Section 2 homework

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Exercises for Section 2.1

A. Write each of the following sets by listing their elements between braces.

1. $\{5x-1 : x \in \mathbb{Z}\} = \{\ldots, -11, -6, -1, 4, 9, \ldots\}$
2. $\{3x+2 : x \in \mathbb{Z}\} = \{\ldots, -4, -1, 2, 5, 8, \ldots\}$
3. $\{x \in \mathbb{Z} : -2 \leq x < 7\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
4. $\{x \in \mathbb{N} : -2 < x \leq 7\} = \{1, 2, 3, 4, 5, 6, 7\}$
5. $\{x \in \mathbb{R} : x^2 = 3\} = \{-\sqrt{3}, \sqrt{3}\}$
6. $\{x \in \mathbb{R} : x^2 = 9\} = \{-3, 3\}$
7. $\{x \in \mathbb{R} : x^2 + 5x = -6\} = \{-3, -2\}$

11 $\{x \in \mathbb{Z} : |x| < 5\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

12 $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2, -1, 0, 1, 2\}$

13 $\{x \in \mathbb{Z} : |6x| < 5\} = \{0\}$

14 $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$

B. Write each of the following sets in set-builder notation.

17 $\{2, 4, 8, 16, 32, 64 \ldots\} = \{2 \cdot 2^x : x \geq 0, x \in \mathbb{Z}\}$

19 $\{\ldots, -6, -3, 0, 3, 6, 9, 12, 15, \ldots\} = \{3x : x \in \mathbb{Z}\}$

24 $\{-4, -3, -2, -1, -0, 1, 2\} = \{x : -4 \leq x \leq 2, x \in \mathbb{Z}\}$

25 $\{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\} = \{2^x : x \in \mathbb{Z}\}$

26 $\{\ldots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \ldots\} = \{3^x : x \in \mathbb{Z}\}$

C. Find the following cardinalities of the following sets.

29 $\{\{1\}, \{2, \{3, 4\}\}, \phi\} = 3$

30 $\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\phi\}\} = 5$

31 $\{\{\{1\}, \{2, \{3, 4\}\}, \phi\}\} = 1$

32 $\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\phi\}\}\} = 1$

33 $\{x \in \mathbb{Z} : |x| < 10\} = 19$

34 $\{x \in \mathbb{N} : |x| < 10\} = 9$

35 $\{x \in \mathbb{Z} : x^2 < 10\} = 7$

36 $\{x \in \mathbb{N} : x^2 < 10\} = 3$

37 $\{x \in \mathbb{N} : x^2 < 0\} = 0$

38 $\{x \in \mathbb{N} : 5x \leq 20\} = 4$

Exercises for Section 2.2

A Write out the indicated sets by listing their elements between braces.

2 Suppose $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$.

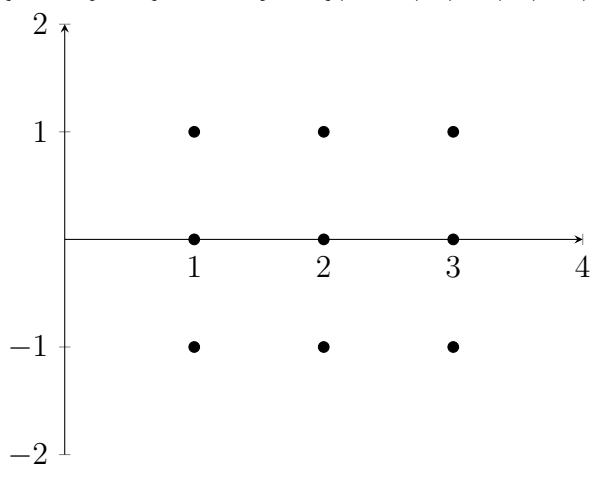
* $A \times B = \{(\pi, 0), (\pi, 1), (e, 0), (e, 1), (0, 0), (0, 1)\}$
* $B \times A = \{(0, \pi), (0, e), (0, 0), (1, \pi), (1, e), (1, 0)\}$
* $A \times A =$
  $\{(\pi, \pi), (\pi, e), (\pi, 0), (e, \pi), (e, e), (e, 0), (0, \pi), (0, e), (0, 0)\}$
* $B \times B = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$
* $A \times \phi = \{(\pi), (e), (0)\}$
* $(A \times B) \times B =$
  $\{((\pi, 0), 0), ((\pi, 0), 1), ((\pi, 1), 0), ((\pi, 1), 1), ((e, 0), 0), ((e, 0), 1),$
  $((e, 1), 0), ((e, 1), 1), ((0, 0), 0), ((0, 0), 1), ((0, 1), 0), ((0, 1), 1)\}$
* $A \times (B \times B) =$
  $\{(\pi, (0, 0)), (\pi, (0, 1)), (\pi, (1, 0)), (\pi, (1, 1)),$
  $(e, (0, 0)), (e, (0, 1)), (e, (1, 0)), (e, (1, 1)),$
  $(0, (0, 0)), (0, (0, 1)), (0, (1, 0)), (0, (1, 1))\}$
* $A \times B \times B =$
  $\{(\pi, 0, 0), (\pi, 0, 1), (\pi, 1, 0), (\pi, 1, 1),$
  $(e, 0, 0), (e, 0, 1), (e, 1, 0), (e, 1, 1),$
  $(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1)\}$

6 $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\} = \{(0, 1), (1, 1)\}$

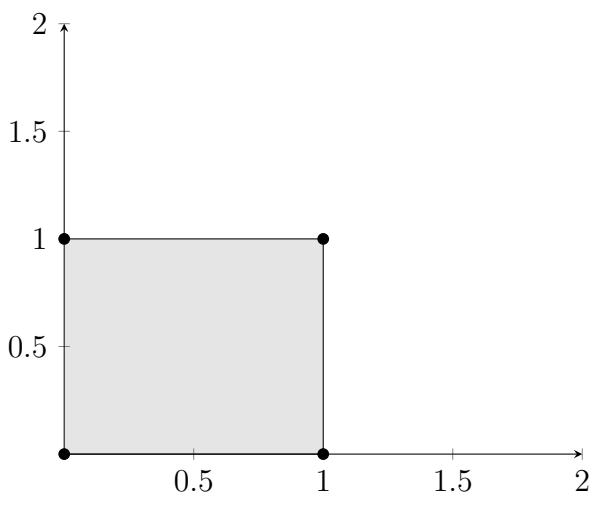
8 $\{0, 1\}^4 =$
$\{(((0, 0), 0), 0), (((0, 0), 0), 1), (((0, 0), 1), 0), (((0, 0), 1), 1), (((0, 1), 0), 0),$
$(((0, 1), 0), 1), (((0, 1), 1), 0), (((0, 1), 1), 1), (((1, 0), 0), 0), (((1, 0), 0), 1),$
$(((1, 0), 1), 0), (((1, 0), 1), 1), (((1, 1), 0), 0), (((1, 1), 0), 1),$
$(((1, 1), 1), 0), (((1, 1), 1), 1)\}$

B Sketch these Cartesian products on the $x - y$ plane $\mathbb{R}^2$ (or $\mathbb{R}^3$for the last two.)

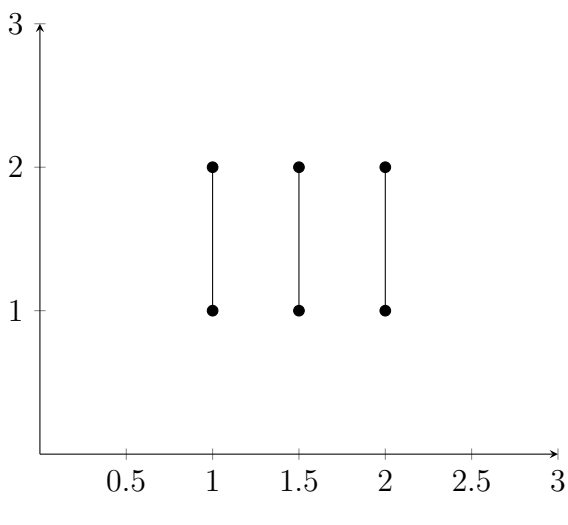
9 $\{1, 2, 3\} \times \{-1, 0, 1\} = \{(1, -1), (1, 0), (1, 1), (2, -1), (2, 0), (2, 1), (3, -1), (3, 0), (3, 1)\}$



11 $[0, 1] \times [0, 1]$



13 $\{1, 1.5, 2\} \times [1, 2]$



15 $\{1\} \times [0, 1]$