Function Homework

Michael Padilla

July 5, 2024

Exercises for Section 17.1

1. domain = {0,1,2, 3, 4}, range = {2,3,4}, f(2) = 4, f(1) = 3
2. domain = A, range = 2,3,4,5, f(b) = 3, f(d) = 5
3. {(a,0), (b, 0)}, {(a, 0), (b,1)}, {(a,1), (b,1)}, {(a,1), (b,0)}
4. {(a,0), (b, 0), (c,0)}, {(a, 0), (b,1), (c, 0)}, {(a,0), (b,1), (c,1)}, {(a,0), (b,0), (c,0)}, {(a,1), (b,0), (c, 0)}, {(a,1), (b, 1), (c,0)}, {(a, 1), (b, 0), (c,1)}, {(a, 1), (b,1), (c,1)}
5. {(a, d)}

Exercises for Section 17.2

1. {(1,a), (2,b), (3,b), (4,a)}
2.

$$\begin{gathered} f(a) - f(b) \neq 0 \\ ln(a) - ln(b) \\ e^{ln(a)} - e^{ln(b)} \\ a - b \neq 0 \\ \text{Therefore, it's Injective} \\ f(a) = b \\ ln(a) = b \\ e^b = a \\ b \in \mathbb{Z} \\ \text{Therefore, it's Surjective} \end{gathered}$$

5

$$\begin{gathered} f(a) - f(b) \neq 0 \\ 2a + 1 - (2b + 1) \\ = 2a + 1 - 2b - 1 \\ = 2a - 2b \neq 0 \\ \text{Therefore, it's Injective} \\ f(a) = b,\ odd \\ 2a + 1 = b \\ 2a = b - 1 \\ a = \frac{b-1}{2} \notin \mathbb{Z} \\ \text{Therefore, it's not Surjective} \end{gathered}$$

6

$$\begin{gathered} f(a1, b1) - f(a2, b2) \neq 0 \\ = (3n_1 - 4m_1) - (3n_2 - 4m_2) \\ = 3n_1 - 4m_1 - 3n_2 + 4m_2 \neq 0 \\ \text{Therefore, it's Injective} \\ f(a, b) = c \\ 3a - 4b = c \\ 3a = c + 4b \\ a = (c + 4b)/3 \\ a = c + 4b \equiv 0(mod3) \\ a = 4b \equiv -c(mod3) \\ \text{Therefore, it's Surjective} \end{gathered}$$

7

$$\begin{gathered} f(0, 2) = f(-1, 0),\ but\ (0, 2) \neq (-1, 0) \\ \text{Therefore, it's not Injective} \\ f(a, b) = k \\ 2b - 4a = k \\ 2(b - 2a) = k,\ even \\ \text{Therefore, it's not Surjective, since if b is odd, the result is not longer even} \end{gathered}$$

9

$$\begin{gathered} f(a) - f(b) \neq 0 \\ (5a + 1)/(a - 2) - (5b + 1)/(b - 2) \\ = (5a + 1)(b - 2) = (5b + 1)(a - 2) \\ = 5ab - 10a + b - 2 = 5ab - 10b + a - 2 \\ = -11a + 11b \neq 0 \\ \text{Therefore, it's Injective} \\ f(a) = b \\ (5a + 1)/(a - 2) = b \\ 5a + 1 = ba - 2b \\ 5a - ba = -2b - 1 \\ a(5 - b) = -2b - 1 \\ a = \frac{-2b-1}{5-b}, \in \mathbb{R} - \{5\} \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \end{gathered}$$

15 There are 7^7 functions. Suppose f is Injective, then there are 7! Injective and Surjective functions. Therefore, there are 7! Bijective functions.

16

17 There are 2^7 functions.

1.
2.
3.
4.
5.
6.
7.

Exercises for Section 17.4

1. (5,1), (6,1), (8,1)

3 $g \circ f$ = (1,1), (2,1), (3,3)
$f \circ g$ = (1,1), (2,2), (3,2)

5 g(f(x)) = $x + 1$
f(g(x)) = $\sqrt[3]{x^3 + 1}$

6 g(f(x)) = $3(\frac{1}{x^2+1}) + 1$
f(g(x)) = $\frac{1}{(3x+2)^2 + 1}$

7 $g \circ f = (mn + 1, mn + m^2)$
$f \circ g = ((m + 1)(m + n), (m + 1)^2)$

8 $g \circ f = (5(3m - 4n) + 2m + n, 3m - 4n)$
$f \circ g = (3(5m + n) - 4m, 2(5m + n) + m)$

9 $g \circ f = (m + n, m + n)$
$f \circ g = m + m = 2m$

i

$$\begin{gathered} f \circ g \circ h = f(g(h(x))) \\ = (\frac{1}{(x^4)^2 + 1})^3 - 4(\frac{1}{(x^4)^2 + 1}) \end{gathered}$$

ii

$$\begin{gathered} f \circ h \circ g = f(h(g(x))) \\ = ((\frac{1}{x^2 + 1})^4)^3 - 4((\frac{1}{x^2 + 1})^4) \end{gathered}$$

iii

$$\begin{gathered} h \circ g \circ f = h(g(f(x))) \\ (\frac{1}{(x^3 - 4x)^2 + 1})^4 \end{gathered}$$

Exercises for Section 17.5

1.

$$\begin{gathered} \text{Injective} \\ f(a) - f(b) \neq 0 \\ 6 - a - 6 + b = -a + b \neq 0 \\ \text{Therefore, it's Injective} \\ \text{Surjective} \\ f(a) = b \\ 6 - a = b \\ a = -b + 6 \\ -b + 6 \in \mathbb{Z} \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \\ \text{Inverse} \\ m = 6 - n \\ m - 6 = -n \\ -m + 6 = n \\ f^{-1}(n) = -n + 6 \end{gathered}$$

2.

$$\begin{gathered} y = \frac{5x+1}{x-2} \\ y(x - 2) = 5x + 1 \\ yx - 2y = 5x + 1 \\ yx - 5x = 1 + 2y \\ x(y - 5) = 1 + 2y \\ x = \frac{1+2y}{y-5} f^{-1}(x) = \frac{1+2x}{x-5} \end{gathered}$$

3.

$$\begin{gathered} \text{Injective} \\ f(a) - f(b) \neq 0 \\ 2^a - 2^b \neq 0 \\ \text{Therefore, it's Injective} \\ \text{Surjective} \\ f(a) = b \\ 2^a = b \\ a = log_2(b) \\ b \in B \\ \text{Therefore, it's Surjective} \\ \text{Therefore, it's Bijective} \\ \text{Inverse} \\ f^{-1}(n) = log_2(n) \end{gathered}$$

5

$$\begin{gathered} y = \pi x - e \\ y + e = \pi x \\ \frac{y+e}{\pi} = x \\ f^{-1}(x) = \frac{x+e}{\pi} \end{gathered}$$