

## Section 2 homework

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### Exercises for Section 2.1

A. Write each of the following sets by listing their elements between braces.

- $\{5x - 1 : x \in \mathbb{Z}\} = \{\dots, -11, -6, -1, 4, 9, \dots\}$
- $\{3x + 2 : x \in \mathbb{Z}\} = \{\dots, -4, -1, 2, 5, 8, \dots\}$
- $\{x \in \mathbb{Z} : -2 \leq x < 7\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
- $\{x \in \mathbb{N} : -2 < x \leq 7\} = \{1, 2, 3, 4, 5, 6, 7\}$
- $\{x \in \mathbb{R} : x^2 = 3\} = \{-\sqrt{3}, \sqrt{3}\}$
- $\{x \in \mathbb{R} : x^2 = 9\} = \{-3, 3\}$
- $\{x \in \mathbb{R} : x^2 + 5x = -6\} = \{-3, -2\}$
- $\{x \in \mathbb{Z} : |x| < 5\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$
- $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2, -1, 0, 1, 2\}$
- $\{x \in \mathbb{Z} : |6x| < 5\} = \{0\}$
- $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$

B. Write each of the following sets in set-builder notation.

- $\{2, 4, 8, 16, 32, 64, \dots\} = \{2 \cdot 2^x : x \geq 0, x \in \mathbb{Z}\}$
- $\{\dots, -6, -3, 0, 3, 6, 9, 12, 15, \dots\} = \{3x : x \in \mathbb{Z}\}$
- $\{\dots, -4, -3, -2, -1, 0, 1, 2\} = \{x : -4 \leq x \leq 2, x \in \mathbb{Z}\}$
- $\{\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \dots\} = \{2^x : x \in \mathbb{Z}\}$
- $\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\} = \{3^x : x \in \mathbb{Z}\}$

C. Find the following cardinalities of the following sets.

- $\{\{1\}, \{2, \{3, 4\}\}, \phi\} = 3$
- $\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\phi\}\} = 5$
- $\{\{\{1\}\}, \{2, \{3, 4\}\}, \phi\} = 1$
- $\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\phi\}\}\} = 1$
- $\{x \in \mathbb{Z} : |x| < 10\} = 19$
- $\{x \in \mathbb{N} : |x| < 10\} = 9$
- $\{x \in \mathbb{Z} : x^2 < 10\} = 7$
- $\{x \in \mathbb{N} : x^2 < 10\} = 3$
- $\{x \in \mathbb{N} : x^2 < 0\} = 0$
- $\{x \in \mathbb{N} : 5x \leq 20\} = 4$

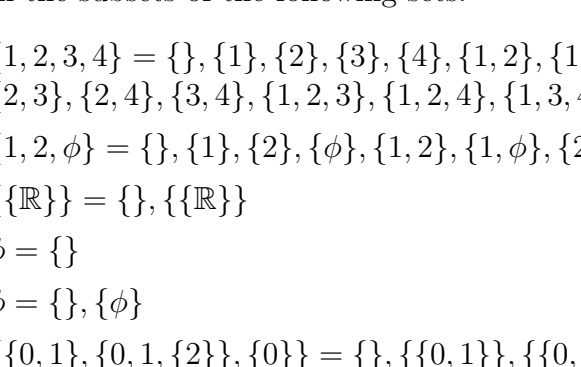
### Exercises for Section 2.2

A Write out the indicated sets by listing their elements between braces.

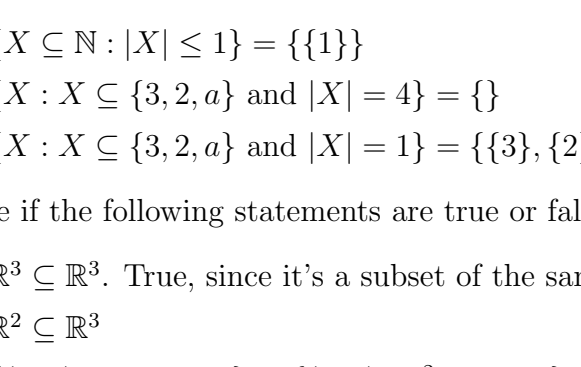
- Suppose  $A = \{\pi, e, 0\}$  and  $B = \{0, 1\}$ .
  - $A \times B = \{(\pi, 0), (\pi, 1), (e, 0), (e, 1), (0, 0), (0, 1)\}$
  - $B \times A = \{(0, \pi), (0, e), (0, 0), (1, \pi), (1, e), (1, 0)\}$
  - $A \times A = \{(\pi, \pi), (\pi, e), (\pi, 0), (e, \pi), (e, e), (e, 0), (0, \pi), (0, e), (0, 0)\}$
  - $B \times B = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$
  - $A \times \phi = \{(\pi), (e), (0)\}$
  - $(A \times B) \times B = \{((\pi, 0), 0), ((\pi, 0), 1), ((\pi, 1), 0), ((\pi, 1), 1), ((e, 0), 0), ((e, 0), 1), ((e, 1), 0), ((e, 1), 1), ((0, 0), 0), ((0, 0), 1), ((0, 1), 0), ((0, 1), 1)\}$
  - $A \times (B \times B) = \{(\pi, (0, 0)), (\pi, (0, 1)), (\pi, (1, 0)), (\pi, (1, 1)), (e, (0, 0)), (e, (0, 1)), (e, (1, 0)), (e, (1, 1)), (0, (0, 0)), (0, (0, 1)), (0, (1, 0)), (0, (1, 1))\}$
  - $A \times B \times B = \{(\pi, 0, 0), (\pi, 0, 1), (\pi, 1, 0), (\pi, 1, 1), (e, 0, 0), (e, 0, 1), (e, 1, 0), (e, 1, 1), (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1)\}$
- $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\} = \{(0, 1), (1, 1)\}$
- $\{0, 1\}^4 = \{((0, 0), 0, 0), ((0, 0), 0, 1), ((0, 0), 1, 0), ((0, 0), 1, 1), ((0, 1), 0, 0), ((0, 1), 0, 1), ((0, 1), 1, 0), ((0, 1), 1, 1), ((1, 0), 0, 0), ((1, 0), 0, 1), ((1, 0), 1, 0), ((1, 0), 1, 1), ((1, 1), 0, 0), ((1, 1), 0, 1), ((1, 1), 1, 0), ((1, 1), 1, 1)\}$

B Sketch these Cartesian products on the  $x - y$  plane  $\mathbb{R}^2$  (or  $\mathbb{R}^3$  for the last two.)

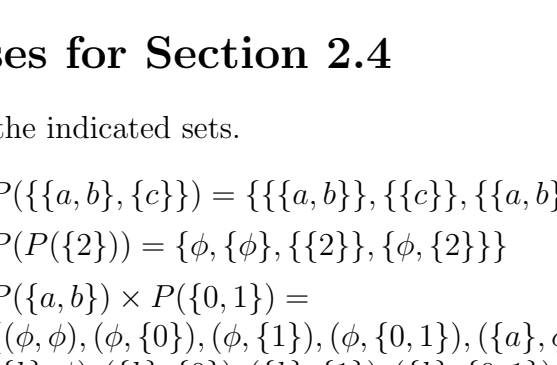
- $\{1, 2, 3\} \times \{-1, 0, 1\} = \{(1, -1), (1, 0), (1, 1), (2, -1), (2, 0), (2, 1), (3, -1), (3, 0), (3, 1)\}$



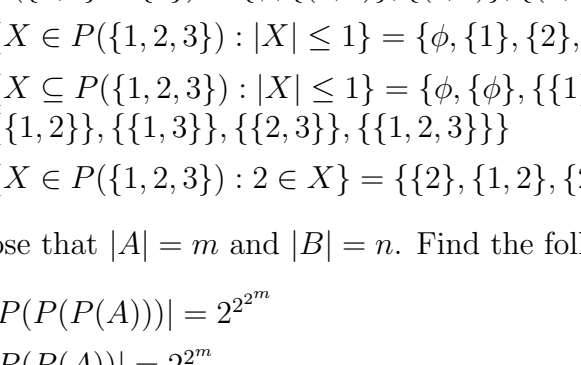
- $[0, 1] \times [0, 1]$



- $\{1, 1.5, 2\} \times [1, 2]$



- $\{1\} \times [0, 1]$



### Exercises for Section 2.3

A List all the subsets of the following sets.

- $\{1, 2, 3, 4\} = \{\}, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}$
- $\{1, 2, \phi\} = \{\}, \{1\}, \{2\}, \{\phi\}, \{1, 2\}, \{1, \phi\}, \{2, \phi\}, \{1, 2, \phi\}$
- $\{\{\mathbb{R}\}\} = \{\}, \{\{\mathbb{R}\}\}$
- $\phi = \{\}$
- $\phi = \{\}, \{\phi\}$
- $\{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\} = \{\}, \{\{0, 1\}\}, \{\{0, 1, \{2\}\}\}, \{\{0\}\}, \{\{0, 1\}, \{0, 1, \{2\}\}\}, \{\{0, 1\}, \{0\}\}, \{\{0, 1, \{2\}\}, \{0\}\}, \{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\}$

B Write out the following sets by listing their elements between braces.

- $\{X \subseteq \mathbb{N} : |X| \leq 1\} = \{\{1\}\}$
- $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 4\} = \{\}$
- $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 1\} = \{\{3\}, \{2\}, \{a\}\}$

C Decide if the following statements are true or false. Explain.

- $\mathbb{R}^3 \subseteq \mathbb{R}^3$ . True, since it's a subset of the same set.
- $\mathbb{R}^2 \subseteq \mathbb{R}^3$
- $\{(x, y) : x - 1 = 0\} \subseteq \{(x, y) : x^2 - x = 0\}$
- $\{(x, y) : x^2 - x = 0\} \subseteq \{(x, y) : x - 1 = 0\}$

### Exercises for Section 2.4

A Find the indicated sets.

- $P(\{\{a, b\}, \{c\}\}) = \{\{\{a, b\}\}, \{\{c\}\}, \{\{a, b\}, \{c\}\}, \phi\}$
- $P(P(\{2\})) = \{\phi, \{2\}, \{\phi, \{2\}\}\}$
- $P(\{a, b\}) \times P(\{0, 1\}) = \{(\phi, \phi), (\phi, \{0\}), (\phi, \{1\}), (\{a\}, \phi), (\{a\}, \{0\}), (\{a\}, \{1\}), (\{a\}, \{0, 1\}), (\{b\}, \phi), (\{b\}, \{0\}), (\{b\}, \{1\}), (\{b\}, \{0, 1\}), (\{a, b\}, \phi), (\{a, b\}, \{0\}), (\{a, b\}, \{1\}), (\{a, b\}, \{0, 1\})\}$
- $P(\{a, b\} \times \{0\}) = \{\phi, \{(a, 0)\}, \{(b, 0)\}, \{(a, 0), (b, 0)\}\}$
- $\{X \in P(\{1, 2, 3\}) : |X| \leq 1\} = \{\phi, \{1\}, \{2\}, \{3\}\}$
- $\{X \subseteq P(\{1, 2, 3\}) : |X| \leq 1\} = \{\phi, \{1\}, \{2\}, \{3\}\}, \{\{1, 2\}\}, \{\{1, 3\}\}, \{\{2, 3\}\}, \{\{1, 2, 3\}\}$
- $\{X \in P(\{1, 2, 3\}) : 2 \in X\} = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

B Suppose that  $|A| = m$  and  $|B| = n$ . Find the following cardinalities.

- $|P(P(P(A)))| = 2^{2^{2^m}}$
- $|P(P(A))| = 2^{2^m}$
- $|P(A \times B)| = 2^{mn}$
- $|P(A) \times P(B)| = 2^{m+n}$
- $|X \in P(A) : |X| \leq 1| = m + 1$
- $|P(A \times P(B))| = 2^{m \cdot 2^n}$
- $|P(P(P(A \times \phi)))| = 4$
- $|X \subseteq P(A) : |X| \leq 1| = 1 + 2^m$

### Exercises for Section 2.5

1. Suppose  $A = \{4, 3, 6, 7, 1, 9\}$ ,  $B = \{5, 6, 8, 4\}$ ,  $C = \{5, 8, 4\}$ . Find:

- $A \cup B = \{4, 3, 5, 6, 7, 1, 9, 8\}$
- $A \cap B = \{4, 6\}$
- $A - B = \{3, 7, 1, 9\}$
- $A - C = \{3, 6, 7, 1, 9\}$
- $B - A = \{5, 8\}$
- $A \cap C = \{4\}$
- $B \cap C = \{5, 4, 8\}$
- $B \cup C = \{5, 4, 8, 6\}$
- $C - B = \{\}$

3. Suppose  $A = \{0, 1\}$ ,  $B = \{1, 2\}$ . Find:

- $(A \times B) \cap (B \times B) = \{(1, 1), (1, 2)\}$
- $(A \times B) \cup (B \times B) = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$
- $(A \times B) - (B \times B) = \{(0, 1), (0, 2)\}$
- $(A \cap B) \times A = \{(1, 0), (1, 1)\}$
- $(A \times B) \cap B = \{\}$
- $P(A) \cap P(B) = \{\phi, \{1\}\}$
- $P(A) - P(B) = \{\{0\}, \{0, 1\}\}$
- $P(A \cap B) = \{\phi, \{1\}\}$
- $P(A \times B) = \{\phi, \{(0, 1)\}, \{(0, 2)\}, \{(1, 1)\}, \{(1, 2)\}, \{(0, 1), (0, 2)\}, \{(0, 1), (1, 1)\}, \{(0, 1), (1, 2)\}, \{(1, 1), (1, 2)\}, \{(0, 2), (1, 1)\}, \{(0, 2), (1, 2)\}, \{(1, 1), (1, 2)\}, \{(0, 1), (0, 2), (1, 1)\}, \{(0, 1), (0, 2), (1, 2)\}, \{(1, 1), (1, 2)\}, \{(0, 1), (0, 2), (1, 1), (1, 2)\}\}$

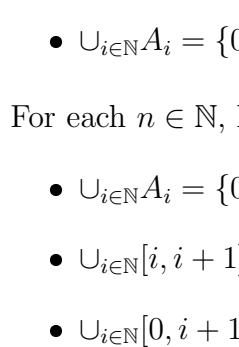
### Exercises for Section 2.6

1. Let  $A = \{4, 3, 6, 7, 1, 9\}$ ,  $B = \{5, 6, 8, 4\}$  have universal set  $U = \{0, 1, 2, \dots, 10\}$ . Find:

- $A^c = \{0, 2, 5, 8, 10\}$
- $B^c = \{0, 1, 2, 3, 7, 9, 10\}$
- $A \cap A^c = \{\}$
- $A \cup A^c = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- $A - A^c = \{1, 3, 4, 6, 7, 9\}$
- $A - B^c = \{4, 6\}$
- $A^c - B^c = \{5, 8\}$
- $A^c \cap B = \{5, 8\}$
- $A^c \cap B^c = \{0, 1, 2, 3, 4, 6, 7, 9, 10\}$

### Exercises for Section 2.7

1. Draw a Venn diagram for  $A^c$



2. Draw a Venn diagram for  $B - A$



3. Draw a Venn diagram for  $(A - B) \cap C$



4. Draw a Venn diagram for  $(A \cup B) - C$



5. Draw a Venn diagram for  $A \cup (B \cap C)$  and  $(A \cup B) \cap (A \cup C)$ .



**They are the same.**

6. Draw a Venn diagram for  $A \cap (B \cup C)$  and  $(A \cap B) \cup (A \cap C)$ .



**They are the same.**

7. Suppose sets A and B are in a universal set U. Draw Venn diagrams for  $A \cap B^c$  and  $A^c \cup B^c$ .



**They are the same.**

8. Suppose sets A and B are in a universal set U. Draw Venn diagrams for  $A \cup B^c$  and  $A^c \cap B^c$ .



**They are the same.**

9. Draw a Venn diagram for  $(A \cap B) - C$ .



10. Draw a Venn diagram for  $(A - B) \cup C$ .



### Exercises for summation

- Write  $1 + 2 + 3 + \dots + 10$  using sigma notation.  
 $\sum_{k=1}^{10} k$
- Write  $1^2 + 4 + 9 + \dots + 49$  using sigma notation.  
 $\sum_{k=1}^7 k^2$
- $\sum_{k=2}^4 k^3 = 99$
- $\sum_{k=2}^6 (k^2 + k) = 110$
- $\sum_{j=-3}^3 (j^2 + j) = 40$
- $\sum_{i=1}^5 (i^2 + 2^i) = 65$
- $\sum_{j=4}^7 j = 840$
- $\prod_{j=2}^5 (k - 1) = 24$
- $\prod_{i=1}^3 \frac{i+1}{i} = 4$
- $\sum_{i=1}^3 \sum_{j=1}^2 (j) = 9$
- $\sum_{i=1}^3 \sum_{j=1}^2 (i) = 12$
- $\prod_{i=1}^2 \prod_{j=3}^6 (i - j) = 1440$
- $\sum_{i=1}^2 \prod_{j=4}^6 (i - j) = -84$
- $\prod_{i=2}^4 \prod_{j=1}^3 (i + j) = 3240$

- Recall from precalculus that a polynomial of degree  $n$  has the form  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ , where  $a_n \neq 0$ . Express the form of a polynomial using summation notation.  
 $= \sum_{i=0}^n (a_n - i x^{n-i})$

### Exercises for section 2.8

1. Suppose  $A_1 = \{a, b, d, e, g, f\}$ ,  $A_2 = \{a, b, c, d\}$ ,  $A_3 = \{b, d, a\}$ ,  $A_4 = \{a, b, h\}$ .

- $\cup_{i=1}^4 A_i = \{a, b, c, d, e, g, f, h\}$
- $\cap_{i=1}^4 A_i = \{a, b\}$

3. For each  $n \in \mathbb{N}$ , let  $A_n = \{0, 1, 2, 3, \dots, n\}$ .

- $\cup_{i \in \mathbb{N}} A_i = \{0, 1, 2, 3, \dots, i\}$
- $\cap_{i \in \mathbb{N}} A_i = \{0, 1\}$

4. For each  $n \in \mathbb{N}$ , let  $A_n = \{-2n, 0, 2n\}$ .

- $\cup_{i \in \mathbb{N}} A_i = \{0, -2, 2, -4, -4, \dots, -2i, 2i\}$
- $\cap_{i \in \mathbb{N}} A_i = \{0\}$

5.  $\cup_{i \in \mathbb{N}} [i, i + 1] = [1, \infty)$

- $\cap_{i \in \mathbb{N}} [i, i + 1] = \phi$

6.  $\cup_{i \in \mathbb{N}} [0, i + 1] = [0, \infty)$

- $\cap_{i \in \mathbb{N}} [0, i + 1] = [0, 2]$

9.  $\cup_{X \in P(\mathbb{N})} X = \mathbb{N}$

- $\cap_{X \in P(\mathbb{N})} X = \phi$