must contain the letter A:
     $|U| = 6x6x6, |X^c| = 5x5x5, |X| = (6x6x6) - (5x5x5)$

5 This problem involves 8-digit binary strings such as 10011011 or 00001010 (i.e., 8-digit numbers composed of 0's and 1's).

   - How many such string are there? **2x2x2x2x2x2x2x2**
   - How many such string end in 0? **2x2x2x2x2x2x2x1**
   - How many such string have 1's for their second and fourth digits? **2x1x2x1x2x2x2x2**
   - How many such string have 1's for their second or fourth digits?
     $|A \cup B| = 2x1x2x1x2x2x2x2$
     $|A| = 2x1x2x2x2x2x2x2$
     $|B| = 2x2x2x1x2x2x2x2$
     $= 2^7 + 2^7 - 2^6 = 192$

7 This problem concerns 4-letter codes made from the letters A, B, C, D, ... , Z.

   - How many such codes can be made? **26x26x26x26**
   - How many such codes have no two consecutive letters the same?
     letter 1: any of all 26
     letter 2: 26 - the first = 25
     letter 3: 26 - the second = 25
     letter 4: 26 - the third = 25
     26x25x25x25

9 A new car comes in a choice of five colors, three engine sizes and two transmissions. How many different combinations are there?
Total length is 3, first is 5 colors, second is 3 engine sizes and last is 2 transmissions.
5x3x2

10 A dice is tossed four times in a row. There are many possible outcomes. How many different outcomes are possible?
Length is 4, a dice has numbers from 1 to 6. **6x6x6x6**

Exercises for Section 4.3

1. Five cards are dealt off of a standard 52-card deck and lined up in a row.

   - How many such lineups are there that have at least one red card?
     $|U| = 52x51x50x49x48$, $|X^c| = 26x25x24x23x22$
     (52x51x50x49x48) - (26x25x24x23x22)
   - How many such lineups are there in which the cards are either all black or all hearts?
     They are not black cards that are hearts, so we use the addition principle:
     All black cards: 26x25x24x23x22
     All hearts: 13x12x11x10x9
     (26x25x24x23x22) + (13x12x11x10x9)

3 Five cards are dealt off of a standard 52-card deck and lined up in a row.

   - How many such lineups are there in which all 5 cards are of the same color (i.e., all black or all red)?
     There can't be black cards that are red, so we use addition principle:
     All black cards: 26x25x24x23x22
     All red cards: 26x25x24x23x22
     (26x25x24x23x22) + (26x25x24x23x22)

5 How many integers between 1 and 9999 have no repeated digits?
1-digit: 9, 2-digit: 9x9, 3-digit: 9x9x8, 4-digit: 9x9x8x7
9+(9x9)+(9x9x8)+(9x9x8x7)

   - How many have at least one repeated digit?
     Using the substraction principle:
     $|U| = 9999, |x^c| = 9+(9x9)+(9x9x8)+(9x9x8x7)$
     $|X| =$ **9999 -(9+(9x9)+(9x9x8)+(9x9x8x7))**

7 A password on a certain site must be five characters long, made from letters of the alphabet, and have at least one upper case letter.

   - How many different passwords are there?
     $|U| = 52^5$, $|X^c| = 26^5|$
     $|X| = 52^5 - 26^5$
   - What if there must be a mix of upper and lower case?

Exercises for Section 4.4

3 ffff

Exercises for Section 4.5

5 ffff

Exercises for Section 4.7

1. ffff

Exercises for Section 4.8

1. ffff