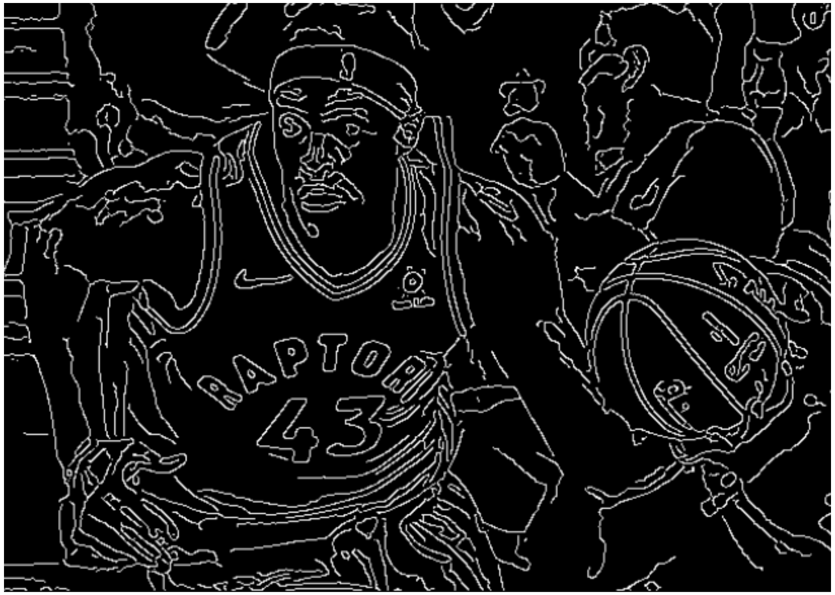
Lecture 4 **Class Activities**

Edge Detection

Wed. Sept 16, 2020

Example: edge detection

(By the way, what are edges?)



edge(I, 'Canny')

Overview of lecture 4 topics

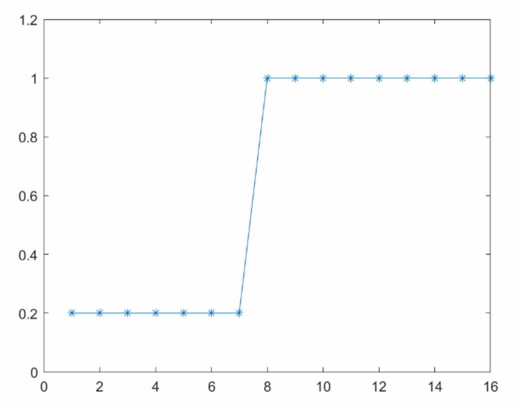
- Image gradient
- Classical filters (Sobel, Prewitt)
- 1^{st} versus 2^{nd} derivative of step edge
- derivative of Gaussian filter
- Laplacian of Gaussian filter
- Marr-Hildreth edge detection
- Canny edge detection

The plan for today is to walk through these topics and pose small problems for you to work on individually.

(No breakout rooms today.)

Example: step edge

$I(x)$



$$\frac{d\,I(x)}{dx} \approx \frac{1}{2}I(x+1) - \frac{1}{2}I(x-1)$$

?

$$\frac{d^2\,I(x)}{dx^2} \approx I(x+1) - 2\,I(x) + I(x-1)$$

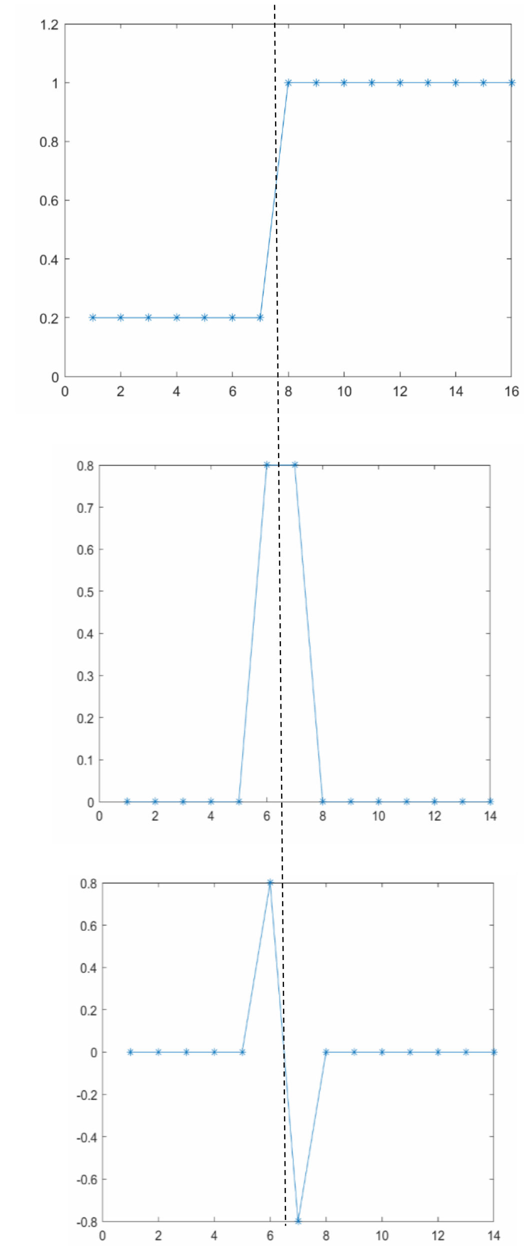
?

Example: step edge

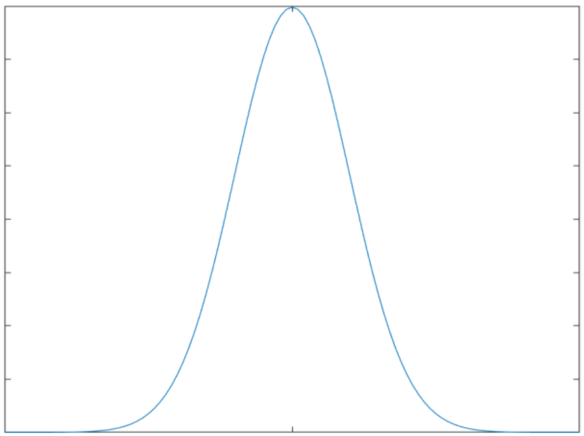
$I(x)$

$$\frac{d\ I(x)}{dx} \approx \frac{1}{2} I(x+1) - \frac{1}{2} I(x-1)$$

$$\frac{d^2\ I(x)}{dx^2} \approx I(x+1) - 2\, I(x) + I(x-1)$$



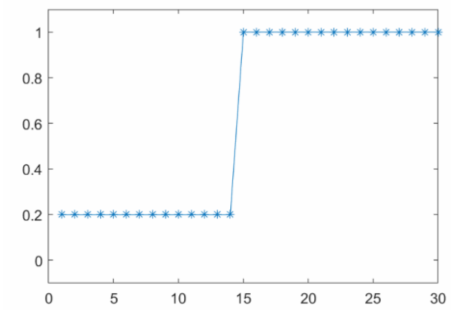
Gaussian function



$$G(x;\ \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Example: blurred step edge

$I(x)$ =



$G(x,\sigma) * I(x)$ = ?

$$\frac{d\ G(x,\sigma) *\ I(x)}{dx} \approx ?$$

$$\frac{d^2 G(x,\sigma) * I(x)}{dx^2} \approx ?$$

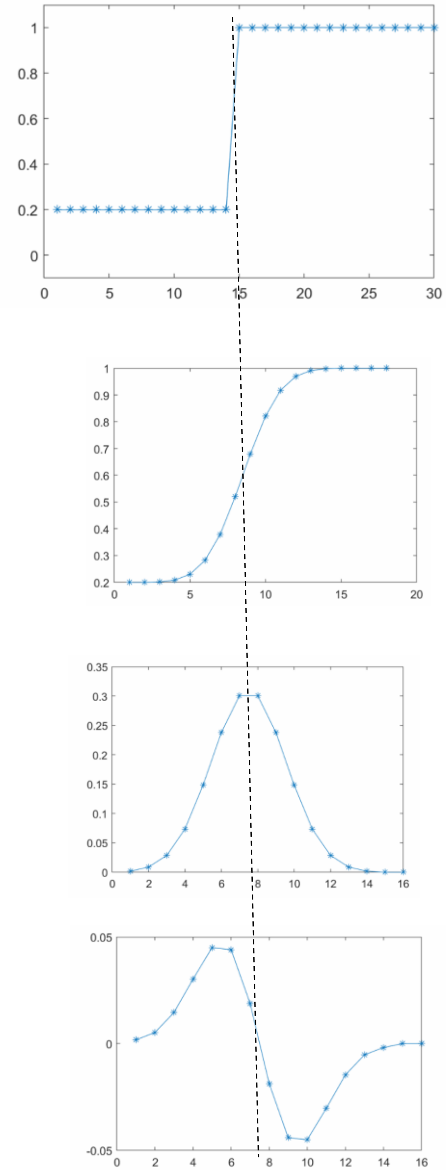
Example: blurred step edge

$$I(x) \quad = $$

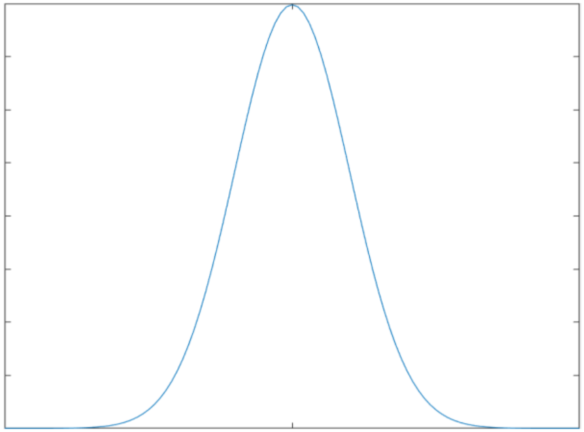
$$G(x,\sigma) * I(x) \quad = $$

$$\frac{d\, G(x,\sigma) * \; I(x)}{dx} \quad \approx $$

$$\frac{d^2 G(x,\sigma) * I(x)}{dx^2} \quad \approx $$



1D Gaussian

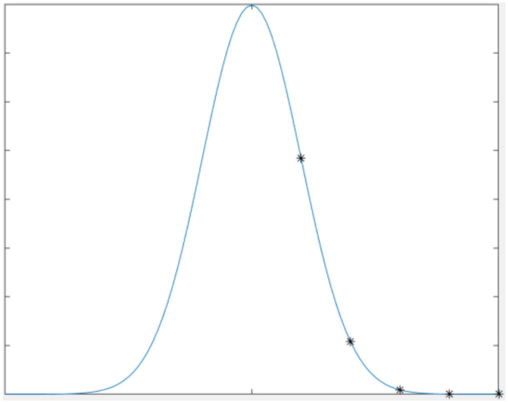


Exercise:

Guess the missing values:

x	$\dfrac{G(x;\,\mu{=}0,\sigma)}{G(0;\,\mu{=}0,\sigma)}$	$\displaystyle\int_{-x}^{x} G(u;\mu=0,\sigma)\,du$
σ		
2σ		
3σ		
4σ		
5σ		

1D Gaussian



Exercise:

x	$\dfrac{G(x;\, \mu=0,\sigma)}{G(0;\, \mu=0,\sigma)}$	$\displaystyle\int_{-x}^{x} \boldsymbol{G(u;\, \mu=0,\sigma)\, d\, u}$
σ	0.61	.68
2σ	0.14	.95
3σ	0.01	.997
4σ	0.003	.999
5σ	0.000003	1

Derivative of Gaussian

Exercise: Sketch

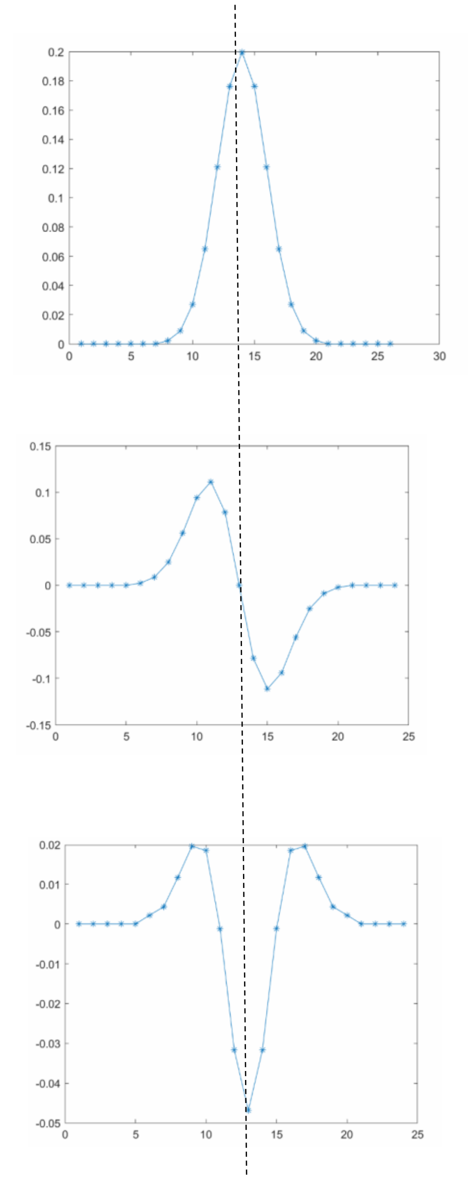
- 1st derivative of a Gaussian
- 2nd derivative of a Gaussian

Derivative(s) of Gaussian

$$G(x,\sigma)$$

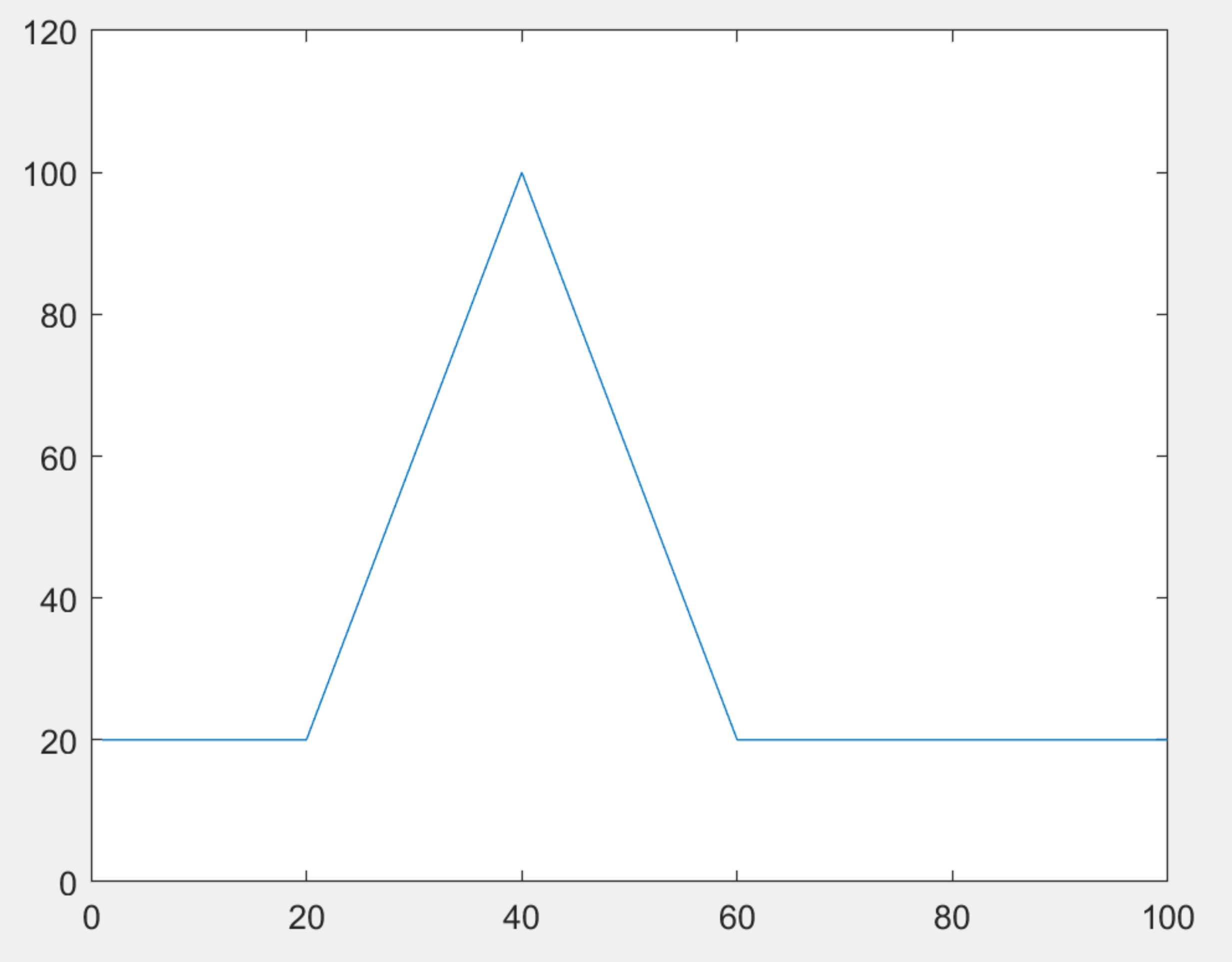
$$\frac{d\,G(x,\sigma)}{dx}$$

$$\frac{d^2\,G(x,\sigma)}{d\,x^2}$$

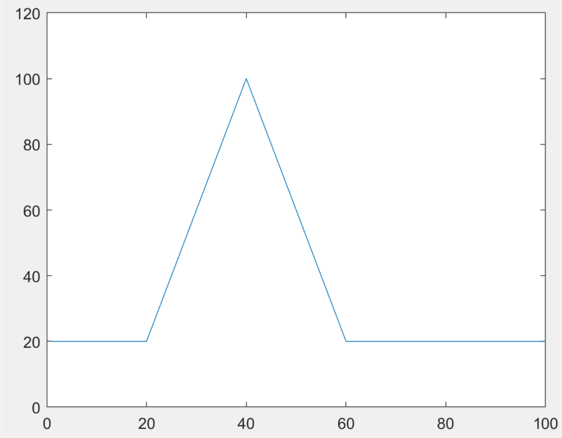


Exercise:

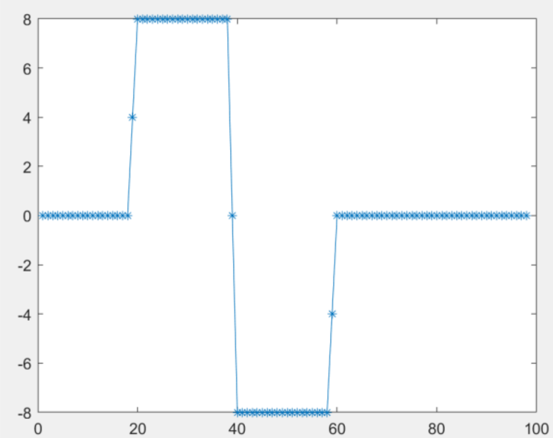
Sketch the first and second derivatives of “roof edge”



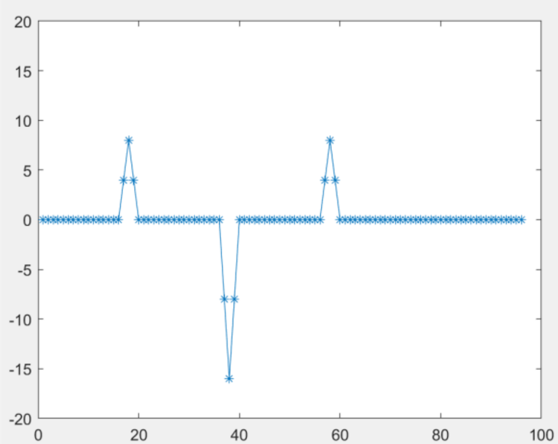
"roof edge"



first derivatives of "roof edge"

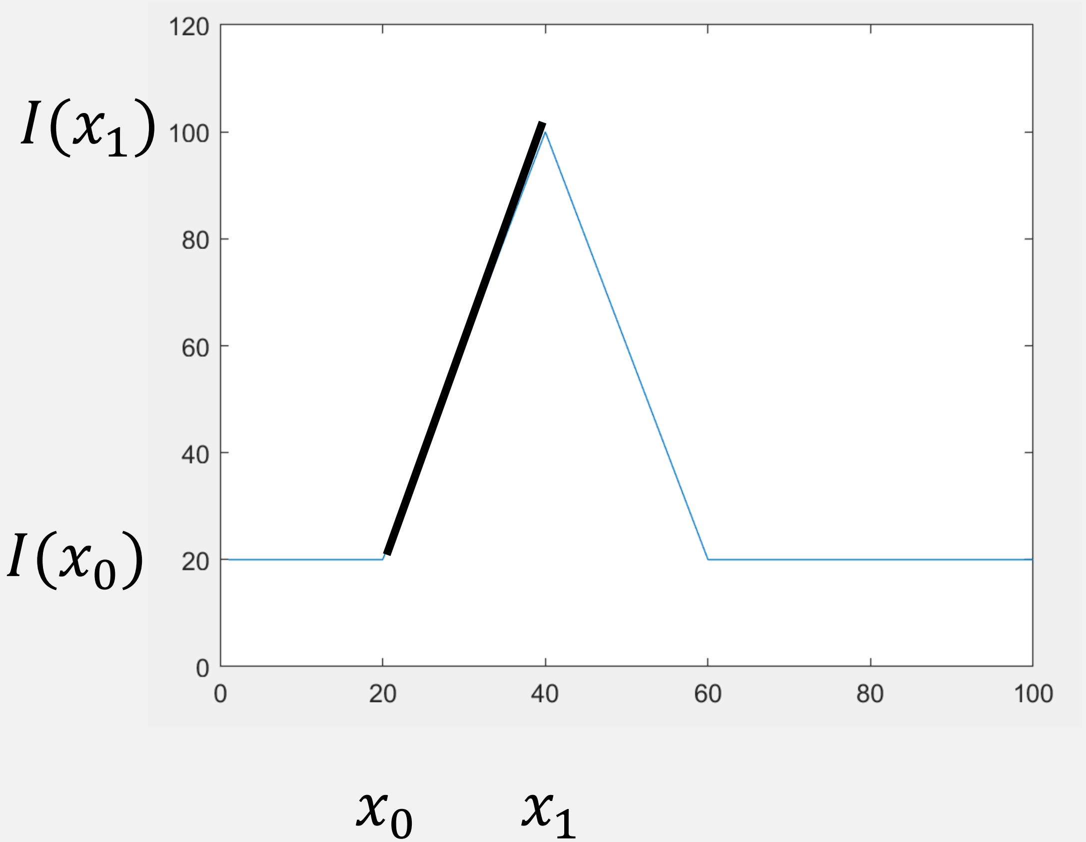


second derivatives of "roof edge"



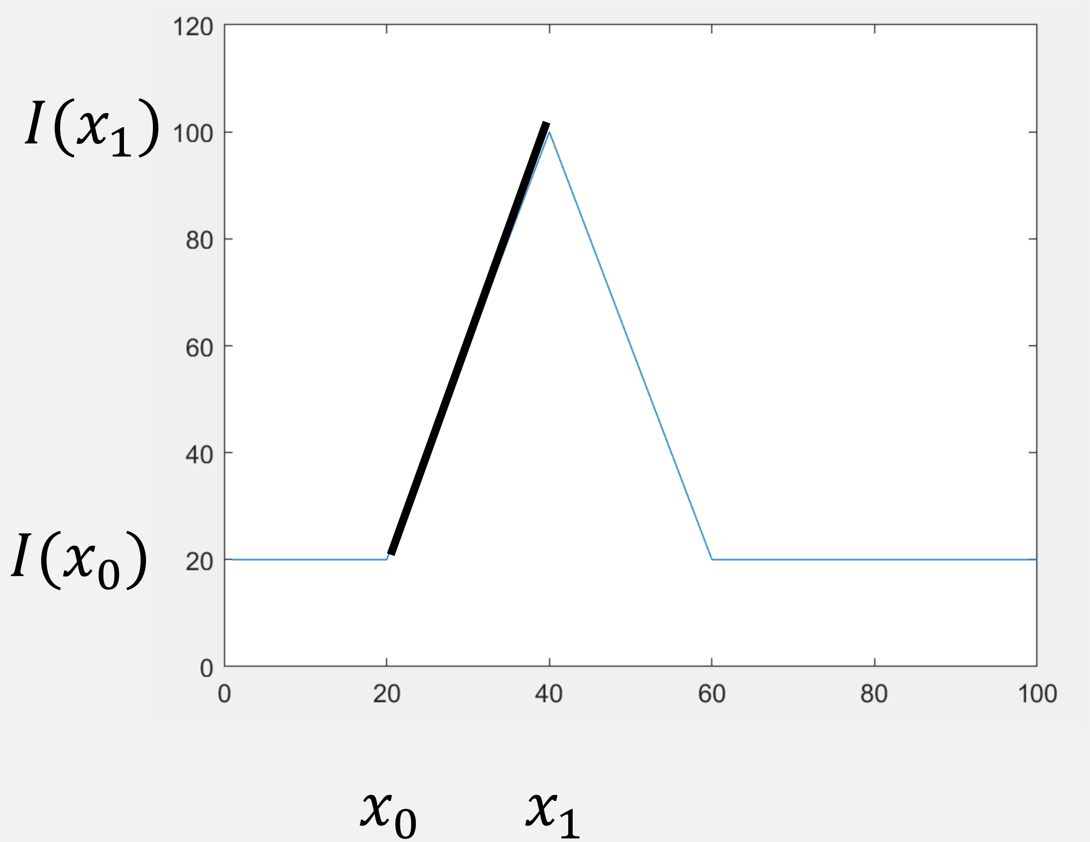
Exercise:

What is the equation of the line below? (high school math)



Exercise:

Write out the equation of a line. (Grade 10 math?)



Can you compute I(x) for this line segment in Matlab without using a `for` loop?

```
N = 100;
I = zeros(N,1);
I(1:x0) = 20;
x0 = 20;
x1 = 40;
Ix0 = 20;
Ix1 = 100;

I(x0:x1) = Ix0 + (Ix1 - Ix0) ./ (x1 - x0) .* ((x0:x1) - x0);

%  called 'vectorization'
```

Image gradient

Exercise:

$$\nabla I(x,y) \equiv \left(\frac{\partial}{\partial x} I(x,y), \frac{\partial}{\partial y} I(x,y)\right)$$

Draw the (discrete) gradient for these images.

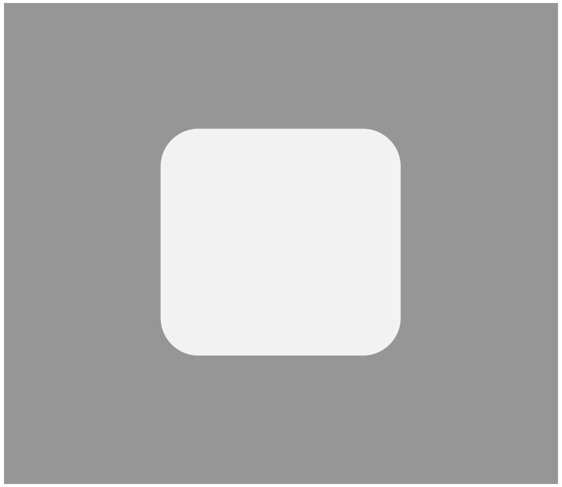


Image gradient

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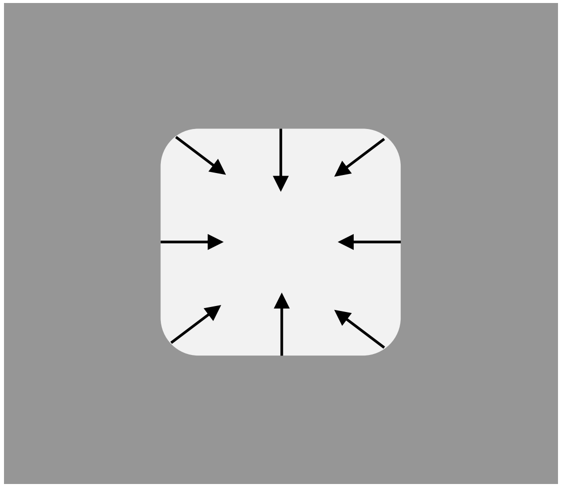
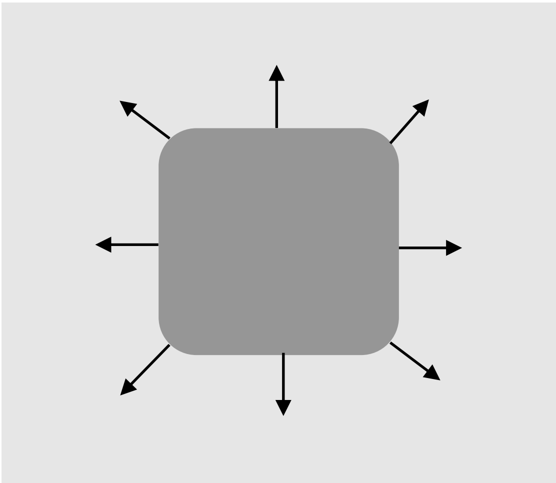
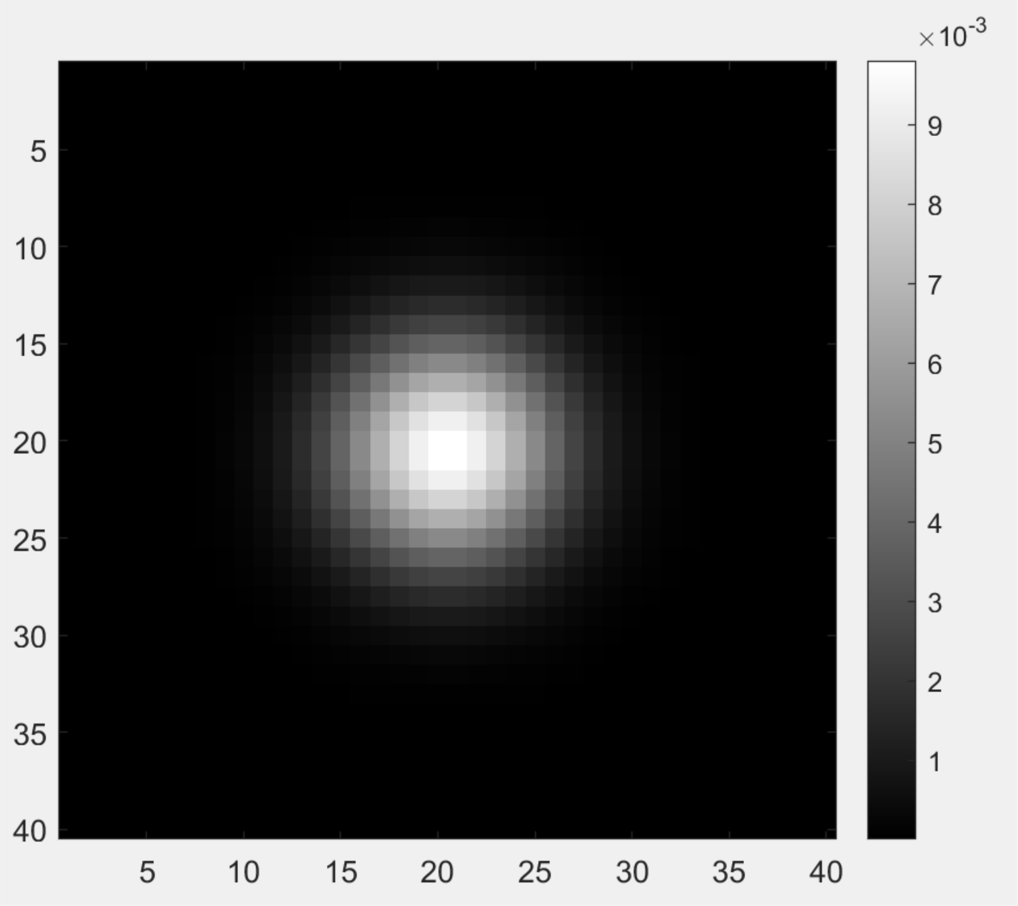




Exercise:

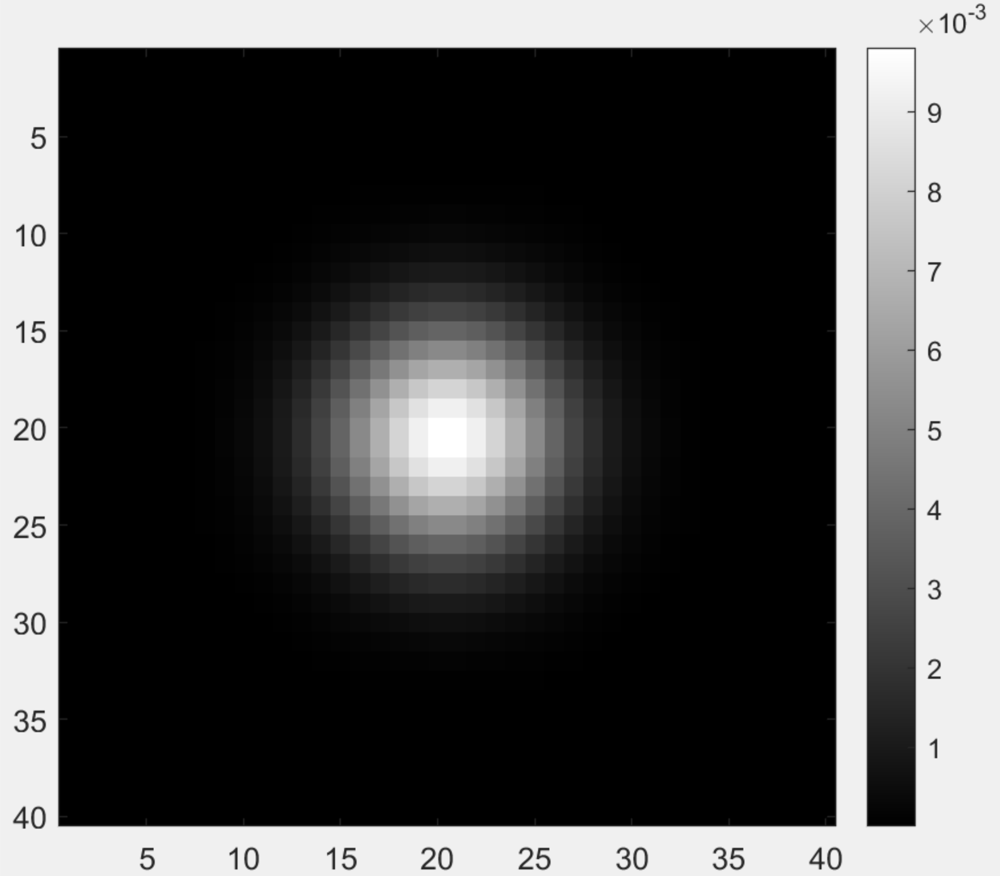
Sketch the gradient of a 2D Gaussian.

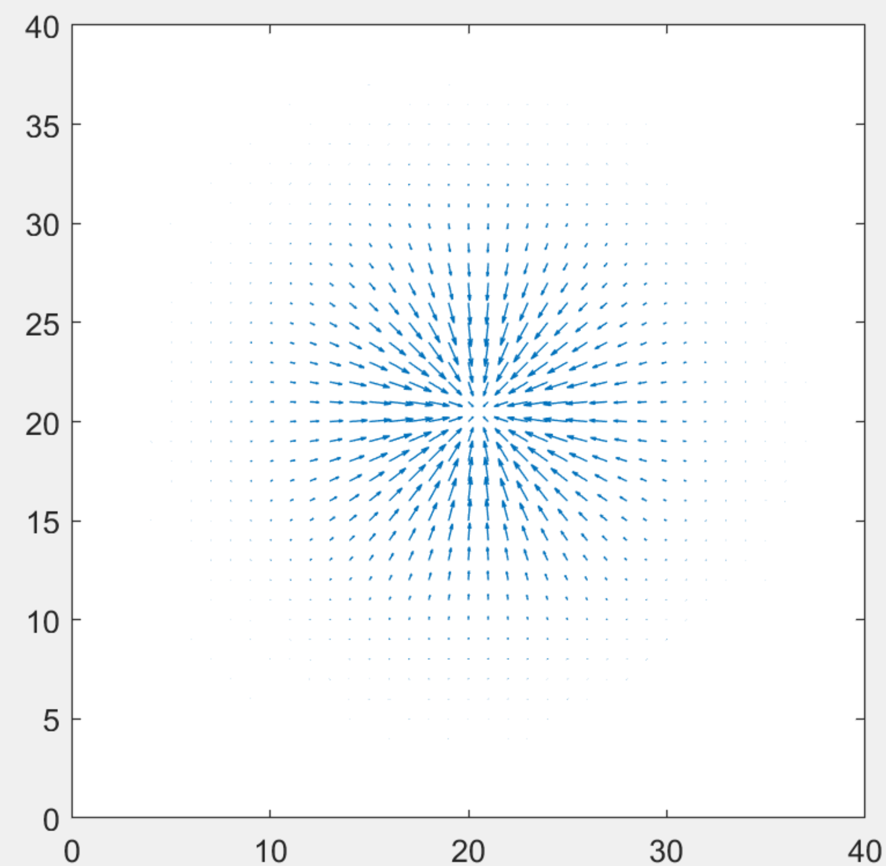


Exercise:

Sketch the gradient of a 2D Gaussian.

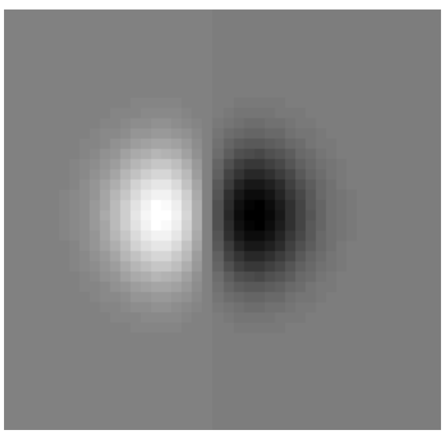
```
g = fspecial('gaussian', 40, 4);
quiver(conv2(g, [1 0 -1], 'same'), conv2(g, [1 0 -1]', 'same'))
```

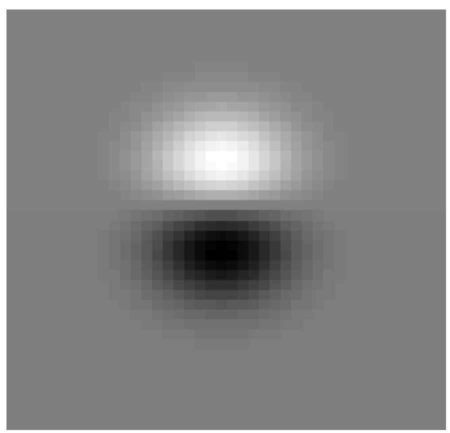


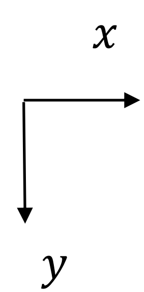


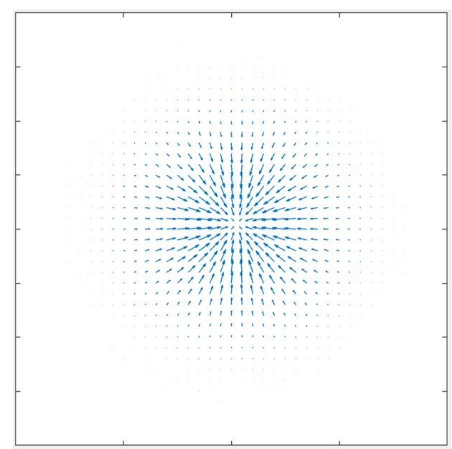
Derivative(s) of 2D Gaussian

$$\frac{\partial G(x, y, \sigma)}{\partial x} \qquad \frac{\partial G(x, y, \sigma)}{\partial y} \qquad \left(\frac{\partial G(x, y, \sigma)}{\partial x}, \frac{\partial G(x, y, \sigma)}{\partial y}\right)$$









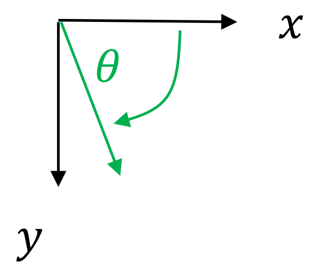
“Steerable filter”

Suppose you are given the responses of the 1st derivative of Gaussian filters on some image $I(x, y)$

$$\left(\frac{\partial G(x,y,\sigma)}{\partial x} * \ I(x,y), \qquad \frac{\partial G(x,y,\sigma)}{\partial y} * I(x,y)\right)$$

We would now like to know the *directional derivative* in some other direction $(\cos\theta, \sin\theta)$.

Exercise: How could we obtain it?



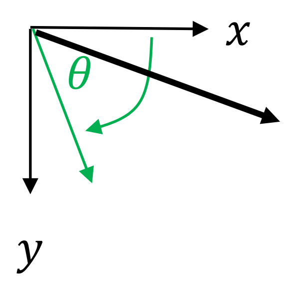
"Steerable filter"

Suppose you are given the responses of the 1st derivative of Gaussian filters on some image $I(x, y)$

$$\left(\frac{\partial G(x,y,\sigma)}{\partial x} * \; I(x,y), \qquad \frac{\partial G(x,y,\sigma)}{\partial y} * I(x,y) \right)$$

The *directional derivative* in direction $(\cos\theta, \sin\theta)$ is the inner product:

$$\cos\theta \left\{ \frac{\partial G(x,y,\sigma)}{\partial x} * \; I(x,y) \right\} + \sin\theta \; \left\{ \frac{\partial G(x,y,\sigma)}{\partial y} * I(x,y) \right\}$$



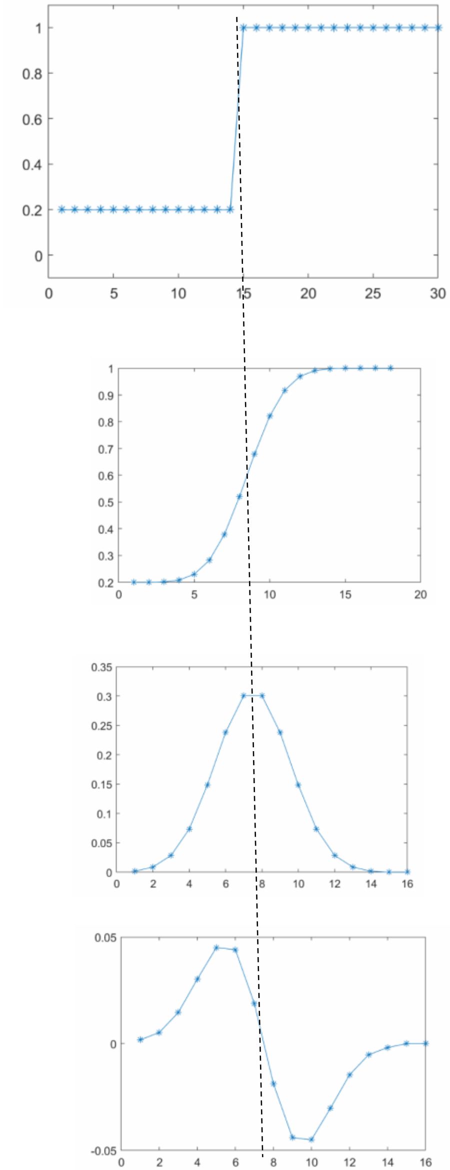
Recall Example: blurred step edge

$$I(x) =$$

$$G(x,\sigma) * I(x) =$$

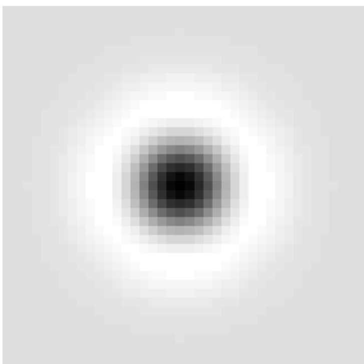
$$\frac{d\, G(x,\sigma) * \ I(x)}{dx} \approx$$

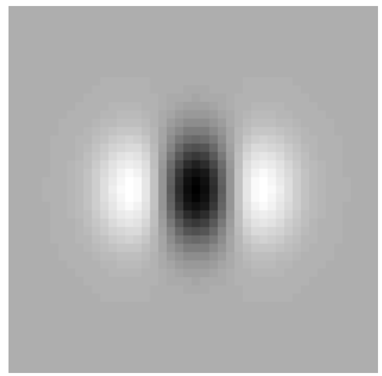
$$\frac{d^2 G(x,\sigma) * I(x)}{dx^2} \approx$$

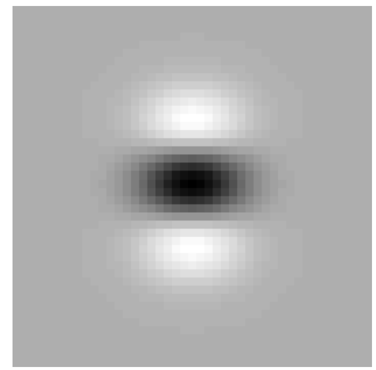


Laplacian of a Gaussian (filter)

$$\nabla^2 G(x,y,\sigma) \equiv \frac{\partial^2 G(x,y,\sigma)}{\partial x^2} + \frac{\partial^2 G(x,y,\sigma)}{\partial y^2}$$

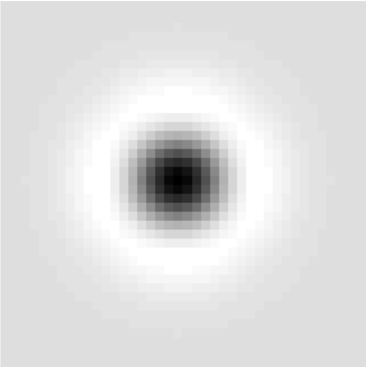


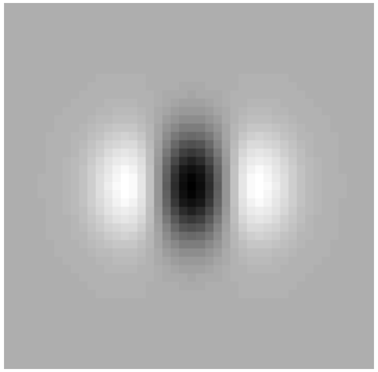


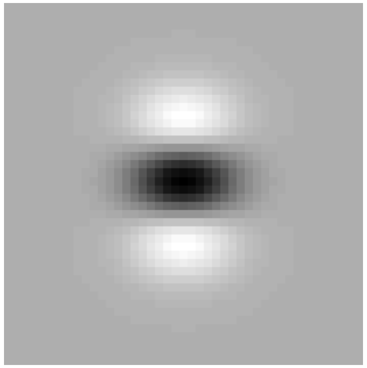


Laplacian of a Gaussian (filter)

$$\nabla^2 G(x,y,\sigma) \equiv \frac{\partial^2 G(x,y,\sigma)}{\partial x^2} + \frac{\partial^2 G(x,y,\sigma)}{\partial y^2}$$

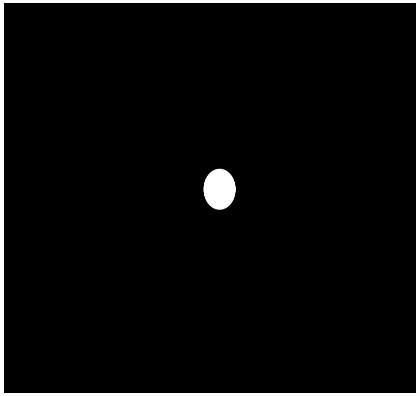






Exercise: (tricky one)

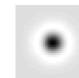
What would be the response of the $\nabla^2 G(x,y,\sigma)$ filter when convolved with this image ?



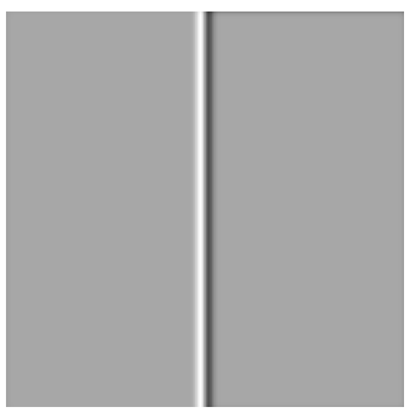
$I(x,y)$

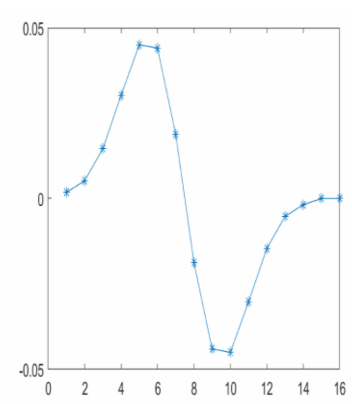
Note that color maps differ in these figures!

Example: vertical step edge

$I(x, y)$   $\nabla^2 G(x, y, \sigma)$

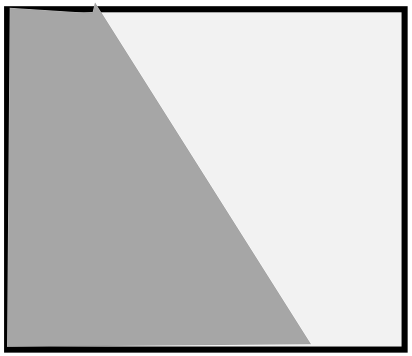
What is $I(x, y) * \nabla^2 G(x, y, \sigma)$?





Marr-Hildreth edge detection (1979)

$I(x, y)$



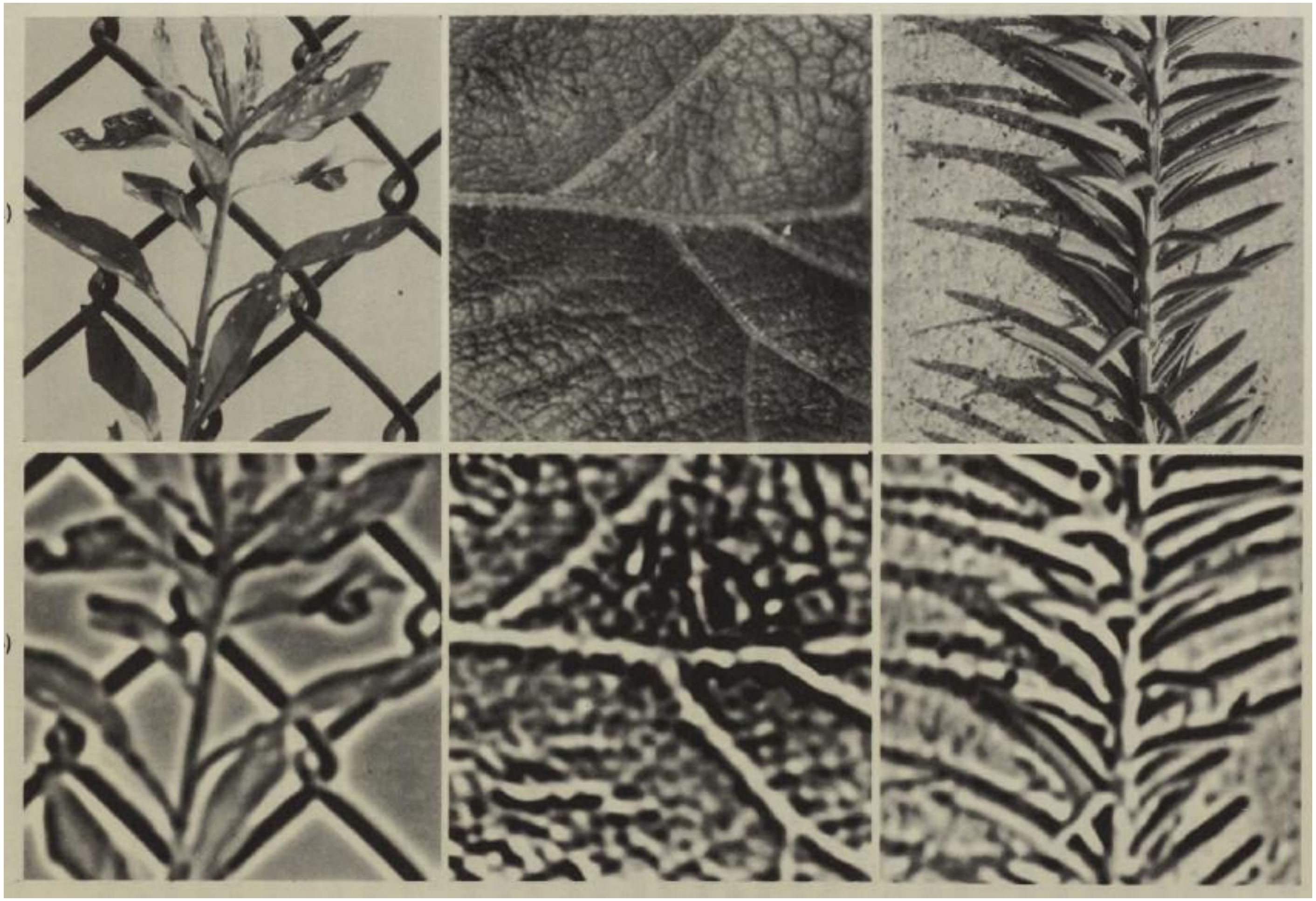
Compute $\nabla^2 G(x, y, \sigma) * I(x, y)$

Then the edges are the points (x, y) where there is a zero-crossing.

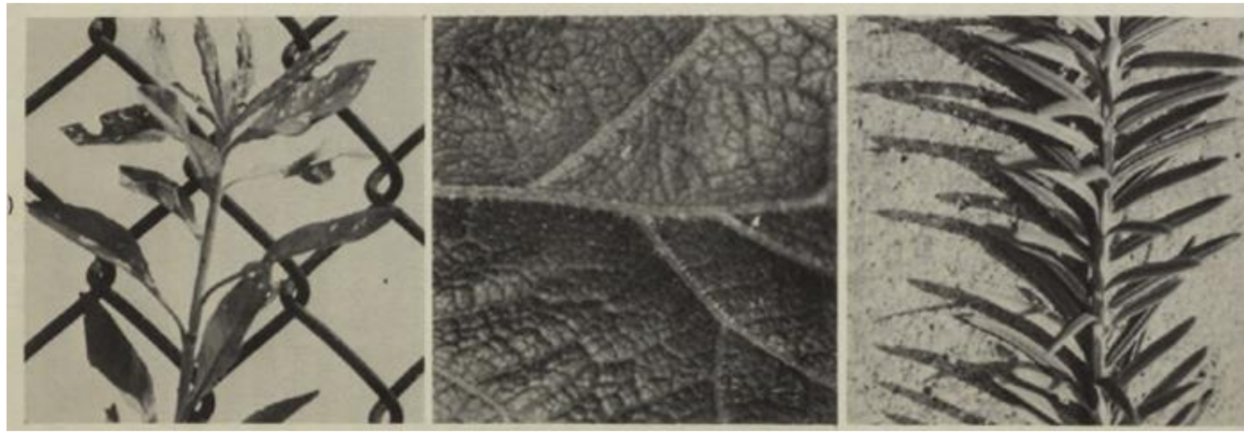
Marr-Hildreth edge detection (1979)

$I(x, y)$

$\nabla^2 G(x, y, \sigma) * I(x, y)$

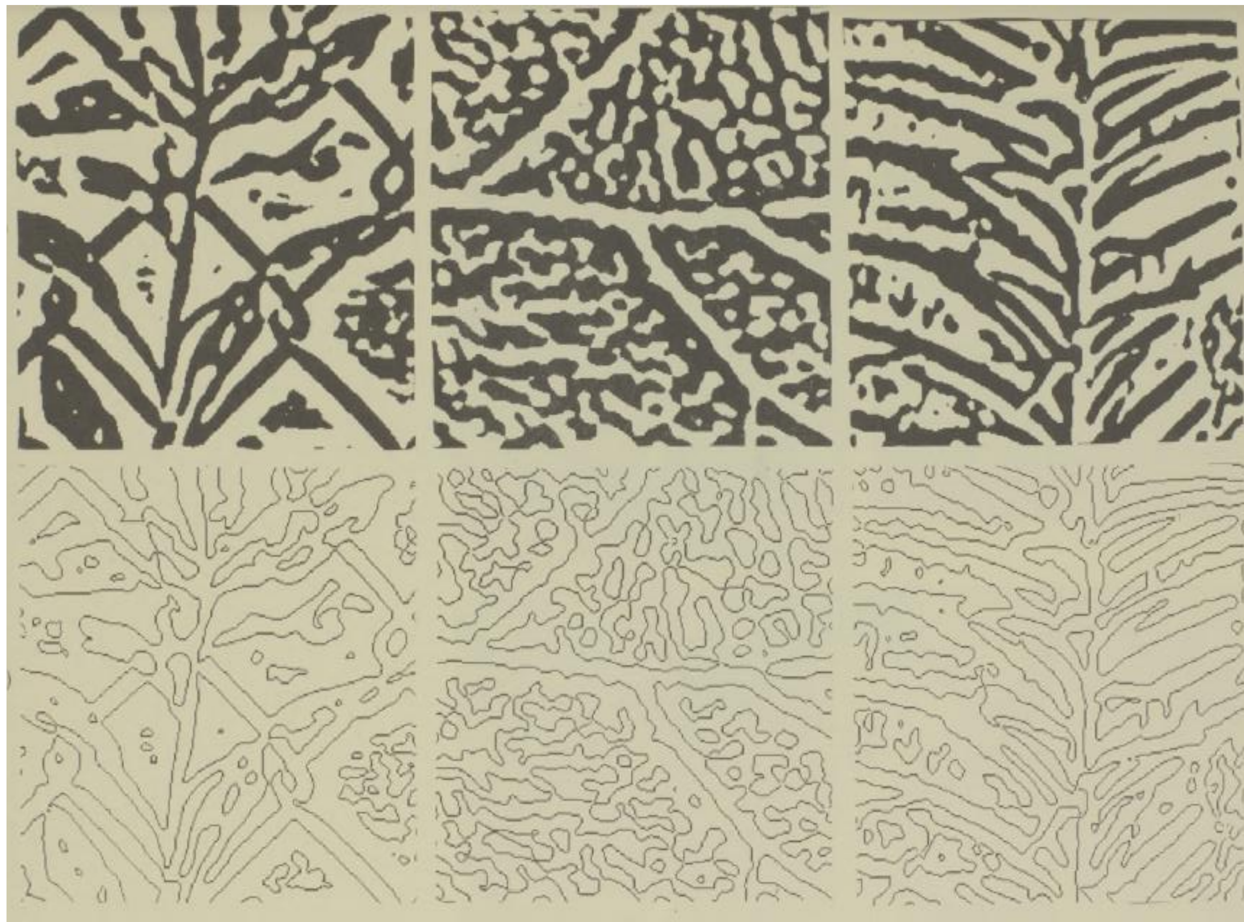


$I(x, y)$



$\nabla^2 G(x, y, \sigma) * I(x, y)$

Positive values white
& negative values black



$\nabla^2 G(x, y, \sigma) * I(x, y)$

Zero crossings

HEADS UP: The locations & number of zero crossing vary with σ.

Canny edge detection

Exercise:

What are the three steps?

Hint:

1. Compute gradient magnitude
2. ?
3. ?

Canny edge detection (discuss?)

1. Compute gradient magnitude

2. Thinning (non-maxima suppression)

3. Hysteresis* (linking edges based on lower threshold)

**"Hysteresis" (definition):   the phenomenon in which the value of a physical property lags behind changes in the effect causing it, as for instance when magnetic induction lags behind the magnetizing force.*