lecture 12

Scale space

Scale

Canny edge detection

$I(x) * f(x)$ vs. $I(x) * f_s(x)$

Edge / Corner detection & Registration

$$\sum_{(x,y) \in Ngd(x_0,y_0)} \left(I(x+h_x, y+h_y) - J(x,y) \right)^2$$

Blurred Images $I(x) * G(x,\sigma)$

368 x 550 pixels

$\sigma = 2$

$\sigma = 4$

$\sigma = 8$

ASIDE

There are formal relationships between blurring and subsampling. (This topic is omitted because of time constraints.)

KEYWORDS: • image pyramids • multiresolution • wavelets

Notation

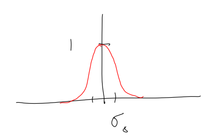
$$f_s(x) \equiv f\left(\frac{x}{s}\right)$$

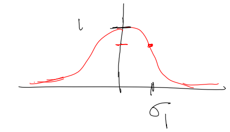
I will change the notes from lecture 9 (Canny) where we used $f_s(x) = f(sx)$

$$g(x) = e^{-\frac{x^2}{2}}$$

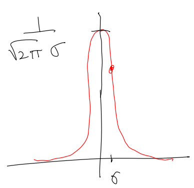
$$g_\sigma(x) = g\left(\frac{x}{\sigma}\right) = e^{-\frac{x^2}{2\sigma^2}}$$

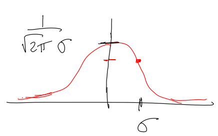
$$g_\sigma(\sigma) = \frac{1}{\sqrt{e}}$$





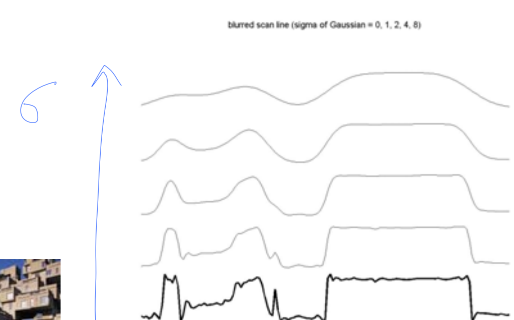
$$G(x;\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot g_\sigma(x)$$



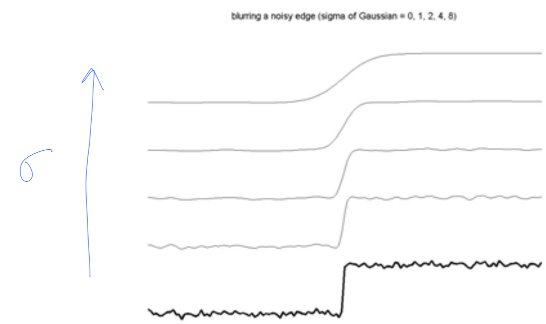


Gaussian Scale Space

$$I(x,\sigma) = I(x) * G(x,\sigma)$$

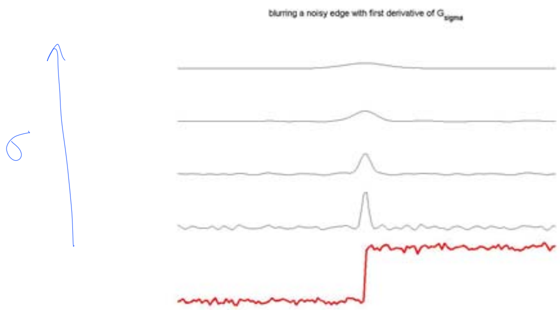


$$I(x,\sigma) = I(x) * G(x,\sigma)$$

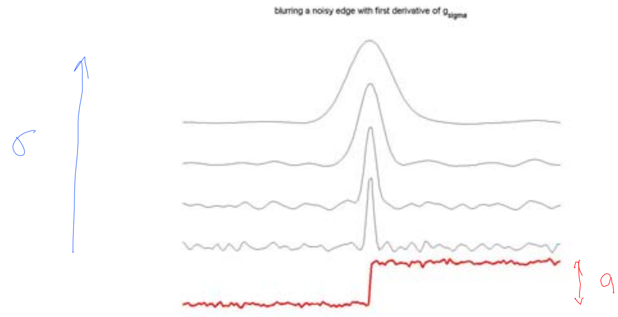


$$I(x) = u(x - x_0) + n(x)$$

$$I(x) * \frac{d}{dx} G(x,\sigma)$$



$$I(x) * \frac{d}{dx} g_\sigma(x)$$



$$I(x) * \frac{d}{dx} g_\sigma(x)$$

$$= a\, u(x-x_0) * \frac{d}{dx} g_\sigma(x)$$

$$= a\, \delta(x-x_0) * g_\sigma(x)$$

$$= a\, g_\sigma(x-x_0)$$

Response at $x = x_0$ is a (independent of σ).

Edge detection

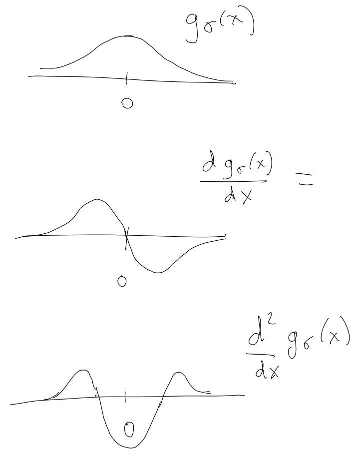
Look for peaks of

$$I(x) * \frac{d}{dx} g_\sigma(x)$$

$\therefore$ Look for zero crossings of

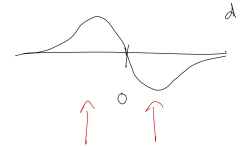
$$\frac{d}{dx}\; I(x) * \frac{d}{dx} g_\sigma(x)$$

$$= I(x) * \frac{d^2}{dx^2} g_\sigma(x)$$



$$\frac{d\, g_\sigma(x)}{dx} = u(x) * \frac{d^2 g_\sigma(x)}{dx^2}$$

$$\frac{d\, g_\sigma(x)}{dx} = u(x) * \frac{d^2 g_\sigma(x)}{dx^2}$$



where do the peaks occur ?

what are the heights of the peaks ?

Where do peaks of $u(x-x_0) * \frac{d^2}{dx^2} g_\sigma(x)$ occur ?

$$u(x-x_0) * \frac{d^2}{dx^2} g_\sigma(x)$$

$$= \delta(x-x_0) * \frac{d\, g_\sigma(x)}{dx}$$

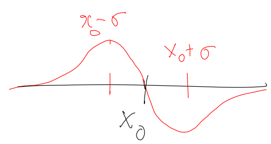
$$= \frac{d}{dx} g_\sigma(x-x_0)$$

$$= \frac{d}{dx} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

$$= -\frac{(x-x_0)}{\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

take derivative and set to 0.

Where do the peaks occur ?



$$\frac{d}{dx}\left(\frac{d}{dx} g_\sigma(x-x_0)\right)$$

$$= -\frac{d}{dx}\left(\left(\frac{x-x_0}{\sigma^2}\right) e^{-\frac{(x-x_0)^2}{2\sigma^2}}\right)$$

$$= \frac{1}{\sigma^2}\left(1 - \frac{(x-x_0)^2}{\sigma^2}\right) e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

$$= 0 \quad \text{when} \quad x = x_0 \pm \sigma$$

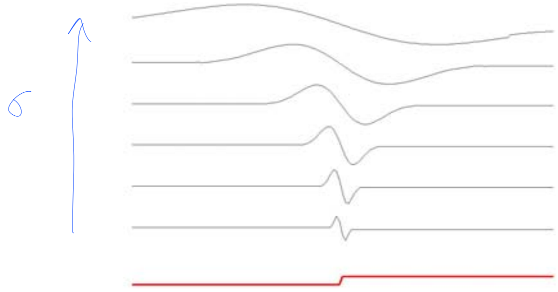
Height of peak of $I(x) * \frac{d^2 g_\sigma(x)}{dx^2}$?

$$-\frac{(x-x_0)}{\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \Bigg|_{x = x_0 \pm \sigma} = \pm \frac{1}{\sigma} e^{-\frac{1}{2}}$$

If we use $\sigma \frac{d^2 g_\sigma(x)}{dx^2}$ then height of peak of $u(x-x_0) * \sigma \frac{d g_\sigma^2(x)}{dx}$ will be $e^{-\frac{1}{2}}$ (independent of σ).

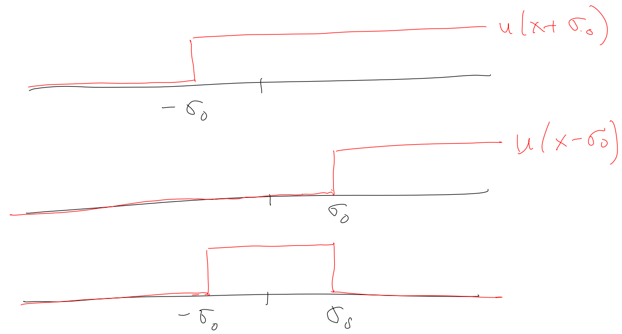
$$u(x-x_0) * \sigma \frac{d^2}{dx^2} g_\sigma(x)$$

blurring a step edge with sigma* $d^2 g_{sigma} / dx^2$

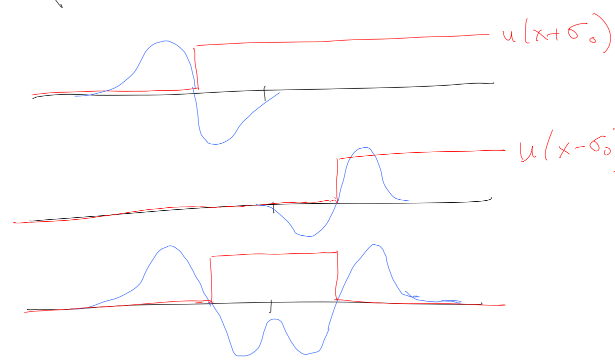


Box function

$$I(x) = u(x+\sigma_0) - u(x-\sigma_0)$$

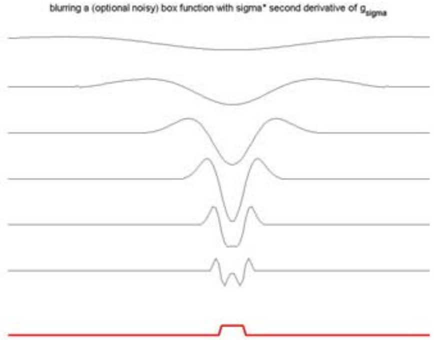


$$\left(u(x+\sigma_0) - u(x-\sigma_0)\right) * \sigma \frac{d^2 g_\sigma(x)}{dx^2}$$

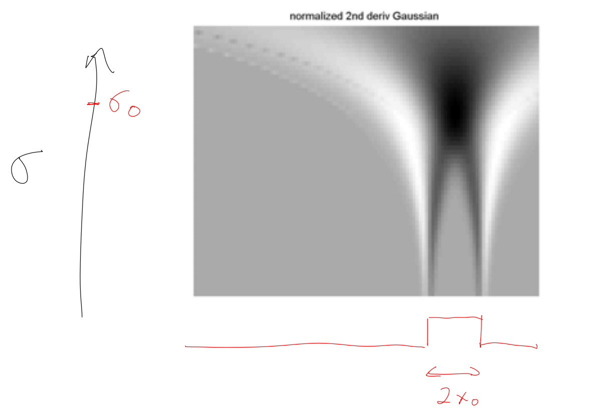


$$\left(u(x+\sigma_0) - u(x-\sigma_0)\right) * \sigma \frac{d^2 g_\sigma(x)}{dx^2}$$

blurring a (optional noisy) box function with sigma* second derivative of g_{sigma}

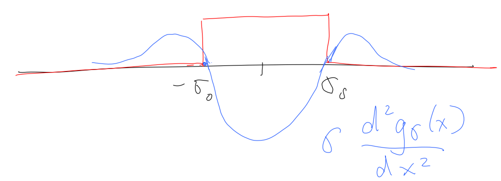


$$\left(u(x+\sigma_0) - u(x-\sigma_0)\right) * \sigma \frac{d^2 g_\sigma(x)}{dx^2}$$



Box function

$$I(x) = u(x+\sigma_0) - u(x-\sigma_0)$$



Minimum occurs when zeros of shifted $\sigma \frac{d^2 g_\sigma(x)}{dx^2}$ align with edges.

Summary

$I(x) * G(x, \sigma)$ ~ blur an image

$I(x) * \frac{d}{dx} g_\sigma(x)$ ~ edge detection

$I(x) * \sigma \frac{d^2}{dx^2} g_\sigma(x)$ ~ box detection