COMP 250

Lecture 29

graph traversal

Nov. 15/16, 2017

Today

- Recursive graph traversal
    - depth first
- Non-recursive graph traversal
    - depth first
    - breadth first

Heads up!

There were a few mistakes in the slides for Sec. 001 for today's lecture. So if you are following the lecture recordings and using these (corrected) slides, then you will notice some differences.

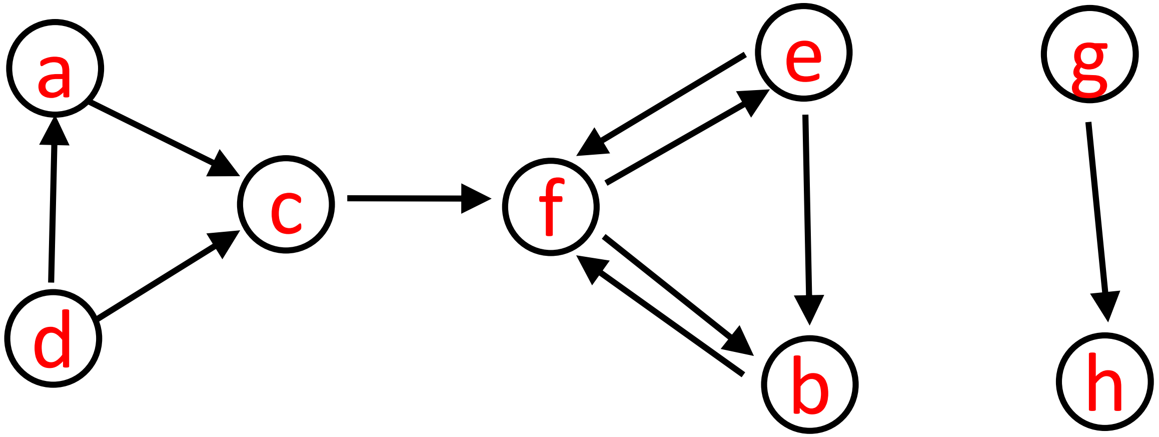
Recall: tree traversal (recursive)

```
depthfirst__Tree (root){
    if (root is not empty){
        root.visited = true                //       "preorder"
        for each child of root
            depthfirst__Tree( child )
    }
}
```

Graph traversal (recursive)

Need to specify a starting vertex.

Visit all nodes that are “reachable” by a path from a starting vertex.



Graph traversal (recursive)

```
depthFirst_Graph(v){
   v.visited = true
   for each w such that (v,w) is in E   //  w in v.adjList
          __________?_______________
}
```

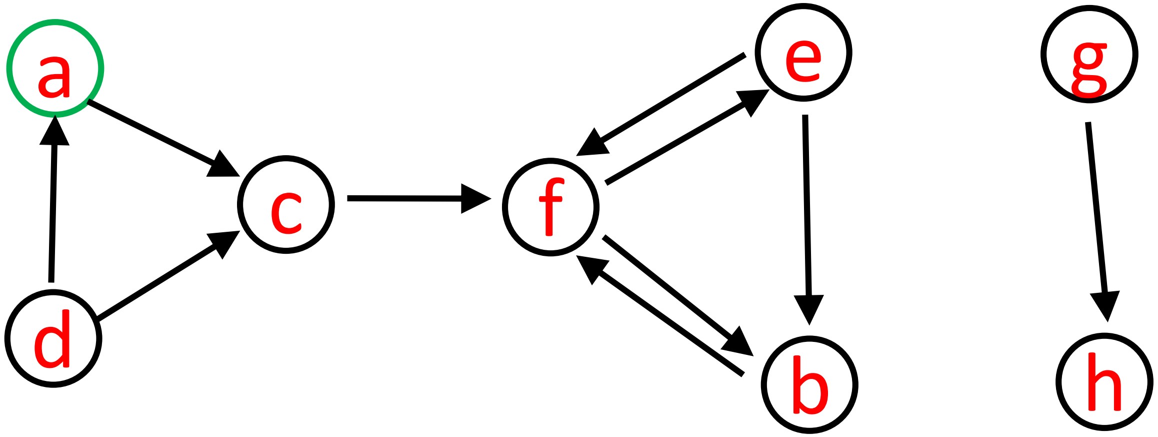
// Here “visiting” just means “reaching”

Graph traversal (recursive)

```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E   //  w in v.adjList
    if  ! (w.visited)                  //  avoids cycles
        depthFirst_Graph(w)
}
```

// Here "visiting" just means "reaching"

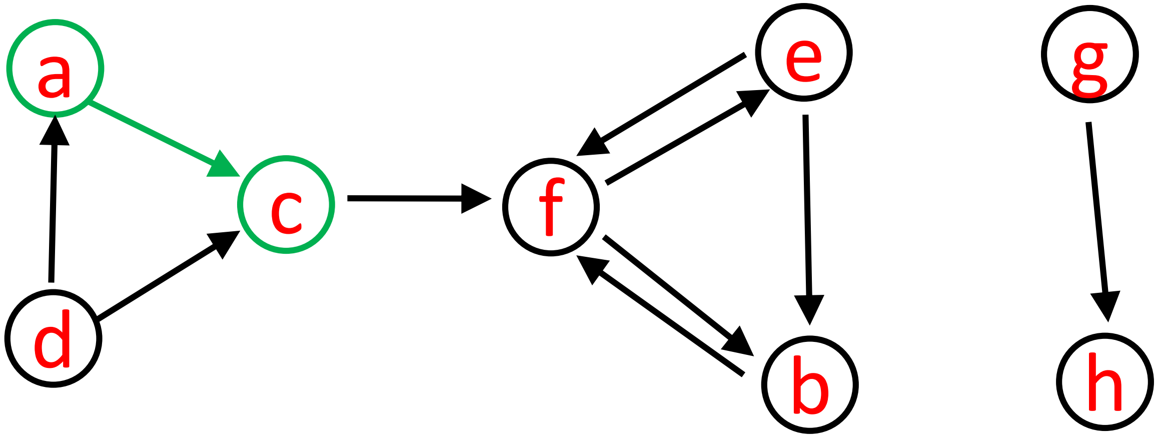
Call Stack for depthFirst(a)



```
depthFirst_Graph(v){
   v.visited = true
   for each w such that (v,w) is in E
      if   ! (w.visited)
         depthFirst_Graph(w)
}
```

a

Call Stack for depthFirst(a)

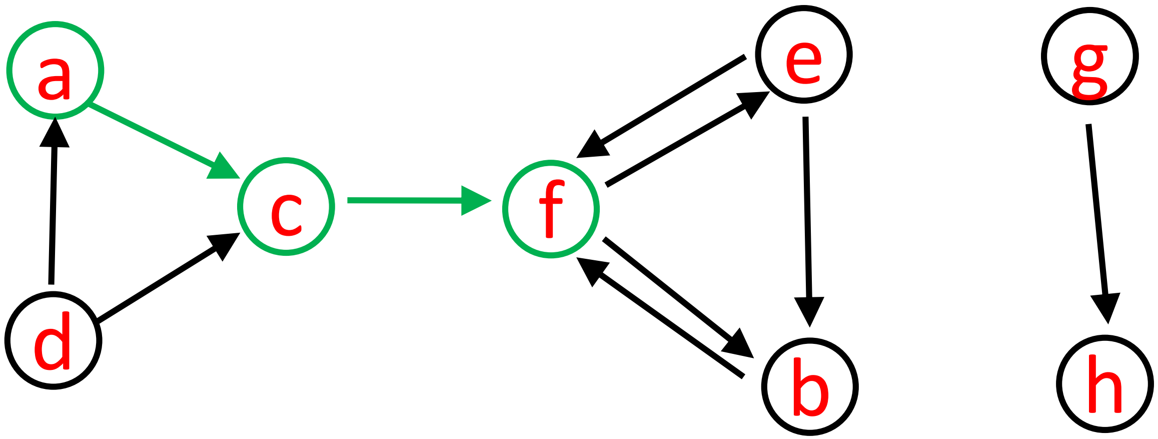


c

a a

```
depthFirst_Graph(v){
   v.visited = true
   for each w such that (v,w) is in E
      if  ! (w.visited)
         depthFirst_Graph(w)
}
```

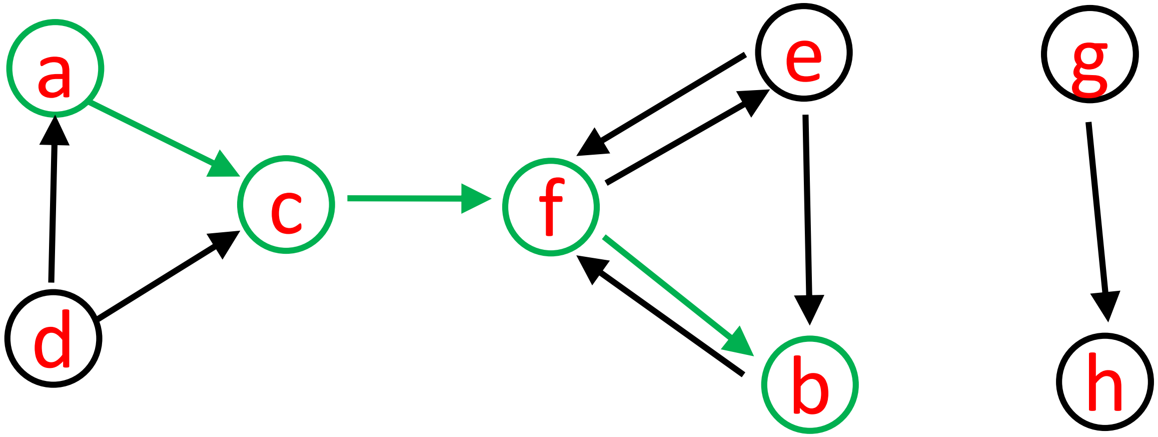
Call Stack for depthFirst(a)



		f
	c	c
a	a	a

```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
      if   ! (w.visited)
          depthFirst_Graph(w)
}
```

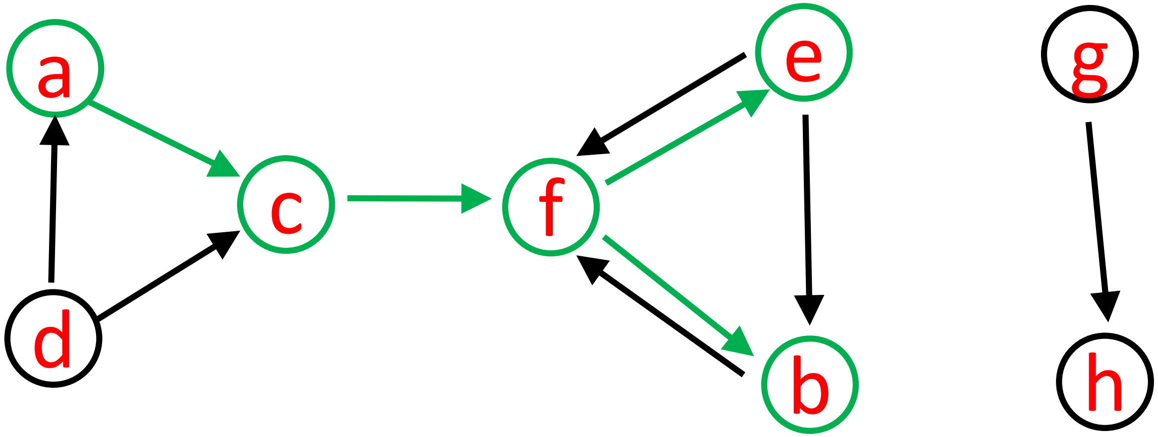
Call Stack for depthFirst(a)



			b
		f	f
	c	c	c
a	a	a	a

```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
     if   ! (w.visited)
          depthFirst_Graph(w)
}
```

Call Stack for depthFirst(a)

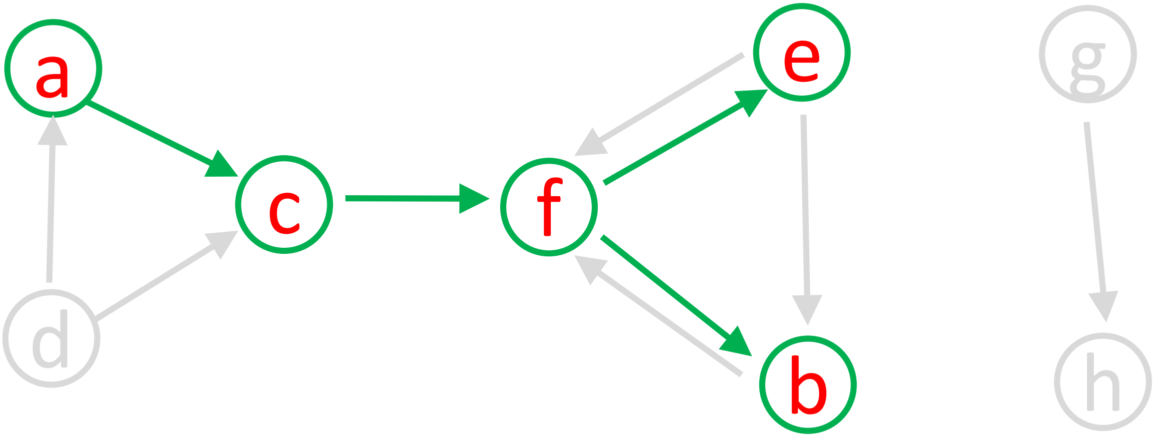


			b		e
		f	f	f	f
	c	c	c	c	c
a	a	a	a	a	a

```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
     if  ! (w.visited)
        depthFirst_Graph(w)
}
```

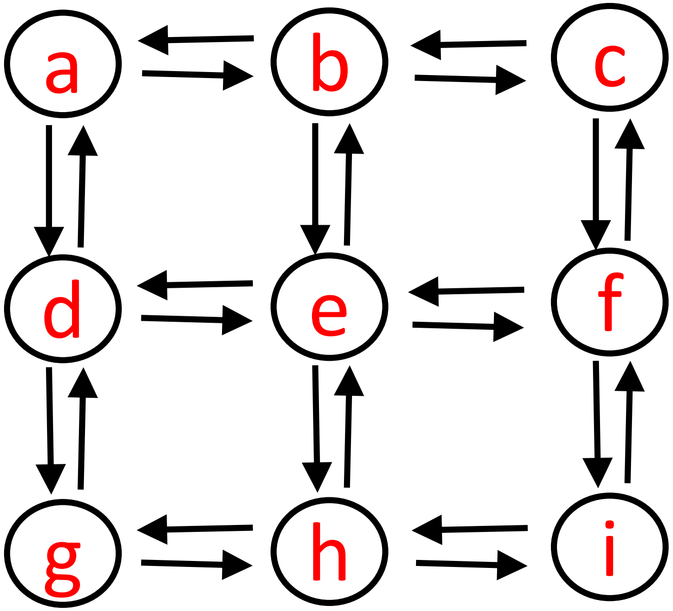
Call Tree

root



			b		e			
		f	f	f	f	f		
	c	c	c	c	c	c	c	
a	a	a	a	a	a	a	a	a

Example 2

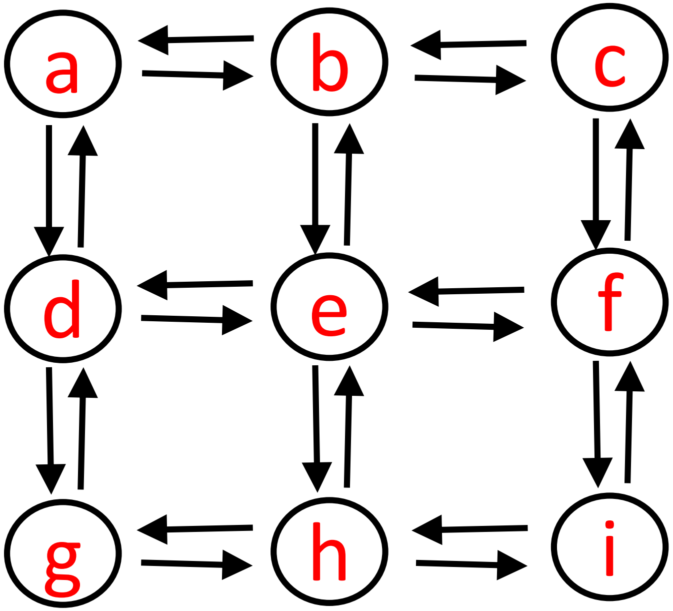


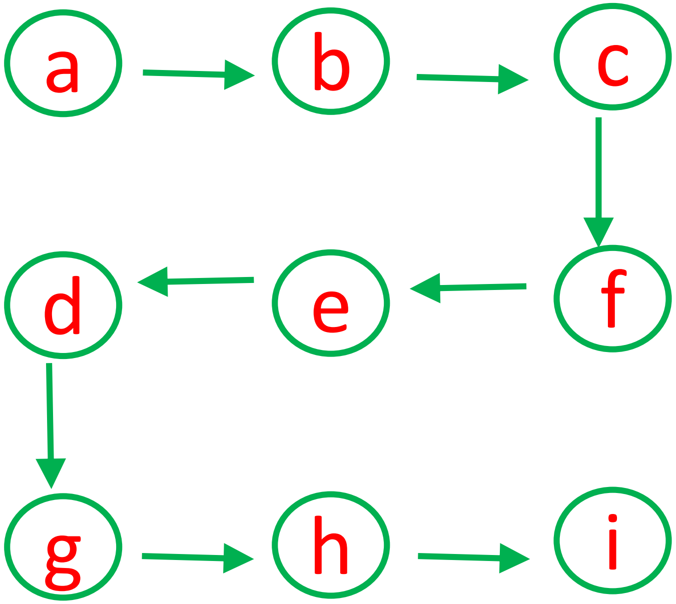
What is the call tree for depthFirst(a) *?*

Adjacency List

a - (b,d)
b - (a,c,e)
c - (b,f)
d - (a,e,g)
e - (b,d,f,h)
f - (c,e,i)
g - (d,h)
h - (e,g,i)
i - (f,h)

Example 2





call tree for depthFirst(a)

Q: Non-recursive graph traversal ?

A: Similar to tree traversal: Use a stack or a queue.

Recall: depth first tree traversal

(with a slight variation)

```
treeTraversalUsingStack(root){
    initialize empty stack s
    visit root
    s.push(root)
    while s is not empty {
        cur = s.pop()
        for each child of cur{
            visit child
            s.push(child)
        }
    }
}
```

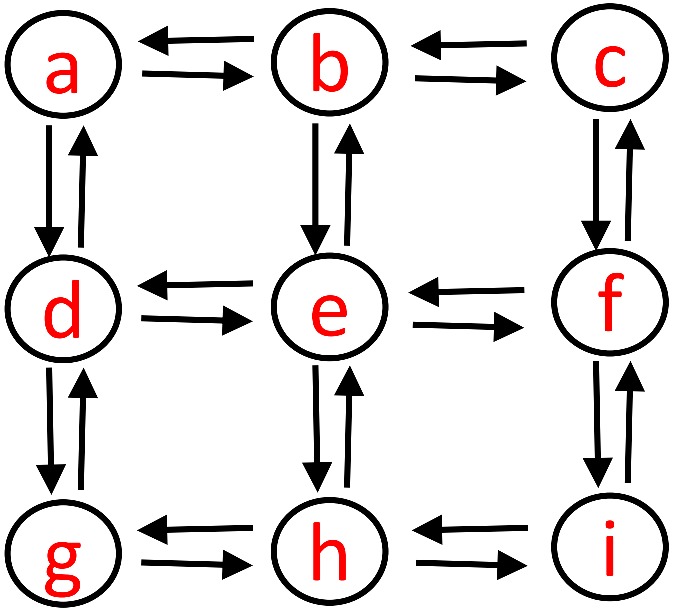
Visit a node *before pushing* it onto the stack.

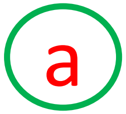
Every node in the tree gets visited, pushed, and then popped.

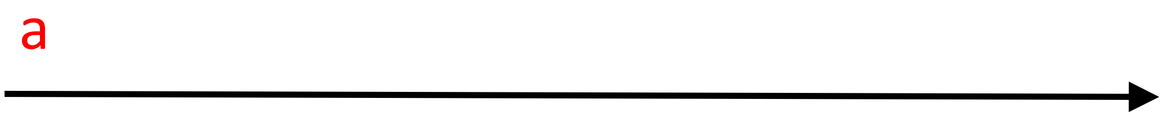
Generalize to graphs…

```
graphTraversalUsingStack(v){
    initialize empty stack s
    v.visited = true
    s.push(v)
    while (!s.empty) {
        u =  s.pop()
        for each w in u.adjList{
            if (!w.visited){                    //  the only new part
                w.visited = true
                s.push(w)
            }
        }
    }
}
```

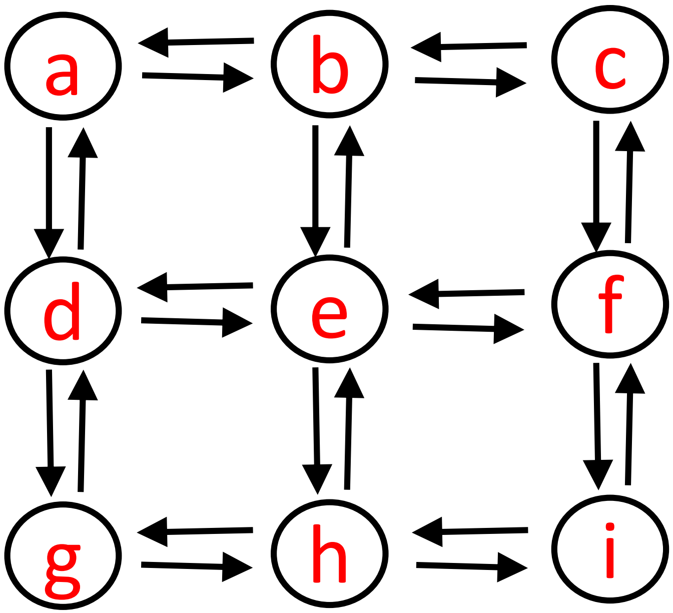
Example: graphTraversalUsingStack(a)

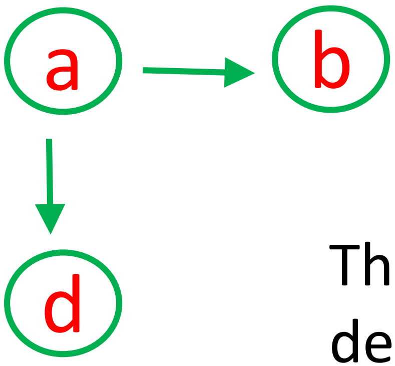




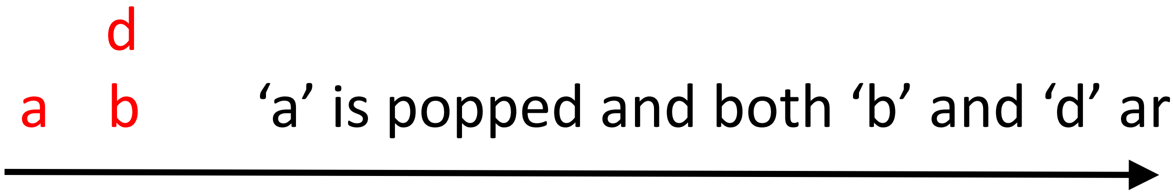


Example: graphTraversalUsingStack(a)



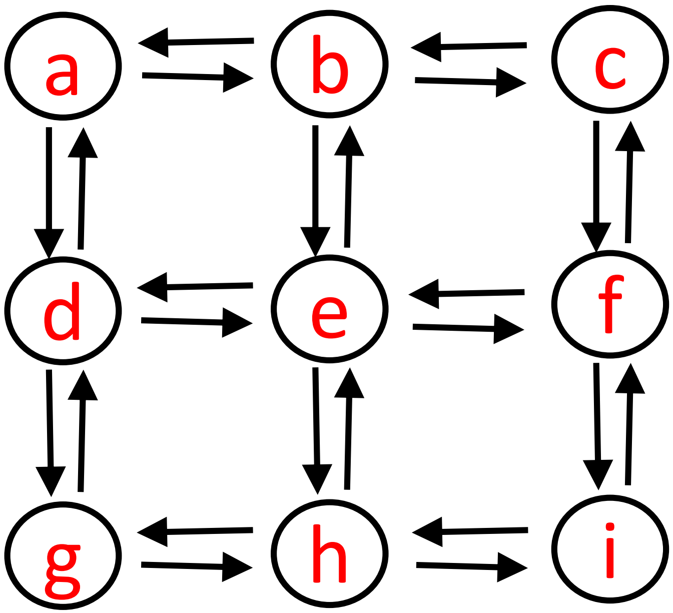


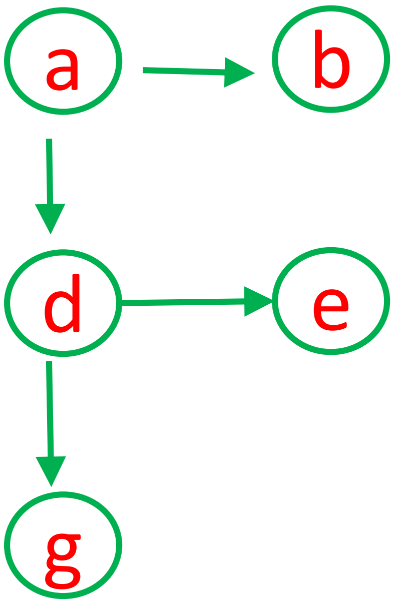
The traversal defines a tree, but it is not a "call tree". Why not?

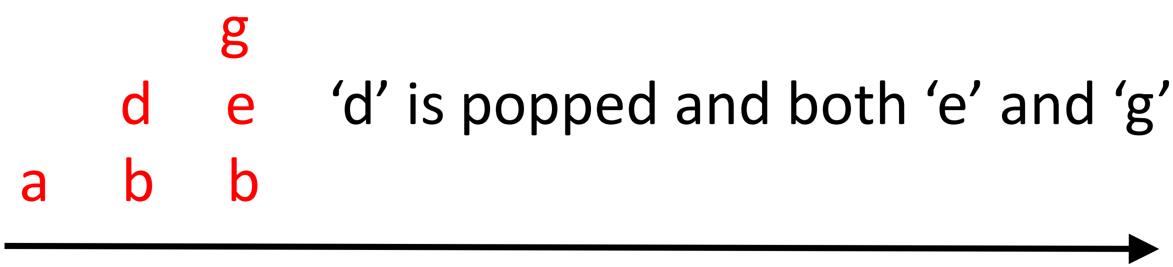


'a' is popped and both 'b' and 'd' are pushed.

Example: graphTraversalUsingStack(a)

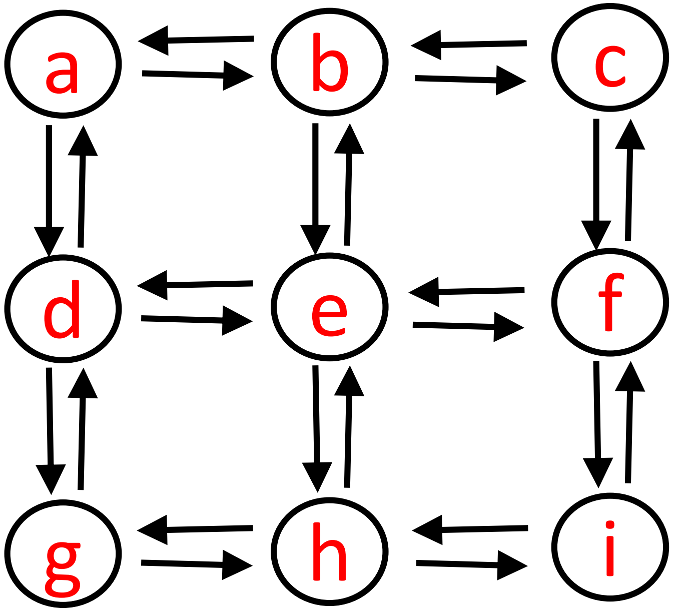


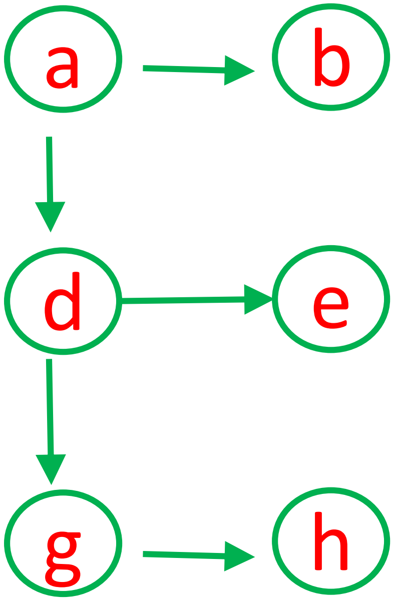




‘d’ is popped and both ‘e’ and ‘g’ are pushed.

Example: graphTraversalUsingStack(a)





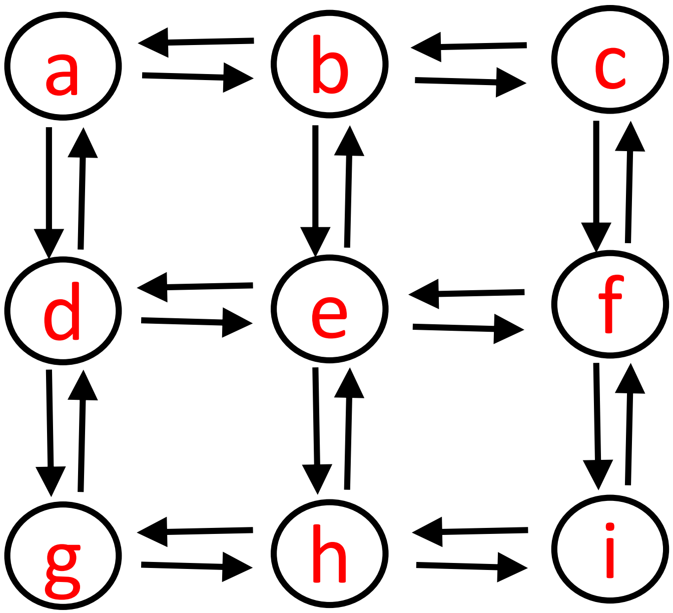
g h

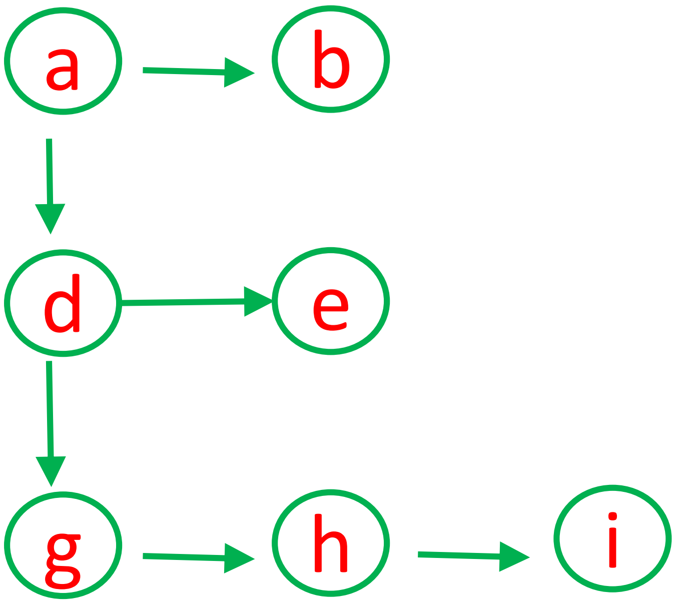
d e e

a b b b

‘g’ is popped and ‘h’ is pushed.

Example: graphTraversalUsingStack(a)

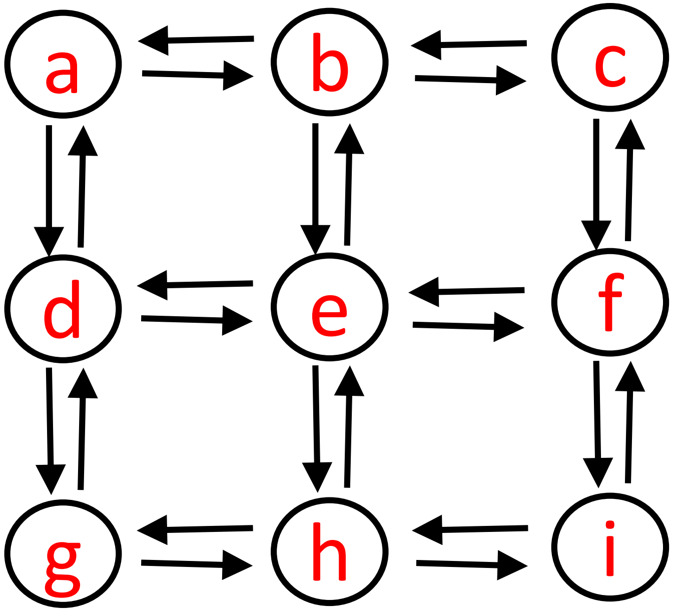


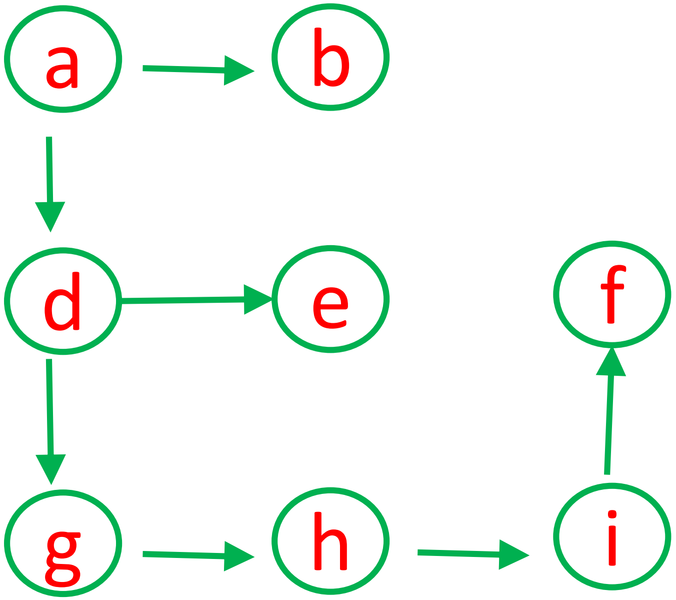


		g	h	i
	d	e	e	e
a	b	b	b	b

‘h’ is popped and ‘i’ is pushed.

Example: graphTraversalUsingStack(a)

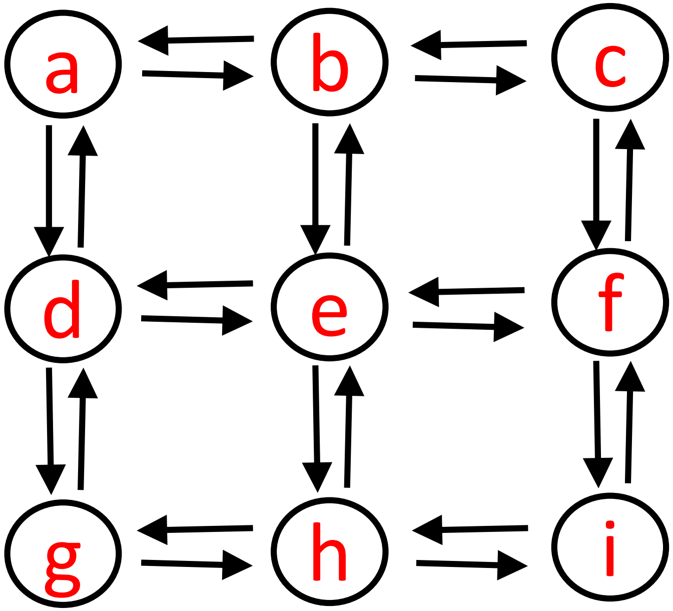


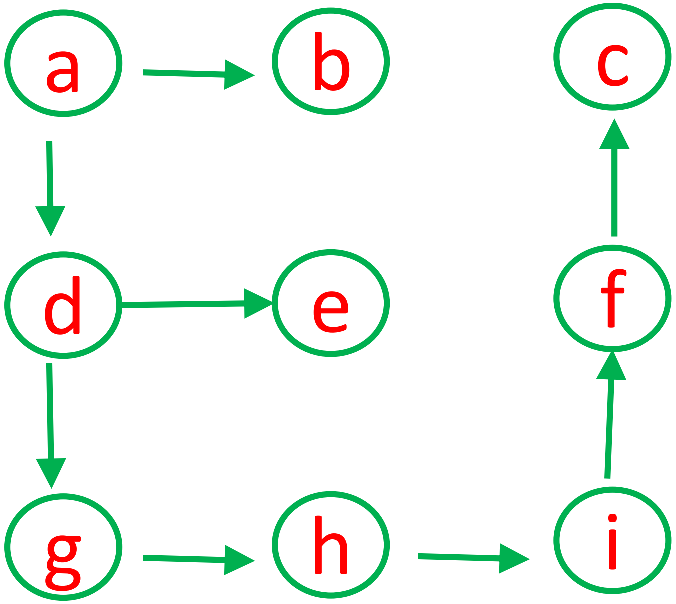


		g	h	i	f
	d	e	e	e	e
a	b	b	b	b	b

‘i’ is popped and ‘f’ is pushed.

Example: graphTraversalUsingStack(a)

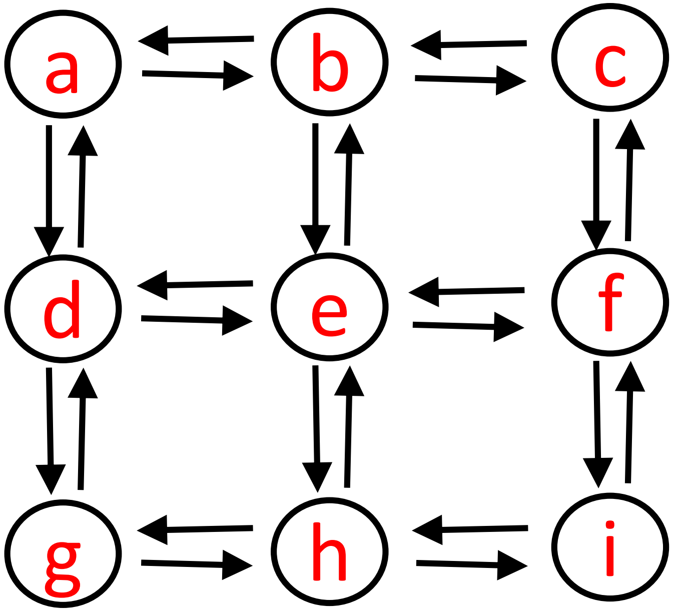


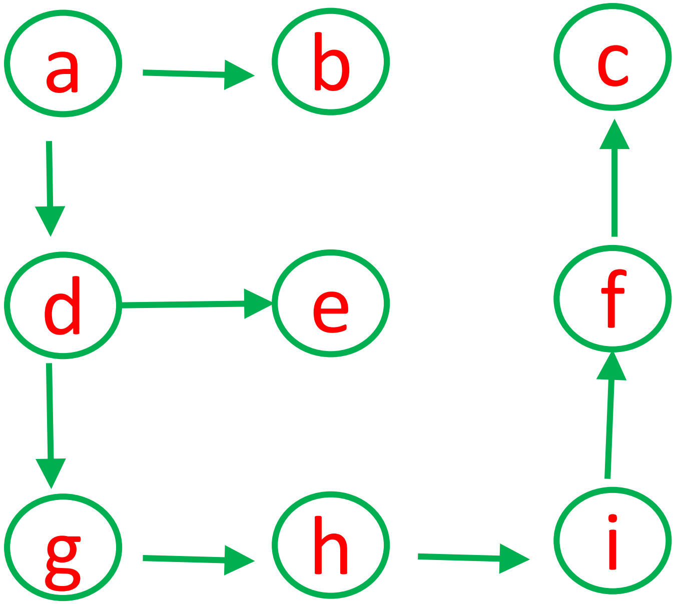


		g	h	i	f	c
	d	e	e	e	e	e
a	b	b	b	b	b	b

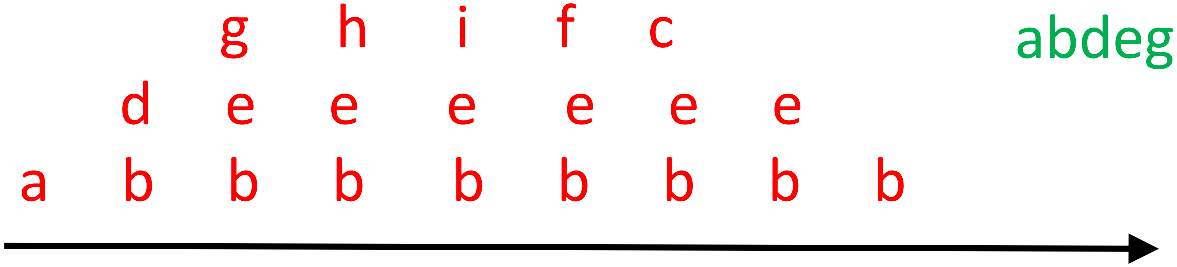
‘f’ is popped and ‘c’ is pushed.

Example: graphTraversalUsingStack(a)





Order of nodes visited:
abdeghifc

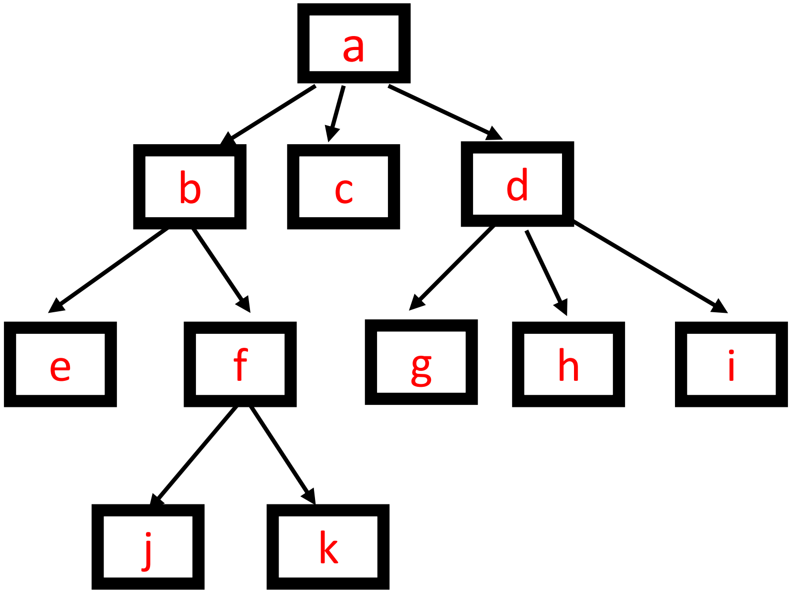


		g	h	i	f	c		
	d	e	e	e	e	e	e	
a	b	b	b	b	b	b	b	b

Recall: breadth first tree traversal

(see lecture 20)

```
for each level i
      visit all nodes at level i
```



```
treeTraversalUsingQueue(root){
    initialize empty queue  q
    q.enqueue(root)
    while q is not empty {
       cur = q.dequeue()
       visit cur
       for each child of cur
            q.enqueue(child)
    }
}
```

Breadth first graph traversal

Given an input vertex, find all vertices that can be reached by paths of length 1, 2, 3, 4, ….

Breadth first graph traversal

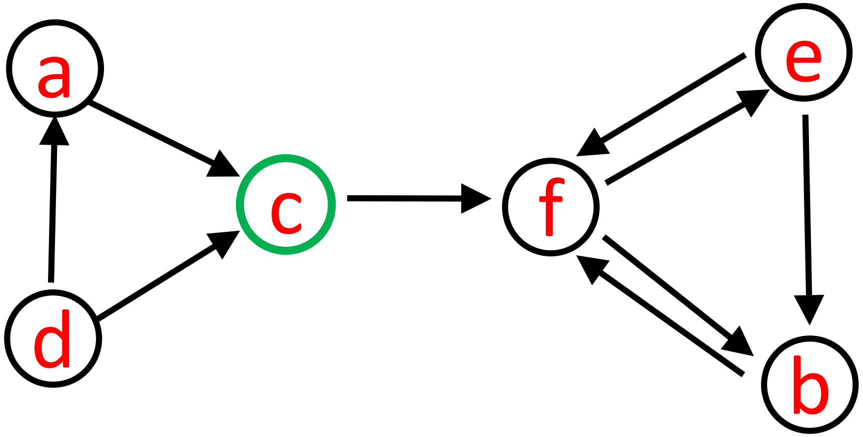
```
graphTraversalUsingQueue(v){
    initialize empty queue q
    v.visited = true
    q.enqueue(v)
    while (! q.empty) {
        u =  q.dequeue()
        for each w in u.adjList{
            if (!w.visited){
                w.visited = true
                q.enqueue(w)
            }
        }
    }
}
```

Example

graphTraversalUsingQueue(c)

queue

c



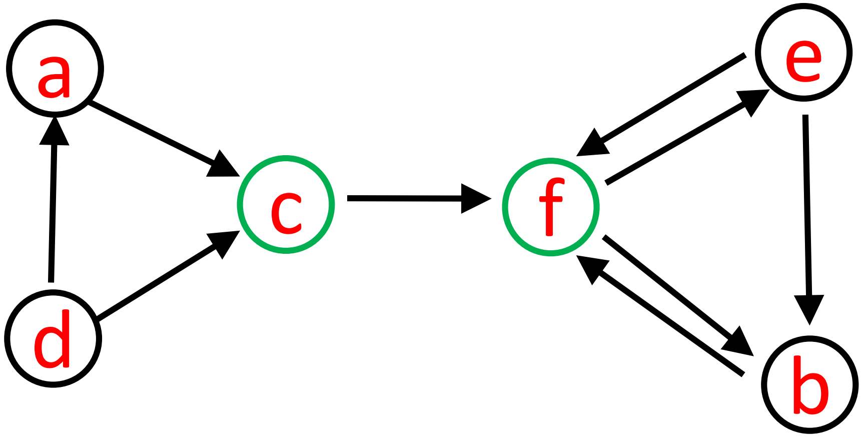
Example

graphTraversalUsingQueue(c)

queue

c

f



Example

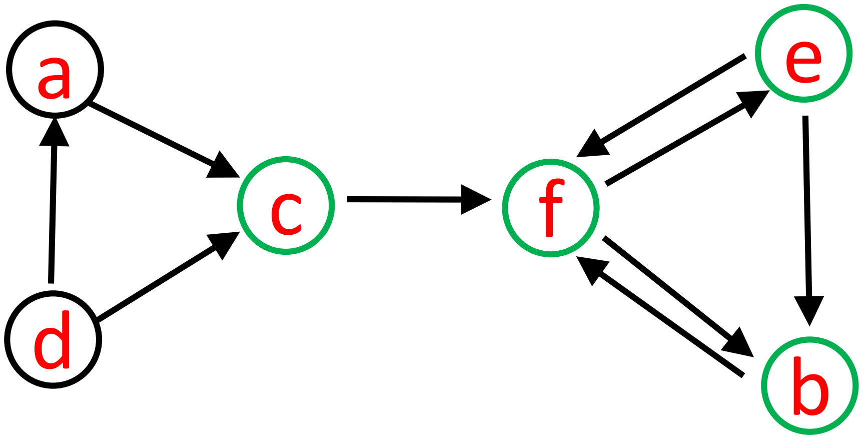
graphTraversalUsingQueue(c)

queue

c

f

be



Both ‘b’, ‘e’ are visited and enqueued before ‘b’ is dequeued.

Example

graphTraversalUsingQueue(c)

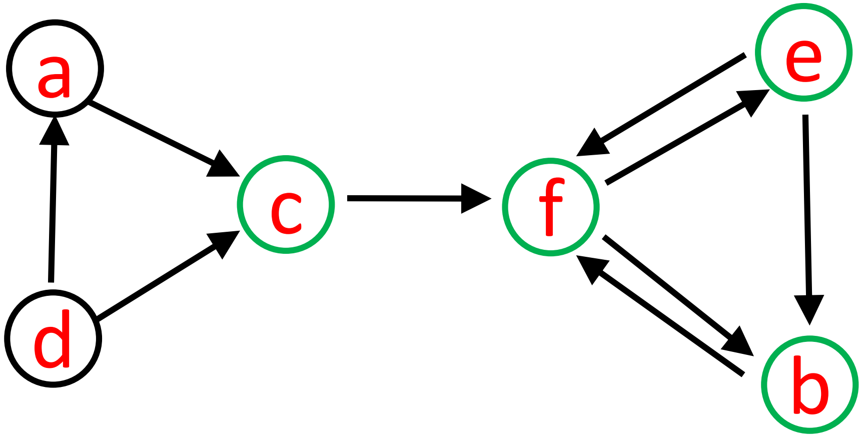
queue

c

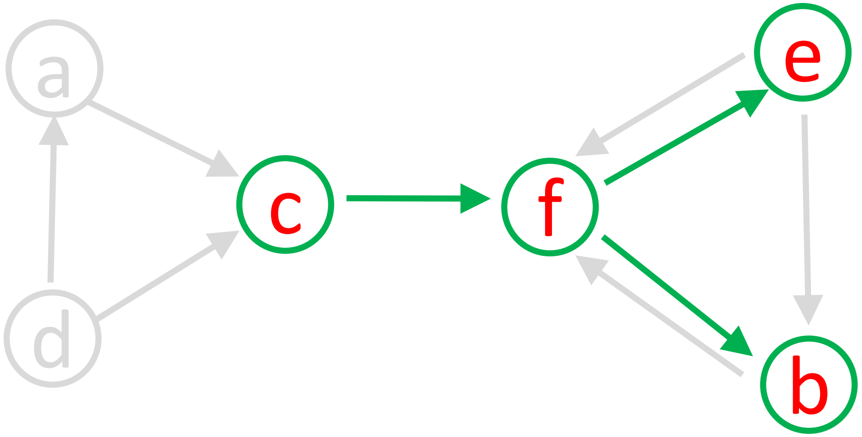
f

be

e

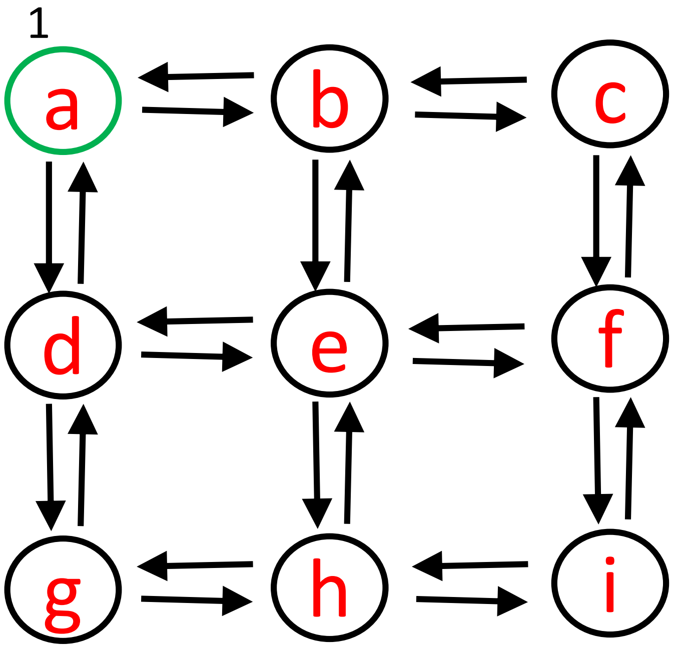


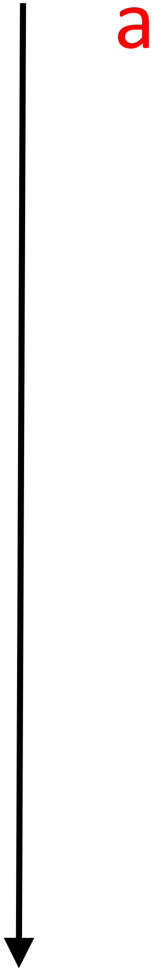
graphTraversalUsingQueue(c)



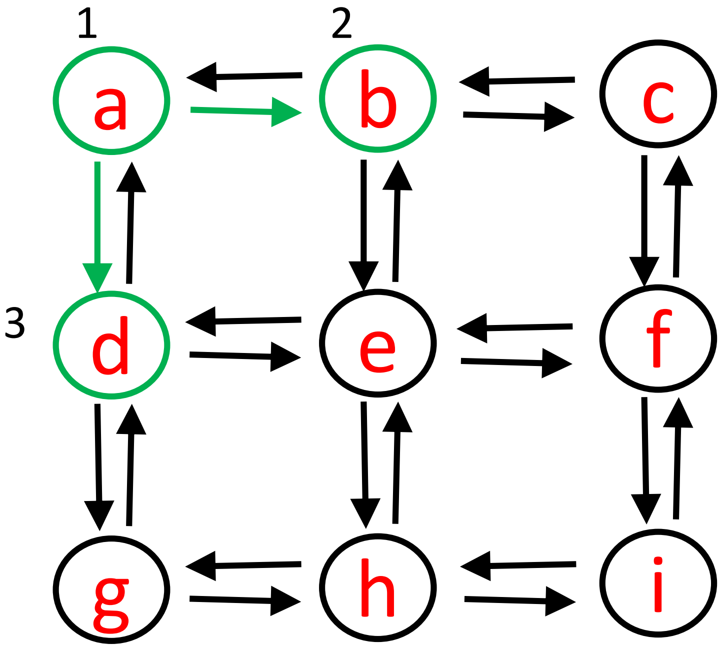
It defines a tree whose root is the starting vertex. It finds the shortest path (number of vertices) to all vertices reachable from starting vertex.

Example: graphTraversalUsingQueue(a)





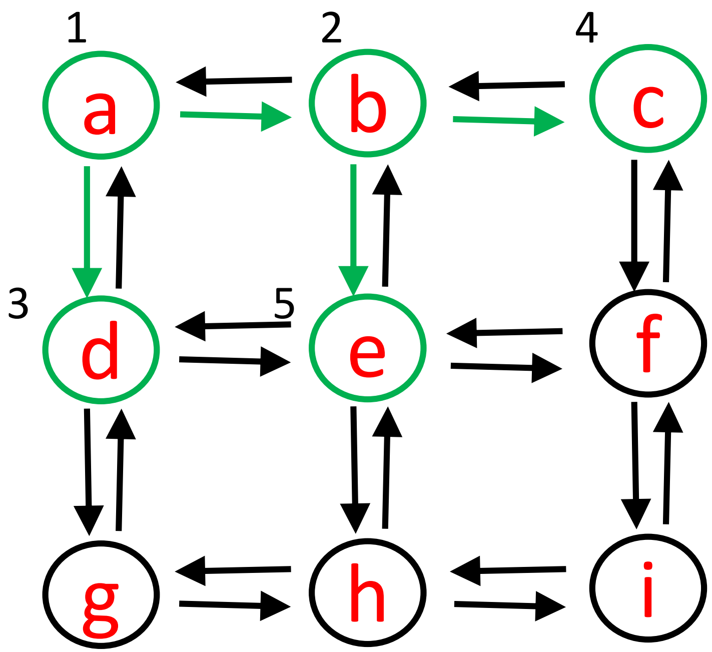
Example: graphTraversalUsingQueue(a)

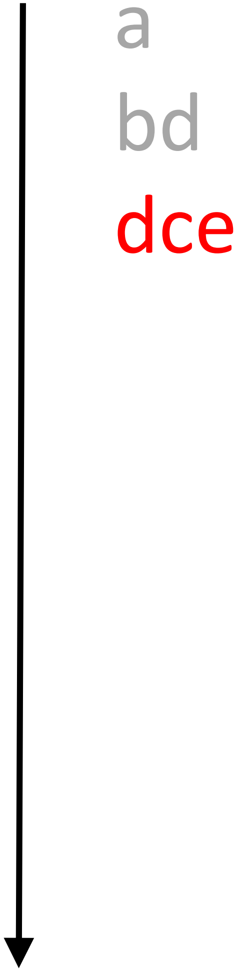


a

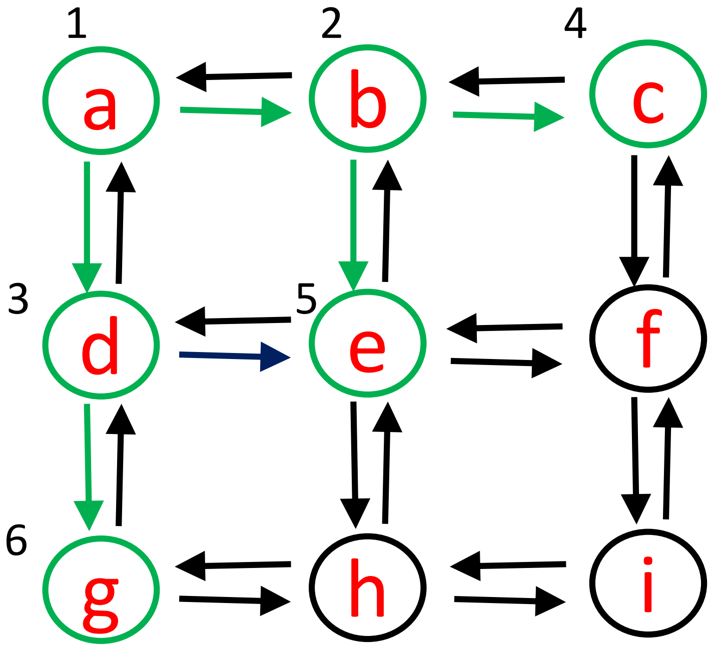
bd

Example: graphTraversalUsingQueue(a)





Example: graphTraversalUsingQueue(a)



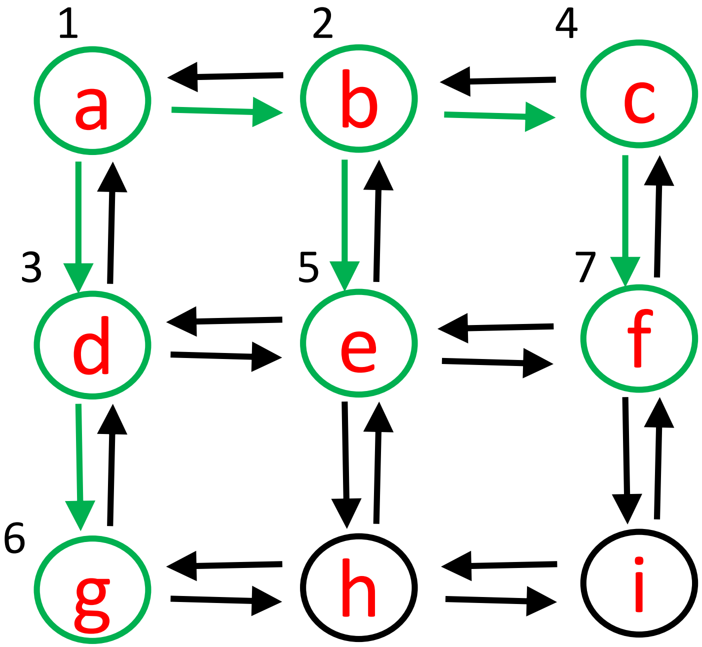
a

bd

dce

ceg

Example: graphTraversalUsingQueue(a)



a

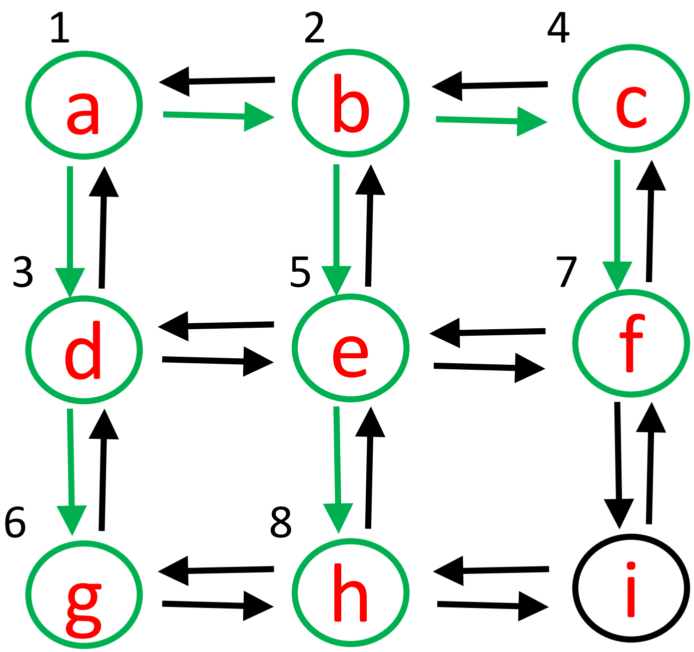
bd

dce

ceg

egf

Example: graphTraversalUsingQueue(a)



a

bd

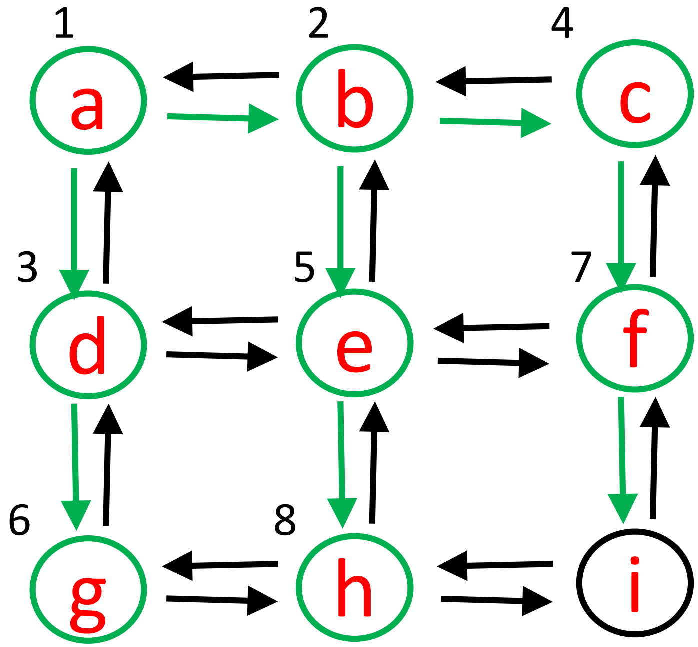
dce

ceg

egf

gfh

Example: graphTraversalUsingQueue(a)



a

bd

dce

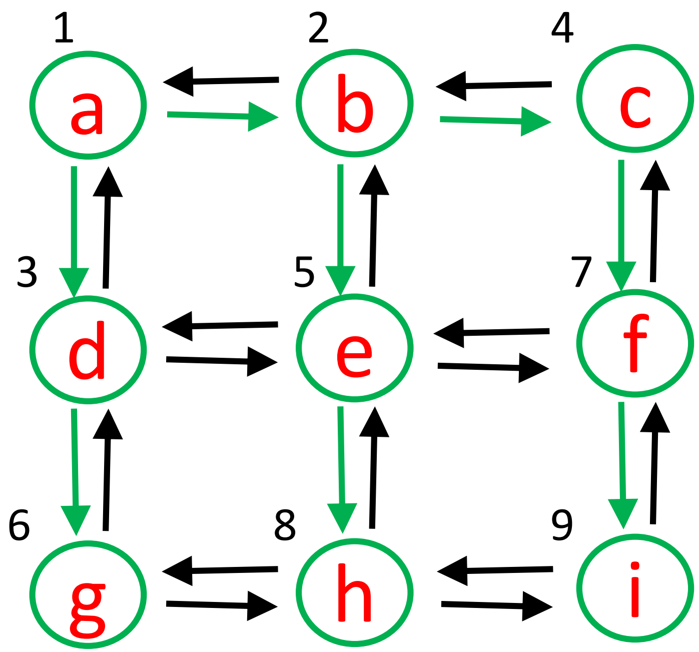
ceg

egf

gfh

fh

Example: graphTraversalUsingQueue(a)



a

bd

dce

ceg

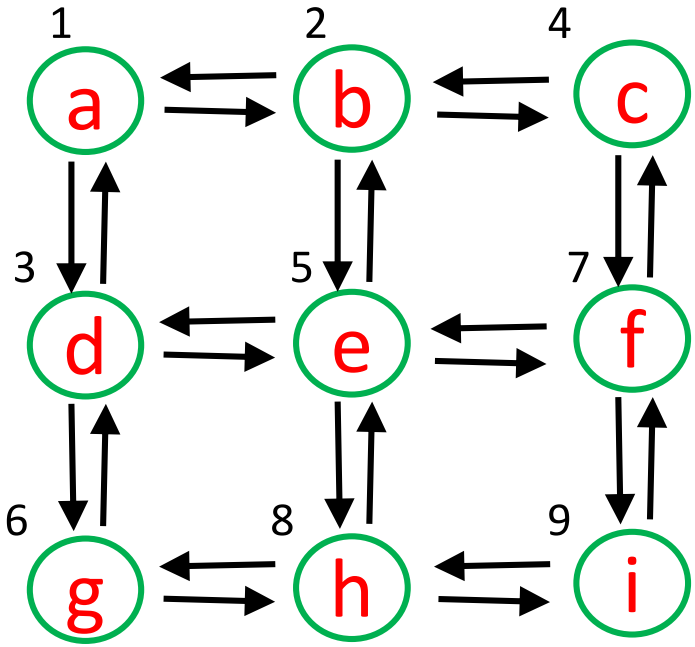
egf

gfh

fh

hi

Example: graphTraversalUsingQueue(a)



a

bd

dce

ceg

egf

gfh

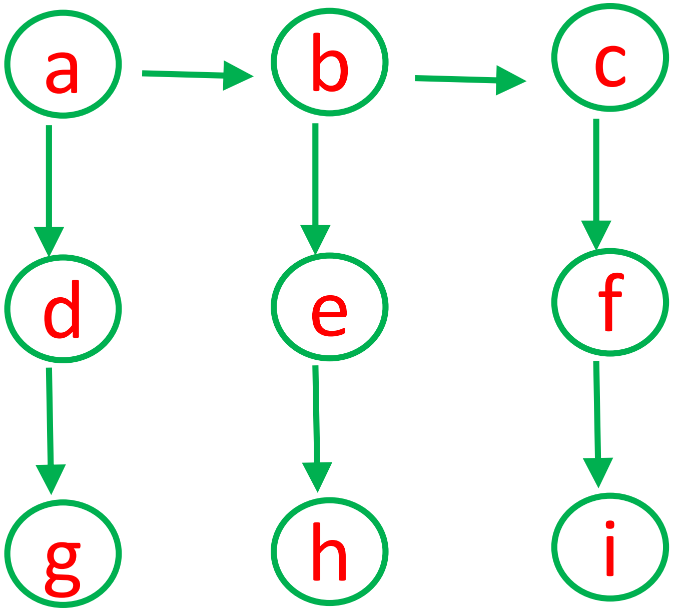
fh

hi

i

Note order of nodes visited:
paths of length 1,2, 3, 4

Example: graphTraversalUsingQueue(a)



The traversal defines a tree, but it is not a “call tree”. Why not?

Recall: How to implement a Graph class in Java?

```
class Graph<T>  {
    HashMap< String, Vertex<T> >    vertexMap;

   class Vertex<T>   {
        ArrayList<Edge>      adjList;
        T                    element;
        boolean              visited;
     }

    class Edge {
       Vertex              endVertex;
       double              weight;
                :
     }
}
```

HEADS UP !   Prior to traversal, ….

```
for each w in V
    w.visited = false
```

How to implement this ?

HEADS UP ! Prior to traversal, ...

```
for each w in V
     w.visited = false
```

How to implement this ?

```
class Graph<T> {
      HashMap< String, Vertex<T> >    vertexMap;
         :
      public void resetVisited() {



      }
}
```

HEADS UP !   Prior to traversal, ….

```
for each w in V
     w.visited = false
```

How to implement this ?

```
class  Graph<T>  {
      HashMap< String, Vertex<T> >    vertexMap;
         :
       public void resetVisited() {
        for( Vertex<T>      v  :      vertexMap.values() ){
                 v.visited = false;
        }
}
```

[ASIDE: I did something unnecessarily complicated on the Sec.001 slides.
What I have above is better.]