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Sistemas Lineares - Regra de Cramer - Escalonamento (Gauss)

Exercício 1.A) e 1.B)

01. Resolva os sistemas pela regra de Cramer

a)
$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$
$$x = \frac{D_x}{D} = \frac{3}{5}$$
$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$
$$y = \frac{D_y}{D} = -\frac{4}{5}$$
$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = -4$$
$$V = \left\{ \left(\frac{3}{5}, -\frac{4}{5} \right) \right\}$$

B)
$$\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} = -10 - 13 = -23$$
$$\begin{aligned} 0 - 12 + 2 &= -10 \\ 0 + 9 - 4 &= 5 \end{aligned}$$
$$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} = -22 - 1 = -23$$
$$\begin{aligned} 0 - 21 - 1 &= -22 \\ 0 + 3 - 2 &= 1 \end{aligned}$$

Continuação Exercício 1.B)

$$\Delta y = \begin{vmatrix} 3 & 1 & 1 & | & 3 & 1 \\ 2 & -1 & 3 & | & 2 & -1 \\ 4 & 7 & -2 & | & 4 & 7 \end{vmatrix} = 32 - 55 = -23 //$$

$$6 \quad 12 \quad 14 = 32$$

$$-4 \quad 63 \quad -4 = 55$$

$$\Delta z = \begin{vmatrix} 3 & -1 & 1 & | & 3 & -1 \\ 2 & 0 & -1 & | & 2 & 0 \\ 4 & 1 & 7 & | & 4 & 1 \end{vmatrix} = 6 - (17) = -11 //$$

$$0 + 4 + 2 = 6$$

$$0 - 3 - 14 = -17$$

$$Dx = \frac{\Delta x}{D} = \frac{-23}{-23} = 1$$

$$y = \frac{\Delta y}{D} = \frac{-23}{-23} = 1 \quad V = \{(1, 1, -1)\}$$

$$z = \frac{\Delta z}{D} = \frac{-11}{-23} = -1$$

Exercício 2

2. (FGV)

$$\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} = 3(15 - 2) - 12(12 - 2) + 8(12 - 31) = 30$$

$$45 + 8 + 8 = 61$$

$$-5 - 12 + 48 = 31$$

$$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} = 3(12 - 8) - 1(12 - 2) + 1(32 - 12) = 30$$

$$108 + 2 - 32 = 78$$

$$-12 + 48 + 12 = 48$$

$$y = \frac{D_y}{D} = \frac{30}{30} = 1 \quad \text{Resposta A}$$

Exercício 3

3(PUCSP)

$$\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} = -36 - (-37) = 1 //$$

$$-1 - 44 + 9 = -36$$

$$2 - 33 - 6 = -37$$

$$D_x = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} = -29 - (-28) = -1 //$$

$$-1 - 22 - 6 = -29$$

$$1 - 33 + 4 = -28$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{vmatrix} = -17 - (-18) = 1 //$$

$$2 - 22 + 3 = -17$$

$$-4 - 11 - 3 = -18$$

$$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 2 - 2 = 0 //$$

$$1 - 8 + 9 = 2$$

$$2 - 6 + 6 = 2$$

Continuação do Exercício 3

$$x = \frac{D_x}{D} = \frac{-1}{1} = -1 //$$

$$y = \frac{D_y}{D} = \frac{1}{1} = 1 //$$

$$z = \frac{D_z}{D} = \frac{0}{1} = 0 //$$

$A + B + C$
 $-1 + 1 + 0 = 0$

Resposta C

Exercício 4

04. (UFRRS)

$$\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} = 1 - (-15) = 16 //$$

$$\begin{aligned} -6 & 4 & 3 & = & 1 \\ -9 & -2 & -4 & = & -15 \end{aligned}$$

$$\Delta_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} = -130 - (-146) = 16 //$$

$$\begin{aligned} -174 & + 32 & + 12 & = & -130 \\ -72 & -58 & -16 & = & -146 \end{aligned}$$

$$\Delta_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} = 26 - (-54) = 80 //$$

$$\begin{aligned} -8 & + 58 & -24 & = & 26 \\ -12 & + 16 & -58 & = & -54 \end{aligned}$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} = 3 - 99 = -96 //$$

$$\begin{aligned} 24 & + 8 & - 29 & = & 3 \\ 87 & - 4 & + 16 & = & 99 \end{aligned}$$

Continuação do Exercício 4 e Exercício 5

$$X = \frac{\Delta x}{D} = \frac{16}{16} = 1$$

$$y = \frac{\Delta y}{D} = \frac{80}{16} = 5$$

$$z = \frac{\Delta z}{D} = \frac{-96}{16} = -6$$

$$X + Y + Z$$

$$1 + 5 - 6 = 0$$

Resposta A

$$5) \begin{cases} 2x + y = 5 \\ \quad + 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 2 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 2(4 - 3) = 2$$

$$4 \ 3 \ 0 = 7$$

$$0 \ 4 \ 0 = 4$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} = 5 \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 1 \\ 7 & 1 \end{vmatrix} = 5(2 - 2) - 1(3 - 7) = 4$$

$$10 \ 7 \ 0 = 17$$

$$0 \ 10 \ 3 = 13$$

Continuação do Exercício 5

$$\Delta y = \begin{vmatrix} 2 & 5 & 0 & 2 & 5 \\ 0 & 3 & 1 & 0 & 3 \\ 3 & 7 & 1 & 3 & 7 \end{vmatrix} = 21 - 14 = 7$$

$$6 \cdot 15 \cdot 0 = 21$$

$$0 \cdot 14 \cdot 0 = 14$$

$$\Delta z = \begin{vmatrix} 2 & 1 & 5 & 2 & 1 \\ 0 & 2 & 3 & 0 & 2 \\ 3 & 2 & 7 & 3 & 2 \end{vmatrix} = 37 - 42 = -5$$

$$28 + 9 \cdot 0 = 37$$

$$30 \cdot 12 \cdot 0 = 42$$

$$x = \frac{\Delta x}{D} = \frac{4}{3}$$

$$y = \frac{\Delta y}{D} = \frac{7}{3}$$

$$z = \frac{\Delta z}{D} = \frac{-5}{3}$$

Resposta D

$$V = \left\{ \left(\frac{4}{3}, \frac{7}{3}, -\frac{5}{3} \right) \right\}$$

Exercício 6

6.
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{cases} 1 \cdot x & 0 \cdot y & 0 \cdot z \\ 2 \cdot x & 1 \cdot y & 0 \cdot z \\ -1 \cdot x & 2 \cdot y & 2 \cdot z \end{cases} = \begin{cases} x + 0 + 0 \\ 2x + y + 0 \\ -1x + 2y + 2z \end{cases}$$

$$D = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} = 2 \cdot 1 = 2 //$$

$$2 \cdot 0 \cdot 0 = 2$$

$$0 \cdot 0 \cdot 0 = 0$$

$$D_x = \begin{vmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} = 6 - 0 = 6 //$$

$$6 \cdot 0 \cdot 0 = 6$$

$$0 \cdot 0 \cdot 0 = 0$$

$$D_y = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ -1 & -1 & 2 \end{vmatrix} = 14 - 0 = 14 //$$

$$14 + 0 + 0 = 14$$

$$0 \cdot 0 \cdot 0 = 0$$

Continuação do Exercício 6

$$Dz = \begin{array}{ccc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & 1 & 7 & 2 & 1 \\ -1 & 2 & -1 & -1 & 2 \end{array} = 11 - 11 = 0 //$$

$$-1 \quad 0 \quad 12 = 11$$

$$-3 \quad 14 \quad 0 = 11$$

$$x = \frac{Dx}{D} = \frac{6}{2} = 3 //$$

$$y = \frac{Dy}{D} = \frac{14}{2} = 7$$

$$z = \frac{Dz}{D} = \frac{0}{2} = 0 //$$

Resposta E

Exercício 1

1) (UFG)

$$S = \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$\begin{array}{l} \rightarrow 3 \left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array} \right) \sim \begin{array}{l} \rightarrow 1 \left(\begin{array}{ccc|c} 7 & 0 & -10 & -4 \\ 1 & 0 & -5 & 3 \end{array} \right) \sim \end{array}$$

$$\left(\begin{array}{ccc|c} & & & \\ & & & \\ 0 & 0 & 25 & -25 \end{array} \right)$$

$$\begin{aligned} 25z &= -25 \\ z &= -25/25 \\ z &= -1 \end{aligned}$$

$$\begin{aligned} 7x - 10z &= -4 \\ 7x &= 10 \cdot (-1) = -4 \\ 7x + 10 &= -4 \\ 7x &= -4 - 10 \\ 7x &= -14 \\ x &= -14/7 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} 2x - y - 3z &= -5 \\ 2 \cdot (-2) - y - 3 \cdot (-1) &= -5 \\ -4 - y + 3 &= -5 \\ -4 + 3 + 3 &= y \\ y &= 4 \end{aligned}$$

$$x = -2, y = 4, z = -1$$

Exercício 2

2. (CGS GRAVIO)

$$\begin{cases} x = 2y \\ 2y = 3z \rightarrow 2y = 3z = z = \frac{2y}{3} \\ x + y + z = 11 \end{cases} \rightarrow x = 2y$$

$$\begin{aligned} x + y + z &= 11 \\ 2y + y + \frac{2y}{3} &= 11 \quad (\times 3) \\ 3y + 3y + 2y &= 33 \end{aligned}$$

$$\begin{aligned} 6y + 3y + 2y &= 33 \\ 11y &= 33 \\ y &= 33/11 \\ y &= 3 \end{aligned}$$

$$\begin{aligned} x &= 2y \\ x &= 2 \cdot 3 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} x + 2y + 3z &= 18 \\ 6 + 2 \cdot 3 + 3 \cdot 2 \\ 6 + 6 + 6 \\ &= 18 \end{aligned}$$

$$z = \frac{2y}{3}$$

$$z = \frac{2 \cdot 3}{3}$$

$$z = \frac{6}{3}$$

$$z = 2$$

Resposta B

Exercício 3

3. (FGV)

$$\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ + 6y + 3z = -12 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{pmatrix} \xrightarrow{+2} \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 0 & -5 & -10 \end{pmatrix}$$

$$-5z = -10$$

$$z = \frac{-10}{-5}$$

$$z = 2, \quad \text{Resposta D}$$

Exercício 4

$$\begin{aligned} 4. (UEL) \quad & \text{Ali} + \text{Bia} + \text{Caco} = 68 \\ & \text{Bia} + 0,2 \cdot \text{Caco} = \text{Ali} \\ & \text{Caco} + 0,2 \cdot \text{Ali} = 3 \text{Bia} \end{aligned}$$

$$\begin{aligned} (\text{Bia} + 0,2) + \text{Bia} + \text{Caco} &= 68 \\ 2 \text{Bia} + 1,2 \text{Caco} &= 68 \end{aligned}$$

$$\text{Bia} = \frac{68 - 1,2 \text{Caco}}{2}$$

$$\text{Bia} = 34 - 0,6 \text{Caco}$$

$$0,2 \text{Ali} + \text{Caco} = 3 \cdot (34 - 0,6)$$

$$0,2 \text{Ali} + \text{Caco} = 102 - 1,8 \text{Caco}$$

$$0,2 \text{Ali} + 2,8 \text{Caco} = 102$$

$$0,2 (\text{Bia} + 0,2 \text{Caco}) + 2,8 \text{Caco} = 102$$

$$0,2 \text{Bia} + 0,4 \text{Caco} + 2,8 \text{Caco} = 102$$

$$0,2 \text{Bia} + 3,2 \text{Caco} = 102$$

$$0,2 (34 - 0,6 \text{Caco}) + 3,2 \text{Caco} = 102$$

$$6,8 - 0,12 \text{Caco} + 3,2 \text{Caco} = 102$$

$$6,8 + 3,08 \text{Caco} = 102$$

$$3,08 \text{Caco} = 102 - 6,8$$

$$3,08 \text{Caco} = 95,2$$

$$\text{Caco} = \frac{95,2}{3,08}$$

$$\text{Caco} = 30,9$$

Continuação do exercício 4

$$Bia = 34 - 0,6 \cdot Caca$$

$$Bia = 34 - 0,6 \cdot 35$$

$$Bia = 34 - 21$$

$$Bia = 13 //$$

$$Ali + Bia + Caca = 68$$

$$Ali + 13 + 35 = 68$$

$$Ali + 48 = 68$$

$$Ali = 68 - 48$$

$$Ali = 20 //$$

Caca - Ali

$$35 - 20 = 15$$

Resposta A

a) R\$ 5,00 a menos que Caca, Não porque Caca tem R\$ 35,00 e Ali R\$ 20,00.

b) R\$ 5,00 a mais que Bia, Não porque Bia tem R\$ 13,00 e Ali R\$ 20,00.

c) R\$ 8,00 a menos que Caca, Não porque ela tem R\$ 15,00 a menos que Caca.

d) R\$ 8,00 a mais que Bia, Não porque Ali tem R\$ 7,00 a mais que Bia.

e) R\$ 6,00 a menos que Caca, Não porque ela tem R\$ 15,00 a menos que Caca.

Exercício 5

$$5. \quad A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{cases} 0 + 3y + 4z = 134 \\ x + 0 + 5z = 115 \\ 2x + y + 0 = 48 \end{cases}$$

$$\begin{pmatrix} 0 & 3 & 4 & : & 134 \\ 1 & 0 & 5 & : & 115 \\ 2 & 1 & 0 & : & 48 \end{pmatrix} \xrightarrow{+1} \begin{pmatrix} 0 & 3 & 4 & : & 134 \\ 1 & 0 & 5 & : & 115 \\ 3 & 0 & 1 & : & -182 \end{pmatrix} \xrightarrow{-3} \begin{pmatrix} 0 & 3 & 4 & : & 134 \\ 1 & 0 & 5 & : & 115 \\ 0 & 0 & -10 & : & -680 \end{pmatrix}$$

$$34z = 680$$

$$z = \frac{680}{34}$$

$$z = 20 //$$

$$x + 5z = 115$$

$$x + 5 \cdot 20 = 115$$

$$x = 100 = 115$$

$$x = 115 - 100 = 15$$

Alfau + Bento + Cintia

$$20 + 15 + 18 = 53 //$$

Resposta A

$$3y + 4z = 134$$

$$3y + 4 \cdot 20 = 134$$

$$3y = 80 = 134$$

$$3y = 134 - 80$$

$$y = 54/3$$

$$y = 18 //$$