Quiz 2 COMP 423 March 5, 2008

Answer all questions in the exam book.
Calculators or other electronic devices are NOT permitted.
The Quiz is marked out of 15.

1. **(2 points)**

   (a) What is the "inverted file" of the following: `aabbabaaaaabbaab` ?

   (b) Transform this inverted file into a file of gap lengths.

2. **(2 points)**

   Briefly describe how run length coding is used in fax compression. Give at least two important details about how the code is defined.

3. **(5 points)**

   (a) Encode the following sequence using LZ2 (sliding window):

   `abaaababababb`

   Assume the window size is $n_w = 4$.
   You answer should include a parsing of the above sequence into phrases.

   (b) In the lecture notes, the number of bits used for Lempel-Ziv version LZ3 is:

   $$totbits = \sum_{i=1}^{\phi(n)} (\lceil \log i \rceil + 1)$$

   where it was assumed there are two symbols in the alphabet. The term $\lceil \log i \rceil$ in the summation implies that, as the sequence length n grows, the number of bits used for phrase $\phi(n)$ is an increasing function of n. Does this imply that LZ3 compresses shorter sequences better than longer sequences (i.e. better = smaller compression ratio) ? Briefly explain.

   (c) Give a formula for *totbits* for the case that there are N symbols in the alphabet, rather than two. Assuming the N symbols in the alphabet do not occur with equal frequency, how could we change the LZ3 code to reduce *totbits* ?

4. **(2 points)**

Consider the following conditional probability matrix for a first order Markov model, where the number of symbols in the alphabet is $N = 3$. Assume that these conditional probabilities are the same for all j.

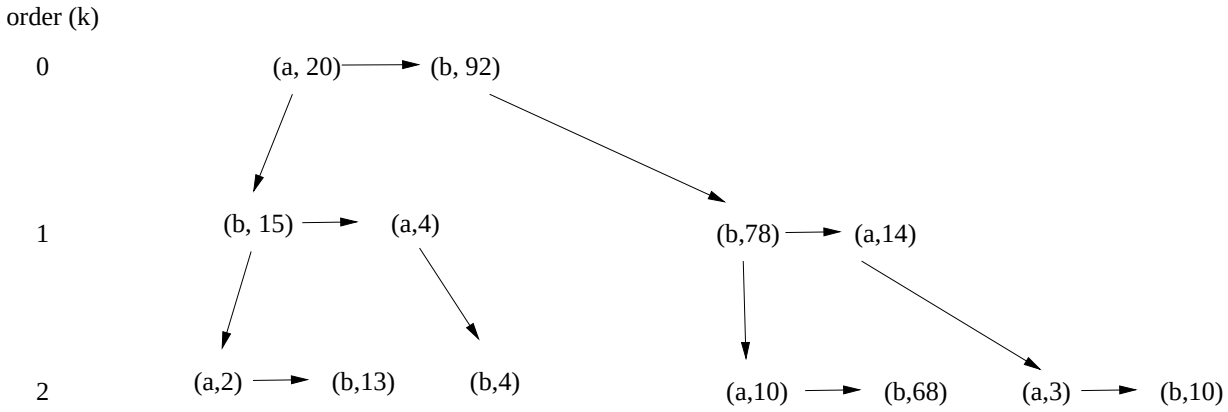
$$P(X_{j+1} \mid X_j) \;=\; \frac{1}{8}\begin{bmatrix} 1 & 6 & 4 \\ 5 & 0 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) What are the values of the function $p(X_2 \mid X_1 = 1)$?

(b) What is the value of $p(X_2 = 2, X_3 = 1 \mid X_1 = 3)$?

5. **(4 points)**

The following data structure represents the frequency counts (in various contexts up to second order) of a sequence, after k symbols have been encoded.



Use these frequency counts to answer the following.

(a) What is k ?

(b) Estimate $p(X_{k+1} \mid X_k = a)$.

(c) Estimate $p(X_{k+1} \mid X_{k-1} = b,\ X_k = a)$.

(d) Suppose PPM were used to encode the sequence, and the next two symbols in the sequence were $X_{k+1} = a, X_{k+2} = a$. How would these two symbols be encoded, i.e. what would the encoder send?

Hint: *Do not* compute probabilities.