Probabily Homework

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Exercises for Section 5.1

1. A card is randomly selected from a deck of 52 cards. What is the chance that the card is red or a king?
   $|S| = 52$, red $= 26$, kings $= 4$, $|E| = 28$
   $P(E) = \dfrac{28}{52}$

5 Toss a dice 5 times in a row. What is the probability that you will get the same number on each roll?
$|S| = 6x6x6x6x6 = 6^5$, $|E| = \{all1, all2, all3, all4, all5, all6\} = 6$ $p(E) = \dfrac{6}{6^5}$

7 You have a pair of dice, a white one and a black one. Toss them both. What is the probability that they show the same number?
$|S| = 6x6, |E| = \{11, 22, 33, 44, 55, 66\} = 6$
$p(E) = \dfrac{6}{6x6}$

11 Toss a coin 8 times. Find the probability that the first and last tosses are heads.
$|S| = 2^8, |E| = 1x2x2x2x2x2x2x1 = 1^2x2^6$
$P(E) = \dfrac{2^6}{2^8}$

13 Five cards are dealt from a shuffled 52-card deck. What is the probability of getting three red cards and two clubs?
$|S| = C^5_{52}, |E| = C^3_{26}xC^2_{13}$
$P(E) = \dfrac{C^3_{26}xC^2_{13}}{C^5_{52}}$

15 Alice and Bob each randomly pick an integer from 0 to 9. $|S| = 10x10$

- What is the probability that they pick the same number?
  $|E| = 10, P(E) = \dfrac{10}{10x10}$
- What is the probability that they pick different numbers?
  $|E^c| = 100 - 10 = 90, P(E^c) = \dfrac{90}{10x10}$

Exercises for Section 5.2

1. A card is taken off the top of a shuffled 52-card deck. What is the probability that it is black or an ace?
   $|S| = 52, p(black) = \dfrac{26}{52}, p(ace) = \dfrac{4}{52}, p(A \cap B) = \dfrac{2}{52}$
   $p(A \cup B) = 4/52 + 26/52 - 2/52 = \dfrac{28}{52}$

3 What is the probability that a 5-card hand dealt off a shuffled 52-card deck contains at least one red card?
$|S| = C^5_{52}, |E| = C^5_{26}, p(E^c) = 1 - \dfrac{C^5_{26}}{C^5_{52}}$

7 Two cards are dealt off a shuffled 52-card deck.What is the probability that the cards are both red or both aces?
$|S| = C^2_{52}, |A| = C^2_{26}, |B| = C^2_4, |A \cap B| = 1$
$p(A \cup B) = \dfrac{C^2_4}{C^2_{52}} + \dfrac{C^2_{26}}{C^2_{52}} - \dfrac{1}{C^2_{52}}$

11 A dice is rolled 5 times. Find the probability that not all of the tosses are even.
$|S| = 6^5, |E| = 3^5, |E^c| = 5^5 - 3^5$
$p(E^c) = \dfrac{6^5 - 3^5}{6^5}$

17 In a shuffled 52-card deck, what is the probability that neither the top nor bottom card is a heart?
$|S| = 52x51, |E| = 39x38$
$p(E) = \dfrac{39x38}{52x51}$

Exercises for Section 5.3

2 A box contains six tickets: A A B B B E . You remove two tickets, one after the other. What is the probability that both tickets are vowels?
$|S| = 6x5, |E| = 3x2, p(E) = \dfrac{3x2}{6x5}$

4 A card is drawn off a 52-card deck. Let A be the event "The card is a heart." Let B be the event "The card is a queen." Are these two events independent or dependent?
$|S| = 52, p(A) = 13/52 = 1/4, p(B) = 4/521/13, p(A|B) = 1/4, p(B|A) = 1/13$
They're independent

7 Say A and B are events with $P(A) = 2/3, P(A|B) = 3/4$, and $P(B|A) = 1/2$. Find $p(B)$.
$p(A) \cdot p(B|A) = p(A \cap B) = p(B) \cdot p(A|B)$
$2/3 \cdot 1/2 = p(B) \cdot 1/4$
$p(B) = \dfrac{4}{9}$

9 A box contains 2 red balls, 3 black balls, and 4 white balls. One is removed, and then another is removed. What is the probability that no black balls were drawn?
$|S| = 9x8, |E| = 6x5, p(E) = \dfrac{6x5}{9x8}$

11 A coin is flipped 5 times, and there are more tails than heads. What is the probability that the first flip was a tail?
$|S| = 2^5 = 32, A = more\ tails, B = first\ tail, p(B|A) = p(A \cap B)/p(A)$
$|A \cap B| = C^2_4 + C^3_4 + C^4_4 = 11, |A| = C^3_5 + C^4_5 + C^5_5 = 16$
$p(B|A) = \dfrac{11}{16}$

Exercises for Section 5.5

1. At a certain college, 40% of the students are male, and 60% are female. Also, 20% of the males are smokers, and 10% of the females are smokers. A student is chosen at random. If the student is a smoker, what is the probability that the student is female?
   S1 = male, S2 = female, $p(s2|E) = p(s2) \cdot p(e|s2)/p(s2) \cdot p(e|s2) + p(s1) \cdot p(e|s1)$
   $p(s2|E) = \dfrac{0.6 \cdot 0.10}{0.6 \cdot 0.10 + 0.4 \cdot 0.2}$

2. At a certain college, 30% of the students are freshmen. Also, 80% of the freshmen live on campus, while only 60% of the non-freshman students live on campus. A student is chosen at random. If the student lives on campus, what is the probability that the student is a freshman?
   S1 = non-freshman, S2 = freshmen, $p(s2|E) = p(s2) \cdot p(e|s2)/p(s2) \cdot p(e|s2) + p(s1) \cdot p(e|s1)$
   $p(s2|E) = \dfrac{0.3 \cdot 0.8}{0.3 \cdot 0.8 + 0.7 \cdot 0.6}$

3. A jar contains 4 red balls and 5 white balls. A random ball is removed, and then another is removed. If the second ball was red, what is the probability that the first ball was red?
   $|S| = 9x8$, A = 1st is red, B = 2nd is red
   $p(A|B) = \dfrac{3x4}{8x4} = 3/8$