Proof Homework

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Exercises for Section 8

1. $x = 2m, m \in \mathbb{Z}$
   $x^2 = 4m^2 = 2(2m^2), n = 2m^2, n \in \mathbb{Z}$
   Therefore x^2 is a even integer.

3 $a = 2m + 1, m \in \mathbb{Z}$
   $a^2 + 3a + 5 = (2m + 1)^2 + 6m + 3 + 5$
   $= 4m^2 + 4m + 9 + 6m = 4m^2 + 10m + 9$
   $= 4m^2 + 10m + 8 + 1 = 2(2m^2 + 5m + 4) + 1$
   Where $(2m^2 + 5m + 4) = n \in \mathbb{Z}$, Therefore $a^2 + 3a + 5 = 2n + 1$, and is an odd integer.

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$$\begin{aligned} x &= 2m + 1, m \in \mathbb{Z} \\ y &= 2n + 1, n \in \mathbb{Z} \\ xy &= (2m + 1)(2n + 1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1 \\ 2mn + m + n &= b \in \mathbb{Z} \\ &= 2b + 1, \text{Therefore it's odd} \end{aligned}$$

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$$\begin{aligned} x &= 2m, m \in \mathbb{Z} \\ y &= 2n, n \in \mathbb{Z} \\ xy &= (2m)(2n) \\ &= 4mn \\ &= 2(2mn) \\ 2mn &= b \in \mathbb{Z} \\ &= 2b, \text{Therefore it's even} \end{aligned}$$

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$$\begin{aligned} b &= ac, c \in \mathbb{Z} \\ b^2 &= a^2c^2, c^2 = d \in \mathbb{Z} \\ b^2 &= a^2d, \text{Therefore} a^2 | b^2 \end{aligned}$$

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$$\begin{aligned} b &= am, m \in \mathbb{Z} \\ d &= cn, n \in \mathbb{Z} \\ bd &= (ac)(mn), \text{Therefore} ac | bd \end{aligned}$$

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$$\begin{aligned} &\text{Case 1, even n} \\ n &= 2a, a \in \mathbb{Z} \\ n^2 + 3n + 4 &= 4a^2 + 6a + 4 \\ &= 2(2a^2 + 3a + 2), (2a^2 + 3a + 2) = c, c \in \mathbb{Z} \\ &= 2c, \text{Therefore, } n^2 + 3n + 4 \text{ is even} \end{aligned}$$

$$\begin{aligned} &\text{Case 2, odd n} \\ n &= 2b + 1, b \in \mathbb{Z} \\ n^2 + 3n + 4 &= 4b^2 + 4b + 1 + 6b + 3 + 4 \\ &= 4b^2 + 10b + 8 = 2(2b^2 + 5b + 4) \\ 2b^2 + 5b + 4 &= d \in \mathbb{Z}, = 2d \\ &\text{Therefore, } n^2 + 3n + 4 \text{ is even} \end{aligned}$$

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$$\begin{aligned} &\text{Case 1: Odd parity} \\ a &= 2x + 1, x \in \mathbb{Z} \\ b &= 2y + 1, y \in \mathbb{Z} \\ a + b &= (2x + 1) + (2y + 1) \\ &= 2x + 2y + 2 = 2(x + y + 1) \\ x + y + 1 &= z \in \mathbb{Z}, \text{Therefore, their sum is even} \end{aligned}$$

$$\begin{aligned} &\text{Case 2: Even parity} \\ a &= 2x, x \in \mathbb{Z} \\ b &= 2y, y \in \mathbb{Z} \\ a + b &= (2x) + (2y) \\ &= 2x + 2y = 2(x + y) \\ x + y &= z \in \mathbb{Z}, \text{Therefore, their sum is even} \end{aligned}$$

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$$\begin{aligned} a &= 2x, x \in \mathbb{Z} \\ b &= 2y + 1, y \in \mathbb{Z} \\ ab &= (2x)(2y + 1) \\ &= 4xy + 2x = 2(2xy + x) \\ 2xy + x &= z \in \mathbb{Z}, \text{Therefore, their product is even} \end{aligned}$$

Exercises for Section 9

1. ffff

3 ffff

5 ffff

7 ffff

9 ffff

11 ffff

15 ffff

17 ffff

19 ffff

20 ffff

23 ffff

Exercises for Section 10

1. ffff

3 ffff

9 ffff

11 ffff

- Prove that the sum of a rational number and an irrational number is always irrational.
- Prove that the product of a nonzero rational number and an irrational number is always an irrational number. (Why "nonzero"?)

Exercises for Section 12

1. ffff

3 ffff

5 ffff

9 ffff

11 ffff

15 ffff

17 ffff

18 ffff

20 ffff

Exercises for Section 13

1. ffff

7 ffff

9 ffff

11 ffff

13 ffff

29 ffff