Counting Homework

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Exercises for Section 4.2

1. Consider lists made from the letters T, H, E, O, R, Y, with repetition allowed.
    - Length 4 lists: **6x6x6x6**
    - Length 4 lists that begin with T: **1x6x6x6**
    - Length 4 lists that do not begin with T: **5x6x6x6**

3 How many lists of length 3 can be made from the symbols A, B, C, D, E, F if...
- repetition is allowed: **6x6x6**
- repetition is not allowed: **6x5x4**
- repetition is not allowed and the list must contain the letter A:
  (A,_,_) = 1x5x4
  (_,A,_) = 5x1x4
  (_,_,A) = 5x4x1
  = 3(5x4)
- repetition is allowed and the list must contain the letter A:
  $|U| = 6x6x6, |X^c| = 5x5x5, |X| = (6x6x6) - (5x5x5)$

5 This problem involves 8-digit binary strings such as 10011011 or 00001010 (i.e., 8-digit numbers composed of 0's and 1's).
- How many such string are there? **2x2x2x2x2x2x2x2**
- How many such string end in 0? **2x2x2x2x2x2x2x1**
- How many such string have 1's for their second and fourth digits? **2x1x2x1x2x2x2x2**
- How many such string have 1's for their second or fourth digits?
  $|A \cup B| = 2x1x2x1x2x2x2x2$
  $|A| = 2x1x2x2x2x2x2x2$
  $|B| = 2x2x2x1x2x2x2x2$
  $= 2^7 + 2^7 - 2^6 = 192$

7 This problem concerns 4-letter codes made from the letters A, B, C, D, ... , Z.
- How many such codes can be made? **26x26x26x26**
- How many such codes have no two consecutive letters the same?
  letter 1: any of all 26
  letter 2: 26 - the first = 25
  letter 3: 26 - the second = 25
  letter 4: 26 - the third = 25
  26x25x25x25

9 A new car comes in a choice of five colors, three engine sizes and two transmissions. How many different combinations are there?
Total length is 3, first is 5 colors, second is 3 engine sizes and last is 2 transmissions.
5x3x2

10 A dice is tossed four times in a row. There are many possible outcomes. How many different outcomes are possible?
Length is 4, a dice has numbers from 1 to 6. **6x6x6x6**

Exercises for Section 4.3

1. Five cards are dealt off of a standard 52-card deck and lined up in a row.
    - How many such lineups are there that have at least one red card?
      $|U|$ =52x51x50x49x48, $|X^c|$ = 26x25x24x23x22
      (52x51x50x49x48) - (26x25x24x23x22)
    - How many such lineups are there in which the cards are either all black or all hearts?
      They are not black cards that are hearts, so we use the addition principle:
      All black cards: 26x25x24x23x22
      All hearts: 13x12x11x10x9
      (26x25x24x23x22) + (13x12x11x10x9)

3 Five cards are dealt off of a standard 52-card deck and lined up in a row.
- How many such lineups are there in which all 5 cards are of the same color (i.e., all black or all red)?
  There can't be black cards that are red, so we use addition principle:
  All black cards: 26x25x24x23x22
  All red cards: 26x25x24x23x22
  (26x25x24x23x22) + (26x25x24x23x22)

5 How many integers between 1 and 9999 have no repeated digits?
1-digit: 9, 2-digit: 9x9, 3-digit: 9x9x8, 4-digit: 9x9x8x7
9+(9x9)+(9x9x8)+(9x9x8x7)
- How many have at least one repeated digit?
  Using the substraction principle:
  $|U| = 9999, |x^c| =$ 9+(9x9)+(9x9x8)+(9x9x8x7)
  $|X|$ = **9999 -(9+(9x9)+(9x9x8)+(9x9x8x7))**

7 A password on a certain site must be five characters long, made from letters of the alphabet, and have at least one upper case letter.
- How many different passwords are there?
  $|U| = 52^5, |X^c = 26^5|$
  $|X| = 52^5 - 26^5$
- What if there must be a mix of upper and lower case?
  $|U| = 52^5, |X^c = 26^5 \cdot 2|$
  $|X| = 52^5 - (26^5 + 26^5)$

Exercises for Section 4.4

3 How many 5-digit positive integers are there in which there are no repeated digits and all digits are odd?
Odds: 1,3,5,7,9, total 5 numbers. **=5!**

5 Using only pencil and paper, find the value of $\frac{120!}{118!}$
$$\frac{120 \cdot 119 \cdot \cancel{118!}}{\cancel{118!}} = 120 \cdot 119$$

9 How many permutations of the letters A, B, C, D, E, F, G are there in which the three letters ABC appear consecutively, in alphabetical order?
n = 7 - ABC = 5
= 5!

13 How many lists of length six (with no repetition) can be made from the 26 letters of the English alphabet?
$P(26, 6)$ = **26x25x24x23x22x21**

15 In a club of 15 people, we need to choose a president, vice-president, secretary, and treasurer. In how many ways can this be done?
$P(15, 4)$ = **15x14x13x12**

17 Three people in a group of ten line up at a ticket counter to buy tickets. How many lineups are possible?
$P(10, 3)$ = **10x9x8**

Exercises for Section 4.5

5 How many 16-digit binary strings contain exactly seven 1's? (Examples of such strings include 01110000111100 and 0011001100110010, etc.)
$C^7_{16} = \frac{16!}{7!9!}$

6 $|X \in P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9) : |X| = 4|$
$= C^4_{10} = \frac{10!}{4!6!}$

7 $|X \in P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9) : |X| < 4|$
We can have a ϕ subset. Therefore $= C^0_{10} + C^1_{10} + C^2_{10} + C^3_{10}$

17 How many 10-digit binary strings are there that have exactly four 1's or exactly five 1's?
$C^4_{10} + C^5_{10}$
- How many do not have exactly four 1's or exactly five 1's?
  $|U| = 2^{10}$
  $2^{10} - C^4_{10} - C^5_{10}$

19 A 5-card poker hand is called a flush if all cards are the same suit. How many different flushes are there?
There are 13 cards in each suit and there are 4 suits. There are $C^5_{13} \cdot 4$ different flushes.

Exercises for Section 4.7

1. At a certain university 523 of the seniors are history majors or math majors (or both). There are 100 senior math majors, and 33 seniors are majoring in both history and math. How many seniors are majoring in history?
   $|A \cup B| = 523, |A|(math) = 100, |B|(history) =?, |A \cap B|(both) = 33$
   $523 = 100 + |B| - 33$
   $523 - 100 + 33 = |B|$
   $|B| = 456$

3 How many 4-digit positive integers are there that are even or contain no 0's?
$|A \cup B| =?, |A|(even) = 9x10x10x5, |B|(no\ 0s) = 9x9x9x9, |A \cap B|(both) = 9x9x9x4$
$|A \cup B| = (9x10x10x5) + (9x9x9x9) - (9x9x9x4)$

13 How many 8-digit binary strings end in 1 or have exactly four 1's?
$|A \cup B| =?, |A|(end\ 1) = 2x2x2x2x2x2x2x1, |B|(four\ 1s) = C^4_8, |A \cap B| = 1 \cdot C^3_7$
$|A \cup B| = (2x2x2x2x2x2x2x1) + (C^4_8) - (C^3_7)$

15 How many 10-digit binary strings begin in 1 or end in 1? $|A \cup B| =?, |A|(beging\ 1) = 1 \cdot 2^9, |B|(end\ 1) = 2^9 \cdot 1, |A \cap B|(both) = 1 \cdot 2^8 \cdot 1$
$|A \cup B| = 2^9 + 2^9 - 2^8$

Exercises for Section 4.8

1. How many 10-element multisets can be made from the symbols $\{1, 2, 3, 4\}$
   Stars = 10, bars = 3, total = 13
   $C^3_{13} = \frac{13!}{10!3!}$

7 In how many ways can you place 20 identical balls into five different boxes?
Stars = 20, bars = 4, total = 24
$C^4_{24} = \frac{24!}{20!4!}$

9 A bag contains 50 pennies, 50 nickels, 50 dimes and 50 quarters. You reach in and grab 30 coins. How many different outcomes are possible?
Stars = 30, bars = 3(4 total boxes), total = 33
$C^{30}_{33} = \frac{33!}{30!3!}$

11 How many integer solutions does the equation $w + x + y + z = 100$ have if $w \geq 4, x \geq 2, y \geq 0 and z \geq 0$?
$\geq$ means at least, so $w = 4, x = 2, y = 0, z = 0$
Stars = 100 - 6 = 94, bars = 3, total = 97
$C^{94}_{97} = \frac{97!}{94!3!}$

20 You distribute 25 identical pieces of candy among five children. In how many ways can this be done?
Stars = 25, bars = 4, total = 29
$C^4_{29} = \frac{29!}{4!25!}$