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Data Structures & Algorithms Sparse Table

```
int a[N], st[LG + 1][N];
void preprocess() {
    for (int i = 1; i <= n; ++i) st[0][i] = a[i];</pre>
    for (int j = 1; j \le LG; ++j)
       for (int i = 1; i + (1 << j) - 1 <= n; ++i)
           st[j][i] = min(st[j-1][i], st[j-1][i+1][i]
                 (1 << (j - 1))]);
}
int query(int 1, int r) {
   int k = _-lg(r - l + 1);
    return min(st[k][1], st[k][r - (1 << k) + 1]);</pre>
//query sum:
int querySum(int 1, int r) {
   int len = r - l + 1;
    int sum = 0;
    for (int j = 0; (1 << j) <= len; ++j)
       if (len >> j & 1) {
           sum = sum + st[j][1];
           1 = 1 + (1 << j);
    return sum;
```

Fenwick Tree

```
void update(int i, int val){
   for (; i <= n; i += i & -i) bit[i] += val;
}
int get(int i){
   int res = 0;
   for (; i; i -= i & -i) res += bit[i];
   return res;
}</pre>
```

Segment Tree

```
struct Segment_tree{
 int st[4 * N], lazy[4 * N];
   void apply(int id, int c){
       update(st[id], c);
       update(lazy[id], c);
 void down(int id, int 1, int r){
   int c = lazy[id]; lazy[id] = 0;
       apply(id << 1, c); apply (id << 1 | 1, c);
 void build(int id, int 1, int r){
   if (1 == r){
     st[id] = a[1];
     return;
   int mid = (1 + r) >> 1;
   build(id << 1, 1, mid);
   build(id << 1 | 1, mid + 1, r);</pre>
   st[id] = merge(st[id << 1], st[id << 1 | 1]);
 }
```

```
void update(int id, int 1, int r, int u, int v, int
       x){
   if (r < u || v < 1) return;</pre>
   if (u <= 1 && r <= v){</pre>
     apply(id, x);
      return;
   down(id, 1, r);
   int mid = (1 + r) >> 1;
   update(id << 1, 1, mid, u, v, x);
   update(id << 1 | 1, mid + 1, r, u, v, x);
    st[id] = merge(st[id << 1], st[id << 1 | 1]);
  int get(int id, int 1, int r, int u, int v){
   if (r < u || v < 1) return -INF;</pre>
   if (u <= 1 && r <= v) return st[id];</pre>
   down(id, 1, r);
   int mid = (1 + r) >> 1;
   return merge(get(id << 1, 1, mid, u, v), get(id</pre>
        << 1 | 1, mid + 1, r, u, v));
 }
} ST;
```

Sigma Tree

```
struct Sigma_Tree{
   int st[2 * N];
   void init(){
       For(i, 1, n) st[i + n - 1] = a[i];
       ForD(i, n - 1, 1) st[i] = merge(st[i << 1],
           st[i << 1 | 1]);
   }
 void update(int p, int val){
   p += n - 1;
   st[p] = val;
   for (; p > 1; p >>= 1) st[p >> 1] = merge(st[p],
        st[p ^ 1]);
   int get(int 1, int r){
       int res = 0;
       for (1 += n - 1, r += n - 1; 1 <= r; 1 >>= 1,
            r >>= 1){
           if (1 & 1) res = merge(res, st[l++]);
           if (!(r & 1)) res = merge(res, st[r--]);
       return res;
   }
} ST;
```

Persistent Segment Tree

```
struct Node {
   int left, right; // ID of left child & right
        child
   long long ln; // Max value of node
   Node() {}
   Node(long long ln, int left, int right) : ln(ln),
        left(left), right(right) {}
```

```
} it[N]; // Each node has a position in this array,
    called ID
int nNode;
int ver[N]; // ID of root in each version
// Update max value of a node
inline void refine(int cur) {
   it[cur].ln = max(it[it[cur].left].ln, it[it[cur].
        right].ln);
}
// Update a range, and return new ID of node
int update(int 1, int r, int u, int x, int oldId) {
   if (1 == r) {
       ++nNode;
       it[nNode] = Node(x, 0, 0);
       return nNode;
   int mid = (1 + r) >> 1;
   int cur = ++nNode;
   if (u <= mid) {</pre>
       it[cur].left = update(1, mid, u, x, it[oldId
            ].left);
       it[cur].right = it[oldId].right;
       refine(cur);
   }
   else {
       it[cur].left = it[oldId].left;
       it[cur].right = update(mid+1, r, u, x, it[
            oldId].right);
       refine(cur);
   return cur;
}
// Get max of range. Same as usual IT
int get(int nodeId, int 1, int r, int u, int v) {
   if (v < 1 || r < u) return -1;</pre>
   if (u <= l && r <= v) return it[nodeId].ln;</pre>
   int mid = (1 + r) >> 1;
   return max(get(it[nodeId].left, 1, mid, u, v),
        get(it[nodeId].right, mid+1, r, u, v));
}
// When update:
   ++nVer;
   ver[nVer] = update(1, n, u, x, ver[nVer-1]);
// When query:
   res = get(ver[t], 1, n, u, v);
```

Hash Map

```
//faster than unordered_map
struct hash_map {
   const static int SZ = 2e4 + 9;
   int nxt[SZ >> 3], val[SZ >> 3];
   int key[SZ >> 3];
   int h[SZ + 5], cnt;
   vector<int>vec;

void clear(){
   for (int i : vec) h[i] = 0;
```

```
for (int i = 1; i <= cnt; i++)</pre>
           val[i] = nxt[i] = 0, key[i] = 0;
       vec.clear();
       cnt = 0;
   }
   int hash(int u) {
       return u % SZ;
   int &operator[](int u) {
       int x = hash(u);
       for (int i = h[x]; i; i = nxt[i])
           if (key[i] == u) return val[i];
       if (!h[x]) vec.push_back(x);
       ++cnt:
       key[cnt] = u;
       val[cnt] = 0;
       nxt[cnt] = h[x];
       h[x] = cnt;
       return val[cnt];
   int qry(int u) {
       int x = hash(u);
       for (int i = h[x]; i; i = nxt[i])
           if (key[i] == u) return val[i];
       return 0:
   }
} hs;
```

Parallel Binary Search

```
while (1){
   bool ok = 1;

For(i, 1, q){
      if (l[i] > r[i]) continue;
      ok = 1;
      queries[(l[i] + r[i]) >> 1].push_back(i);
}

reset();
For(t, 1, n){
      //update(t,...);
      for (int i: queries[t]){
            if (check(i)) r[i] = t - 1;
            else l[i] = t + 1;
            }
      }
}
```

Ternary Search

Find the maximum Point in a Graph like: /\

```
double max_f(double left, double right) {
   int N_ITER = 100;
   for (int i = 0; i < N_ITER; i++) {
       double x1 = left + (right - left) / 3.0;
       double x2 = right - (right - left) / 3.0;
       if (f(x1) > f(x2)) right = x2;
       else left = x1;
   }
```

```
return f(left);
}
```

String

Trie 1

```
struct node{
  node *g[26];
  node(){
    rep(i, 26) g[i] = NULL;
  }
} *root = new node();

void Insert(string s){
  node *p = root;
  for (char t: s){
    if (p->g[t - 'a'] == NULL)
       p->g[t - 'a'] = new node();

    p = p->g[t - 'a'];
}
}
```

$s = "\$" + s + "^"$: vector<int> p(n + 2); int 1 = 0, r = 1;for(int i = 1; i <= n; i++) {</pre> p[i] = min(r - i, p[1 + (r - i)]);while(s[i - p[i]] == s[i + p[i]]) { p[i]++; } $if(i + p[i] > r) {$ 1 = i - p[i], r = i + p[i];return vector<int>(begin(p) + 1, end(p) - 1); vector<int> manacher(string s) { string t; for(auto c: s) { t += string("#") + c; auto res = manacher_odd(t + "#"); return vector<int>(begin(res) + 1, end(res) - 1);

Trie 2

```
int nNode = 0;
int g[N][26];

void Insert(string s){
   int p = 0;
   for (char t: s){
      if (!g[p][t - 'a']) g[p][t - 'a'] = ++nNode;
      p = g[p][t - 'a'];
   }
}
```

Hash

\mathbf{KMP}

```
//prefix function: length of the longest prefix of
    the substring s[1..i] that is also a suffix of
    this same substring
int k = 0;
For(i, 2, n){ //1-indexed
    while (k && s[k + 1] != s[i]) k = kmp[k];
    kmp[i] = (s[k + 1] == s[i]) ? ++k : 0;
}
```

Manacher

```
vector<int> manacher_odd(string s) {
  int n = s.size();
```

Aho - Corasick

```
namespace Trie{
 struct Node{
   int child[26], p = -1, cnt = 0;
   char pch;
   int link = -1, go[26];
   Node(int p = -1, char ch = '#'): p(p), pch(ch){
     fill(begin(child), end(child), -1);
         fill(begin(go), end(go), -1);
 };
  vector<Node> g(1);
  void add(string s){
   int v = 0;
   for (char t: s){
     int c = t - 'a';
     if (g[v].child[c] == -1){
       g[v].child[c] = g.size();
       g.emplace_back(v, t);
     v = g[v].child[c];
   g[v].cnt++;
  int go(int v, char c);
  int get_link(int v){
   if (g[v].link == -1){
     if (!v | | !g[v].p) g[v].link = 0;
     else g[v].link = go(get_link(g[v].p), g[v].pch)
   }
   return g[v].link;
 int go(int v, char t){
   int c = t - 'a';
   if (g[v].go[c] == -1){
     if (g[v].child[c] != -1) g[v].go[c] = g[v].
          child[c];
```

Aho - Corasick (BFS)

```
struct trie{
 struct Node{
   Node *child[26], *link;
   int cnt = 0;
   Node(){
     cnt = 0;
     rep(i, 26) child[i] = NULL;
     link = NULL;
 } *root = new Node();
  void add(string &s){
   Node* p = root;
   for (char &t: s){
     int c = t - 'a';
     if (p->child[c] == NULL) p->child[c] = new Node
     p = p->child[c];
   p->cnt++;
  void AhoCorasick(){
   root->link = root;
   queue<Node*> q; q.push(root);
   while (!q.empty()){
     Node* p = q.front(); q.pop();
     rep(i, 26) if (p->child[i]){
       Node* k = p - link;
       while (k != root && k->child[i] == NULL) k =
           k->link:
       if (k->child[i] && k != p) p->child[i]->link
            = k->child[i];
       else p->child[i]->link = root;
       p->child[i]->cnt += p->child[i]->link->cnt;
       q.push(p->child[i]);
   }
 }
};
```

SQRT Decomposition MO

```
struct query{
   int 1, r, id;
}
bool cmp(const query &a, const query &b){
   if(a.1 / S != b.1 / S) return a.1 < b.1;

   if((a.1 / S) & 1)
      return a.r < b.r;
   else
      return a.r > b.r
}
```

Graph

Joint and Bridge

```
void dfs(int u, int pre) {
    int child = 0;
    num[u] = low[u] = ++timer;
    for (int v: g[u]) {
        if (v == pre) continue;
        if (!num[v]) {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] == num[v]) bridge++;
            child++;
            if (u == pre) {
                if (child > 1) joint[u] = true;
            }
            else if (low[v] >= num[u]) joint[u] = true
            ;
        }
        else low[u] = min(low[u], num[v]);
    }
}
```

SCC

```
void dfs(int u) {
   num[u] = low[u] = ++timer;
   st.push(u);
   for (int v : g[u]) {
       if (!num[v]){
           dfs(v);
           low[u] = min(low[u], low[v]);
       else low[u] = min(low[u], num[v]);
   }
   if (low[u] == num[u]) {
       scc++;
       int v;
       do {
           v = st.top();
           st.pop();
           num[v] = INF;
       while (v != u);
   }
```

Topology Sort 1

```
//u -> v
```

```
//++deg[v]
for (int u = 1; u <= n; ++u)
    if (!deg[u]) q.push(u);

while (!q.empty()) {
    int u = q.front();
    q.pop();
    topo.push_back(u);
    for (auto v : g[u]) {
        deg[v]--;
        if (!deg[v]) q.push(v);
    }
}</pre>
```

Topology Sort 2

```
void dfs(int u) {
    visit[u] = 1;
    for (auto v : g[u]) {
        assert(visit[v] != 1);
        //graph contains a cycle
        if (!visit[v]) dfs(v);
    }
    topo.push(u);
    visit[u] = 2;
}
```

Max Flow

```
struct edge{
 int to, rev, flow, cap;
};
void add_edge(int u, int v, int cap){
 edge e1 = \{v, sz(g[v]), 0, cap\};
    edge e2 = \{u, sz(g[u]), 0, 0\};
    g[u].pb(e1); g[v].pb(e2);
bool bfs(){
 memset(dist, 0x3f, sizeof dist);
  queue<int> q;
  q.push(source); dist[source] = 0;
  while (!q.empty()){
   int u = q.front(); q.pop();
   for (edge e: g[u]){
     int v = e.to, flow = e.flow, cap = e.cap;
     if (flow < cap && minimize(dist[v], dist[u] +</pre>
          1))
       q.push(v);
   }
  return dist[sink] < INF;</pre>
int dfs(int u, int mn){
  if (u == sink) return mn;
  for (int &i = lazy[u]; i < sz(g[u]); ++i){</pre>
    auto &[v, rev, flow, cap] = g[u][i];
    if (dist[v] == dist[u] + 1 && flow < cap){</pre>
     int cur = dfs(v, min(mn, cap - flow));
     if (cur > 0){
       flow += cur;
       g[v][rev].flow -= cur;
       return cur;
   }
  }
```

```
return 0;
}
int main(){
    //...
    int res = 0;
    while (bfs()){
        memset(lazy, 0, sizeof lazy);
        while (int del = dfs(source, INF))
            res += del;
}
cout << res;
return 0;
}</pre>
```

Bipartite Matching

```
bool dfs(int u){
   if (seen[u]) return 0;
   seen[u] = 1;

  for (int v: g[u])
    if (!mt[v] || dfs(mt[v]))
      return mt[v] = u, 1;

  return 0;
}

//memset(mt, 0, sizeof mt);

//For(i, 1, n){
      //memset(seen, 0, sizeof seen);
      //dfs(i);
//}
```

Matching 2

```
bool bfs(){
  bool res = 0;
  queue<int> q;
  For(i, 1, n){
    if (!mx[i]){
     q.push(i), dist[i] = 0;
    }
   else{
     dist[i] = -1;
   }
  while (!q.empty()){
   int u = q.front(); q.pop();
    for (int v: g[u]){
     if (!my[v]) res = 1;
      else if (dist[my[v]] < 0){</pre>
       dist[my[v]] = dist[u] + 1;
       q.push(my[v]);
   }
 }
  return res;
bool dfs(int u){
  for (int v: g[u]){
    if (!my[v]){
     mx[u] = v; my[v] = u;
```

```
return 1;
   }
   else if (dist[my[v]] == dist[u] + 1 && dfs(my[v])
    mx[u] = v; my[v] = u;
    return 1;
  }
 }
 return 0;
while (bfs()){
   For(i, 1, n) if (!mx[i]){
      dfs(i);
}
```

HLD

```
void dfs(int u){
 sz[u] = 1;
 for (int v: g[u]) if (v != par[u]){
   par[v] = u;
   dfs(v);
   sz[u] += sz[v];
 }
}
void hld(int u){
 if (!Head[nChain]) Head[nChain] = u;
  idChain[u] = nChain;
 pos[u] = ++timer;
 node[timer] = u;
  int bigC = 0;
  for (int v: g[u]) if (v != par[u])
   if (!bigC || sz[v] > sz[bigC])
     bigC = v;
  if (bigC) hld(bigC);
  for (int v: g[u]) if (v != par[u] && v != bigC){
   ++nChain;
   hld(v);;
 }
}
//LCA
int LCA(int u, int v){
 while (idChain[u] != idChain[v]){
   if (idChain[u] > idChain[v])
     u = par[Head[idChain[u]]];
   else
     v = par[Head[idChain[v]]];
 }
 if (h[u] < h[v]) return u;</pre>
 return v;
}
int get(int u, int v){
 int res = 0;
 while (idChain[u] != idChain[v]){
   if (idChain[u] > idChain[v]){
       maximize(res, ST.get(pos[Head[idChain[u]]],
           pos[u]));
       u = par[Head[idChain[u]]];
   }
```

```
elsef
     maximize(res, ST.get(pos[Head[idChain[v]]]),
          pos[v]));
     v = par[Head[idChain[v]]];
 }
}
if (pos[u] < pos[v])</pre>
 maximize(res, ST.get(pos[u], pos[v]));
 maximize(res, ST.get(pos[v], pos[u]));
return res;
```

DSU on tree

```
void dfs(int u, int prev = -1){
  in[u] = ++timer; node[timer] = u;
 for (int v: g[u]) if (v != prev)
   dfs(v, u);
 out[u] = timer;
#define sz(u) out[u] - in[u]
void calc(int u, int prev = -1){
 int bigC = 0;
 for (int v: g[u]) if (v != prev)
   if (sz(v) > sz(bigC))
     bigC = v;
 for (int v: g[u]) if (v != prev && v != bigC){
       calc(v, u);
       //reset(v)...
   }
  if (bigC) calc(bigC, u);
 for (int v: g[u]) if (v != prev && v != bigC){
   For(t, in[v], out[v]){
     int x = node[t];
     //...
 }
}
```

Centroid Decomposition

```
int size(int u, int prev){
 sz[u] = 1;
 for (int v: g[u]) if (!del[v] && v != prev)
   sz[u] += size(v, u);
 return sz[u];
int centroid(int u, int prev){
 for (int v: g[u]) if (!del[v] && v != prev)
   if (sz[v] > n/2)
     return centroid(v, u);
 return u;
void dfs(int u, int prev){
   in[u] = ++timer; node[timer] = u;
   for (int v: g[u]) if (!del[v] && v != prev){
       dfs(v, u);
       //...
```

Centroid Tree (CT)

Centroid Tree properties:

- Centroid tree height $\leq \log(n)$
- LCA(u, v) in CT lies on the path from u to v in the original tree

```
int size(int u, int prev){
 sz[u] = 1;
 for (int v: g[u]) if (v != prev && !del[v]){
   sz[u] += size(v, u);
   return sz[u];
int centroid(int u, int prev, int m){
 for (int v: g[u]) if (v != prev && !del[v])
   if (sz[v] > m/2)
     return centroid(v, u, m);
 return u;
int cd(int u){
 int m = size(u);
 u = centroid(u, 0, m);
 del[u] = 1;
 for (int v: g[u]) if (!del[v]){
   v = cd(v);
   par[v] = u;
 return u;
//example problems:
void solve(){
   dfs(1, 0); init(); //to calculate the dist(u, v)
        from the original tree
 cd(1):
 memset(d, 0x3f, sizeof d);
 c[1] = 1; //color
 int pp = 1;
 while (pp){
   minimize(d[pp], dist(pp, 1));
   pp = par[pp];
```

```
}
while (q--){
  int t; cin >> t;
  if (t == 1){
    int u; cin >> u;
    c[u] = 1;
    int p = u;
    while (p){
      minimize(d[p], dist(p, u));
      p = par[p];
  }
  else{
    int u; cin >> u;
    if (c[u]) {
             cout << 0 << endl; continue;</pre>
    int p = u, res = INF;
    while(p){
     minimize(res, dist(u, p) + d[p]);
     p = par[p];
    cout << res << endl;</pre>
}
```

Virtual Tree

```
void dfs(int u){
  in[u] = ++timer;
 for (int v: g[u]) if (v != up[u][0]){
   up[v][0] = u;
   For(j, 1, 17) up[v][j] = up[up[v][j - 1]][j - 1];
   dfs(v);
 }
 out[u] = timer;
bool is_anc(int u, int v){
 if (!u) return 1;
 return in[u] <= in[v] && in[v] <= out[u];</pre>
//short LCA
int lca(int u, int v){
 if (is_anc(u, v)) return u;
 ForD(j, 17, 0){
       if (!is_anc(up[u][j], v)){
           u = up[u][j];
 return up[u][0];
bool cmp(int u, int v){
 return in[u] < in[v];</pre>
void query(){
 cin >> k;
 For(i, 1, k) cin >> a[i], sz[a[i]] = 1;
  sort(a + 1, a + k + 1, cmp);
 For(i, 1, k - 1) a[i + k] = lca(a[i], a[i + 1]);
```

```
sort(a + 1, a + k + k, cmp);
k = unique(a + 1, a + k + k) - a - 1;

stack<int> st; st.push(a[1]);
For(i, 2, k){
   while (!is_anc(st.top(), a[i])) st.pop();
   g[st.top()].pb(a[i]);
   st.push(a[i]);
}

res = 0; calc(a[1]);
cout << res << endl;

For(i, 1, k) sz[a[i]] = 0, g[a[i]].clear();
}

void solve(){
   //...
   dfs(1);
   For(i, 1, n) g[i].clear();
   while (q--) query();
}</pre>
```

\mathbf{DP}

Digit DP

```
int f(int id, bool sml, ...){
   if (id < 0) return ...;</pre>
   if (!sml && dp[id][...] != -1) return dp[id
        ][...];
   int lim = sml ? a[id] : 9;
   int res = ...;
   For(c, 0, 1im){
       update(res, f(id - 1, sml && c == lim, ...));
   if (!sml) dp[id][...] = res;
   return res;
int get(int x){
   int n = 0;
   while (x){
       a[n++] = x \% 10;
       x /= 10;
   return f(n - 1, 1, ...);
```

SOS DP

```
for (int k = 0; k < n; k++)
  for (int mask = 0; mask < (1 << n); mask++)
    if (mask & (1 << k))
        dp[mask] += dp[mask ^ (1 << k)];</pre>
```

DP DNC

Solving problems that have the form:

$$dp(i,j) = \min_{0 \le k \le j} \left(dp(i-1,k-1) + C(k,j) \right)$$

The cost function has to satisfie the quadrangle inequality: $C(a,c) + C(b,d) \le C(a,d) + C(b,c)$ for all $a \le b \le c \le d$.

```
Example:

C(j,i) = f(i-j) where f is convex.

C(j,i) = (i-j)

C(j,i) = (i-j)^2
```

```
vector<int> dp_before(n + 1), dp_cur(n + 1);
// cost function
int C(int 1, int r);
// calculate dp_cur[1], ..., dp_cur[r]
void compute(int 1, int r, int optl, int optr) {
   if (1 > r) return;
   int mid = (1 + r) >> 1;
   pair<int, int> best = {INT_MAX, -1};
   //calculate dp_cur[mid] & opt[i][mid] depend on
        dp_before and cost func
   for (int k = optl; k <= min(mid, optr); ++k) {</pre>
       minimize(best, {dp_before[k] + C(k, mid), k})
   dp_cur[mid] = best.first;
   int opt = best.second;
    compute(1, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
int solve() {
   for (int i = 0; i <= n; ++i)</pre>
       dp_before[i] = C(0, i);
   for (int i = 1; i < m; ++i) {</pre>
       compute(0, n, 0, n);
       dp_before = dp_cur;
   return dp_before[n];
```

Convex Hull Trick

Adding lines y = kx + m and querying minimum values at integer x.

```
struct Line {
    int k, m;
   mutable int p;
    int eval(int x){
     return k * x + m;
    bool operator < (const Line& 1) const {</pre>
       return k < 1.k;</pre>
    bool operator < (const int &x) const {</pre>
       return p < x;</pre>
   }
};
struct ConvexHull : multiset<Line, less<>>> {
    int div(int a, int b) {
        return a / b - ((a ^ b) < 0 && a % b);
    bool bad(iterator x, iterator y) {
        if(y == end()) {
```

```
x->p = LINF;
           return 0;
       if(x->k == y->k) x->p = x->m > y->m ? LINF :
       else x->p = div(y->m - x->m, x->k - y->k);
       return x->p >= y->p;
   }
   void add(int k, int m) {
       auto z = insert(\{k, m, 0\}), y = z++, x = y;
       while (bad(y, z)) z = erase(z);
       if(x != begin() && bad(--x, y)) bad(x, y =
       while((y = x) != begin() && (--x)->p >= y->p)
             bad(x, erase(y));
   }
   int query(int x) {
       assert(!empty());
       Line 1 = *lower_bound(x);
       return l.eval(x);
   }
} CH;
//If you want maximum, just flip signs:
CH.add(-k, -m);
int res = -CH.query(x);
```

1D1D Optimization

Solving problems that have the form:

$$dp(i) = \min_{0 \le j < i} \Big(dp(j) + C(j, i) \Big),$$

```
struct item {
  int 1, r, p;
};
long long w(int j, int i) {
   //cost function
void solve() {
  deque<item> dq;
  dq.push_back({1, n, 0});
 for (int i = 1; i <= n; ++i) {
   f[i]=f[dq.front().p]+w(dq.front().p,i);
   ++dq.front().1;
   if (dq.front().1 > dq.front().r) {
     dq.pop_front();
   while (!dq.empty()) {
     auto [1, r, p] = dq.back();
     if (f[i] + w(i, 1) < f[p] + w(p, 1)) {
       dq.pop_back();
     else break;
   if (dq.empty()) {
     dq.push_back({i + 1, n, i});
     // h[i+1]=h[i+2]=...=h[n]=i
```

```
}
    else {
      auto& [1, r, p] = dq.back();
      int low = 1, high = r;
      int pos = r + 1, mid;
      while (low <= high) {
       mid = (low + high) / 2;
       if (f[i] + w(i, mid) < f[p] + w(p, mid)) {
         pos = mid, high = mid - 1;
       else {
         low = mid + 1;
     r = pos - 1;
      if (pos <= n) {</pre>
       dq.push_back({pos, n, i});
        // h[pos]=h[pos+1]=...=h[n]=i
   }
 }
}
```

Knapsack on Tree

Problem: Given a tree T with N vertices rooted at vertex 1 ($1 \le N \le 5000$). The i-th vertex has a value C_i and a constraint K_i ($|C_i| \le 10^9$, $1 \le K_i \le N$). Choose a subset of vertices such that, in the subtree of every vertex i, there are at most K_i chosen vertices, and the total sum of the chosen vertices' values is maximized.

```
void calc(int V){
   int n = child[V].size();
   for(int v_i: child[V]) {
       calc(v_i);
   for(int i = 0; i <= n; i++)fill(fV[i], fV[i] + N</pre>
        + 1, -INF);
   fV[0][0] = 0;
    for(int i = 1; i <= n; i++){</pre>
       int v_i = child[V][i - 1];
       for(int a = 0; a <= sz[V]; a++){</pre>
           for(int b = 0; b <= sz[v_i]; b++){</pre>
               fV[i][a+b] = max(fV[i][a+b], fV[i-1][a
                    ] + dp[v_i][b]);
       sz[V] += sz[v_i];
   for(int k = 0; k \le N; k++){
       if(k > K[V])dp[V][k] = -INF;
       else {
           if(k > 0)dp[V][k] = max(fV[n][k], fV[n][k]
                -1] + C[V]);
           else dp[V][k] = fV[n][k];
       }
   }
   sz[V]++;
long long solve() {
    calc(1);
```

```
return *max_element(dp[1], dp[1] + N + 1);
}
```

DP on Broken Profile

count the number of ways you can fill an $n \times m$ grid using 1×2 and 2×1 tiles. $(1 \le n \le 10, 1 \le m \le 1000)$

```
dp[0][0] = 1;
for (int j = 0; j < m; j++)
   for (int i = 0; i < n; i++) {</pre>
       for (int mask = 0; mask < (1 << n); mask++) {</pre>
           dp[mask][1] = dp[mask ^ (1 << i)][0]; //</pre>
                Horizontal or no tile
           if (i && !(mask & (1 << i)) && !(mask & (1</pre>
                 << i - 1))) // Vertical tile
               dp[mask][1] += dp[mask ^ (1 << i - 1)
                    1[0]:
           if (dp[mask][1] >= MOD) dp[mask][1] -= MOD
       for (int mask = 0; mask < (1 << n); mask++)</pre>
            dp[mask][0] = dp[mask][1];
cout << dp[0][0] << '\n';
```

Number Theory

Euler's totient function

```
int phi(int n) {
   int res = n;
   for (int i = 2; i * i <= n; i++) {
       if (n % i == 0) {
          while (n \% i == 0) n /= i;
          res -= res / i;
   }
   if (n > 1) res -= res / n;
   return res;
```

Euler's totient function from 1 to N

```
void preCompute(int n) {
    iota(phi, phi + N, 0); //phi[i] = i
    for (int i = 2; i <= n; i++) {
       if (phi[i] == i) {
           for (int j = i; j <= n; j += i)</pre>
               phi[j] -= phi[j] / i;
       }
   }
}
```

Modular Inverse

```
//if MOD is a prime number then phi(MOD) = MOD - 1
int inv(int x, int MOD){
   return Pow(x, phi(MOD) - 1);
```

Extended Euclidean Algorithm

```
//computing gcd(a, b) and finding (x, y) that
//ax + by = gcd(a, b)
//recursive version
int gcd(int a, int b, int& x, int& y) {
```

```
if (b == 0) {
       x = 1; y = 0;
       return a;
   }
   int x1, y1;
   int d = gcd(b, a % b, x1, y1);
   x = y1;
   y = x1 - y1 * (a / b);
   return d;
//iterative version
int gcd(int a, int b, int& x, int& y) {
   x = 1, y = 0;
   int x1 = 0, y1 = 1, a1 = a, b1 = b;
   while (b1) {
       int q = a1 / b1;
       tie(x, x1) = make_tuple(x1, x - q * x1);
       tie(y, y1) = make_tuple(y1, y - q * y1);
       tie(a1, b1) = make_tuple(b1, a1 - q * b1);
   }
   return a1;
```

Diophantine

```
bool find_any_solution(int a, int b, int c, int &x0,
    int &y0, int &g) {
   g = gcd(abs(a), abs(b), x0, y0);
   if (c % g) return false;
   x0 *= c / g; y0 *= c / g;
   if (a < 0) x0 = -x0;
   if (b < 0) y0 = -y0;
   return true;
//all the solutions have the form:
//x = x0 + k * b/g
//y = y0 - k * b/g
//IN A GIVEN INTERVAL:
void shift(int & x, int & y, int a, int b, int cnt) {
   x += cnt * b;
   y -= cnt * a;
int find_all_solutions(int a, int b, int c, int minx,
    int maxx, int miny, int maxy) {
   <u>int</u> x, y, g;
   if (!find_any_solution(a, b, c, x, y, g)) return
        0;
   a /= g; b /= g;
   int sign_a = a > 0 ? +1 : -1;
   int sign_b = b > 0 ? +1 : -1;
   shift(x, y, a, b, (minx - x) / b);
   if (x < minx) shift(x, y, a, b, sign_b);</pre>
   if (x > maxx) return 0;
   int lx1 = x;
   shift(x, y, a, b, (maxx - x) / b);
   if (x > maxx) shift(x, y, a, b, -sign_b);
   int rx1 = x;
   shift(x, y, a, b, -(miny - y) / a);
   if (y < miny) shift(x, y, a, b, -sign_a);</pre>
```

```
if (y > maxy) return 0;
int lx2 = x;

shift(x, y, a, b, -(maxy - y) / a);
if (y > maxy) shift(x, y, a, b, sign_a);
int rx2 = x;

if (lx2 > rx2) swap(lx2, rx2);
int lx = max(lx1, lx2);
int rx = min(rx1, rx2);

if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
}
```

Chinese Remainder Theorem

```
// Combine two congruences:
// x = a1 \pmod{m1}, x = a2 \pmod{m2}
// Returns (x, lcm) or (-1,-1) if no solution
pair<11, 11> crt2(11 a1, 11 m1, 11 a2, 11 m2) {
   int x, y;
   11 g = gcd(m1, m2, x, y);
   if ((a2 - a1) % g != 0) {
       return {-1, -1}; // no solution
   11 \ 1cm = m1 / g * m2;
   11 k = (a2 - a1) / g;
   11 mult = (1LL * x * k) % (m2 / g);
   11 ans = (a1 + m1 * mult) % lcm;
   if (ans < 0) ans += 1cm;
   return {ans, lcm};
}
//solve a system of congruences:
//x = a1 \pmod{m1}
//x = a2 \pmod{m2}
//...
//x = ak \pmod{mk}
pair<11, 11> crt(vector<11> a, vector<11> m) {
   pair<11,11> res = {a[0], m[0]};
   for (int i = 1; i < sz(a); i++) {</pre>
       res = crt2(res.first, res.second, a[i], m[i])
       if (res.first == -1) return {-1,-1};
   }
   return res;
//x = sol.first (mod sol.second)
```

Rabin-Miller primality test

```
bool test(ll a, ll n, ll k, ll m){
    ll mod = Pow(a, m, n);
    if (mod == 1 || mod == n - 1) return 1;
    for (int l = 1; l < k; ++l){
        mod = (mod * mod) % n;
        if (mod == n - 1) return 1;
    }
    return 0;
}</pre>
```

```
//check if n is a prime number
bool RabinMiller(ll n){
   if (n == 2 || n == 3 || n == 5 || n == 7) return
        1;
   if (n < 11) return 0;

   ll k = 0, m = n - 1;
   while (!(m & 1)){
        m >>= 1;
        k++;
   }

   const static int repeatTime = 3;
   for (int i = 0; i < repeatTime; ++i){
        ll a = rand() % (n - 3) + 2;
        if (!test(a, n, k, m)) return 0;
   }
   return 1;
}</pre>
```

Pollard's Rho prime factorization

- Average Time Complexity: $O(\sqrt[4]{N}log^2(N))$
- Space Complexity: O(log N)

```
using ll = long long;
using ull = unsigned long long;
using ld = long double;
11 mult(11 x, 11 y, 11 md) {
   ull q = (1d)x * y / md;
   11 res = ((ull)x * y - q * md);
   if (res >= md) res -= md;
   if (res < 0) res += md;</pre>
   return res;
11 powMod(11 x, 11 p, 11 md) {
   if (p == 0) return 1;
   if (p & 1) return mult(x, powMod(x, p - 1, md),
   return powMod(mult(x, x, md), p / 2, md);
bool checkMillerRabin(ll x, ll md, ll s, int k) {
   x = powMod(x, s, md);
   if (x == 1) return true;
   while (k--) {
       if (x == md - 1) return true;
       x = mult(x, x, md);
       if (x == 1) return false;
   return false;
bool isPrime(11 x) {
   if (x == 2 || x == 3 || x == 5 || x == 7) return
   if (x % 2 == 0 || x % 3 == 0 || x % 5 == 0 || x %
        7 == 0) return false;
   if (x < 121) return x > 1;
   11 s = x - 1;
   int k = 0;
   while (s \% 2 == 0) {
       s >>= 1;
   if (x < 1LL << 32) {
```

```
for (11 z : {2, 7, 61}) {
           if (!checkMillerRabin(z, x, s, k)) return
       }
   } else {
       for (11 z : {2, 325, 9375, 28178, 450775,
            9780504, 1795265022}) {
           if (!checkMillerRabin(z, x, s, k)) return
   }
   return true;
mt19937_64 rng(chrono::steady_clock::now().
    time_since_epoch().count());
long long get_rand(long long r) {
   return uniform_int_distribution<long long>(0, r -
         1)(rng);
void pollard(l1 x, vector<l1> &ans) {
   if (isPrime(x)) {
       ans.push_back(x);
       return;
   11 c = 1;
   while (true) {
       c = 1 + get_rand(x - 1);
       auto f = [\&](11 y) {
           11 res = mult(y, y, x) + c;
           if (res >= x) res -= x;
           return res;
       };
       11 y = 2;
       int B = 100;
       int len = 1;
       11 g = 1;
       while (g == 1) {
           11 z = y;
           for (int i = 0; i < len; i++) {</pre>
               z = f(z);
           11 zs = -1;
           int lft = len;
           while (g == 1 && lft > 0) {
               zs = z;
              11 p = 1;
               for (int i = 0; i < B && i < lft; i++)</pre>
                  p = mult(p, abs(z - y), x);
                  z = f(z);
               }
               g = gcd(p, x);
               1ft -= B;
           if (g == 1) {
               y = z;
               len <<= 1;
               continue;
           if (g == x) {
               g = 1;
               z = zs;
               while (g == 1) {
                  g = gcd(abs(z - y), x);
                  z = f(z);
               }
```

```
if (g == x) break;
           assert(g != 1);
           pollard(g, ans);
           pollard(x / g, ans);
           return;
       }
   }
// return list of all prime factors of x (can have
    duplicates)
vector<11> factorize(11 x) {
   vector<11> ans;
   for (11 p : {2, 3, 5, 7, 11, 13, 17, 19}) {
       while (x \% p == 0) {
           x /= p;
           ans.push_back(p);
       }
   if (x != 1) {
       pollard(x, ans);
   sort(ans.begin(), ans.end());
   return ans;
// return pairs of (p, k) where x = product(p^k)
vector<pair<11, int>> factorize_pk(11 x) {
   auto ps = factorize(x);
   11  last = -1, cnt = 0;
   vector<pair<11, int>> res;
   for (auto p : ps) {
       if (p == last)
           ++cnt;
       else {
           if (last > 0) res.emplace_back(last, cnt);
           last = p;
           cnt = 1;
       }
   }
   if (cnt > 0) {
       res.emplace_back(last, cnt);
   }
   return res;
```

Multiplicative Function (Sieve)

- For all coprimes $n, m \in N$, we have f(m, n) =f(m)f(n)
- ex: I(N) = 1, id(n) = n, $id_k(n) = n^k$, $gcd(n, const), \varphi(n), \mu(n), f_k(n) = \sum_{d|n} d^k,$ $\mu(n)$
- Dirichlet Convolution: $h(n) = \sum_{d|n} f(d) \times$ g(n/d) is also a multiplicative function

```
const int MN = 1e6 + 11;
int sieve[MN];
pair<int,int> pk[MN];
int ndiv[MN];
int main() {
```

```
for (int i = 2; i <= 1000; i++)</pre>
    if (!sieve[i]) {
       for (int j = i*i; j <= 1000000; j += i)</pre>
           sieve[j] = i;
    }
ndiv[1] = 1;
for (int i = 2; i <= 1000000; i++) {</pre>
    if (!sieve[i]) {
       pk[i] = make_pair(i, 1);
       ndiv[i] = 2;
    else {
       int p = sieve[i];
       if (pk[i/p].first == p) {
           pk[i] = make_pair(p, pk[i/p].second +
           ndiv[i] = pk[i].second + 1;
       }
       else {
           pk[i] = make_pair(-1, 0);
           int u = i, v = 1;
           while (u % p == 0) {
               u /= p;
               v = v * p;
           ndiv[i] = ndiv[u] * ndiv[v];
       }
   }
}
```

Multiplicative Function

```
int n;
int res = 1;
for (int i = 2; i*i <= n; i++) {
   if (n % i == 0) {
      int u = 1, k = 0;
      while (n % i == 0) {
        n /= i;
        u = u * i;
        k += 1;
    }
   res = res * f(i, k);
}

if (n > 1) {
   res = res * f(n, 1);
}
```

Discrete Logarithm

- To find min x that $a^x \equiv b \pmod{m}$
- Find $ord_m(a) = \min x \text{ that } a^x \equiv 1 \pmod{m}$ in $O(\sqrt[4]{m}logm)$

```
//gcd(a, m) == 1
int discrete_log_BSGS_coprime(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;
    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q) {
        vals[cur] = q;
    }
}</pre>
```

```
cur = 1LL * cur * a % m;
   }
    int step = binpow(a, n, m);
   for (int p = 1, f1 = 1; p <= n; p++) {
       f1 = 1LL * f1 * step % m;
       if (vals.count(f1)) {
           return n * p - vals[f1];
   }
    return -1;
//gcd != 1
int discrete_log_BSGS(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;
    int k = 1, add = 0, g;
    while ((g = \_gcd(a, m)) > 1)  {
       if (b == k) return add;
       if (b % g) return -1;
       b /= g, m /= g, ++add;
       a %= m;
       k = (k * 111 * a / g) % m;
   unordered_map<int, int> vals;
    for (int q = 0, cur = b; q \le n; ++q) {
       vals[cur] = q;
       cur = 1LL * cur * a % m;
    int step = binpow(a, n, m);
    for (int p = 1, f1 = k; p \le n; p++) {
       f1 = 1LL * f1 * step % m;
       if (vals.count(f1)) {
           int ans = n * p - vals[f1] + add;
           return ans;
       }
    return -1;
//ord_m(a):
using ll = long long;
11 powMod(11 x, 11 p, 11 md);
11 gcd(l1 x, l1 y);
vector<11> factorize(11 x); //Pollard's Rho
11 phi(11 n) {
   auto ps = factorize(n);
   11 res = n;
   11  last = -1;
    for (auto p : ps) {
       if (p != last) {
           res = res / p * (p - 1);
           last = p;
       }
   return res;
11 ord(11 a, 11 m) {
    if (gcd(a, m) != 1) return -1;
    ll res = phi(m);
    auto ps = factorize(res);
    for (auto p : ps)
       if (powMod(a, res / p, m) == 1) res /= p;
   return res;
}
```

Geometry

Dot Product

```
//remember that u * v = 0 -> u is perdendicular with
v
//or (u, v) = pi/2
double dotProduct(Vector u, Vector v){
   return u.x * v.x + u.y * v.y;
}
```

Angle

```
//u * v = |u| * |v| * cos(theta)
//-> theta = acos (u * v / (|u| * |v|))
double Cos(Vector u, Vector v){
    return dotProduct(u, v)/(u.len * v.len);
}
double theta(Vector u, Vector v){
    return acos(Cos(u, v));
}
```

Cross Product

```
//u * v = |u| * |v| * sin(theta)
//u * v = u.x * v.y - u.y * v.x
//|u * v| = area of a parallelogram formed by
    adjacent vectors u and v
//= double the area of the triangle
double crossProduct(Vector u, Vector v){
    return u.x * v.y - u.y * v.x;
}
```

Distance from a Point to a line

```
d(C, AB) = |\vec{AB} * \vec{AC}|/AB
```

```
// Compute the distance from AB to C
// if isSegment is true, AB is a segment, not a line.
double linePointDist(Point A, Point B, Point C, bool
   isSegment) {
    double res = abs(cross(A, B, C)) / dist(A, B);
    if (isSegment) {
        int dot1 = dot(B, A, C);
        if (dot1 < 0) return distance(B, C);
        int dot2 = dot(A, B, C);
        if (dot2 < 0) return distance(A, C);
   }
   return res;
}</pre>
```

Template 1

```
struct vec {
    db x, y;
    vec(db _x = 0, db _y = 0) : x(_x), y(_y) {}
    db dot(const vec &other) { // Compute the dot
        product
        return x * other.x + y * other.y;
    }
    db cross(const vec &other) { // Compute the cross
        product
        return x * other.y - y * other.x;
    }
    db length() const {
        return sqrt(x * x + y * y);
    }
};
```

```
using point = vec; // or use 'typedef vec point'
vec operator - (const point &B, const point &A) { //
    vecAB = B - A
    return vec(B.x - A.x, B.y - A.y);
}

// if isSegment is true, AB is a segment, not a line.
db linePointDist(const point &A, const point &B,
    const point &C, bool isSegment) {
    db dist = abs((B - A).cross(C - A)) / (A - B).
        length();
    if (isSegment) {
        db dot1 = (A - B).dot(C - B);
        if (dot1 < 0) return (B - C).length();
        db dot2 = (B - A).dot(C - A);
        if (dot2 < 0) return (A - C).length();
    }
    return dist;
}</pre>
```

Intersection of 2 lines and bla bla (I have no time bro)

```
Lines will have the form: ax + by = c.

\vec{AB} \times \vec{AC} > 0 \Rightarrow A, B, C are counterclockwise.

\vec{AB} \times \vec{AC} < 0 \Rightarrow A, B, C are clockwise.

\vec{AB} \times \vec{AC} = 0 \Rightarrow A, B, C are collinear.
```

```
const double eps = 1e-9;
int sign(double x) {
   if (x > eps) return 1;
   if (x < -eps) return -1;</pre>
   return 0;
double cross(Vec AB, Vec AC) {
   return AB.x * AC.y - AC.x * AB.y;
}
double dot(Vec AB, Vec AC) {
   return AB.x * AC.x + AB.y * AC.y;
//intersection of 2 segments
bool intersect(Point A, Point B, Point C, Point D) {
   int ABxAC = sign(cross(B - A, C - A));
   int ABxAD = sign(cross(B - A, D - A));
   int CDxCA = sign(cross(D - C, A - C));
   int CDxCB = sign(cross(D - C, B - C));
   if (ABxAC == 0 \mid \mid ABxAD == 0 \mid \mid CDxCA == 0 \mid \mid
        CDxCB == 0) {
       // C on segment AB if ABxAC = 0 and CA.CB <=
       if (ABxAC == 0 && sign(dot(A - C, B - C)) <=</pre>
            0) return true;
       if (ABxAD == 0 && sign(dot(A - D, B - D)) <=</pre>
            0) return true;
       if (CDxCA == 0 && sign(dot(C - A, D - A)) <=</pre>
            0) return true;
        if (CDxCB == 0 && sign(dot(C - B, D - B)) <=</pre>
            0) return true;
       return false;
   return (ABxAC * ABxAD < 0 && CDxCA * CDxCB < 0);
```

Circle passing through 3 points

```
struct Point {
   double x, y;
   Point() { x = y = 0.0; }
```

```
Point(double x, double y) : x(x), y(y) {}
   Point operator + (const Point &a) const { return
        Point(x + a.x, y + a.y); }
   Point operator - (const Point &a) const { return
        Point(x - a.x, y - a.y); }
   Point operator * (double k) const { return Point(
        x * k, y * k); }
   Point operator / (double k) const { return Point(
        x / k, y / k); }
};
struct Line { // Ax + By = C
   double a, b, c;
   Line(double a = 0, double b = 0, double c = 0):
        a(a), b(b), c(c) {}
   Line(Point A, Point B) {
       a = B.y - A.y;
       b = A.x - B.x;
       c = a * A.x + b * A.y;
   }
};
Line Perpendicular_Bisector(Point A, Point B) {
   Point M = (A + B) / 2;
   Line d = Line(A, B);
   // the equation of a perpendicular line has the
        form: -Bx + Ay = D
   double D = -d.b * M.x + d.a * M.y;
   return Line(-d.b, d.a, D);
//Intersection of 2 Perpendicular Bisector is the
    center of the circle
```

Symmetry

```
struct Line { // Ax + By = C
   double a, b, c;
   Line(double a = 0, double b = 0, double c = 0):
        a(a), b(b), c(c) {}
};
Point intersect(Line d1, Line d2) {
   double det = d1.a * d2.b - d2.a * d1.b;
   // det != 0 because d1 is perpendicular to d2
   return Point((d2.b * d1.c - d1.b * d2.c) / det, (
        d1.a * d2.c - d2.a * d1.c) / det);
Point Symmetry (Point X, Line d) {
   \ensuremath{//} the equation of a perpendicular line has the
        form: -Bx + Ay = D
   double D = -d.b * X.x + d.a * X.y;
   Line d2 = Line(-d.b, d.a, D);
   Point Y = intersect(d, d2);
   Point X2 = Point(2 * Y.x - X.x, 2 * Y.y - X.y);
   return X2;
```

Rotation

To rotate A(x, y) counterclockwise by an angle theta around the origin, we can easily use this formula:

```
x' = x\cos\theta - y\sin\theta
y' = x\sin\theta + y\cos\theta
```

```
| Point Rotations(Point A, Point C, double rad) {
```

```
Point A2 = A - C;
Point B2 = Point(A2.x * cos(rad) - A2.y * sin(rad
    ), A2.x * sin(rad) + A2.y * cos(rad));
Point B = B2 + C;
return B;
```

Area of a Polygon

```
double polygonArea(const vector<Point>& poly) {
   int n = poly.size();
   double area = 0.0;
   for (int i = 0; i < n; i++) {</pre>
       int j = (i + 1) \% n;
       area += poly[i].x * poly[j].y - poly[j].x *
           poly[i].y;
   }
   return fabs(area) / 2.0;
```

Relative position of a point to a polygon O(N)

```
//using Area
//Time Complexity: O(N) per query
PointPolygonPosition position(Polygon plg, Point p) {
   long long sSumTris = 0;
   for (int i = 0; i < plg.nVertices; i++) {</pre>
       int i1 = (i + 1) % plg.nVertices;
       Polygon tri(p, plg.vertices[i], plg.vertices[
            i1]);
       auto sTri = tri.area2(); //2 * area
       if (!sTri) {
           return BOUNDARY;
       sSumTris += sTri;
   return (sSumTris == plg.area2() ? INSIDE :
        OUTSIDE);
```

Relative position of a point to a polygon O(log N)

```
//using binary search
bool isCW(Point a, Point b, Point c) {
    return (Vector(a, b) ^ Vector(a, c)) < 0;</pre>
PointPolygonPosition position(Polygon plg, Point p) {
    // Check if P is on A_1A_n
    Vector pa1(p, plg.vertices[0]);
    Vector pan(p, plg.vertices[plg.nVertices - 1]);
    if (pa1 ^ pan == 0) { //cross product
        if (111 * pa1.x * pan.x <= 0) {</pre>
             return BOUNDARY;
        return OUTSIDE;
    int l = 1, r = plg.nVertices;
    while (r - 1 > 1) {
        int mid = (1 + r) >> 1;
         \begin{tabular}{ll} \textbf{if} & (is CW (plg. vertices [0], p, plg. vertices [mid \\ \end{tabular} ) \label{table constraints} 
             ])) {
             1 = mid;
        } else {
             r = mid;
```

```
}
int k = 1;
if (k == plg.nVertices - 1) {
   return OUTSIDE;
// Check if P is on the triangle
if (Vector(p, plg.vertices[k]) ^ Vector(p, plg.
    vertices[k + 1]) == 0) {
   return BOUNDARY;
}
long long ss = 0;
ss += Polygon(p, plg.vertices[0], plg.vertices[k
    ]).area2();
ss += Polygon(p, plg.vertices[k], plg.vertices[k
    + 1]).area2();
ss += Polygon(p, plg.vertices[k + 1], plg.
    vertices[0]).area2();
if (ss == Polygon(plg.vertices[0], plg.vertices[k
                plg.vertices[k + 1]).area2()) {
   return INSIDE;
return OUTSIDE;
```

Pick Theorem

A = I + B/2 - 1

- A =Area of the Polygon
- I = Number of interior lattice points (strictly inside the polygon)
- B = Number of boundary lattice points (on the polygon edges)

Convex Hull (Graham scan)

```
// Cross Product of AB and AC
long long cross(const Point &A, const Point &B, const
     Point &C) {
   return 1LL * (B.x - A.x) * (C.y - A.y) - 1LL * (C
        .x - A.x) * (B.y - A.y);
// A -> B -> C clockwise (-1), collinear (0),
    counterclockwise (1)
int ccw(const Point &A, const Point &B, const Point &
    C) {
   long long S = cross(A, B, C);
   if (S < 0) return -1;</pre>
   if (S == 0) return 0;
   return 1;
}
//convex hull listed in counterclockwise order
vector<Point> convexHull(vector<Point> p, int n) {
   for (int i = 1; i < n; ++i) {</pre>
       if (p[0].y > p[i].y || (p[0].y == p[i].y && p
            [0].x > p[i].x)) {
           swap(p[0], p[i]);
       }
   sort(p.begin() + 1, p.end(), [&p](const Point &A,
         const Point &B) {
       int c = ccw(p[0], A, B);
```

Convex Hull (Monotone chain algorithm)

```
bool ccw(const Point &A, const Point &B, const Point
    &C) {
   return 1LL * (B.x - A.x) * (C.y - A.y) - 1LL * (C
        .x - A.x) * (B.y - A.y) > 0;
vector<Point> convexHull(vector<Point> p, int n) {
   sort(p.begin(), p.end(), [](const Point &A, const
         Point &B) {
       if (A.x != B.x) return A.x < B.x;
       return A.y < B.y;</pre>
   });
   vector<Point> hull;
   hull.push_back(p[0]);
   for (int i = 1; i < n; ++i) {
       while (hull.size() >= 2 && ccw(hull[hull.size
           () - 2], hull.back(), p[i])) {
           hull.pop_back();
       hull.push_back(p[i]);
   }
   for (int i = n - 2; i >= 0; --i) {
       while (hull.size() >= 2 && ccw(hull[hull.size
           () - 2], hull.back(), p[i])) {
           hull.pop_back();
       hull.push_back(p[i]);
   }
   if (n > 1) hull.pop_back();
   return hull;
```

More

- Number of triangles with no polygon side (no two vertices adjacent): n(n-4)(n-5)/6.
- Number of intersection points of diagonals in a convex n-gon: n(n-1)(n-2)(n-3)/24
- Number of triangles formed by one diagonal + a third vertex: $\binom{n}{2}(n-4)$

- Partitions of the polygon into triangles using non-crossing diagonals: $C_{n-2} = \frac{1}{n-1} {2n-4 \choose n-2}$ where C_k is the k-th Catalan number.
- Number of ways to choose a set of non-crossing diagonals C_{n-2}
- Number of ways to choose k non-crossing diagonals in a convex $= N(n, k) = \frac{1}{k+1} \binom{n-3k}{k}$.
- You are given a convex polygon with n sides. You then connect all of the points by drawing lines among the diagonals of the polygon, then cut through the lines all at once. The maximum number of pieces: $R(N) = (n-1)(n-2)(n^2 3n + 12)/24$
- With a circle: $+ = n \ (n \text{ edges}).$
- a/sinA = b/sinB = c/sicC = R
- Menelaus: a line that cross BC, AC, AB at points D, E, F-> AF/FB*BD/DC*CE/EA = 1
- Ceva: AX, BY, CZ will be concurrent, if: BX/XC * CY/YA * AZ/ZB = 1
- Apollonius: Median line AD: $AB^2 + AC^2 = 2(AD^2 + BC^2/4)$
- Euler: $OI^2 = R(R 2r)$
- Cyclic quadrilateral: $\angle A + \angle C = 180^{\circ}$, AB * CD = AD * BC, AC * BD = AB * CD + AD * BC
- Simson line: the feet of the perpendiculars from a point on a triangle's circumcircle to the sides of the triangle are collinear.
- Triangle: $S = \sqrt{p(p-a)(p-b)(p-c)}$
- Cyclic quadrilateral:
- $S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$

Algebra

Matrix Multiplication

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot 8 \\ 3 \cdot 5 + 4 \cdot 7 & 3 \cdot 6 + 4 \cdot 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

```
#define vi vector<int>
struct Matrix{
  vector<vi> a;
  int r, c;
  Matrix(){
    a.clear(); r = c = 0;
  }
  Matrix(vector<vi> a, int r, int c): a(a), r(r), c(c)
  ){}
```

```
Matrix operator * (const Matrix &B) const{
   vector<vi> res(r, vi(B.c, 0));
   vector<vi> b = B.a;
   rep(i, r){
     rep(j, B.c){
       rep(k, c){
         res[i][j] += a[i][k] * b[k][j] % MOD;
         res[i][j] %= MOD;
     }
   return Matrix(res, r, B.c);
Matrix Pow(Matrix A, int b){
 vector<vi> a(A.r, vi(A.r, 0));
 rep(i, A.r) a[i][i] = 1;
 Matrix res(a, A.r, A.c);
 while (b){
   if (b & 1) res = res * A;
   A = A * A;
   b >>= 1:
 return res;
```

Fast Fourier Transform

```
namespace FFT{
   #define cd complex<long double>
   #define vc vector<cd>
   const long double PI = acosl(-1.0L);
   const int N = 1e6 + 5;
   int rev[N];
   void fft(vc &a, bool inverse = 0){
       int n = sz(a);
       rep(i, n) if (i < rev[i]){
           swap(a[i], a[rev[i]]);
       for (int len = 2; len <= n; len <<= 1){</pre>
           cd wn = polar(1.0L, PI/len * (inverse ? -2
                 : 2));
           for (int i = 0; i < n; i += len){</pre>
               cd w = 1;
               rep(j, len/2){
                   cd u = a[i + j];
                   cd v = a[i + j + len/2] * w;
                  a[i + j] = u + v;
                  a[i + j + len/2] = u - v;
                  w *= wn;
              }
           }
       }
       if (inverse){
           for (cd &x: a){
```

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rep(i, n) res[i] = (int)(real(fa[i]) + 0.5);

Number Theory Transform

rep(i, n) fa[i] *= fb[i];

n = sz(a) + sz(b) - 1;

fft(fa, 1);

vi res(n);

return res;

}

```
namespace NTT{
   const int MOD = 998244353;
   const int g = 3; //primitive root
   int rev[N];
   void ntt(vi &a, bool inverse = 0){
       int n = sz(a);
       rep(i, n) if (i < rev[i]){
           swap(a[i], a[rev[i]]);
       for (int len = 2; len <= n; len <<= 1){</pre>
           int wn = Pow(g, (MOD - 1)/len);
           if (inverse) wn = Pow(wn, MOD - 2);
           for (int i = 0; i < n; i += len){</pre>
              int w = 1;
              rep(j, len/2){
                  int u = a[i + j];
                  int v = mul(w, a[i + j + len/2]);
                  a[i + j] = sum(u, v);
                  a[i + j + len/2] = dif(u, v);
                  w = mul(w, wn);
              }
           }
       }
       if (inverse){
           int div_n = Pow(n, MOD - 2);
           rep(i, n) a[i] = mul(a[i], div_n);
       }
```

```
}
   vi operator * (const vi &a, const vi &b){
       if (a.empty() || b.empty()) return {};
       vi fa(all(a)), fb(all(b));
       int n = 1, L = 0;
       while (n < sz(a) + sz(b) - 1) n <<= 1, ++L;
       rep(i, n){
           rev[i] = (rev[i >> 1] | (i & 1) << L) >>
               1:
       fa.resize(n); fb.resize(n);
       ntt(fa); ntt(fb);
       rep(i, n) fa[i] = mul(fa[i], fb[i]);
       ntt(fa, 1);
       fa.resize(sz(a) + sz(b) - 1);
       return fa:
   }
}
```

Combinatoric

Formula

```
//DP version:
void preCompute(){
    for (int i = 0; i <= n; i++){
        C[i][0] = 1;
        for (int k = 1; k <= i; k++){
            C[i][k] = C[i - 1][k - 1] + C[i - 1][k];
        }
    }
}

//"you know what it is" version:
int C(int n, int k){
    if (n < k || k < 0) return 0;
    return mul(fact[n], mul(ifact[n - k], ifact[k]))
}</pre>
```

Catalan

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} = \frac{1}{n+1} {2n \choose n}$$

Applications

- Number of ways to triangulate a convex polygon with n + 2 vertices.
- Number of Dyck words of length 2n (strings of $n \times X$ and $n \times Y$, every prefix has $\#X \ge \#Y$).
- Number of valid parentheses sequences with *n* pairs.

```
For n = 3: ((())), (()()), (())(), ()(()), ()(()).
```

• Number of ways to parenthesize (n+1) factors. Example (n=3):

```
((ab)c)d, (a(bc))d, (ab)(cd), a((bc)d), a(b(cd))
```

Team: HCMUS-PeacefulPear

• Number of full binary trees with *n* internal nodes.

Derangement

```
//Principle of Inclusion-Exclusion
int c = 1;
for (int i = 1; i <= n; i++) {
    c = (c * i) + (i % 2 == 1 ? -1 : 1);
    cout << c << ' ';
}
//DP
//dp[n] = (n - 1)(dp[n - 2] + dp[n - 1])</pre>
```

Classical Sums

$$\sum_{k=0}^{n} \binom{n}{k} = 2^{n}$$

$$\sum_{k=0}^{n} k \binom{n}{k} = n \cdot 2^{n-1}, \quad \sum_{k=0}^{n} k^{2} \binom{n}{k} = n(n+1) \cdot 2^{n-2}$$

$$\sum_{k=0}^{n} (-1)^{k} \binom{n}{k} = 0 \quad (n > 0)$$

$$\sum_{k=0}^{n} \binom{n}{k}^{2} = \binom{2n}{n}$$

$$\sum_{k=0}^{r} \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r} \quad \text{(Vandermonde)}$$

Useful Identities

$$\sum_{k=r}^{n} {n \choose r} = {n+1 \choose r+1} \quad \text{(Hockey-stick)}$$

$$\sum_{k=0}^{r} (-1)^k {r \choose k} {n+k \choose m} = {n \choose m-r}$$

$${n \choose k_1, k_2, \dots, k_m} = \frac{n!}{k_1! k_2! \cdots k_m!}, \quad \sum_{k=0}^{m} k_i = n$$

Stirling Numbers

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$x^n = \sum_{k=0}^n S(n,k)(x)_k, \quad (x)_k = x(x-1)\cdots(x-k+1)$$

Others

Test?

```
int c = 0;
ios_base::sync_with_stdio(false); cin.tie(0);
    cout.tie(0);
for (int i = 1; i<=1000; i++){
    ofstream fo("a.inp");
    int n = 1000;
    fo<<n<<endl;
    REP(i, n) fo<<rand(1, 100000000)<<<endl;
    REP(i, n) fo<<rand(1, 100000000)<<endl;
    system("ac.exe");
    system("trau.exe");
    if (system("fc al.out a2.out") == 1) exit(0);
}
return 0;</pre>
```

Big Int

```
#include <bits/stdc++.h>
using namespace std;
struct BigInt {
   static const int base = 1000000000; // 1e9
   static const int base_digits = 9;
   vector<int> a; // little-endian (a[0] is least
        significant block)
   int sign;
   // Constructors
   BigInt(): sign(1) {}
   BigInt(long long v) { *this = v; }
   BigInt(const string &s) { read(s); }
   // Assign from integer
   BigInt& operator=(long long v) {
       sign = 1;
       if (v < 0) sign = -1, v = -v;
       a.clear();
       for (; v > 0; v /= base) a.push_back(v % base
       return *this;
   }
   // Remove leading zeroes
   void trim() {
       while (!a.empty() && a.back() == 0) a.
           pop_back();
       if (a.empty()) sign = 1;
   }
   // Read from string
   void read(const string &s) {
       sign = 1;
       a.clear();
       int pos = 0;
       while (pos < (int)s.size() && (s[pos] == '-'</pre>
            || s[pos] == '+')) {
           if (s[pos] == '-') sign = -sign;
           pos++;
       }
       for (int i = s.size()-1; i >= pos; i -=
           base_digits) {
           int x = 0;
           for (int j = max(pos, i - base_digits + 1)
               ; j <= i; j++)
               x = x * 10 + (s[j]-'0');
           a.push_back(x);
       }
       trim();
```

```
}
// Output
friend ostream& operator<<(ostream &os, const</pre>
    BigInt &v) {
    if (v.sign == -1 && !v.isZero()) os << '-';</pre>
    if (v.a.empty()) { os << 0; return os; }</pre>
    os << v.a.back();
    for (int i = (int)v.a.size()-2; i >= 0; i--)
       os << setw(base_digits) << setfill('0') <<
             v.a[i]:
    return os;
}
// Compare absolute values
static int cmpAbs(const BigInt &a, const BigInt &
    if (a.a.size() != b.a.size()) return a.a.size
        () < b.a.size() ? -1 : 1;
    for (int i = (int)a.a.size()-1; i >= 0; i--)
        if (a.a[i] != b.a[i]) return a.a[i] < b.a[</pre>
            i] ? -1 : 1;
    return 0;
}
// Comparison operators
bool operator<(const BigInt &v) const {</pre>
    if (sign != v.sign) return sign < v.sign;</pre>
    int cmp = cmpAbs(*this, v);
    return sign == 1 ? cmp < 0 : cmp > 0;
bool operator==(const BigInt &v) const { return
    sign == v.sign && a == v.a; }
bool operator!=(const BigInt &v) const { return
    !(*this == v); }
bool operator>(const BigInt &v) const { return v
    < *this; }
bool operator<=(const BigInt &v) const { return</pre>
    !(v < *this); }
bool operator>=(const BigInt &v) const { return
    !(*this < v); }
bool isZero() const { return a.empty(); }
// Addition
BigInt operator+(const BigInt &v) const {
    if (sign == v.sign) {
       BigInt res = v;
       int carry = 0;
       for (size_t i = 0; i < max(a.size(), v.a.</pre>
            size()) || carry; i++) {
           if (i == res.a.size()) res.a.push_back
           long long sum = res.a[i] + carry + (i
               < a.size() ? a[i] : 0);</pre>
           carry = sum >= base;
           if (carry) sum -= base;
           res.a[i] = sum;
       return res;
    return *this - (-v);
}
// Negation
BigInt operator-() const {
    BigInt res = *this;
    if (!res.isZero()) res.sign = -sign;
```

```
return res;
}
// Subtraction
BigInt operator-(const BigInt &v) const {
    if (sign == v.sign) {
        if (cmpAbs(*this, v) >= 0) {
           BigInt res = *this;
           int carry = 0;
           for (size_t i = 0; i < v.a.size() ||</pre>
                carry; i++) {
               res.a[i] -= carry + (i < v.a.size()
                    ? v.a[i] : 0);
               carry = res.a[i] < 0;</pre>
               if (carry) res.a[i] += base;
           res.trim();
           return res;
       }
       return -(v - *this);
    }
    return *this + (-v);
}
// Multiplication
BigInt operator*(const BigInt &v) const {
    BigInt res;
    res.sign = sign * v.sign;
    res.a.assign(a.size()+v.a.size(), 0);
    for (size_t i = 0; i < a.size(); i++) {</pre>
       long long carry = 0;
       for (size_t j = 0; j < v.a.size() || carry</pre>
            ; j++) {
           long long cur = res.a[i+j] + carry +
               1LL * a[i] * (j < v.a.size() ? v.a[
                   j]: 0);
           res.a[i+j] = int(cur % base);
           carry = cur / base;
       }
    }
    res.trim();
    return res;
// Division and modulo
BigInt divmod(const BigInt &v, BigInt &rem) const
    int norm = base / (v.a.back() + 1);
    BigInt a = abs() * norm;
    BigInt b = v.abs() * norm;
    BigInt q; q.a.assign(a.a.size(), 0);
    rem = 0;
    rem.a.resize(a.a.size());
    for (int i = (int)a.a.size()-1; i >= 0; i--)
       rem.shiftRight();
       rem.a[0] = a.a[i];
        rem.trim();
        int s1 = rem.a.size() <= b.a.size() ? 0 :</pre>
            rem.a[b.a.size()];
        int s2 = rem.a.size() <= b.a.size()-1 ? 0</pre>
            : rem.a[b.a.size()-1];
        long long d = ((long long)base * s1 + s2)
            / b.a.back();
       BigInt tmp = b * d;
        while (rem < tmp) { d--; tmp = tmp - b; }</pre>
       rem = rem - tmp;
        q.a[i] = d;
```

```
}
       q.sign = sign * v.sign;
       rem.sign = sign;
       q.trim();
       rem.trim();
       return q;
   }
   BigInt operator/(const BigInt &v) const {
       BigInt rem;
       return divmod(v, rem);
   BigInt operator%(const BigInt &v) const {
       BigInt rem;
       divmod(v, rem);
       return rem;
   // Helpers
   BigInt abs() const {
       BigInt res = *this;
       res.sign = 1;
       return res;
   void shiftRight() {
       if (a.empty()) a.push_back(0);
       a.insert(a.begin(), 0);
   }
};
```

- Ctrl + Shift + J \rightarrow Insert new line below current line.
- $Ctrl + G \rightarrow Go to line$.
- Ctrl + Shift + V → Paste without indentation (useful when pasting code from outside).

Multiplication Function

```
11 mult(11 x, 11 y, 11 md) {
    ull q = (1d)x * y / md;
    ll res = ((ull)x * y - q * md);
    if (res >= md) res -= md;
    if (res < 0) res += md;
    return res;
}</pre>
```

Random Function

CODE::BLOCK Set Up

*

Settings -> Compiler... -> Selected compiler: GNU GCC Compiler

Have g++ follow the C++17 ISO C++ language standard [-std=c++17]

Enable all common compiler warnings (-Wall) Enable extra compiler warnings (-Wextra) -O2

Template

Settings -> Editor -> Abbreviations -> Add Ctrl + J to use

CODE::BLOCK shortcuts

*

- Ctrl + Space \rightarrow Autocomplete (symbols, functions, variables).
- $Ctrl + Shift + C \rightarrow Comment selected block.$
- $Ctrl + Shift + X \rightarrow Uncomment selected block.$
- $Ctrl + D \rightarrow Duplicate current line/selection.$
- Ctrl + Shift + ↑ / ↓ → Move current line/selection up or down.
- $Ctrl + L \rightarrow Delete current line.$
- Ctrl + Shift + K \rightarrow Insert new line above current line.