$$t(n) = 2t\left(\frac{n}{2}\right) + c_1 n + c_2 n \log n$$

5 points 5 points

The c1 term accounts for the work in the "for loop", namely, for each point in the central strip, check the next 7 points as sorted by y value. The c2 term accounts for the work done in sorting by y value.

To solve the recurrence by backsubstitution, you should notice that the two terms give rise to two separate sums. So we can solve for the two sums on their own. The first one is handled by the Master Method and I don't expect you to write out that solution, though I do include it for completeness on the next slide. The second one is what this question is mainly about.

$$\begin{aligned} t(n) &= 2t\left(\tfrac{n}{2}\right) + cn \qquad \text{i.e. } c = c_1 \\ &= 2\left\{2t\left(\tfrac{n}{4}\right) + c\tfrac{n}{2}\right\} + cn \\ &= 4t\left(\tfrac{n}{4}\right) + cn + cn \\ &= 4\left\{2t\left(\tfrac{n}{8}\right) + c\tfrac{n}{4}\right\} + 2cn \\ &= 8t\left(\tfrac{n}{8}\right) + 3cn \\ &= n\,t(1) + \log n \cdot c \cdot n \end{aligned}$$

No points for this. Its just the Master Method.

$$\begin{aligned} t(n) &= 2t\left(\tfrac{n}{2}\right) + cn\log n \qquad \text{i.e. } c = c_2 \\ &= 2\left\{2t\left(\tfrac{n}{4}\right) + c\tfrac{n}{2}\log\tfrac{n}{2}\right\} + cn\log n \\ &= 4t\left(\tfrac{n}{4}\right) + cn\log\tfrac{n}{2} + cn\log n \\ &= 4\left\{2t\left(\tfrac{n}{8}\right) + c\tfrac{n}{4}\log\tfrac{n}{4}\right\} + cn\log\tfrac{n}{2} + cn\log n \\ &= 8t\left(\tfrac{n}{8}\right) + cn\log\tfrac{n}{4} + cn\log\tfrac{n}{2} + cn\log n \\ &\;\vdots \\ &= 2^k t\left(\tfrac{n}{2^k}\right) + cn\sum_{i=0}^{k-1}\log\tfrac{n}{2^i} \end{aligned}$$

but $\log\frac{n}{2^i} = \log n - i$

let $k = \log n$

$$\begin{aligned} &= n\,t(1) + cn(\log n)^2 - cn\sum_{i=0}^{\log n - 1} i \\ &= n\,t(1) + cn(\log n)^2 - cn\frac{\log n(\log n - 1)}{2} \end{aligned}$$

10 points. 10 points

So putting it all together...

$$\begin{aligned} t(n) &= 2t\left(\tfrac{n}{2}\right) + c_1 n + c_2 n\log n \\ &= 2n\,t(1) + c_1 n\log n \\ &\quad + c_2 n(\log n)^2 - cn\frac{\log n(\log n - 1)}{2} \end{aligned}$$