Function Homework

Michael Padilla

July 6, 2024

Exercises for Section 17.1

1. domain = {0,1,2, 3, 4}, range = {2,3,4}, f(2) = 4, f(1) = 3
2. domain = A, range = 2,3,4,5, f(b) = 3, f(d) = 5
3. {(a,0), (b, 0)}, {(a, 0), (b,1)}, {(a,1), (b,1)}, {(a,1), (b,0)}
4. {(a,0), (b, 0), (c,0)}, {(a, 0), (b,1), (c, 0)}, {(a,0), (b,1), (c,1)}, {(a,0), (b,0), (c,0)}, {(a,1), (b,0), (c, 0)}, {(a,1), (b, 1), (c,0)}, {(a, 1), (b, 0), (c,1)}, {(a, 1), (b,1), (c,1)}
5. {(a, d)}

Exercises for Section 17.2

1. {(1,a), (2,b), (3,b), (4,a)}
2.

$$\begin{aligned} f(a) - f(b) &\neq 0 \\ ln(a) - ln(b) \\ e^{ln(a)} - e^{ln(b)} \\ a - b &\neq 0 \\ \text{Therefore, it's Injective} \\ f(a) &= b \\ ln(a) &= b \\ e^b &= a \\ b &\in \mathbb{Z} \\ \text{Therefore, it's Surjective} \end{aligned}$$

5

$$\begin{aligned} f(a) - f(b) &\neq 0 \\ 2a + 1 - (2b + 1) \\ = 2a + 1 - 2b - 1 \\ = 2a - 2b &\neq 0 \\ \text{Therefore, it's Injective} \\ f(a) &= b,\ odd \\ 2a + 1 &= b \\ 2a &= b - 1 \\ a = \frac{b-1}{2} &\notin \mathbb{Z} \\ \text{Therefore, it's not Surjective} \end{aligned}$$

6

$$\begin{aligned} f(a1, b1) - f(a2, b2) &\neq 0 \\ = (3n_1 - 4m_1) - (3n_2 - 4m_2) \\ = 3n_1 - 4m_1 - 3n_2 + 4m_2 &\neq 0 \\ \text{Therefore, it's Injective} \\ f(a, b) &= c \\ 3a - 4b &= c \\ 3a &= c + 4b \\ a &= (c + 4b)/3 \\ a = c + 4b &\equiv 0(mod3) \\ a = 4b &\equiv -c(mod3) \\ \text{Therefore, it's Surjective} \end{aligned}$$

7

$$\begin{aligned} f(0, 2) = f(-1, 0),\ but\ (0, 2) &\neq (-1, 0) \\ \text{Therefore, it's not Injective} \\ f(a, b) &= k \\ 2b - 4a &= k \\ 2(b - 2a) &= k,\ even \\ \text{Therefore, it's not Surjective, since if b is odd, the result is not longer even} \end{aligned}$$

9

$$\begin{aligned} f(a) - f(b) &\neq 0 \\ (5a + 1)/(a - 2) - (5b + 1)/(b - 2) \\ = (5a + 1)(b - 2) &= (5b + 1)(a - 2) \\ = 5ab - 10a + b - 2 &= 5ab - 10b + a - 2 \\ = -11a + 11b &\neq 0 \\ \text{Therefore, it's Injective} \\ f(a) &= b \\ (5a + 1)/(a - 2) &= b \\ 5a + 1 &= ba - 2b \\ 5a - ba &= -2b - 1 \\ a(5 - b) &= -2b - 1 \\ a &= \frac{-2b-1}{5-b}, \in \mathbb{R} - \{5\} \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \end{aligned}$$

15 There are 7^7 functions. Suppose f is Injective, then there are 7! Injective and Surjective functions. Therefore, there are 7! Bijective functions.

16 There are 7^5 functions. There are not any Surjective functions because $|A| < |B|$. If the are not any Surjective functions, then there are not any Bijective functions. There are $P(7, 5) = 7x6x5x4x3$ Injective functions.

17 There are 2^7 functions. There are not any Injective functions because $|A| > |B|$. If there are not Injective functions, then there are not Bijective functions. There are $2^7 - 2$ Surjective functions because $\{(a, 1),(b, 1),(c, 1),(d, 1),(e, 1),(f, 1),(g, 1)\}, \{(a, 2),(b, 2),(c, 2),(d, 2),(e, 2),(f, 2),(g, 2)\}$ are not Surjective.

1.

$$\begin{aligned} f(a) - f(b) &\neq 0, a \neq b \\ \lfloor a \rfloor - \lfloor b \rfloor &\neq 0 \\ \text{By the counter example, if a is 3.5 and b is 3, the result is 0} \\ \text{Therefore, it's not Injective} \\ f(a) &= b \\ \lfloor a \rfloor &= b \\ b &\in \mathbb{Z} \\ \text{Therefore, it's Surjective} \end{aligned}$$

2.

$$\begin{aligned} f(a) - f(b) &\neq 0, a \neq b \\ \lceil a \rceil - \lceil b \rceil &\neq 0 \\ \text{By the counter example, if a is 3.5 and b is 4, the result is 0} \\ \text{Therefore, it's not Injective} \\ f(a) &= b \\ \lceil a \rceil &= b \\ b &\in \mathbb{Z} \\ \text{Therefore, it's Surjective} \end{aligned}$$

3.

$$\begin{aligned} f(a) - f(b) &\neq 0, a \neq b \\ \lfloor a \rfloor - \lfloor b \rfloor &\neq 0 \\ \text{Therefore, it's Injective} \\ f(a) &= b \\ \lfloor a \rfloor &= b \\ b &\in \mathbb{Z} \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \end{aligned}$$

4.

$$\begin{aligned} f(a) - f(b) &\neq 0, a \neq b \\ \lceil a \rceil - \lceil b \rceil &\neq 0 \\ \text{Therefore, it's Injective} \\ f(a) &= b \\ \lceil a \rceil &= b \\ b &\in \mathbb{Z} \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \end{aligned}$$

5.

$$\begin{aligned} f(a_1, b_1) - f(a_2, b_2) &\neq 0 \\ max(a_1, b_1) - max(a_1, b_1) &\neq 0 \\ \text{Therefore, it's Injective} \\ f(a, b) &= k \\ max(a, b) &= k \\ k &\in \mathbb{R} \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \end{aligned}$$

6.

$$\begin{aligned} f(a) - f(b) &\neq 0 \\ min(a, 100) - min(b, 100) &\neq 0 \\ \text{By a counter example, when a and b are both greater than 100, the result is 0} \\ \text{Therefore, it's not Injective} \\ f(a) &= k \\ min(a, 100) &= k \\ k &\in \mathbb{N} \\ \text{Therefore, it's Surjective} \end{aligned}$$

7.

$$\begin{aligned} f(a) - f(b) &\neq 0 \\ max(1, a - 1) - max(1, b - 1) &\neq 0 \\ \text{By a counter example, when a = 1, b = 2, ther result is 1-1=0} \\ \text{Therefore, it's not Injective} \\ f(a) &= k \\ max(1, a - 1) &= k \\ \text{If a = 1, the result is 1} \\ \text{If a ¿ 1, the result is a-1} \\ k &\in \mathbb{N} \\ \text{Regardless, the result will always be from the set of Natural numbers} \\ \text{Therefore, it's Surjective} \end{aligned}$$

Exercises for Section 17.4

1. (5,1), (6,1), (8,1)

3 $g \circ f$ = (1,1), (2,1), (3,3)
$f \circ g$ = (1,1), (2,2), (3,2)

5 g(f(x)) = $x + 1$
f(g(x)) = $\sqrt[3]{x^3 + 1}$

6 g(f(x)) = $3(\frac{1}{x^2+1}) + 1$
f(g(x)) = $\frac{1}{(3x+2)^2+1}$

7 $g \circ f = (mn + 1, mn + m^2)$
$f \circ g = ((m + 1)(m + n),(m + 1)^2)$

8 $g \circ f = (5(3m - 4n) + 2m + n, 3m - 4n)$
$f \circ g = (3(5m + n) - 4m, 2(5m + n) + m)$

9 $g \circ f = (m + n, m + n)$
$f \circ g = m + m = 2m$

i

$$\begin{aligned} f \circ g \circ h &= f(g(h(x))) \\ &= (\frac{1}{(x^4)^2+1})^3 - 4(\frac{1}{(x^4)^2+1}) \end{aligned}$$

ii

$$\begin{aligned} f \circ h \circ g &= f(h(g(x))) \\ &= ((\frac{1}{x^2+1})^4)^3 - 4((\frac{1}{x^2+1})^4) \end{aligned}$$

iii

$$\begin{aligned} h \circ g \circ f &= h(g(f(x))) \\ (\frac{1}{(x^3-4x)^2+1})^4 \end{aligned}$$

Exercises for Section 17.5

1.

$$\begin{aligned} \text{Injective} \\ f(a) - f(b) &\neq 0 \\ 6 - a - 6 + b = -a + b &\neq 0 \\ \text{Therefore, it's Injective} \\ \text{Surjective} \\ f(a) &= b \\ 6 - a &= b \\ a &= -b + 6 \\ -b + 6 &\in \mathbb{Z} \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \\ \text{Inverse} \\ m &= 6 - n \\ m - 6 &= -n \\ -m + 6 &= n \\ f^{-1}(n) &= -n + 6 \end{aligned}$$

2.

$$\begin{aligned} y &= \frac{5x+1}{x-2} \\ y(x - 2) &= 5x + 1 \\ yx - 2y &= 5x + 1 \\ yx - 5x &= 1 + 2y \\ x(y - 5) &= 1 + 2y \\ x = \frac{1+2y}{y-5} f^{-1}(x) &= \frac{1+2x}{x-5} \end{aligned}$$

3.

$$\begin{aligned} \text{Injective} \\ f(a) - f(b) &\neq 0 \\ 2^a - 2^b &\neq 0 \\ \text{Therefore, it's Injective} \\ \text{Surjective} \\ f(a) &= b \\ 2^a &= b \\ a &= log_2(b) \\ b &\in B \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \\ \text{Inverse} \\ f^{-1}(n) &= log_2(n) \end{aligned}$$

5

$$\begin{aligned} y &= \pi x - e \\ y + e &= \pi x \\ \frac{y+e}{\pi} &= x \\ f^{-1}(x) &= \frac{x+e}{\pi} \end{aligned}$$