Math Induction Homework

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Exercises for Section 14

1.

Step 1:
$n = 1, (n^2 + n)/2 = 1, 1 = 1$
Step 2: Suppose
$k \geq 1, k \in \mathbb{N}$
$s(k) = (k^2 + k)/2$
We want to proof this:
$s(k+1) = ((k+1)^2 + k + 1)/2$
$(1 + 2 + 3 + 4 + ... + k) + (k+1) = (k^2 + k)/2 + (k+1)$
$= (k^2 + k + 2k + 1 + 1)/2$
$= (k^2 + 2k + 1 + (k+1))/2$
$= ((k+1)^2 + k + 1)/2$
Therefore, we proofed the proposition is true

3

Step 1:
$n = 1, (n^2(n+1)^2)/4 = 1, 1 = 1$
Step 2: Suppose
$k \geq 1, k \in \mathbb{N}$
$s(k) = (k^2(k+1)^2)/4$
We want to proof this:
$s(k+1) = ((k+1)^2(k+2)^2)/4$
$1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3$
$= (k^2(k+1)^2)/4 + (k+1)^3$
$= (k^2(k+1)^2 + 4(k+1)^3)/4$
$= ((K+1)^2(k^2 + 4k + 4))/4$
$= ((k+1)^2(k+2)^2)/4$
Therefore, we proofed the proposition is true

4

Step 1:
$n = 1, (n(n+1)(n+2))/3 = 2, 2 = 2$
Step 2: Suppose
$k \geq 1, k \in \mathbb{N}$
$s(k) = (k(k+1)(k+2))/3$
We want to proof this:
$s(k+1) = ((k+1)(k+2)(k+3))/3$
$1x2 + 2x3 + 3x4 + ... + k(k+1) + (k+1)(k+2)$
$= (k(k+1)(k+2))/3 + (k+1)(k+2)$
$= ((k(k+1)(k+2)) + 3(k+1)(k+2))/3$
$= ((k+1)(k+2)(k+3))/3$
Therefore, we proofed the proposition is true

5

Step 1:
$n = 1, 2^{n+1} - 2 = 2, 2 = 2$
Step 2: Suppose
$k \geq 1, k \in \mathbb{N}$
$s(k) = 2^{k+1} - 2$
We want to proof this:
$s(k+1) = 2^{k+2} - 2$
$2^1 + 2^2 + 2^3 + ... + 2^k + 2^{k+1}$
$= 2^{k+1} - 2 + 2^{k+1}$
$= 2 \cdot 2^{k+1} - 2 = 2^{k+2} - 2$
Therefore, we proofed the proposition is true

6

Step 1:
$n = 1, 3 = 4(1) - 1, 3 = 3$
Step 2: Suppose
$k \geq 1, k \in \mathbb{N}$
$$s(k) = \sum_{i=1}^{k}(8i - 5) = 4k^2 - k$$
We want to proof this:
$$s(k+1) = \sum_{i=1}^{k+1}(8i - 5) = 4(k+1)^2 - k - 1 = 4k^2 + 7k + 3$$
$$\sum_{i=1}^{k+1}(8i - 5) = \sum_{i=1}^{k}(8i - 5) + (8k + 3)$$
$$\sum_{i=1}^{k+1}(8i - 5) = 4k^2 - k + 8k + 3$$
$$\sum_{i=1}^{k+1}(8i - 5) = 4k^2 + 7k + 3$$
Therefore, we proofed the proposition is true

7

Step 1:
$n = 1, 3 = ((2)(9))/6, 3 = 3$
Step 2: Suppose
$k \geq 1, k \in \mathbb{N}$
$s(k) = (k(k+1)(2k+7))/6$
We want to proof this:
$s(k+1) = ((k+1)(k+2)(2k+9))/6$
$1x3 + 2x4 + 3x5 + ... + k(k+2) + (k+1)(k+3)$
$= (k(k+1)(2k+7))/6 + (k+1)(k+3)$
$= ((k(k+1)(2k+7)) + 6(k+1)(k+3))/6$
$= ((k+1)(2k^2 + 7k + 6k + 18))/6$
$= ((k+1)(2k^2 + 13k + 18))/6$
$= ((k+1)(k+2)(2k+9))/6$
Therefore, we proofed the proposition is true

11

Step 1:
$n = 0, 3|(0 + 0 + 6), 3|6$
Step 2: Suppose
$k \geq 0, k \in \mathbb{Z}$
$3|(k^3 + 5k + 6)$
$(k^3 + 5k + 6) = 3x, x \in \mathbb{Z}$
We want to proof this:
$((k+1)^3 + 5k + 11) = 3y, y \in \mathbb{Z}$
$= 3|(k^2 + 2k + 1)(k+1) + 5k + 11$
$= 3|k^3 + 2k^2 + k + k^2 + 2k + 1 + 5k + 11$
$= 3|(k^3 + 5k + 6) + 3k^2 + 3k + 6$
$= 3|3x + 3k^2 + 3k + 6$
$= 3|3(x + k^2 + k + 2)$
$= (x + k^2 + k + 2) = y, y \in \mathbb{Z}$
Therefore, we proofed the proposition is true

13

Step 1:
$n = 0, 6|(0) = 0$
Step 2: Suppose
$k \geq 0, k \in \mathbb{Z}$
$k^3 - k = 6x, x \in \mathbb{Z}$
We want to proof this:
$6|(k+1)^3 - (k+1)$
$= (k^2 + 2k + 1)(k+1) - k - 1$
$= (k^3 - k) + 3k^2 + 3k$
$= 6x + 3k^2 + 3k$
$= 6x + 3k(k+1)$
Therefore, we proofed the proposition is true, since k(k+1) is even

Additional Questions

A

$k^2 + 5k + 1 \; even$
$p(k+1) = (k+1)^2 + 5(k+1) + 1 \; even$
$= k^2 + 2k + 1 + 5k + 5 + 1 = k^2 + 7k + 7$
$= (k^2 + 5k + 1) + 2k + 6$
$2k + 6 = 2(k+3), \in \mathbb{Z} \; even$
a) Therefore, we proofed the proposition is true
Case 1: n is even
$n = 2m, m \in \mathbb{Z}$
$n^2 + 5n + 1 = (2m)^2 + 5(2m) + 1 = 4m^2 + 10m + 1$
$= 2(2m^2 + 5m) + 1 \; = \; odd$
Case 2: n is odd
$n = 2m + 1, m \in \mathbb{Z}$
$n^2 + 5n + 1 = (2m+1)^2 + 10m + 6$
$= 4m^2 + 14m + 2$
$= 2(2m^2 + 7m + 2) \in \mathbb{Z} = even$
b) Therefore, when n is even, p(n) is odd and when n is odd, p(n) is even
c) Therefore, $p(n) \Rightarrow p(n+1)$ is True, but it doesn't mean that p(n) is always even.

B

Step 1:
$n = 1, 5 = 3 + 2$
Step 2: Suppose
$k > 1, k \in \mathbb{Z}$
$f(k) = 3k + 2$
We want to proof this:
$f(k+1) = 3(k+1) + 2 = 3k + 5$
$f(k+1) = f(k) + 3$
$= 3k + 5$Therefore, we proofed the proposition is true

C

Step 1:
$n = 0, 3 = 8 - 5$
Step 2: Suppose
$k > 0, k \in \mathbb{Z}$
$h(k) = 2^{k+3} - 5$
We want to proof this:
$h(k+1) = 2^{k+4} - 5$
$h(k+1) = 2h(k) + 5$
$= 2(2^{k+3} - 5) + 5$
$= 2^{k+4} - 5$
Therefore, we proofed the proposition is true

D

Step 1:
$n = 0, 1 = 2/2$
Step 2: Suppose
$k > 0, k \in \mathbb{Z}$
$g(k) = (3^k + 1)/2$
We want to proof this:
$g(k+1) = (3^{k+1} + 1)/2$
$g(k+1) = 3^k + g(k)$
$= 3^k + (3^k + 1)/2$
$= (2 \cdot 3^k + 3^k + 1)/2$
$= (3 \cdot 3^k + 1)/2$
$= (3^{k+1} + 1)/2$
Therefore, we proofed the proposition is true

E

Step 1:
$n = 1, t(1) = 2, 2 = 2$
Step 2: Suppose
$k > 1, k \in \mathbb{Z}$
$t(k) = (k+1)!$
We want to proof this:
$t(k+1) = (k+2)! = (k+1)(k+2)k! t(k+1) = (k+2)t(k)$
$= (k+2)(k+1)!$
$= (k+2)(k+1)k!$
Therefore, we proofed the proposition is true