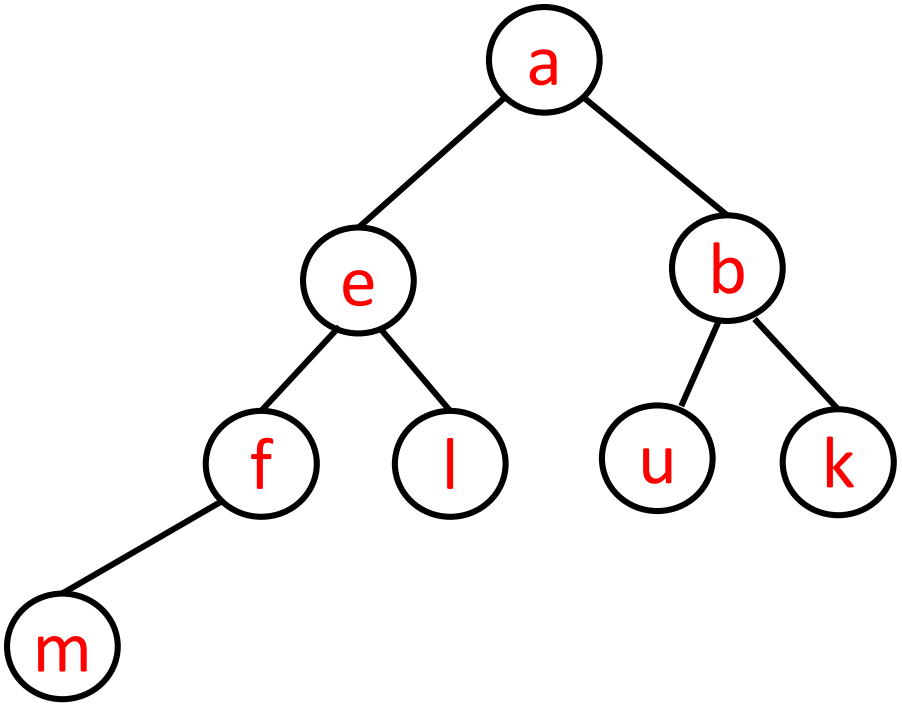
COMP 250

Lecture 24

heaps 2

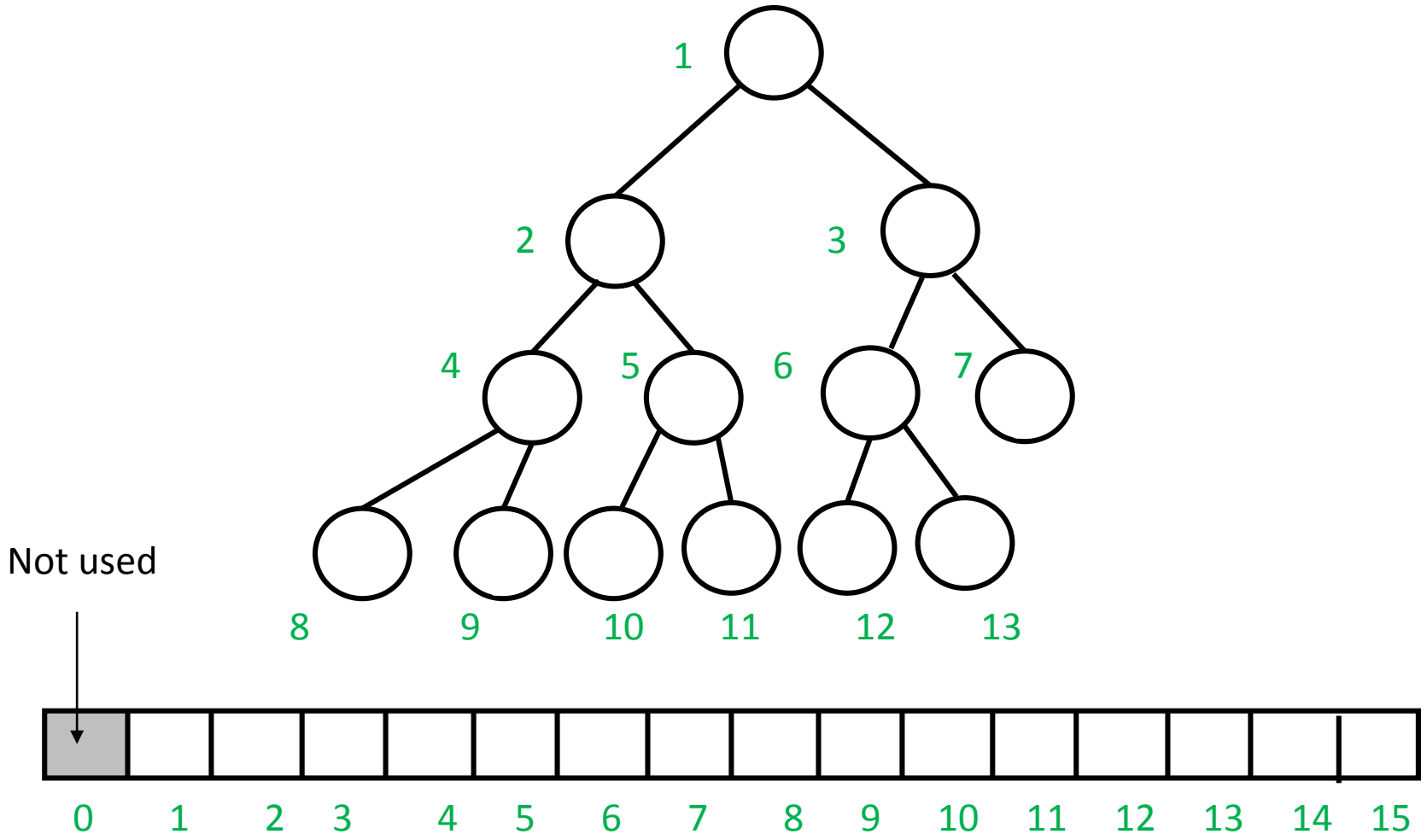
Nov. 3, 2017

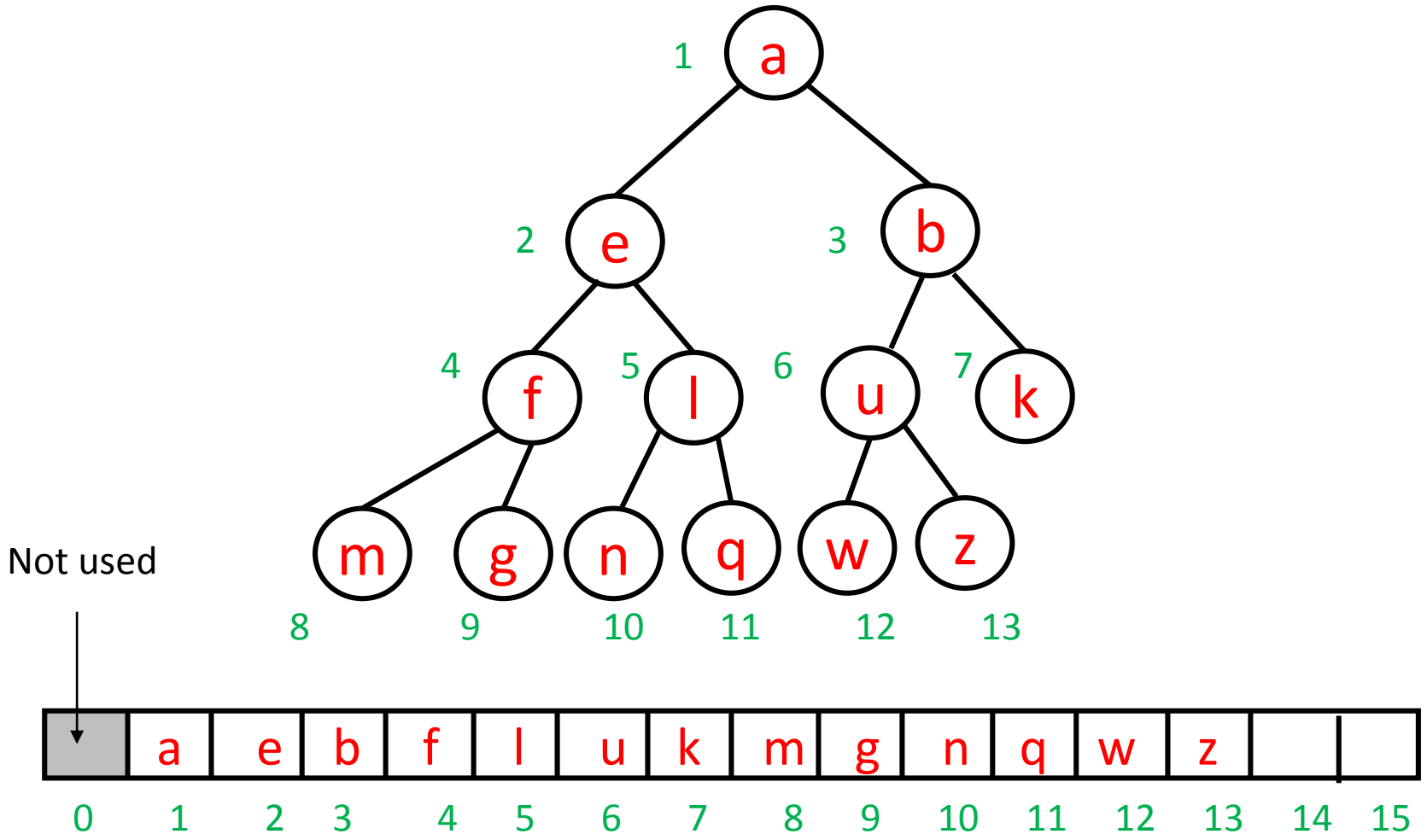
RECALL: min Heap (definition)



Complete binary tree with (unique) comparable elements, such that each node's element is less than its children's element(s).

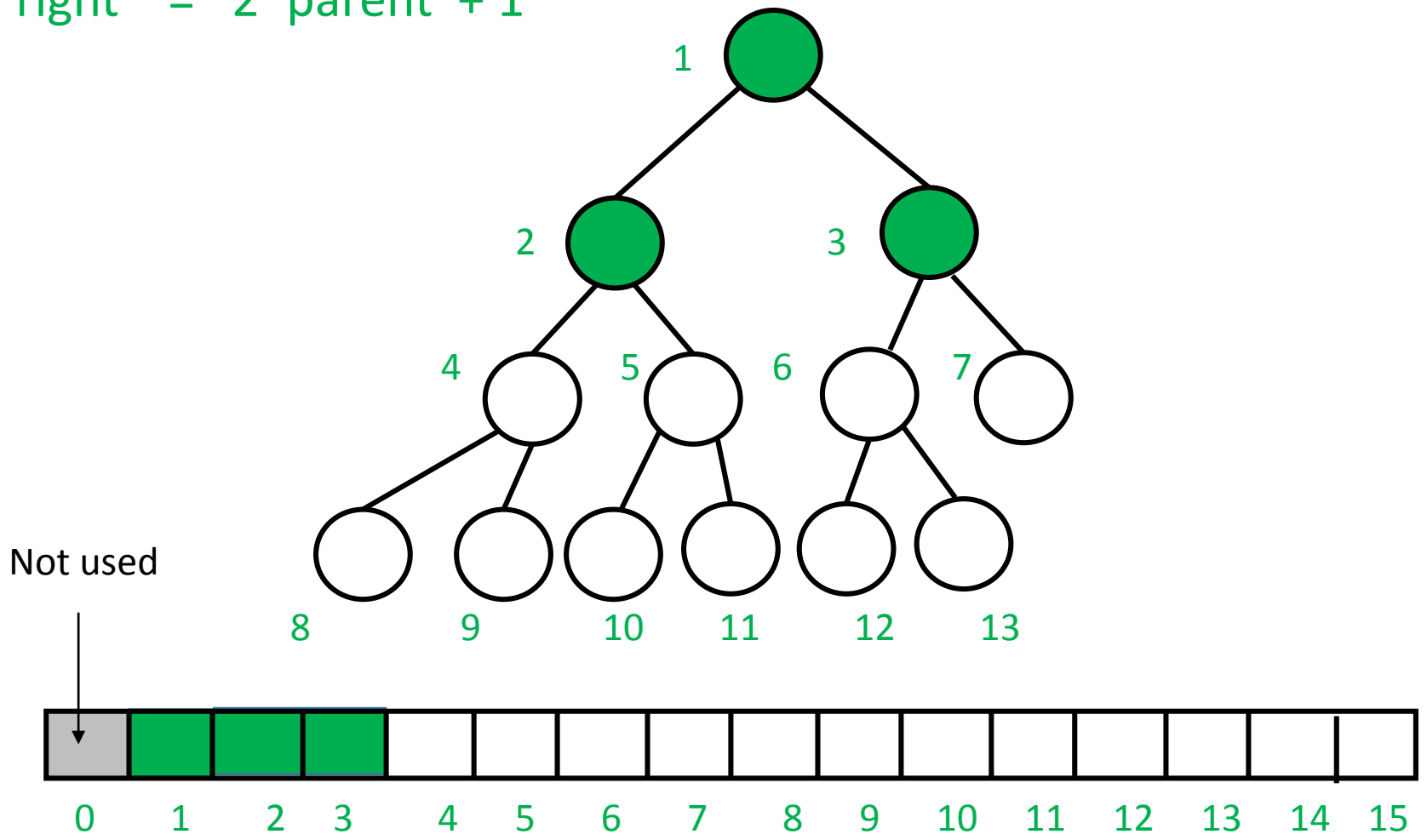
Heap (array implementation)




1
a
2
e
3
b
4
f
5
l
6
u
7
k
m
g
n
q
w
z
8
9
10
11
12
13
Not used
a
e
b
f
l
u
k
m
g
n
q
w
z
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

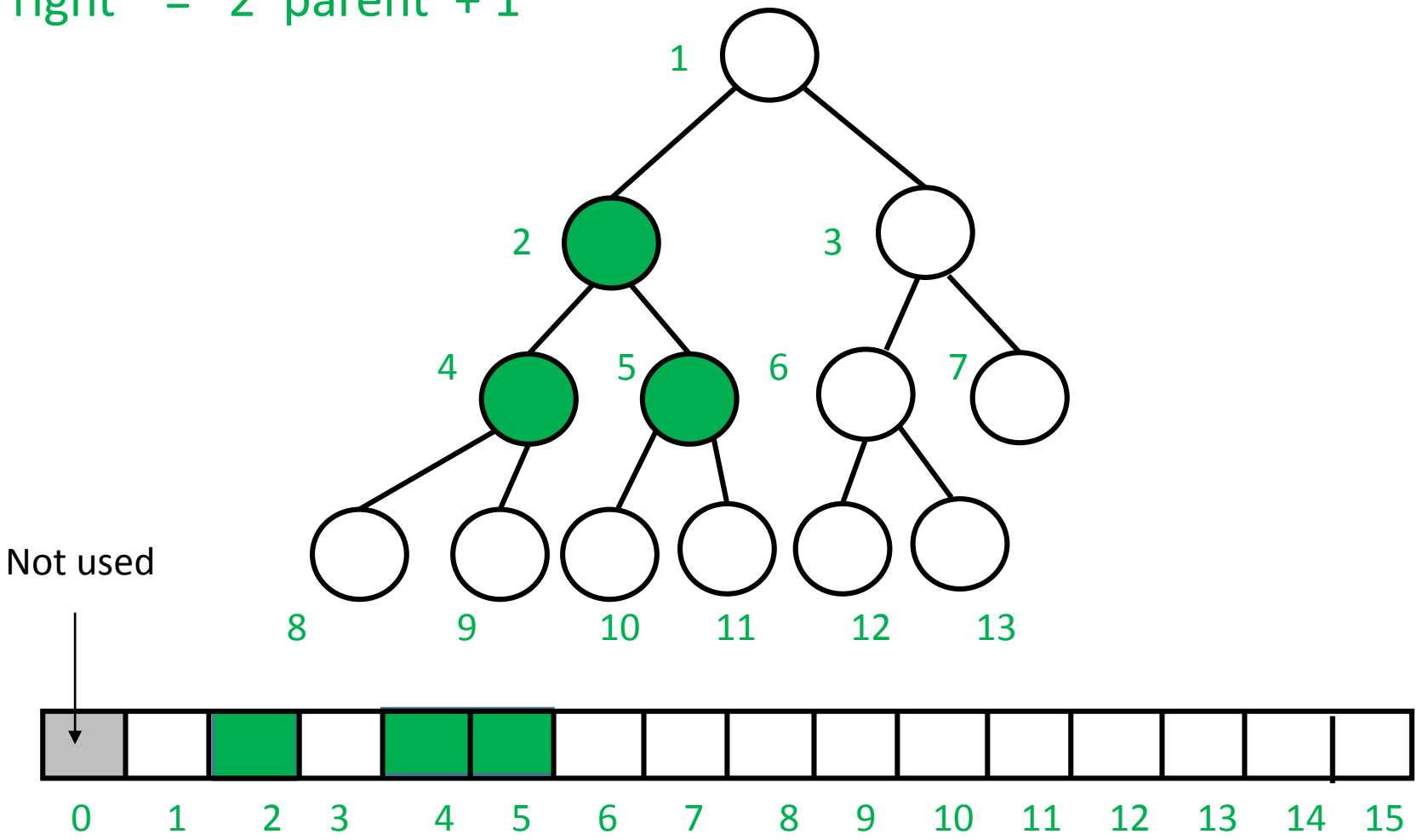
Heap index relations

parent = child / 2
left = 2*parent
right = 2*parent + 1



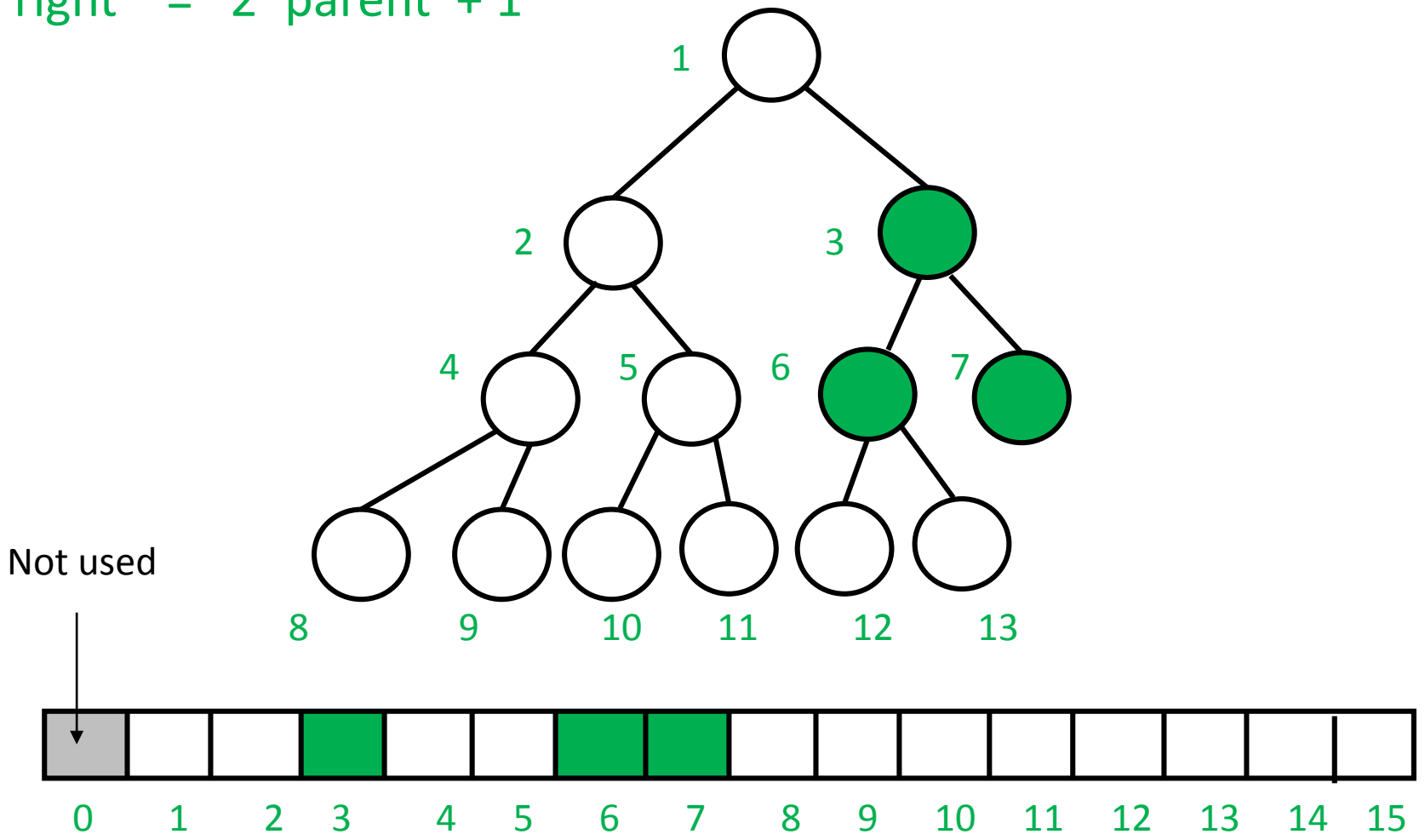
Heap index relations

parent = child / 2
left = 2*parent
right = 2*parent + 1



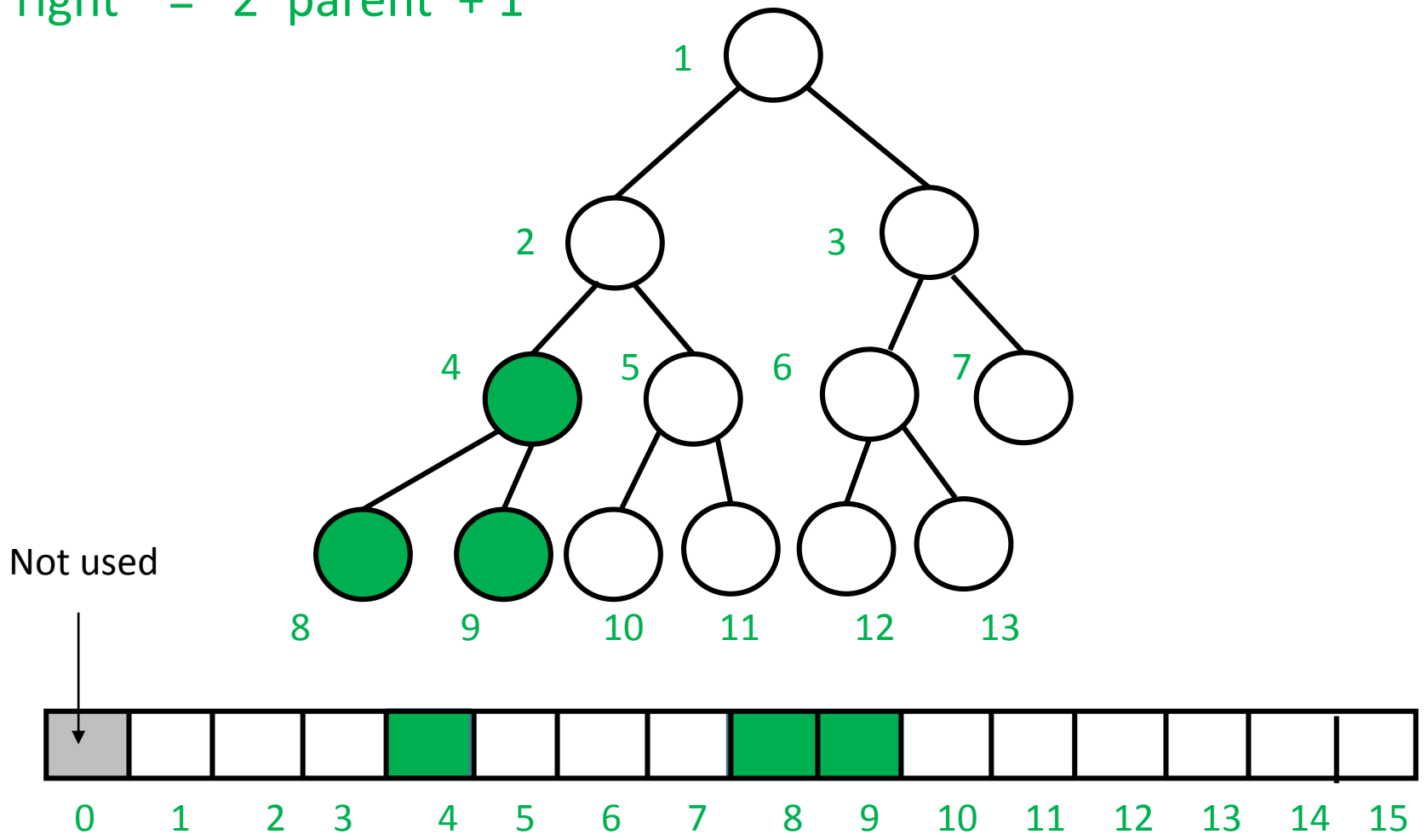
Heap index relations

parent = child / 2
left = 2*parent
right = 2*parent + 1

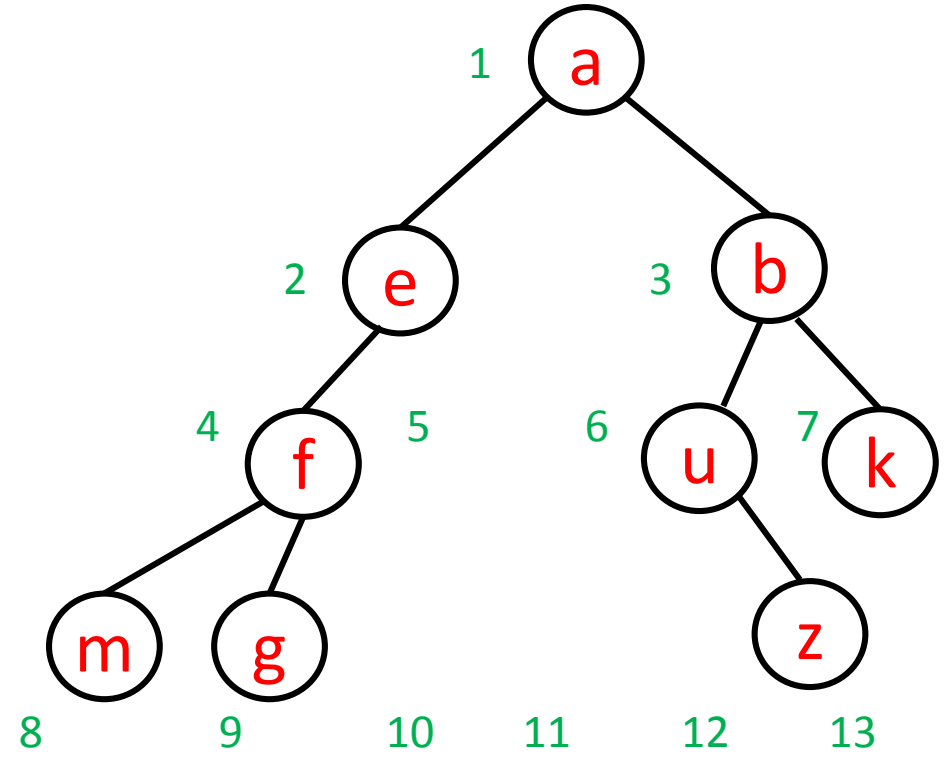


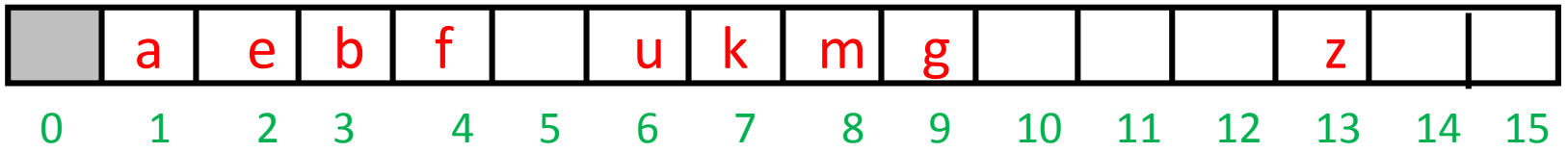
Heap index relations

parent = child / 2
left = 2*parent
right = 2*parent + 1

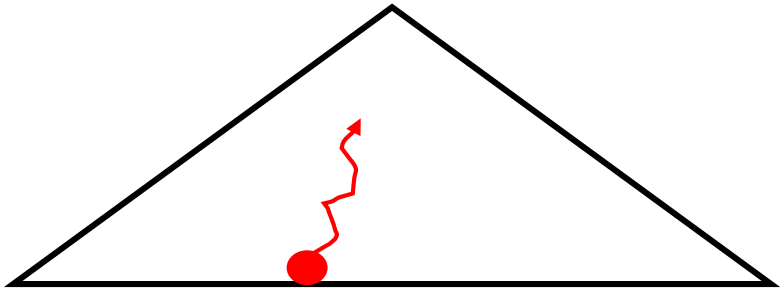


ASIDE: an array data structure can be used for *any* binary tree. But this is uncommon and often inefficient.



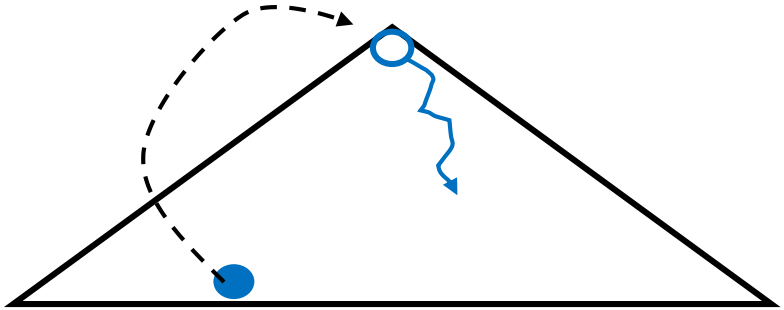


add(element)



“upHeap”

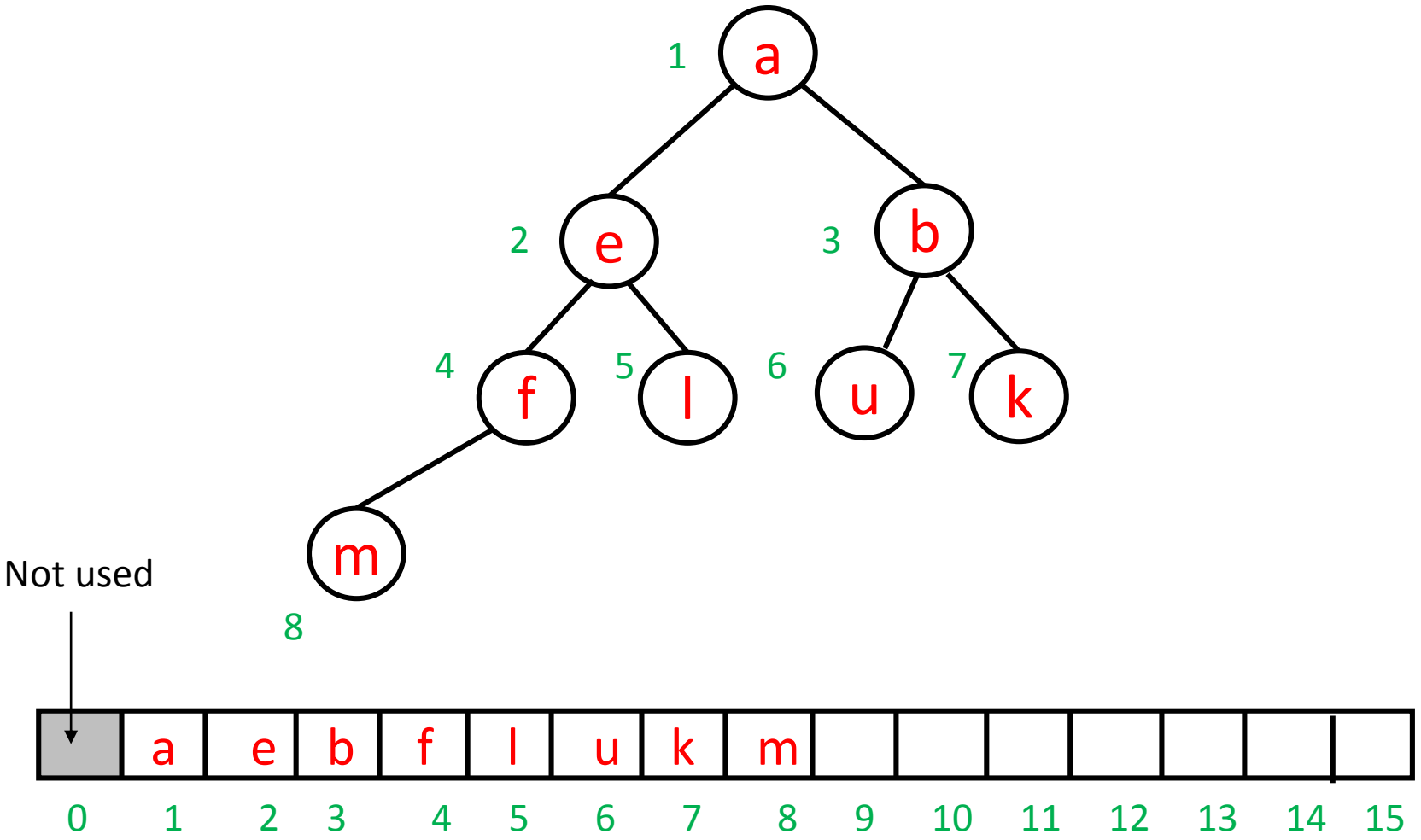
removeMin()



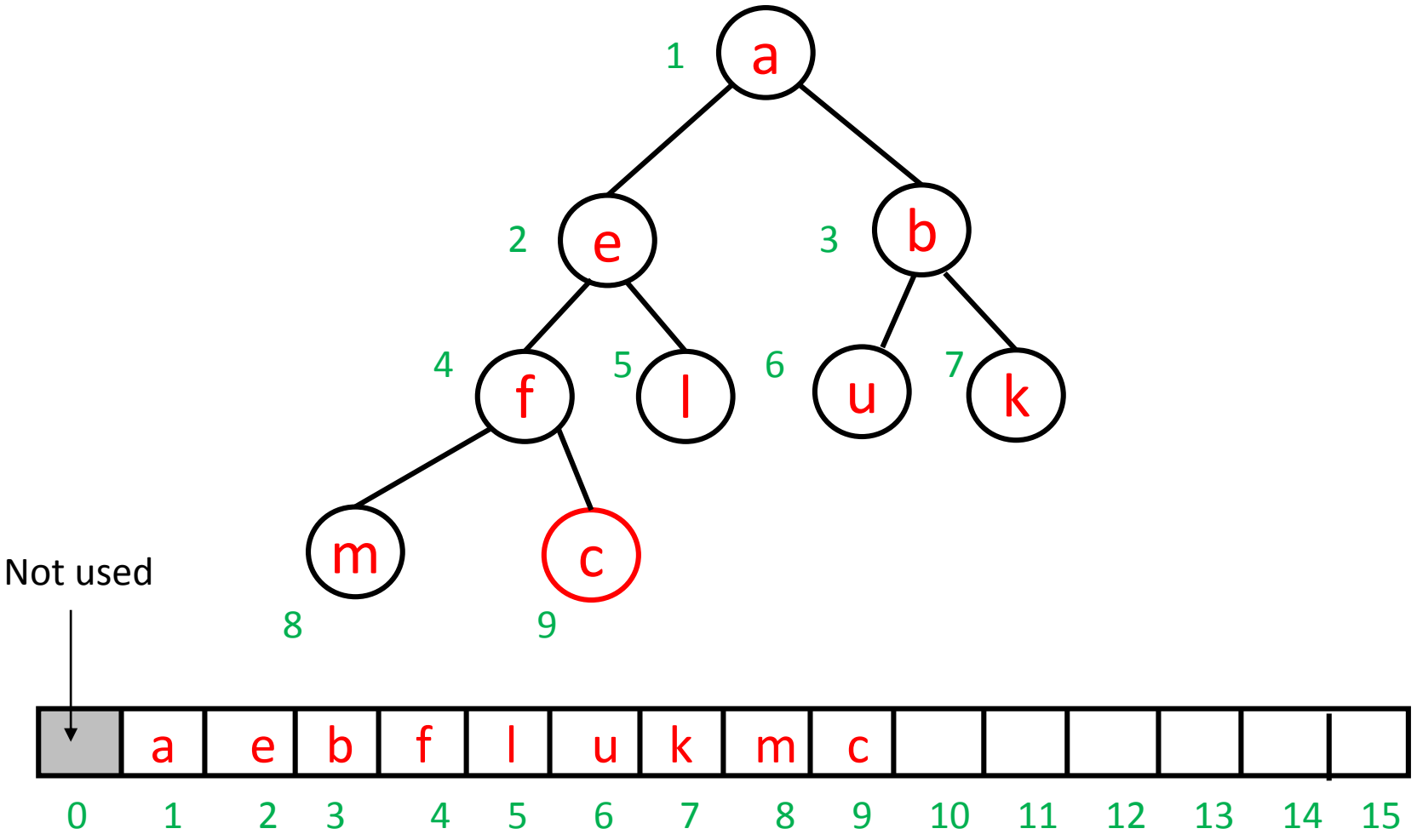
“downHeap”

```
add(element ){
   size = size + 1          // number of elements in heap
   heap[ size ] = element   // assuming array
                            //  has room for another element
   i = size

  //  the following is sometimes called "upHeap"

   while ( i > 1  and  heap[i] < heap[ i/2 ]){
       swapElements( i, i/2 )
       i = i/2
  }
}
```

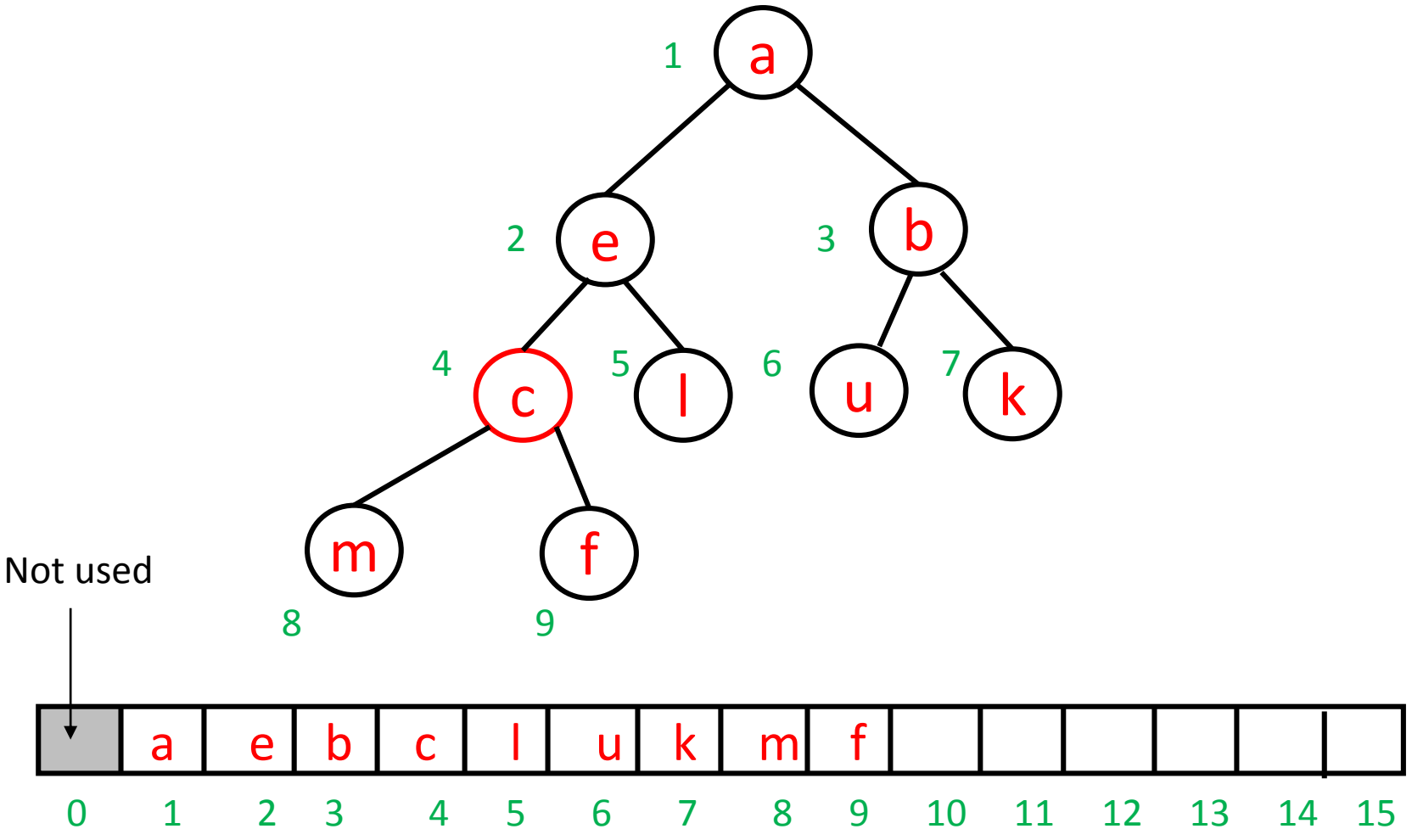
e.g. add(c)


1
a
2
e
3
b
4
f
5
l
6
u
7
k
m
8
Not used
a
e
b
f
l
u
k
m
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

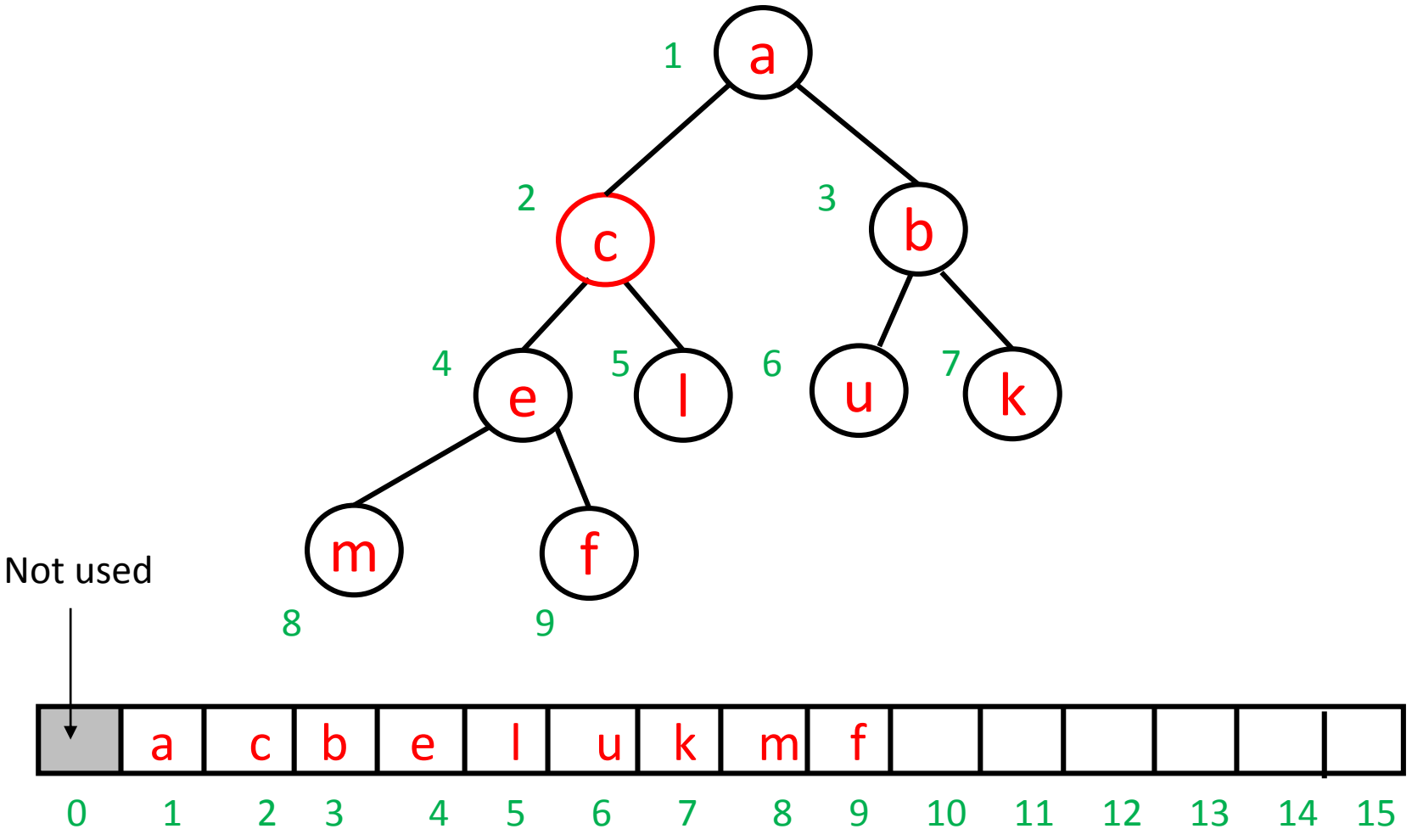
e.g. add(c)


1
a
2
e
3
b
4
f
5
l
6
u
7
k
m
c
8
9
Not used
a
e
b
f
l
u
k
m
c
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

e.g. add(c)


1
a
2
e
3
b
4
c
5
l
6
u
7
k
m
f
8
9
Not used
a
e
b
c
l
u
k
m
f
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

e.g. add(c)

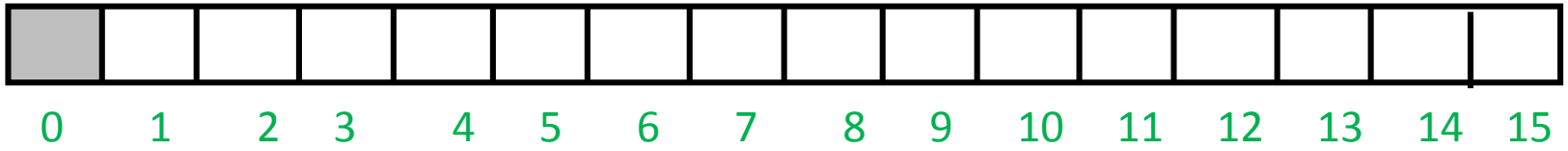

1
a
2
c
3
b
4
e
5
l
6
u
7
k
m
f
8
9
Not used
a
c
b
e
l
u
k
m
f
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Given a list with size elements:

```
buildHeap(list){
    create new heap array              // length >  list.size
    for (k = 0; k < list.size; k++)
        add( list[k] )                 //  add to heap[ ]
}
```

You could write the buildHeap algorithm slightly differently by putting all the list elements into the array at the beginning, and then `upheaping’ each one.

Best case of buildHeap is … ?



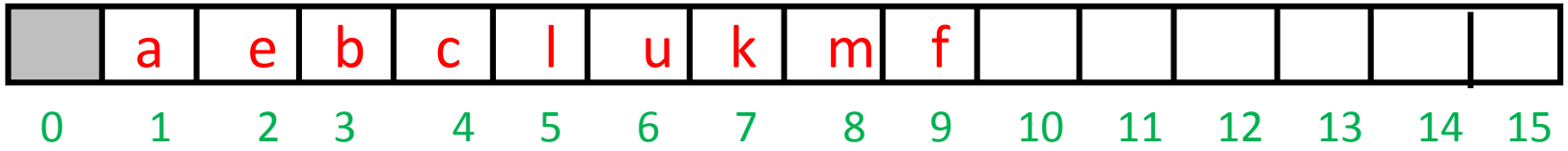
Suppose we want to add some elements to an empty heap:

a e b c l u k m f

How many swaps do we need to add each element?

In the best case, …

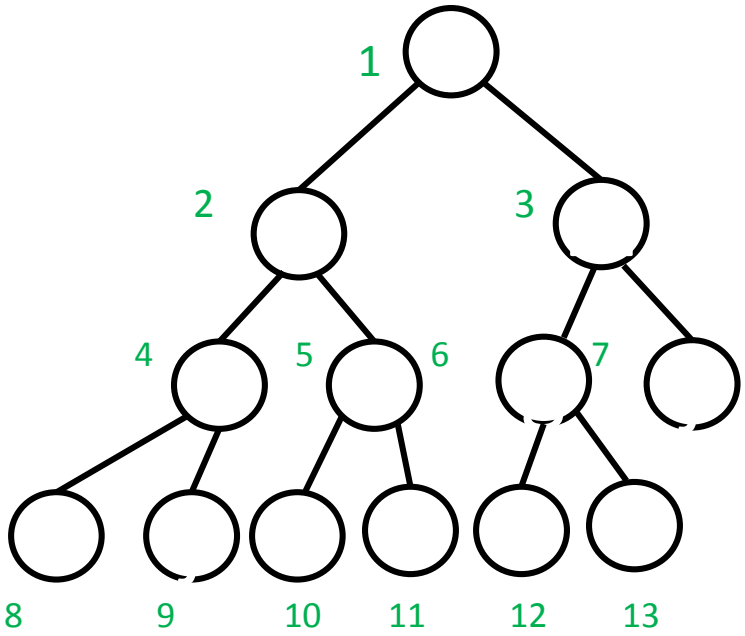
Best case of buildHeap is $\Theta(n)$



How many swaps do we need to add each element?

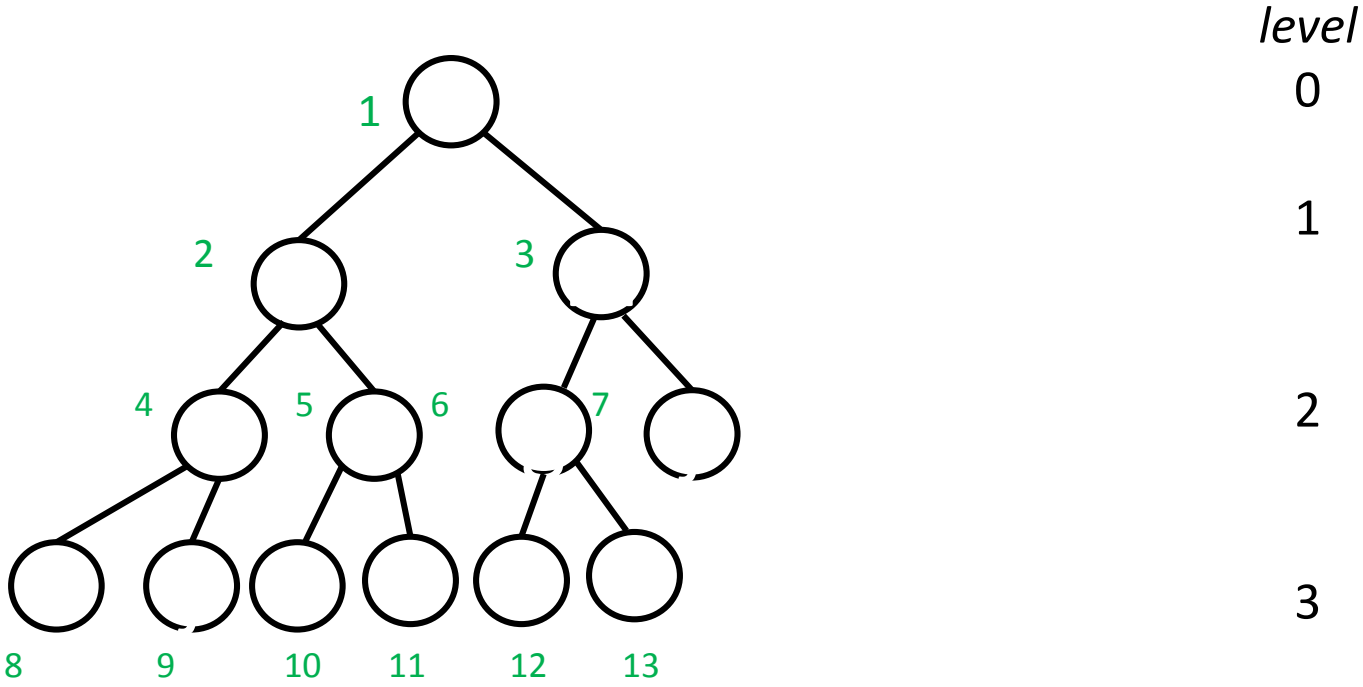
In the best case, the order of elements that we add is already a heap, and no swaps are necessary.

Worse case of buildHeap ?



How many swaps do we need to add the i-th element?

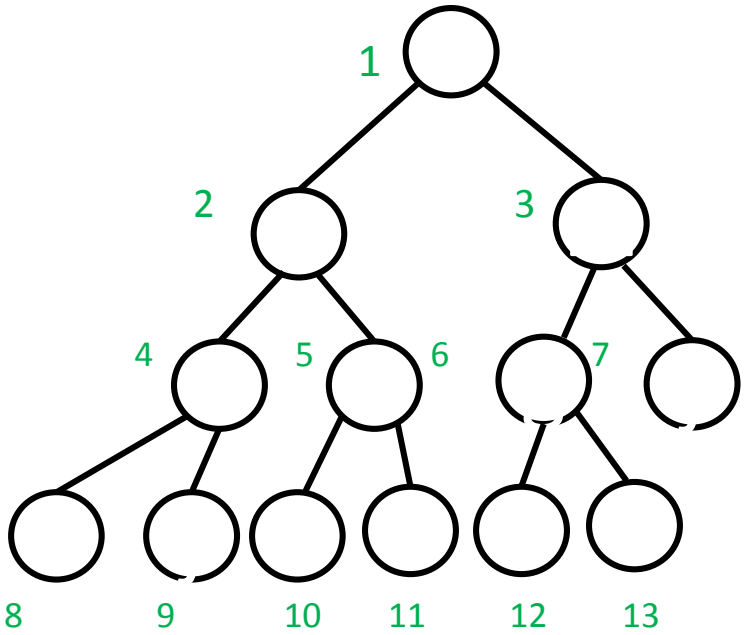
Worse case of buildHeap ?



How many swaps do we need to add the i-th element?
Element i gets added to some level, such that:

$$2^{level} \leq i < 2^{level+1}$$

Worse case of buildHeap ?



level

0

1

2

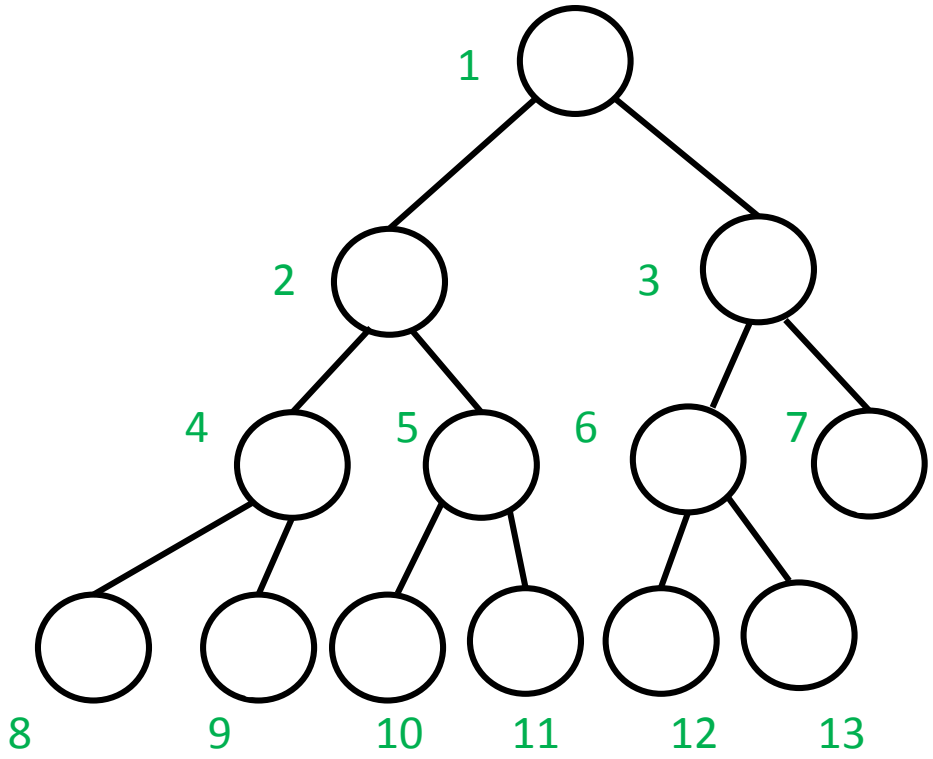
3

$$2^{level} \leq i < 2^{level+1}$$

$$level \leq \log_2 i < level + 1$$

Thus, $level = floor(\log_2 i)$

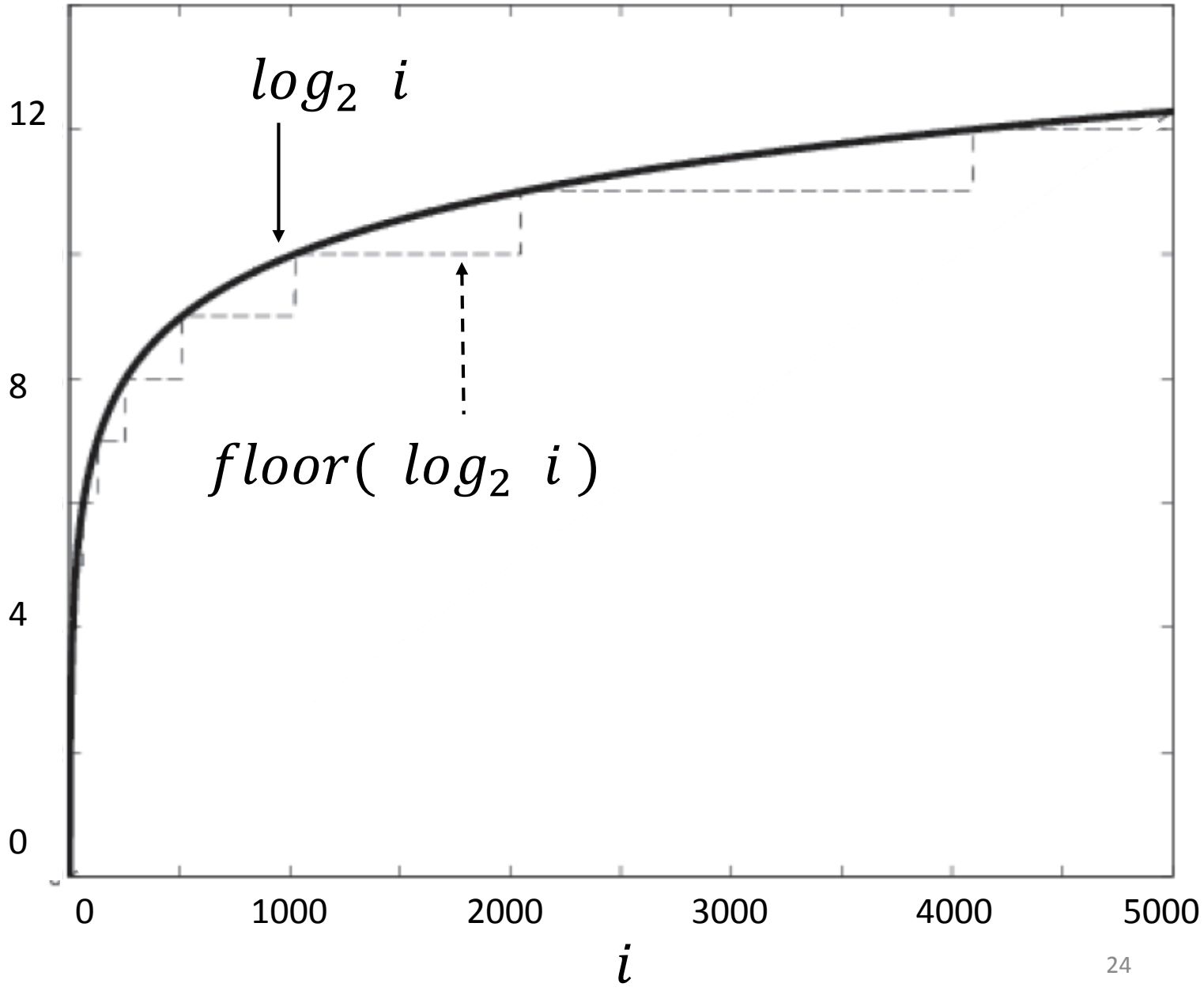
Worse case of buildHeap

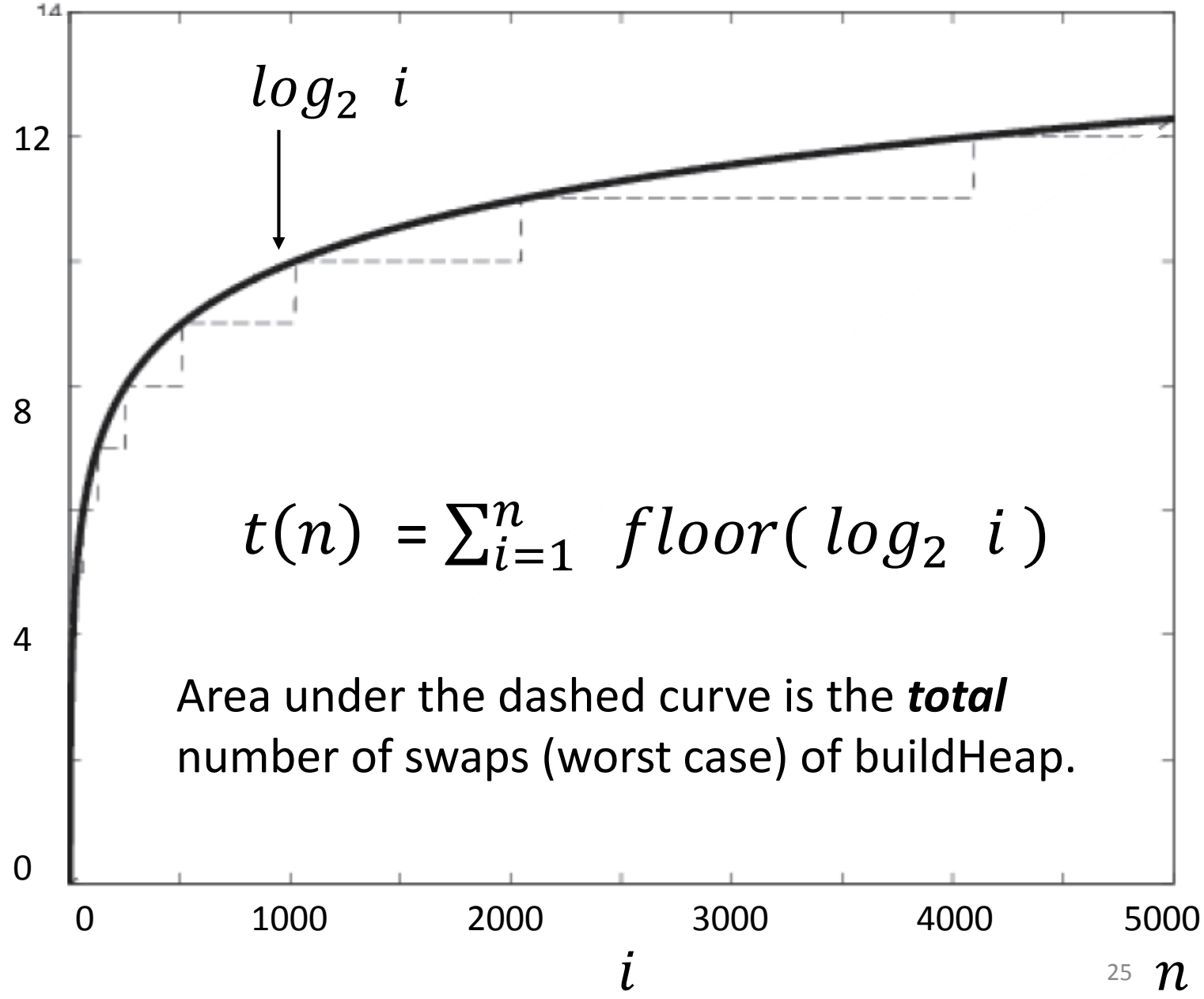


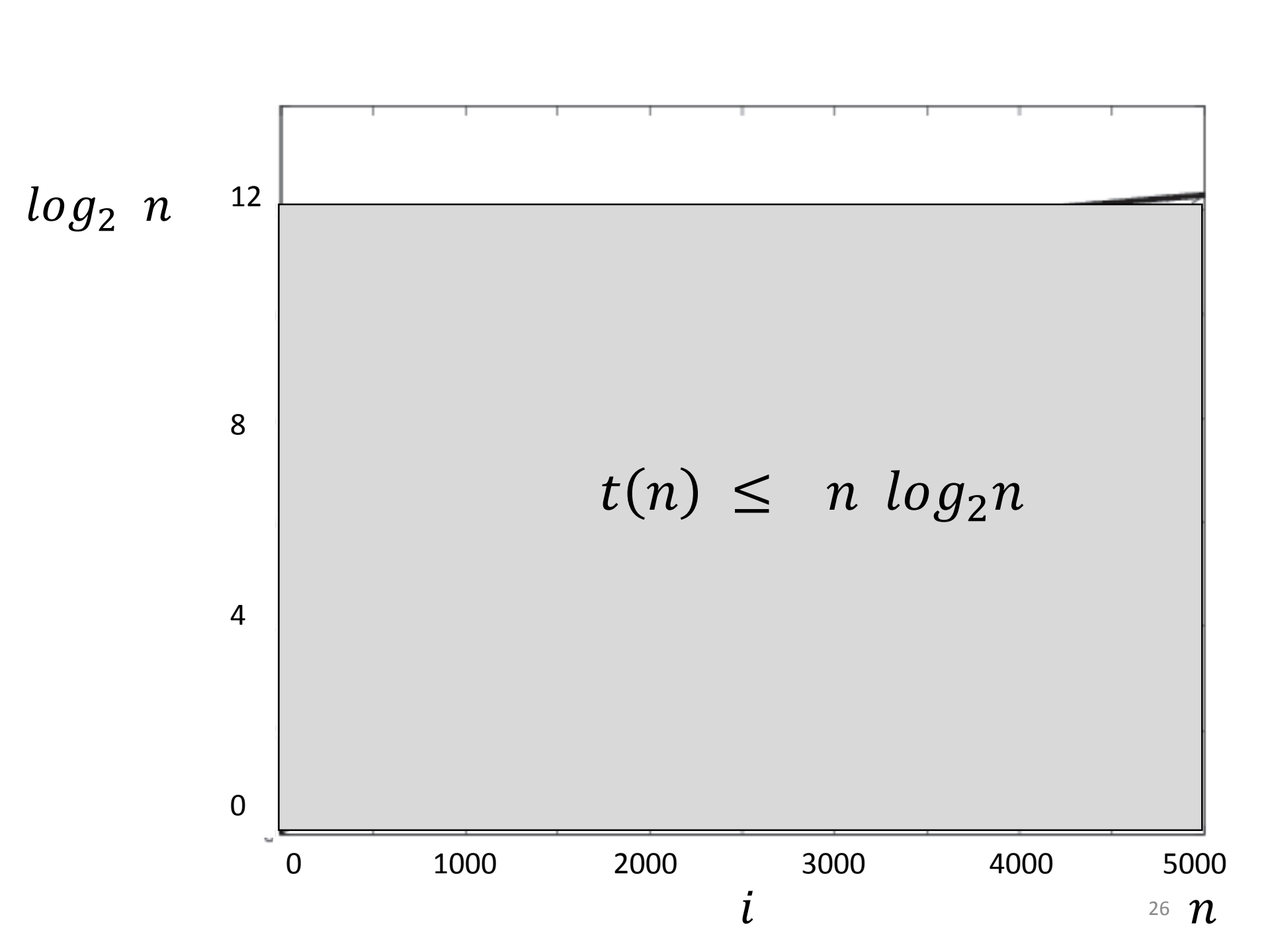
Suppose there are i. n elements to add.

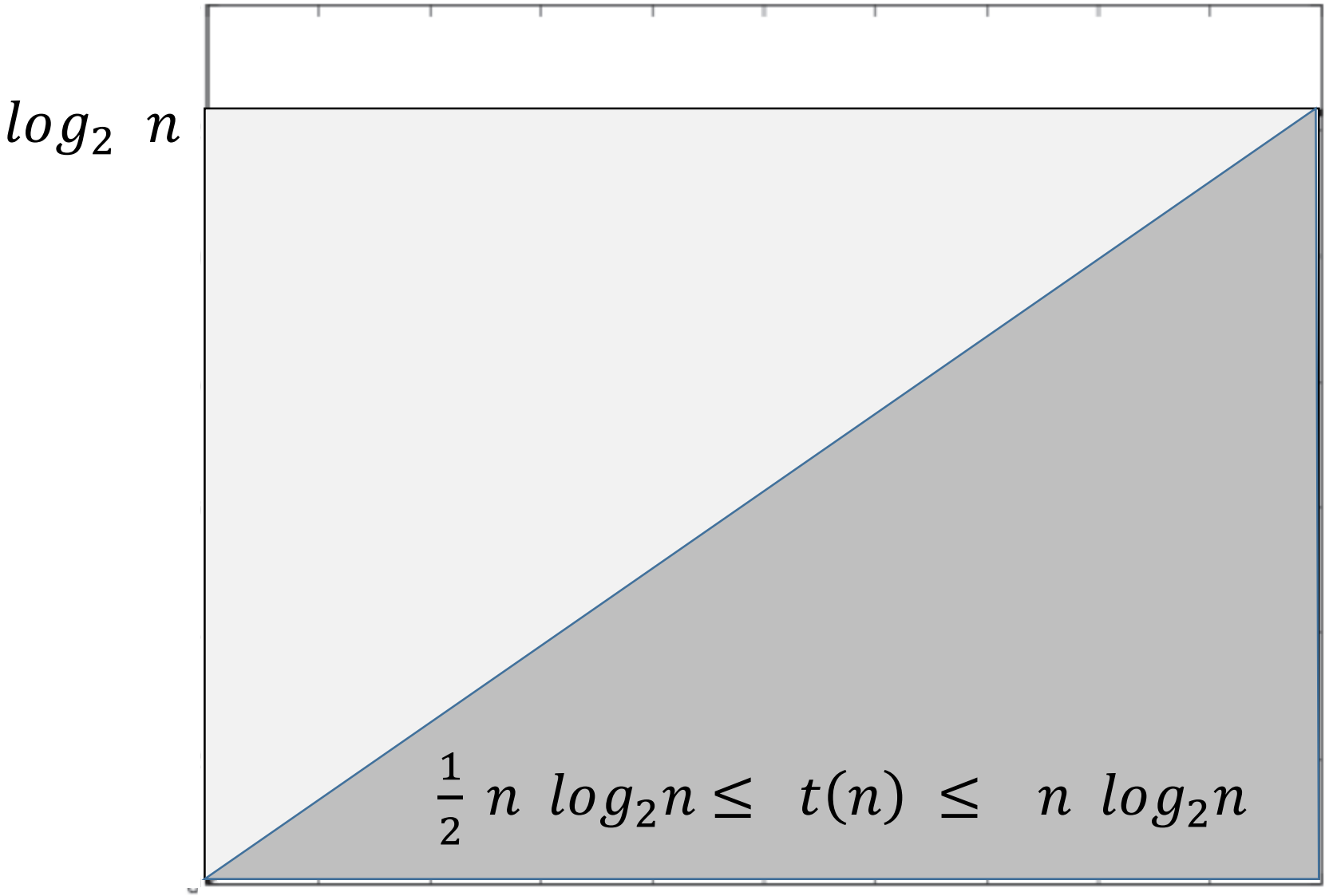
Worst case number of swaps needed to add node i.

$$t(n) = \sum_{i=1}^{n} floor(\log_2 i)$$


$log_2\ i$
$floor(\ log_2\ i\ )$
12
8
4
0
0
1000
2000
3000
4000
5000
i


$log_2\ i$
$t(n) = \sum_{i=1}^{n} floor(\ log_2\ i\ )$
Area under the dashed curve is the ***total*** number of swaps (worst case) of buildHeap.
14
12
8
4
0
0
1000
2000
3000
4000
5000
i
n


$log_2\ n$
12
8
4
0
$t(n) \leq\ \ n\ log_2 n$
0
1000
2000
3000
4000
5000
i
n


$log_2\ n$
$\frac{1}{2}\ n\ log_2 n \leq\ t(n)\ \leq\ \ n\ log_2 n$
n

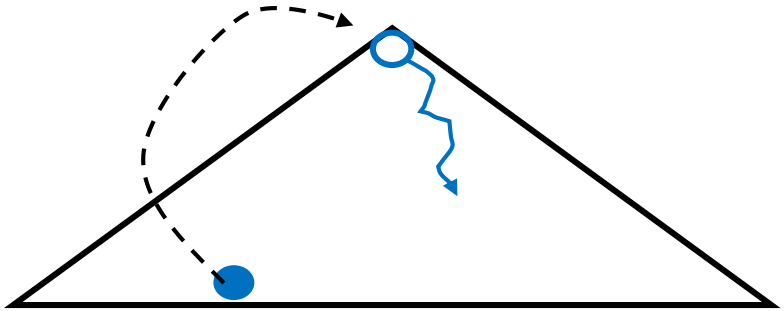
Thus, worst case: buildHeap is $\Theta(n \log_2 n)$

Next lecture I will show you a $\Theta(n)$ algorithm.

add(element)

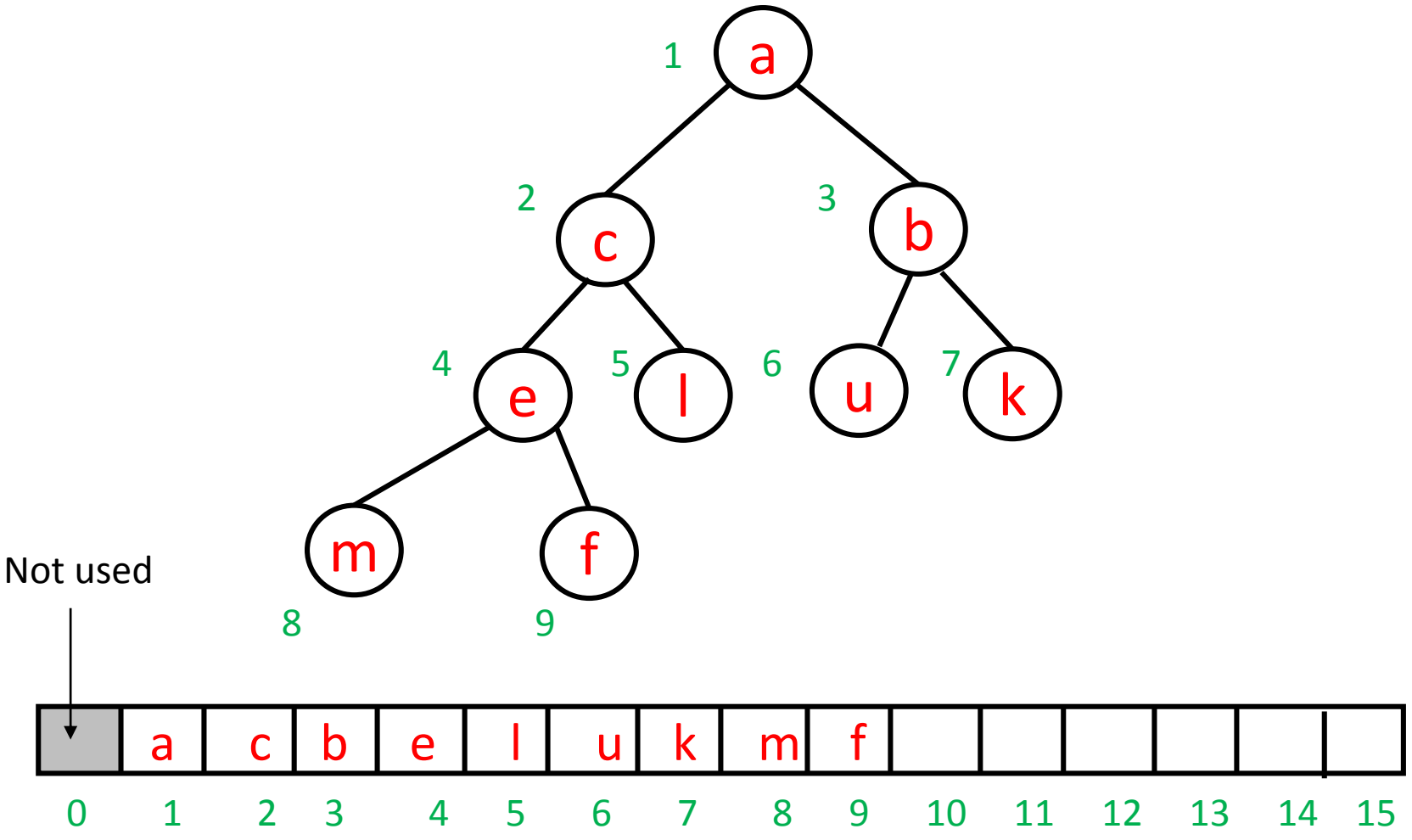
“upHeap”

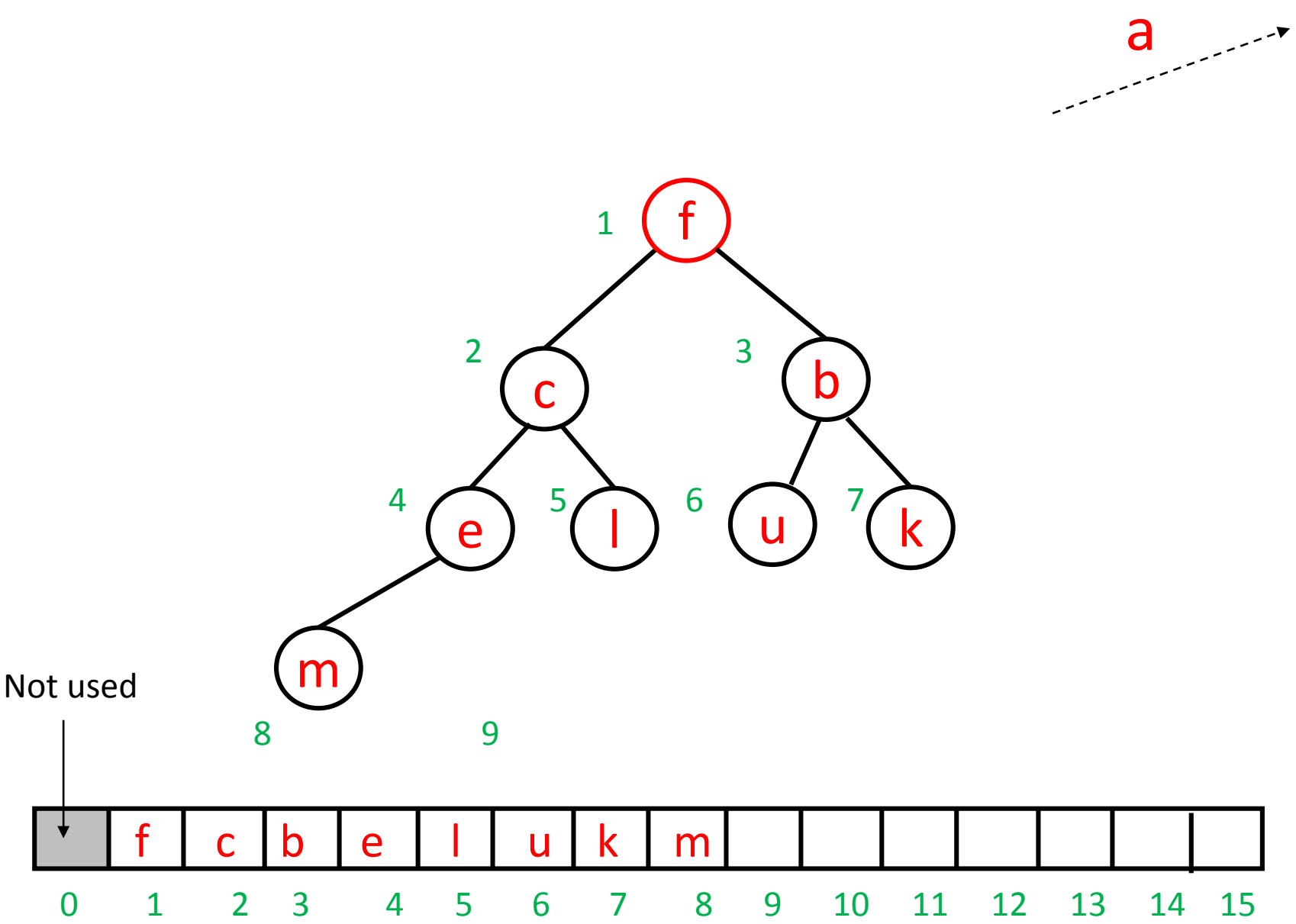
removeMin()

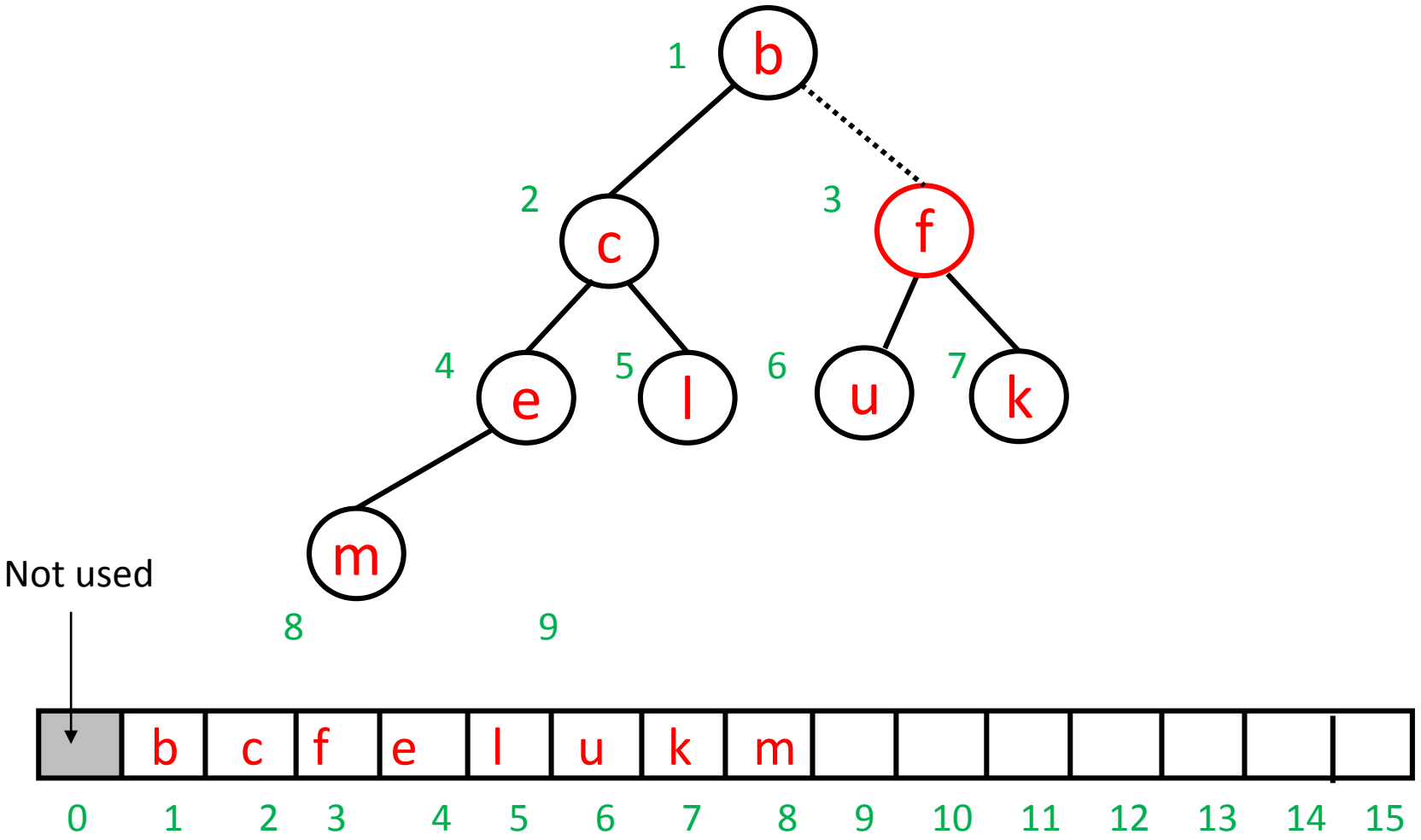


“downHeap”

e.g. removeMin()




a
1
f
2
c
3
b
4
e
5
l
6
u
7
k
m
8
9
Not used
f
c
b
e
l
u
k
m
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15


1
b
2
c
3
f
4
e
5
l
6
u
7
k
m
8
9
Not used
b
c
f
e
l
u
k
m
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

removeMin()

Let heap[ ] be the array.
Let size be the number of elements in the heap.

```
removeMin( ){
    tmpElement = heap[1]          //   heap[0] not used.
    heap[1] = heap[size]
    
}
```

removeMin()

Let heap[ ] be the array.
Let size be the number of elements in the heap.

```
removeMin( ){
    tmpElement = heap[1]          //  heap[0] not used.
    heap[1] = heap[size]
    heap[size] = null
    size = size - 1
    downHeap(1, size)             //   next slide
    return  tmpElement
}
```

```
downHeap( startIndex , maxIndex ){

   i = startIndex
   while (2*i <= maxIndex){          // if there is a left child
      child = 2*i
      Find the smaller child  (left or right?)
   }
}
```

```
downHeap( startIndex , maxIndex ){

   i = startIndex
   while (2*i <= maxIndex){            // if there is a left child
      child = 2*i
      if child < size {                // if there is a right sibling
         if (heap[child + 1] < heap[child])    //  if rightchild < leftchild ?
         child = child + 1
      }
      
    }
}
```

```
downHeap( startIndex , maxIndex ){

  i = startIndex
  while (2*i <= maxIndex){            // if there is a left child
    child = 2*i
    if child < size {                 // if there is a right sibling
      if (heap[child + 1] < heap[child])    //  if rightchild < leftchild ?
      child = child + 1
    }
    if (heap[child] < heap[ i ]){     // Do we need to swap with child?
      swapElements(i , child)
      i = child
    }
    else return                       //   otherwise we have an infinite loop.
  }
}
```

Announcements

- Mycourses survey about MATH 240/235 and COMP 251
- Update on final exam grading policy

Final Exam grading policy

- Multiple Choice with 50 questions
- Four choices on each question
- *No penalty for incorrect answers*

  (so don't leave any question blank)

- **Grade out of 50**

  = max(0, -10 + 6/5 * raw number correct)

Raw number correct for pure guessing ?

(binomial distribution, n=50, p=.25)

Hey, me and all my buddies averaged 25% raw scores on the final.