

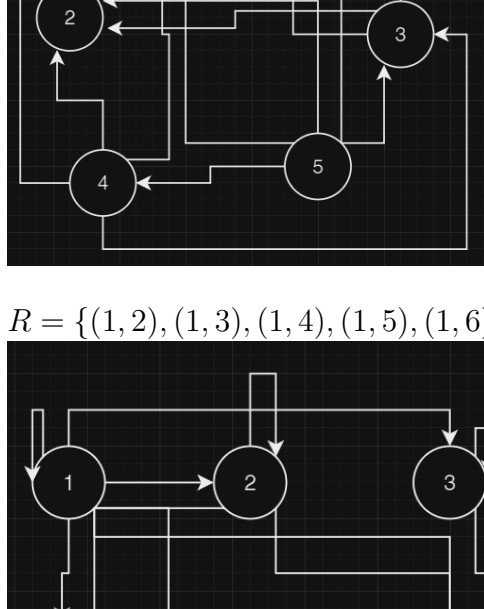
# Relation Homework

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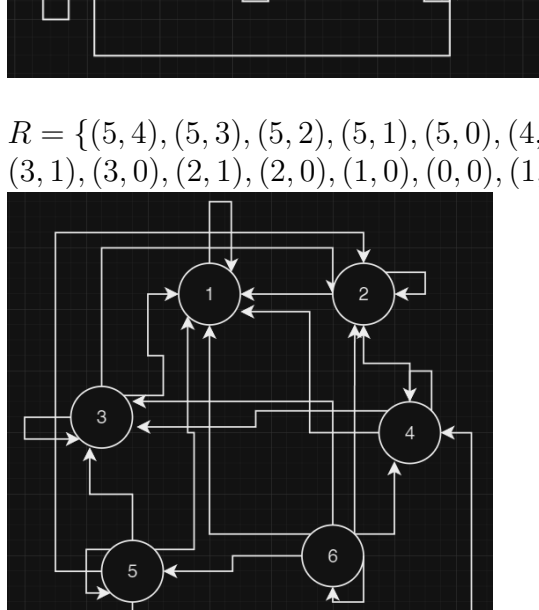
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## Exercises for Section 16.1

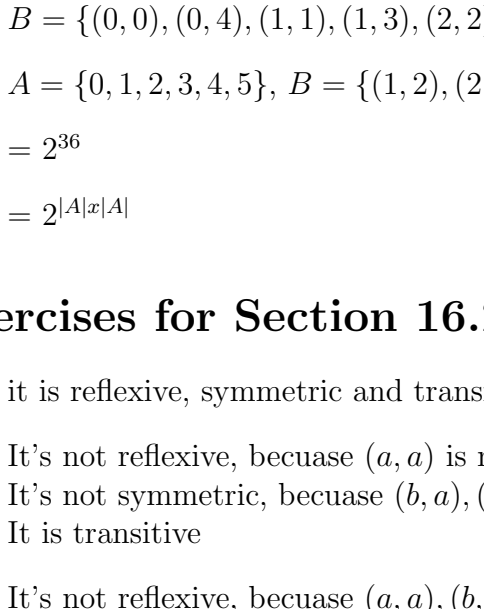
1.  $R = \{(5, 4), (5, 3), (5, 2), (5, 1), (5, 0), (4, 3), (4, 2), (4, 1), (4, 0), (3, 2), (3, 1), (3, 0), (2, 1), (2, 0), (1, 0)\}$



2.  $R = \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 1), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (4, 6), (5, 5), (6, 6)\}$



3.  $R = \{(5, 4), (5, 3), (5, 2), (5, 1), (5, 0), (4, 3), (4, 2), (4, 1), (4, 0), (3, 2), (3, 1), (3, 0), (2, 1), (2, 0), (1, 0), (0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$



4.  $A = \{0, 1, 2, 3, 4, 5\}$ ,  
 $B = \{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (2, 4), (3, 3), (3, 1), (4, 4), (4, 0), (4, 2), (5, 5), (5, 1)\}$

5.  $A = \{0, 1, 2, 3, 4, 5\}$ ,  $B = \{(1, 2), (2, 5), (3, 3), (4, 3), (4, 2), (5, 0)\}$

9  $= 2^{36}$

11  $= 2^{|A||x|A|}$

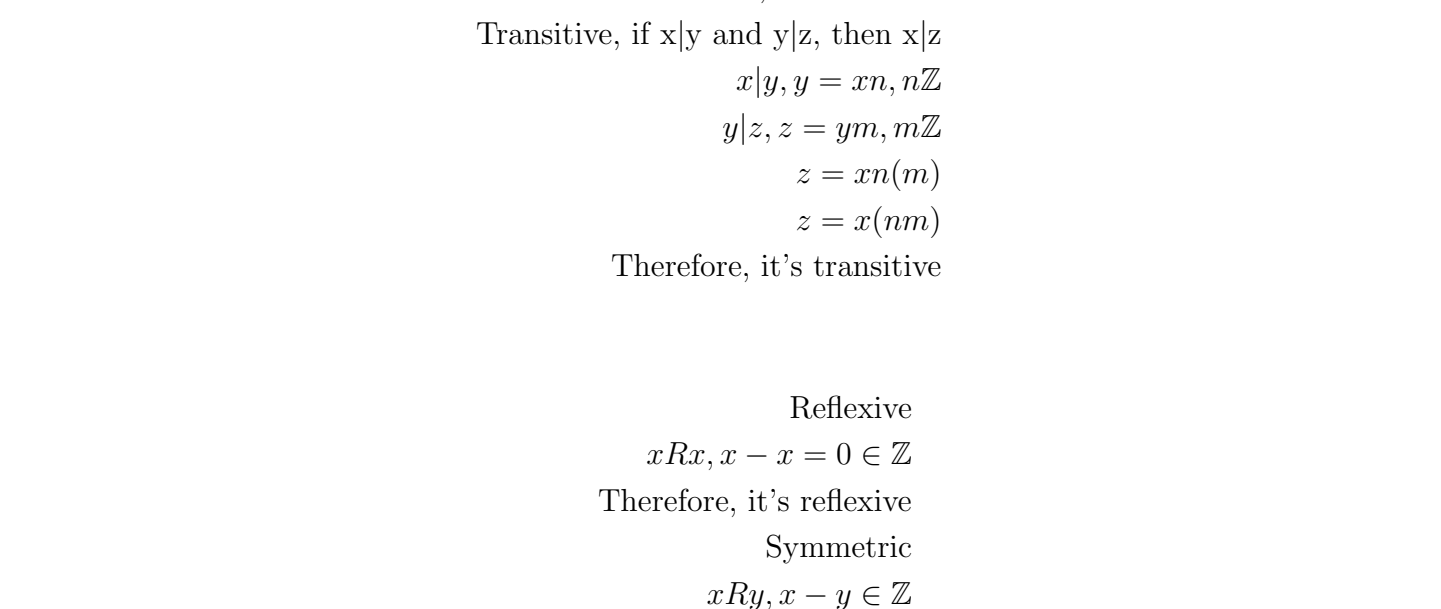
## Exercises for Section 16.2

1. it is reflexive, symmetric and transitive.  
2. It's not reflexive, because  $(a, a)$  is missing.  
It's not symmetric, because  $(b, a), (c, a)$  is missing.  
It is transitive  
3. It's not reflexive, because  $(a, a), (b, b), (c, c)$  is missing.  
It's not symmetric, because  $(b, a), (c, a)$  is missing.  
It's not transitive, because  $(b, b), (c, c)$  is missing.

4. it is reflexive, symmetric and transitive.

5. It's not reflexive, because only 0 and  $\sqrt{2}$  are in the form of  $(x, x)$   
It is symmetric, and transitive

- 7 R=reflexive, T = transitive and S= symmetric



- 11 It's reflexive, symmetric and transitive

- 12

Reflexive

$$x|x, x = xn, n \in \mathbb{Z}$$

$$x/x = n, n = 1$$

$$x = x(1)$$

Therefore, it's reflexive

Transitive, if  $x|y$  and  $y|z$ , then  $x|z$

$$x|y, y = xn, n \in \mathbb{Z}$$

$$y|z, z = ym, m \in \mathbb{Z}$$

$$z = xn(m)$$

$$z = x(nm)$$

Therefore, it's transitive

- 13

Reflexive

$$xRx, x - x = 0 \in \mathbb{Z}$$

Therefore, it's reflexive

Symmetric

$$xRy, x - y \in \mathbb{Z}$$

$$= -(x - y) = y - x$$

Therefore, it's symmetric

Transitive

$$xRy \wedge yRz \Rightarrow xRz$$

$$x - y \in \mathbb{Z}, y - z \in \mathbb{Z}$$

$$(x - y) + (y - z) = x - z \in \mathbb{Z}$$

Therefore, it's transitive

- 15 By counterexample, A = 1,2,3, R = (1,1), (1,2), (2,1), (2,2), it's not reflexive.

- 16

$$R = \{(x, y) \in \mathbb{Z}x\mathbb{Z} : xRy \Leftrightarrow x^2 \equiv y^2 \pmod{4}\}$$

Reflexive

$$xRx, x^2 \equiv x^2 \pmod{4}, 4|(x^2 - x^2)$$

$$4|0, true$$

$$4|0, x^2 - x^2 = (0)$$

Therefore, it's reflexive

Symmetric

$$xRy, x^2 \equiv y^2 \pmod{4}, 4|(x^2 - y^2)$$

$$4|(y^2 - x^2), true, yRx$$

Therefore, it's symmetric

Transitive

$$xRy, x^2 \equiv y^2 \pmod{4}, 4|(x^2 - y^2), x^2 = y^2 + 4a, a \in \mathbb{Z}$$

$$yRz, y^2 \equiv z^2 \pmod{4}, 4|(y^2 - z^2), y^2 = z^2 + 4b, b \in \mathbb{Z}$$

$$x^2 = z^2 + 4a + 4b$$

$$x^2 = z^2 + 4(a + b)$$

$$a + b = c, c \in \mathbb{Z}, c \text{ is a multiple of } 4$$

$$4|x^2 - z^2$$

Therefore, it's transitive

## Exercises for Section 16.3

- 1.

$$[1] = \{1\}$$

$$[2] = \{2, 3\}$$

$$[3] = \{3, 2\}$$

$$[4] = \{4, 5, 6\}$$

$$[5] = \{4, 5, 6\}$$

$$[6] = \{4, 5, 6\}$$

- 3 R = (a, d),(b, c),(a, a),(c, c),(b, b),(e, e),(d, d),(d, a),(c, b)

- 5 R1 = (a, a), (b, b), (a, b), (b, a), R2 = (a, a), (b, b)

- 6 R1 = (a,a),(b,b),(c,c)  
R2 = (a,a),(b,b),(c,c),(a,b),(b,a),(c,a),(a,c),(c,b),(b,c)  
R3 = (a,a),(b,b),(c,c), (a,b), (b, a)  
R4 = (a,a),(b,b),(c,c), (c, a), (a, c)  
R5 = (a,a),(b,b),(c,c), (b, c), (c, b)

- 7

Reflexive

$$xRx, 3x - 5x = -2x, even$$

Therefore, it's reflexive

Symmetric

$$xRy, 3x - 5y = 2a$$

$$3x - 5y + 8y - 8x = 2a + 8y - 8x$$

$$3y - 5x = 2(a + 4y - 4x)$$

Therefore, it's symmetric

Transitive

$$xRy, 3x - 5y = 2a$$

$$yRz, 3y - 5z = 2a$$

$$(3x - 5y) + (3y - 5z) = 2a + 2b$$

$$3x - 5z = 2a + 2b + 2y$$

$$3x - 5z = 2(a + b + y)$$

Therefore, it's transitive

Equivalence classes

$$[0] = \{x \in \mathbb{Z} : xR0\} = \{x \in \mathbb{Z} : 3x - 0 \text{ even}\}$$

$$= \{x \in \mathbb{Z} : x \text{ even}\}$$

$$[0] = \text{All even integers}$$

$$[1] = \{x \in \mathbb{Z} : xR1\} = \{x \in \mathbb{Z} : 3x - 5 \text{ even}\}$$

$$= \{x \in \mathbb{Z} : x \text{ odd}\}$$

$$[1] = \text{All odd integers}$$

- 9

Reflexive

$$xRx, 4|(4x) = 4|4, true$$

Therefore, it's reflexive

Symmetric

$$xRy, 4|(x + 3y), x + 3y = 4n, n \in \mathbb{Z}$$

$$3x + 9y = 12n$$

$$y + 3x = 12n - 8y$$

$$y + 3x = 4(3n - 2y)$$

$$3n - 2y \in \mathbb{Z}, 4|(y + 3x)$$

Therefore, it's symmetric

$$xRy, 4|(x + 3y), x + 3y = 4n, n \in \mathbb{Z}$$

$$yRz, 4|(y + 3z), y + 3z = 4m, m \in \mathbb{Z}$$

$$x + 3y + y + 3z = 4n + 4m$$

$$x + 3z = 4(n + m - y)$$

$$n + m - y \in \mathbb{Z}, 4|(x + 3z)$$

Therefore, it's transitive

Equivalence classes

$$[0] = \{x \in \mathbb{Z} : 4|x\} = \{\dots, -4, 0, 4, 8, 12, \dots\}$$

$$[1] = \{x \in \mathbb{Z} : 4|x + 3\} = \{\dots, -3, 1, 5, 9, 13, \dots\}$$

$$[2] = \{x \in \mathbb{Z} : 4|x + 6\} = \{\dots, -2, 2, 6, 10, 14, \dots\}$$

$$[3] = \{x \in \mathbb{Z} : 4|x + 9\} = \{\dots, -1, 3, 7, 11, 15, \dots\}$$

- 11 This is not true. By the counterexample on the Relation based on Z, where  $xRy$  such that  $3x-5y$  is even. It has 2 equivalence classes.

## Exercises for Section 16.4

1. a, b, a, b

- 2.

- 3.

- 5

## Exercises for Section 16.5

- 1.

- 3

- 4

- 5

- 6