Questions

1. Consider the following statement. “Let $E_1(\lambda)$ and $E_2(\lambda)$ be any two color spectra. If a color blind person who is missing the L cones can distinguish these two spectra, then a normal color vision person can distinguish these spectra.”

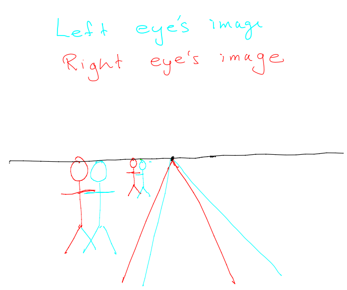
   Is the statement true or false? Justify your answer.

2. Suppose we have two different monitors with RGB emission spectra $\mathbf{P}_1$ and $\mathbf{P}_2$ respectively. (The matrices $\mathbf{P}$ are $N \times 3$ as sketched in the lecture.) These two monitors could be made by different companies, for example.

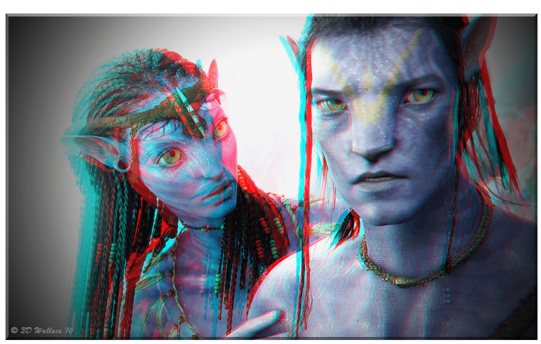
   Suppose that we want corresponding pixels on the two monitors to be a color match, so that when we place the two monitors side-by-side, the images appear identical.

   What mapping we can perform on the RGB values of the image displayed on the second monitor so that we get a color match to the first monitor? Hint: it is a 3×3 matrix.

3. (a) In the lecture slides, I showed the sketch below and said that the anaglyph glasses should have a red filter on the left eye and a cyan filter on the right eye. However, the sketch below seems to indicate the opposite: it shows the words ”left eye’s image” in cyan and ”right eye’s image” in red. If the left eye’s image is the one shown in cyan color, then shouldn’t we put the cyan filter on the left eye?



   (b) In the image below, the cyan now appears to be to the left of the red – which is the opposite of the above. Why?



4. The company Sharp produces a television called the Quattron that has four different types of color emitters (RGBY) at each pixel, instead of the standard three (RGB). The extra color Y stands for yellow. Sharp claims it "delivers color never seen before on TV".

   Give your best technical interpretation of what Sharp means by this claim. Your answer should be in terms of how light is emitted by the display and measured by the eye.

5. In the 1850's, James Clerk Maxwell proposed a method for helping color blind people to discriminate red from green, using colored glasses that have one glass tinted red and the other tinted green. Maxwell claimed that although the color blind person would not acquire our experience of the colors red and green using such glasses, they would at least be able to decide which objects are red and which are green.

   Explain how this ability would be possible, using the fact that pigments in a tinted lens selectively absorb and transmit different amounts of light at different wavelengths.

6. Suppose you are shopping for clothes and you try on a shirt and pants and you find in the store that the colors go well together. But then when you bring the clothes home and try them on, you find that the colors no longer go well together. How can this happen? To simplify the question, suppose you pick a shirt and pants that appear to have exactly the same color when viewed in the store. (In general, even I am aware that one should not wear a shirt and pants of exactly the same color.)

7. Many people with color blindness are simply missing one of the three cone types. However, there are many other people that have all three cone types but one cone type is defective ("anomolous"), namely its absorption spectrum has more overlap than usual with one of the other two cone types. Recently the company EnChroma began selling glasses for improving the color vision of color anomalous people. The glasses have lenses that filter out (remove) the center of the overlap regions between the three cone spectra.

   (a) Why do color *anomolous* people have limited color vision, relative to those with normal color vision, but still have better color vision than color blind people?

   (b) Why do the EnChroma lenses indeed improve the color vision of color anomalous people?

   Here are a few references to check out.

   `http://enchroma.com/technology/`

   `http://www.nytimes.com/2015/08/16/business/enchromas-accidental-spectacles-find-niche-among- html?_r=1`

Answers

1. The statement is true. If a person is missing say the L cones, then the color sensitivity matrix $\mathbf{C}$ has only two rows instead of three. In this case, two spectra $E_1(\lambda)$ and $E_2(\lambda)$ are indistinguishable (metameric) for this person if and only if

$$\begin{bmatrix} C_M \\ C_S \end{bmatrix}_{2\times N} E_1(\lambda) \;=\; \begin{bmatrix} C_M \\ C_S \end{bmatrix}_{2\times N} E_2(\lambda)$$

If a person missing the L cones can discriminate the two, then the pairs are not the same. In this case, a person with normal color vision can also discriminate the two spectra since

$$\begin{bmatrix} C_L \\ C_M \\ C_S \end{bmatrix}_{3\times N} E_1(\lambda) \;\neq\; \begin{bmatrix} C_L \\ C_M \\ C_S \end{bmatrix}_{3\times N} E_2(\lambda)$$

and so a person with all three types of cones can discriminate the two spectra also.

Notice that the converse does not hold. There may be spectrum pairs that a person with all three types of cones can discriminate, but a person with only two cone systems cannot discriminate. (And that is what we mean by color blindness.)

2. Let $\mathbf{s}_1$ be the RGB values of the original image at some pixel. Let $\mathbf{s_2}$ be the RGB values that we need to use at a pixel on the second monitor so that we get a color match to the corresponding pixel on the first monitor. We want

$$\mathbf{C}\;\mathbf{P}_1\;\mathbf{s}_1 \;=\; \mathbf{C}\;\mathbf{P}_2\;\mathbf{s_2}$$

Since $\mathbf{CP}_1$ and $\mathbf{CP}_2$ are 3×3 matrices, we require

$$(\mathbf{C}\;\mathbf{P_2})^{-1}\;(\mathbf{C}\;\mathbf{P_1})\;\mathbf{s}_1 \;=\; \mathbf{s_2}$$

Notice that this equation does not guarantee that for any $\mathbf{s}_1$, the three corresponding $\mathbf{s_2}$ will be positive numbers. Indeed, if the $\mathbf{s_2}$ are not all positive numbers for some $\mathbf{s}$, then we cannot produce a match for that pixel.

3. (a) Assume for simplicity that the red filter passes the display's R channel but completely blocks (absorbs) the G and B channels. This would make sense: the reason the red filter looks red is that it lets long wavelength light through and blocks the medium and short wavelengths. Similarly, assume the cyan filter passes the G and B channels but blocks the R channel.

The red stickmen in the figure appear red in the sketch because they are coded with RGB values of $(255, 0, 0)$. The background is white, which means the background is coded with value $(255, 255, 255)$. If the red filter is in front of the left eye, then the red stick man will *not* be visible in the left eye, because both it and the background will pass the same value through the red filter namely (255,0,0).

The cyan stickman, however, has RGB values $(0, 255, 255)$. When it passes through the red filter, it appears the same as $(0, 0, 0)$ i.e. black since the red filter doesn't let B and

G channels through. Thus, the left eye with the red filter sees the cyan stickman, which appears as a black stickman on a red background.

The same argument applies to the right eye looking through the cyan filter. The stickmen appear as black lines on a cyan background.

(b) In the image from Avatar, the R and GB images are aligned on the foreground face on the right. We know that because there are no red/cyan colors visible in the face. So if the eye's converge on the display plane (the screen) then the disparity will be zero for these points.

Next, consider the hair of that avatar. Its depth is slightly beyond the face of the avatar. The hair should be shifted left in the left (eye's) image, relative to its position in the right (eye's) image.[1]

The left eye sees the image that passes through the red filter. Note that the left eye will see the dark hair at the location where the unfiltered image looks cyan, since the cyan does not pass through the red filter and hence the image looks dark there. Similarly, the red part in the unfiltered image will pass through the red filter without attenuation, and will appear the same as the background (white). I told you this was subtle!

4. A display with RGBY has a P matrix which is N x 4 and thus produces a 4D vector space of emitted light spectra, instead of 3D space.

   Now the issue is whether RGBY "delivers more colors" than RGB. The answer is YES. Why ? Its subtle. The cone space is 3D and so you might think that an RGB display is the best you could do. However, this is not true. For example, if the column of P were linearly independent but the columns were very similar to each other, then you would only be able to reach LMS values that were very similar to each other. The subtlety (in terms of linear algegra) is that you are limited because you can multiply the P columns by positive weights only. Having a 4D space allows you (potentially) to reach more points in the LMS space with positive weights on the P columns.

5. The idea here is that the light reflected from a red object will pass more of its energy through the red lens than through the green lens, and vice versa for a green object. Thus, if the red and green objects look about the same to the colorblind observer without the lens, then the red object will look brighter through the red lens than through the green lens (and vice-versa for green objects). Maxwell's idea was that a colorblind person could *discriminate* red vs. green objects on this basis, i.e. objects that appeared brighter in the left (right) eye would be reddish (greenish), respectively. The red object wouldn't appear red, but at least it would appear different than a green object – so the observer could see that there are two different colors present.

6. The spectrum of light reflected from an object is the *product* of two spectra: the illumination spectrum and the reflectance spectrum. If the two reflected spectra are metameric under the lighting used in the store, there is no guarentee they will be metameric under the lighting that you have at home since the illuminance spectra could be quite different.

[1] To understand this, raise your figure in front of your face, converge your eyes on your finger, and then alternately close your left and right eye. Any object that is beyond your finger will jump to the left when you close your right eye and will jump to the right when you close your left eye.

7. (a) If two cone absorption spectra have a large overlap, then the responses of these two cone types tend to be very similar, since the center of the overlap region will contribute the same component response to each cone type. (Technically, two of the rows of the **C** matrix will be similar, so the matrix will be near rank 2 rather than 3). In particular, the differences in cone response types will tend to be small, relative to the total response of either cone type. Such small differences will sometimes be difficult to detect.

   (b) If the central overlap regions between L/M and M/S cones are filtered away, then the spectra that pass through the lenses will be isolated in three non-overlapping wavelength (or frequency) bands. The chromaticities of the light passing through these bands now will be well separated. Thus if two light spectra have different luminance or chromaticity, then this would produce a greater difference in response of two neighboring cone types (relative to their mean response, e.g. $(L - M) : (L + M)$) than it would if the lenses were not there.