Artificial Intelligence - Lab 3

Part II: Inference in an existing Bayesian network

Q5

- a) P(melt-down) = **0.02578** P(melt-down | icy weather) = **0.03472**
- b) P(meltdown | WaterLeakWarning = T and PumpFailureWarning = T) = **0.14535** P(meltdown | WaterLeak = T and PumpFailure = T) = **0.2**

The conditional probability P(meltdown | WaterLeakWarning = T and PumpFailureWarning = T) = 0.14535 indicates that the meltdown is more likely when there is a real WaterLeak and PumpFailure compared to that there are only true warnings. This could indicate that the warning sensors are very sensitive and indicate failure more often than they really happen.

- c) P (WaterLeak | IcyWeather) is quite hard to estimate because IcyWeather is not accurate enough. IcyWeather could mean 0 degree weather or -20 degree weather and therefore have a completely different impact on the water leak. For a boolean variable we need a precise definition of the conditions that does not have such a wide span.
- d) Since temperature is a continuous measure we have to transform it into some kind of attribute that can be true or false. The different alternatives for the variable temperature could be intervals of temperatures (e.g. [0 to -1 Celsius], [-1 to -2 Celsius],...). This would lead to many different alternatives (depending on how narrow you make the intervals, which would lead to very low conditional probabilities for each P(WaterLeak | specific temperature interval).

Q6

- a) The conditional distributions are shown as a conditional probability table, or CPT. Each row in a CPT contains the conditional probability of each node value for a conditioning case.
- b) Given random variables X,Y,... that are defined on a probability space, the joint probability distribution for X,Y,... is a probability distribution that gives the probability that each of X,Y,... falls in any particular range or discrete set of values specified for that variable Chain Rule:

```
Chain Rule:

P(everything false) = P(IcyWeather = F | WaterLeak = F, WaterLeakWarning = F, PumpFailure

= F, PumpFailureWarning = F, Meltdown = F)

* P (WaterLeak = F | IcyWeather = F, WaterLeakWarning = F, PumpFailure = F,
PumpFailureWarning = F, Meltdown = F)

* P(PumpFailure = F | IcyWeather = F, WaterLeakWarning = F, PumpFailure = F,
PumpFailureWarning = F, Meltdown = F, WaterLeak = F)

* P(PumpFailureWarning = F | PumpFailure = F, IcyWeather = F, WaterLeakWarning = F,
PumpFailure = F, PumpFailureWarning = F, Meltdown = F, WaterLeak = F)

* P(Meltdown = F | PumpFailure = F, WaterLeak = F, IcyWeather = F, WaterLeakWarning = F,
PumpFailureWarning = F)

* P(WaterLeakWarning = F | WaterLeak = F, IcyWeather = F, PumpFailure = F,
PumpFailureWarning = F, Meltdown = F)

Semantics of BN:
p(ALL = F) = P(IcyWeather = F) * P (WaterLeak = F | IcyWeather = F)
```

* P(PumpFailure = F) * P(PumpFailureWarning = F | PumpFailure = F)

* P(Meltdown = F | PumpFailure = F, WaterLeak = F)

* P(WaterLeakWarning = F | WaterLeak = F)

= 0.95*0.9*0.9*0.95*0.999*0.95 =**0.6938**

Thus it's quite a common state.

```
c) P(meltdown | WaterLeak = T and PumpFailure = T) = 0.2.
       The state of any other variable does not matter since they have no direct influence on
      Meltdown. Because WaterLeak is already True, the state of IcyWeather has no influence.
d) Exact Inference:
      Conditioning on the variables we know (evidence): PumpFailureWarning=F, WaterLeak=F,
      WaterLeakWarning=F and IcyWeather=F
      Marginalizing (summing) over the variable we do not know (unobserved or hidden):
      PumpFailure
      P ( Meltdown | PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, IcyWeather=F)
       = a*P ( Meltdown, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F,
      IcyWeather=F)
       = \alpha * \Sigma(over PumpFailure) P ( Meltdown, PumpFailureWarning=F, WaterLeak=F,
      WaterLeakWarning=F, IcyWeather=F, PumpFailure)
       = \alpha*[P (Meltdown, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, WaterLeak=F, WaterLeakWarning=F, WaterLeak=F, W
      IcyWeather=F, PumpFailure=T) + P (Meltdown, PumpFailureWarning=F, WaterLeak=F,
       WaterLeakWarning=F, IcyWeather=F, PumpFailure=F)]
      P(Meltdown, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, IcyWeather=F,
      PumpFailure=T) = \langle 0,001218375, 0,006904125 \rangle
      ⟨P(Meltdown=T, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, IcyWeather=F,
      PumpFailure=T)= P(IcyWeather = F) * P (WaterLeak = F | IcyWeather = F)
                                 * P(PumpFailure = T) * P(PumpFailureWarning = F | PumpFailure = T)
                                 * P(Meltdown = T | PumpFailure = T, WaterLeak = F)
                                 * P(WaterLeakWarning = F | WaterLeak = F)
                                 = 0.95*0.9*0.1*0.1*0.15*0.95 = 0.001218375
       P(Meltdown=F, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, IcyWeather=F,
       PumpFailure=T) = P(IcyWeather = F) * P(WaterLeak = F | IcyWeather = F)
                                 * P(PumpFailure = T) * P(PumpFailureWarning = F | PumpFailure = T)
                                 * P(Meltdown = F | PumpFailure = T, WaterLeak = F)
                                 * P(WaterLeakWarning = F | WaterLeak = F)
                                 = 0.95*0.9*0.1*0.1*0.85*0.95 = 0.006904125
      P(Meltdown, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, IcyWeather=F,
      PumpFailure=F) = \langle 0.00069447375, 0.6938 \rangle
      ⟨P(Meltdown=T, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, IcyWeather=F,
      PumpFailure=F) = P(IcyWeather = F) * P (WaterLeak = F | IcyWeather = F)
                                 * P(PumpFailure = F) * P(PumpFailureWarning = F | PumpFailure = F)
                                 * P(Meltdown = T | PumpFailure = F, WaterLeak = F)
                                 * P(WaterLeakWarning = F | WaterLeak = F)
                                 = 0.95*0.9*0.9*0.95*0.001*0.95 = 0.00069447375
      P(Meltdown=F, PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F, IcyWeather=F,
       PumpFailure=F) = P(IcyWeather = F) * P(WaterLeak = F | IcyWeather = F)
                                 * P(PumpFailure = F) * P(PumpFailureWarning = F | PumpFailure = F)
                                 * P(Meltdown = F | PumpFailure = F, WaterLeak = F)
                                 * P(WaterLeakWarning = F | WaterLeak = F)
                                  = 0.95*0.9*0.9*0.95*0.999*0.95 = 0.693733927625
```

```
\begin{split} &\alpha[\langle 0.001218375,\, 0.006904125\rangle + \langle 0.00069447375,\, 0.693733927625] = \alpha[\langle 0.00191284875,\, 0.70068340125\rangle] \\ &\text{Normalization:} \\ &\alpha = 1 \ / \ (0.00191284875 + \ 0.70068340125) \\ &= 1 \ / \ 0.70259625 \\ &= 1.423292538210957 \\ &\text{Normalized probability:} \\ &\alpha[\langle 0.00191284875,\, 0.70068340125\rangle] = \textbf{0.0027225433526012,} \ \textbf{0.9972774566473989} \end{split}
```

Part III: Extending a network

Given probabilities:

- P(battery | icyWeather) = 0.8
- P(battery | ¬icyWeather) = 0.95
- P(radio | battery) = 0.95
- P(ignition | battery) = 0.95
- P(gas) = 0.95
- P(starts | gas ∧ ignition) = 0.95
- P(moves | starts) = 0.95
- P(survives | moves ∧ melt-down) = 0.8
- P(survives | moves ∧ ¬melt-down) = P(survives | ¬moves ∧ ¬melt-down) = 1.0
- P(survives | ¬moves ∧ melt-down) = 0.0

Rest of the probabilities:

- P(¬battery | icyWeather) = 0.2
- P(¬battery | ¬icyWeather) = 0.05
- P(¬radio | battery) = 0.05
- P(radio | ¬battery) = 0 (logic)
- $P(\neg radio \mid \neg battery) = 1 (logic)$
- P(¬ignition | battery) = 0.05
- P(ignition | ¬battery) = 0 (logic)
- P(¬ignition | ¬battery) = 1 (logic)
- $P(\neg gas) = 0.05$
- $P(\neg starts \mid gas \land ignition) = 0.05$
- P(starts | \neg gas \land ignition) = 0.00
- P(starts | \neg gas $\land \neg$ ignition) = 0.00
- P(starts | gas $\land \neg ignition$) = 0.00
- $P(\neg starts \mid \neg gas \land ignition) = 1.00$
- $P(\neg starts \mid gas \land \neg ignition) = 1.00$
- $P(\neg starts \mid \neg gas \land \neg ignition) = 1.00$
- P(¬moves | starts) = 0.05
- P(moves | ¬starts) = 0 (logic)
- P(¬moves | ¬starts) = 1 (logic)
- P(¬survives | moves ∧ melt-down) = 0.2
- P(¬survives | moves ∧ ¬melt-down) = 0
- P(¬survives | ¬moves ∧ ¬melt-down) = 0
- P(\neg survives | \neg moves \land melt-down)= 1
- P(¬survives | moves ∧ ¬melt-down)
- 1) During the lunch break, the owner tries to show off for his employees by demonstrating the many features of his car stereo. To everyone's disappointment, it doesn't work. How did the owner's chances of surviving the day change after this observation?

His chance to survive goes down from 99% to 98%.

2) The owner buys a new bicycle that he brings to work every day. The bicycle has the following properties:

 $P(bicycle_works) = 0.9$

P(survives | \neg moves \land melt-down \land bicycle_works) = 0.6

P(survives | moves \land melt-down \land bicycle_works) = 0.9

How does the bicycle change the owner's chances of survival?

It increases his chance of survival to 99.5%.

3) It is possible to model any function in propositional logic with Bayesian Networks. What does this fact say about the complexity of exact inference in Bayesian Networks? What alternatives are there to exact inference?

It is possible to model any function in propositional logic with Bayesian Networks. Therefore, the representation of exact inference in Bayesian Networks will grow exponentially, because the tree (or table) grows exponentially with every added function / node. The complexity is #P-hard which is strictly harder than NP-complete problems. An alternative is to use approximate inference.

Part IV: More extensions

1) The owner had an idea that instead of employing a safety person, to replace the pump with a better one. Is it possible, in your model, to compensate for the lack of Mr H.S.'s expertise with a better pump?

Yes, a better pump will lower the probability of a pump failure and thus the probability of a meltdown and by this increase his chance to survive.

2) Mr H.S. fell asleep on one of the plant's couches. When he wakes up he hears someone scream: "There is one or more warning signals beeping in your control room!". Mr H.S. realizes that he does not have time to fix the error before it is too late (we can assume that he wasn't in the control room at all). What is the chance of survival for Mr H.S. if he has a car with the same properties as the owner? Hint: This question involves a disjunction (A or B) which can not be answered by querying the network as is. How could you answer such questions? Maybe something could be added or modified in the network.

We can add a node "Warning" which represents the OR function between PumpFailureWarning and WaterLeakWarning. Then we ask the network. See Appendix for our modeled network.

P(survives | warning = True) = **0.98320**

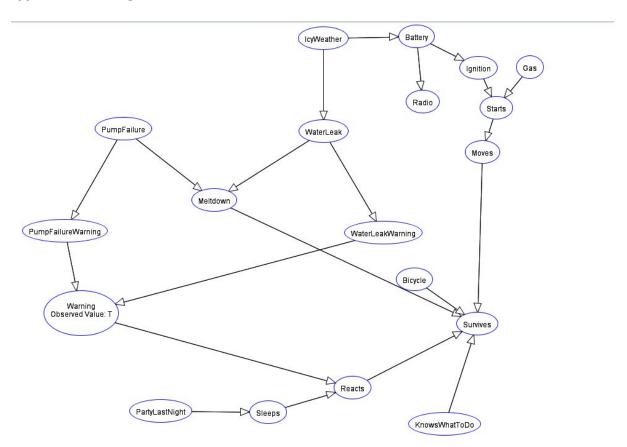
3) What unrealistic assumptions do you make when creating a Bayesian Network model of a person?

We make the unrealistic assumption that the person we model always behaves in the same (more or less) rational way. In fact human behavior is influenced by much more factors and is dynamic (e.g. improvement of competence with experience).

4) Describe how you would model a more dynamic world where for example the "IcyWeather" is more likely to be true the next day if it was true the day before. You only have to consider a limited sequence of days.

One could apply Markov chain Monte Carlo (MCMC) algorithms to consider precedent values in our samples and take the dynamic world into account.

Appendix: Modeling of the Network



```
<?xml version="1.0" encoding="UTF-8"?>
<BIF VERSION="0.3" xmlns="http://www.cs.ubc.ca/labs/lci/fopi/ve/XMLBIFv0_3"
      xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
      xsi:schemaLocation="http://www.cs.ubc.ca/labs/lci/fopi/ve/XMLBIFv0_3
http://www.cs.ubc.ca/labs/lci/fopi/ve/XMLBIFv0_3/XMLBIFv0_3.xsd">
<NETWORK>
<NAME>Nuclear Power Station</NAME>
<PROPERTY>detailed = </PROPERTY>
<PROPERTY>short = </PROPERTY>
<VARIABLE TYPE="nature">
      <NAME>WaterLeak</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7520.52294921875, 5294.0087890625)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>WaterLeakWarning</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7617.27294921875, 5459.63134765625)</PROPERTY>
</VARIABLE>
```

```
<VARIABLE TYPE="nature">
      <NAME>IcyWeather</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7520.52294921875, 5143.14453125)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>PumpFailure</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7195.83740234375, 5290.72900390625)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>PumpFailureWarning</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7100.72705078125, 5456.3515625)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Meltdown</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7343.22265625, 5406.85546875)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Battery</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7671.0, 5140.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Radio</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7681.0, 5246.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Ignition</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7772.0, 5191.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Starts</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7801.0, 5256.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Gas</NAME>
```

```
<OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7855.0, 5190.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Moves</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7778.0, 5328.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Survives</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7770.0, 5607.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Bicycle</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7668.0, 5535.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Sleeps</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7430.0, 5751.0)/PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Reacts</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7567.0, 5711.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>KnowsWhatToDo</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7717.0, 5770.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>PartyLastNight</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
      <PROPERTY>position = (7256.0, 5749.0)</PROPERTY>
</VARIABLE>
<VARIABLE TYPE="nature">
      <NAME>Warning</NAME>
      <OUTCOME>T</OUTCOME>
      <OUTCOME>F</OUTCOME>
```

```
<OBS>T</OBS>
      <PROPERTY>position = (7127.0, 5590.0)</PROPERTY>
</VARIABLE>
<DEFINITION>
      <FOR>WaterLeak</FOR>
      <GIVEN>IcyWeather</GIVEN>
      <TABLE>0.2 0.8 0.1 0.9</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>WaterLeakWarning</FOR>
      <GIVEN>WaterLeak</GIVEN>
      <TABLE>0.9 0.1 0.05 0.95</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>IcyWeather</FOR>
      <TABLE>0.05 0.95</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>PumpFailure</FOR>
      <TABLE>0.1 0.9</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>PumpFailureWarning</FOR>
      <GIVEN>PumpFailure</GIVEN>
      <TABLE>0.9 0.1 0.05 0.95</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Meltdown</FOR>
      <GIVEN>WaterLeak</GIVEN>
      <GIVEN>PumpFailure</GIVEN>
      <TABLE>0.2 0.8 0.1 0.9 0.15 0.85 0.001 0.999</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Battery</FOR>
      <GIVEN>IcyWeather</GIVEN>
      <TABLE>0.8 0.2 0.95 0.05</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Radio</FOR>
      <GIVEN>Battery</GIVEN>
      <TABLE>0.95 0.05 0.0 1.0</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Ignition</FOR>
      <GIVEN>Battery</GIVEN>
      <TABLE>0.95 0.05 0.0 1.0</TABLE>
</DEFINITION>
<DEFINITION>
```

```
<FOR>Starts</FOR>
      <GIVEN>Ignition</GIVEN>
      <GIVEN>Gas</GIVEN>
      <TABLE>0.95 0.05 0.0 1.0 0.0 1.0 0.0 1.0</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Gas</FOR>
      <TABLE>0.95 0.05</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Moves</FOR>
      <GIVEN>Starts</GIVEN>
      <TABLE>0.95 0.05 0.0 1.0</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Survives</FOR>
      <GIVEN>Meltdown</GIVEN>
      <GIVEN>Moves</GIVEN>
      <GIVEN>Bicycle</GIVEN>
      <GIVEN>Reacts</GIVEN>
      <GIVEN>KnowsWhatToDo</GIVEN>
      <TABLE>0.95 0.05 0.9 0.1 0.9 0.1 0.9 0.1 0.8 0.2 0.8 0.2 0.8 0.2 0.8 0.2 0.7 0.3 0.6 0.4
0.6\ 0.4\ 0.6\ 0.4\ 0.0\ 1.0\ 0.0\ 1.0\ 0.0\ 1.0\ 0.0\ 1.0\ 0.0\ 1.0\ 0.0\ 1.0\ 0.0\ 1.0\ 0.0\ 1.0\ 0.0\ 1.0
0.0 1.0 0.0 1.0 0.0 1.0 0.0 1.0 0.0 1.0 0.0 1.0 0.0 1.0 0.0 1.0 0.0 1.0 0.0 /TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Bicycle</FOR>
      <TABLE>0.9 0.1</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Sleeps</FOR>
      <GIVEN>PartyLastNight</GIVEN>
      <TABLE>0.95 0.05 0.7 0.3</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>Reacts</FOR>
      <GIVEN>Sleeps</GIVEN>
      <GIVEN>Warning</GIVEN>
      <TABLE>0.0 1.0 0.0 1.0 0.9 0.1 0.9 0.1</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>KnowsWhatToDo</FOR>
      <TABLE>0.5 0.5</TABLE>
</DEFINITION>
<DEFINITION>
      <FOR>PartyLastNight</FOR>
      <TABLE>0.5 0.5</TABLE>
</DEFINITION>
<DEFINITION>
```