Logic Homework

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Exercises for Section 3.1

1. Every real number is an even integer. **False**
2. Every even number is a real number. **True**
3. If x and y are real numbers and $5x = 5y$, then $x = y$. **True**
4. Sets $\mathbb{Z}$ and $\mathbb{N}$. **Not a statement**
5. Sets $\mathbb{Z}$ and $\mathbb{N}$ are infinite. **True**
6. Some sets are finite. **True**

8 $\mathbb{N} \notin P(\mathbb{N})$. **True**

11 The integer x is a multiple of 7. **Not a statement**

12 If the integer x is a multiple of 7, then it is divisible by 7. **True**

13 Either x is a multiple of 7, or it is not. **True**

14 Call me Ishmael. **Not a statement**

Exercises for Section 3.2

1. The number 8 is both even and a power of 2.
   p = The number 8 is even
   q = The number 8 is a power of 2
   $p \wedge q$

2. The matrix A is not invertible.
   p = matrix A is invertible
   $\neg p$

3. $x \neq y$
   p = $(x = y)$
   $\neg p$

5 $y \geq x$
p = $(y < x)$
$\neg p$

7 The number x equals zero, but the number y does not.
p = The number x equals zero
q = The number y equals zero
$p \wedge \neg q$

8 At least one of the numbers x and y equals 0.
p = The number x equals zero
q = The number y equals zero
$p \vee q$

9 $x \in A - B$
p = $x \in A$
q = $x \in B$
$p \wedge \neg q$

10 $x \in A \cup B$
p = $x \in A$
q = $x \in B$
$p \vee q$

13 Human beings want to be good, but not too good, and not all the time.
p = Human beings want to be good
q = Human beings want to be too good
r = Human beings want to be good all the time
$p \wedge \neg q \wedge \neg r$

14 A man should look for what is, and not for what he thinks should be.
p = A man should look for what is
q = A man should look for what he thinks should be
$p \wedge \neg q$

Exercises for Section 3.3

1. A matrix is invertible provided that its determinant is not zero.
   If a matrix determinant is not zero, then it's invertible.
2. For a function to be continuous, it is sufficient that it is differenciable.
   If a function is differenciable, then it's continuous.
3. For a function to be integrable, it is necessary that it is continuous.
   If a function is integrable, then it's continuous.
4. A function is rational if it is a polynomial
   If a function is a polynomial, then it's rational.
5. An integer is divisible by 8 only if it is divisible by 4
   If an integer is divisible by 8, then it's divisible by 4.
6. Whenever a surface has only one side, it is non-orientable
   If a surface has only one side, then it is non-orientable.
7. A series converges whenever it converges absolutely
   If a series converges absolutely, then it converges.
8. A geometric series with ratio r converges if $|r| < 1$
   If the ratio r of a geometric series is $|r| < 1$, then it converges.
9. A function is integrable provided the function is continuous
   If a function is continuous, then it's integrable.
10. The discriminant is negative only if the quadratic equation has no real solutions.
    If the discriminant of a quadratic equation is negative, then it has no real solutions.
11. You fail only if you stop writing. (Ray Bradbury)
    If you fail, then you stopped writing.
12. People will generally accept facts as truth only if the facts agree with what they already believe. (Andy Rooney)
    If people will generally accept facts as truth, then the facts agree with what they already believe.
13. Whenever people agree with me I feel I must be wrong. (Oscar Wilde)
    If people agree with me, then I feel I must be wrong.

Exercises for Section 3.4

1. For matrix A to be invertible, it is necessary and sufficient that det(A)$\neq 0$.
   A matrix A is invertible if and only if det(A)$\neq 0$
2. If a function has a constant derivative then it is linear, and conversely.
   A function has a constant derivative, if and only if it's linear.
3. If $xy = 0$ then $x = 0$ or $y = 0$, and conversely.
   $xy = 0$ if an only if $x = 0$ or $y = 0$
4. If $a \in \mathbb{Q}$ then $5a \in \mathbb{Q}$, and if $5a \in \mathbb{Q}$ then $a \in \mathbb{Q}$.
   $a \in \mathbb{Q}$ if and only if $5a \in \mathbb{Q}$
5. For an occurrence to become an adventure, it is necessary and sufficient for one to recount it.
   An occurrence becomes an adventure if and only if one recounts it.

Exercises for Section 3.5

1. $P \vee (Q \Rightarrow R)$

P	Q	R	$(Q \Rightarrow R)$	$P \vee (Q \Leftrightarrow R)$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

2. $(Q \vee R) \Leftrightarrow (R \wedge Q)$

P	Q	R	$(Q \vee R)$	$(R \wedge Q)$	$(Q \vee R) \Leftrightarrow (R \wedge Q)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	F	T

7 $(P \wedge \neg P) \Rightarrow Q$

P	Q	$(\neg P)$	$(P \wedge \neg P)$	$(P \neg P) \Rightarrow Q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

10 Suppose the statement $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R, S
R = false, S = false, P = true, Q = true

11 Suppose P is false and that the statement $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true. Find the truth values of R and S.
R = true, S = false

Exercises for Section 3.6

1 $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

P	Q	R	$(Q \vee R)$	$(P \wedge Q)$	$(P \wedge R)$	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

3 $P \Rightarrow Q \equiv (\neg P) \vee Q$

P	Q	$(\neg P)$	$(\neg P \vee Q)$	$(P \Rightarrow Q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

5 $\neg(P \vee Q \vee R) \equiv \neg P \wedge \neg Q \wedge \neg R$

P	Q	R	$(\neg P)$	$(\neg Q)$	$(\neg R)$	$\neg(P \vee Q \vee R)$	$\neg P \wedge \neg Q \wedge \neg R$
T	T	T	F	F	F	F	F
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	F
T	F	F	F	T	T	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	T	T	T

7 $P \Rightarrow Q \equiv (P \wedge \neg Q) \Rightarrow (Q \wedge \neg Q)$

P	Q	$(\neg Q)$	$(P \wedge \neg Q)$	$(Q \wedge \neg Q)$	$(P \wedge \neg Q) \Rightarrow (Q \wedge \neg Q)$	$P \Rightarrow Q$
T	T	F	F	F	T	T
T	F	T	T	F	F	F
F	T	F	F	F	T	T
F	F	T	F	F	T	T

9 $P \wedge Q$ and $\neg(\neg P \vee \neg Q)$
$\equiv \neg\neg P \wedge \neg\neg Q$
They are logically equivalent by DeMorgan's Law.

11 $(\neg P) \wedge (P \Rightarrow Q)$ and $\neg(Q \Rightarrow P)$
$\neg P \wedge (\neg P \vee Q) \neq \neg P \wedge Q$
They are not logically equivalent.

13 $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$
$(P \vee Q) \wedge (P \vee R) \neq (R \wedge P) \vee (R \wedge Q)$
They are not logically equivalent.

A Prove or disprove: $(P \oplus Q) \oplus R$ and $P \oplus (Q \oplus R)$
$(P \oplus Q) \oplus R \equiv P \oplus (Q \oplus R)$
Using the associative laws, we can see they're logically equivalent.

B Prove or disprove: $(P \oplus Q) \Rightarrow (P \oplus R)$ and $P \oplus (Q \Rightarrow R)$

P	Q	R	$(P \oplus Q)$	$(P \oplus R)$	$(P \oplus Q) \Rightarrow (P \oplus R)$	$Q \Rightarrow R$	$P \oplus (Q \Rightarrow R)$
T	T	T	F	F	T	T	F
T	T	F	F	T	T	F	T
T	F	T	T	F	F	T	F
T	F	F	T	T	T	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F
F	F	T	F	F	T	T	T
F	F	F	F	F	T	T	T

They are not logically equivalent, based on the truth table.

Exercises for Section 7.1

1. $\forall x \in \mathbb{R}, x^2 > 0$
   For every Real number x, x^2 is positive. **False**
2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$
   For every Real number x, there's at least one Natural number n, that x^n is zero or positive. **True**
3. $\exists a \in \mathbb{R}, \forall x \in \mathbb{R}, ax = x$
   There's at least one Real number a, that $ax = x$ for any Real number x. **True**
4. $\forall X \in P(\mathbb{N}), X \subseteq \mathbb{R}$
   For every set X in $P(\mathbb{N})$, X is a subset of Real numbers. **False**
5. $\forall n \in \mathbb{N}, \exists X \in P(\mathbb{N}), |X| < n$
   For every Natural number n, there's at least one subset X of $\mathbb{N}$, that it's cardinatily is less than n. **True**
6. $\exists n \in \mathbb{N}, \forall X \in P(\mathbb{N}), |X| < n$
   There's at least one Natural number n, that $|x| < n$ for every subset X of $\mathbb{N}$. **True**
7. $\forall X \subseteq \mathbb{N}, \exists n \in \mathbb{Z}, |X| = n$
   For every subset X of the Natural numbers, there's at least one integer n, that $|X| = n$. **False**
8. $\forall n \in \mathbb{Z}, \exists X \subseteq \mathbb{N}, |X| = n$
   For every integer n, there's at least one subset X of the Natural numbers, that the cardinatily of X is equal to n. **True**
9. $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m = n + 5$
   For every integer n, there's at least one integer m, that $m = n + 5$. **True**
10. $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$
    There's at least one integer m, that $m = n + 5$ for every integer n. **True**

A $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, y - x = y$
True

B $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, y - x = y$
True

C $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, y - x = y$
True

D $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y - x = y$
False

E $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, y \cdot x = y$
True

F $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, y \cdot x = y$
True

G $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, y \cdot x = y$
False

H $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y \cdot x = y$
True

Exercises for Section 7.3

1. If f is a polynomial and its degree is greater than 2, then f' is not constant.
   p = f is a polynomial
   q = its degree is greater than 2
   r = f' is constant
   $(p \wedge q) \Rightarrow \neg r$
2. The number x is positive, but the number y is not positive.
   p = x is positive
   q = y is positive
   $p \wedge \neg q$
3. If x is prime then $\sqrt{x}$ is not a rational number.
   p = x is prime
   q = $\sqrt{x}$ is a rational number
   $p \Rightarrow \neg q$
4. For every prime number p there is another prime number q with $q > p$
   $\forall p \in primes, \exists q \in primes, q > p$
5. For every positive number ε, there is a positive number δ for which $|x - a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$
   $\forall \varepsilon \in \mathbb{R}, \varepsilon > 0, \exists \delta \in \mathbb{R}, \delta > 0, (|x - a| < \delta) \Rightarrow (|f(x) - f(a)| < \varepsilon)$

7 There exists a real number a for which $a + x = x$ for every real number x.
$\exists a \in \mathbb{R}, \forall x \in \mathbb{R}, a + x = x$

8 I don't eat anything that has a face.
p = I eat some things that have a face
$\neg p$

9 If x is a rational number and $x \neq 0$, then tan(x) is not a rational number.
$((x \in \mathbb{Q}) \wedge (x \neq 0)) \Rightarrow \tan x \notin \mathbb{Q}$

10 If sin(x) ¡ 0, then it is not the case that $0 \leq x \leq \pi$.
$(sin(x) < 0) \Rightarrow \neg(0 \leq x \leq \pi)$

Exercises for Section 7.4

1. The number x is positive, but the number y is not positive.
   The number x is not positive, or the number y is positive.
2. If x is prime, then $\sqrt{x}$ is not a rational number.
   x is prime and $\sqrt{x}$ is a rational number
3. For every prime number p, there is another prime number q with $q > p$.
   There is at least one prime number p that $q \leq p$ for every prime number q.
4. For every positive number ε, there is a positive number δ such that $|x - a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$
   There is at least one positive number ε, that $(x - a < d) \wedge |f(x) - f(a) \geq \varepsilon|$ for every positive number δ

6 There exists a real number a for which $a + x = x$ for every real number x.
For every real number a, there is another real number x such that $a + x \neq x$

7 I don't eat anything that has a face.
I will eat some things that have a face.

8 If x is a rational number and $x \neq 0$, then tan(x) is not a rational number.
X is a rational number and $x \neq 0$ and tan(x) is a rational number.

9 If sin(x) < 0, then it is not the case that $0 \leq x \leq \pi$.
sin(x) ≥ 0 and $0 \leq x \leq \pi$

10 If f is a polynomial and its degree is greater than 2, then f' is not constant.
F is a polynomial and its degree is greater than 2 and f' is constant.

A Write $\neg(\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, z > y \Rightarrow z > x^2)$ without using the $\neg$ symbol.
$\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \forall z \in \mathbb{R}, (z > y \wedge z \leq x^2)$

B Write $\neg(\forall x, y \in \mathbb{R}, x < y \Rightarrow \exists z \in \mathbb{R}, x < z < y)$ without using the $\neg$ symbol.
$\exists x, y \in \mathbb{R}, x < y \wedge \forall z \in \mathbb{R}, z \leq x \vee z \geq y$