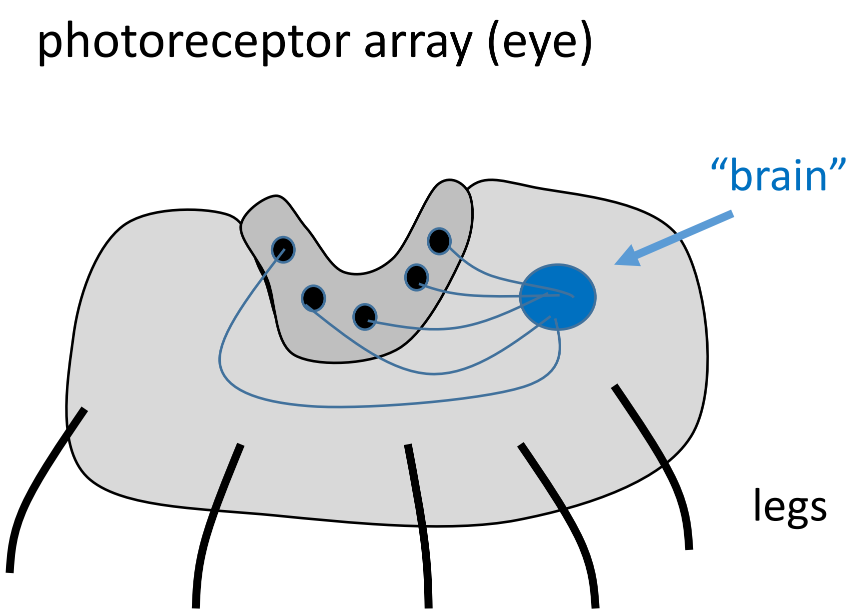
COMP 546

Lecture 1

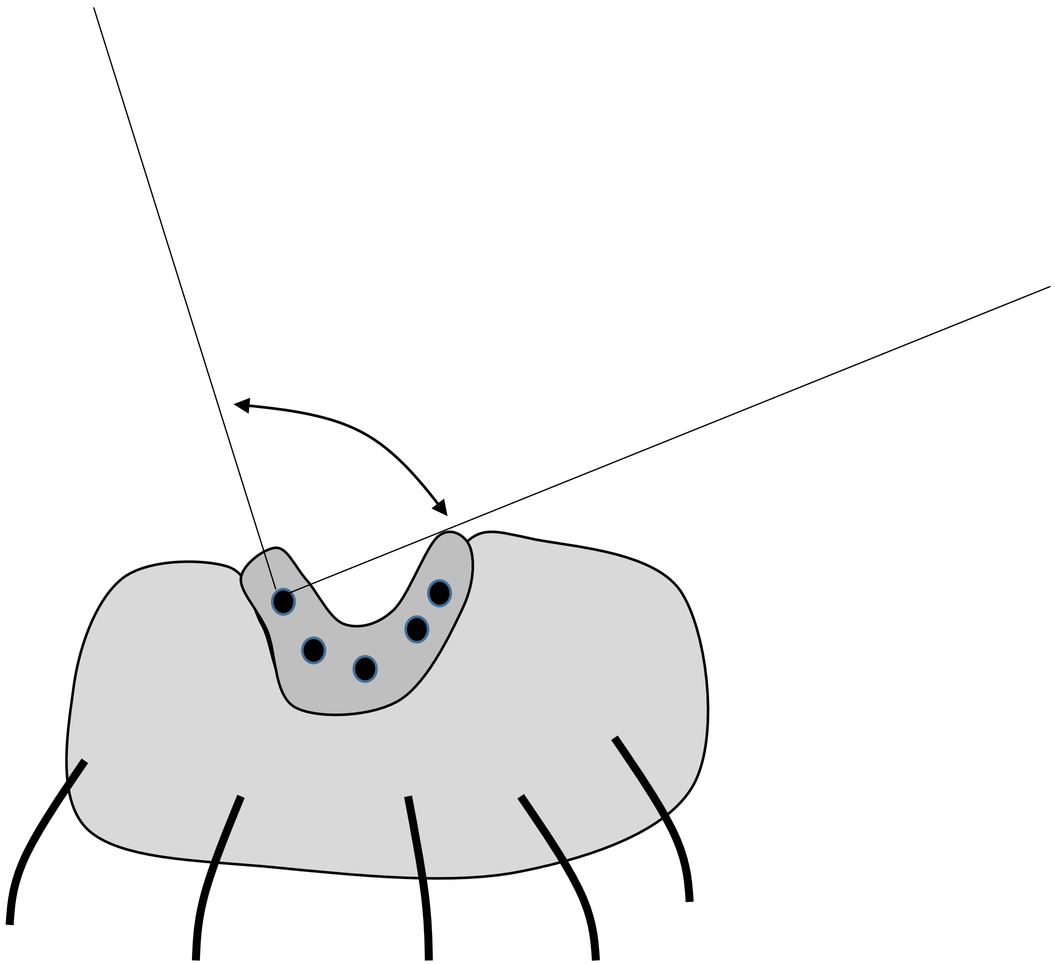
Image Formation: Geometry

Thurs. Jan. 11, 2018

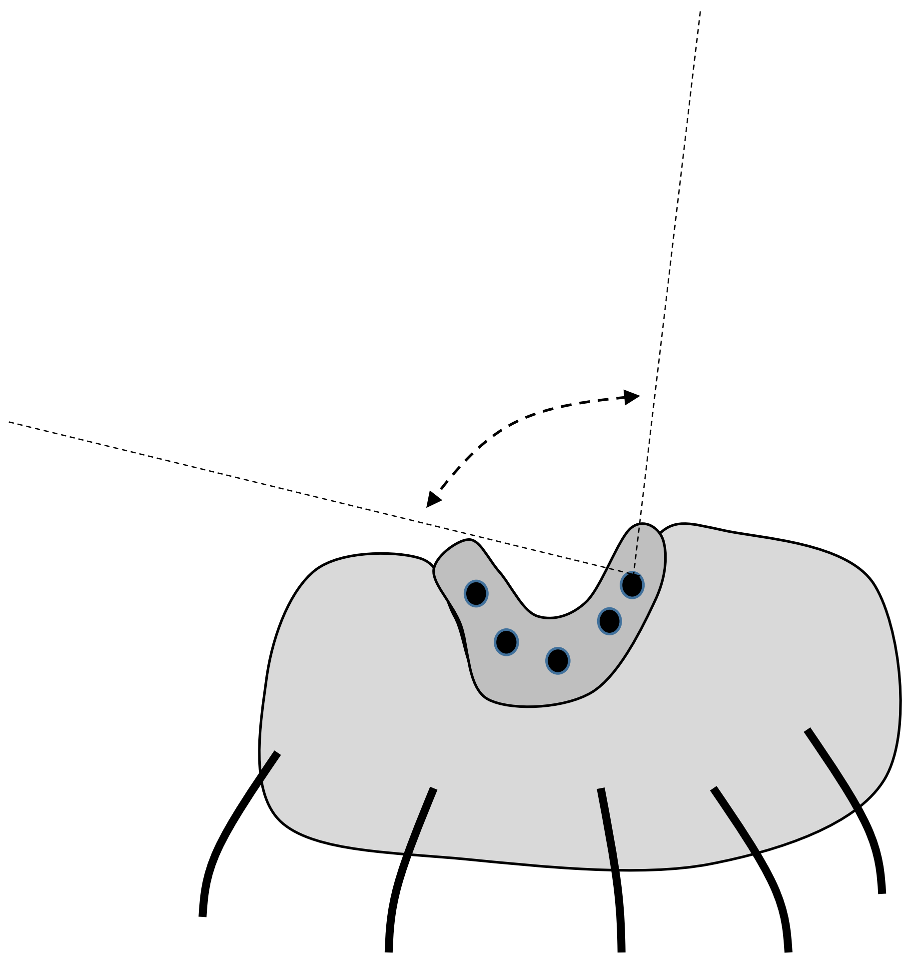
Origins of spatial vision (500 million years ago?)



Origins of spatial vision

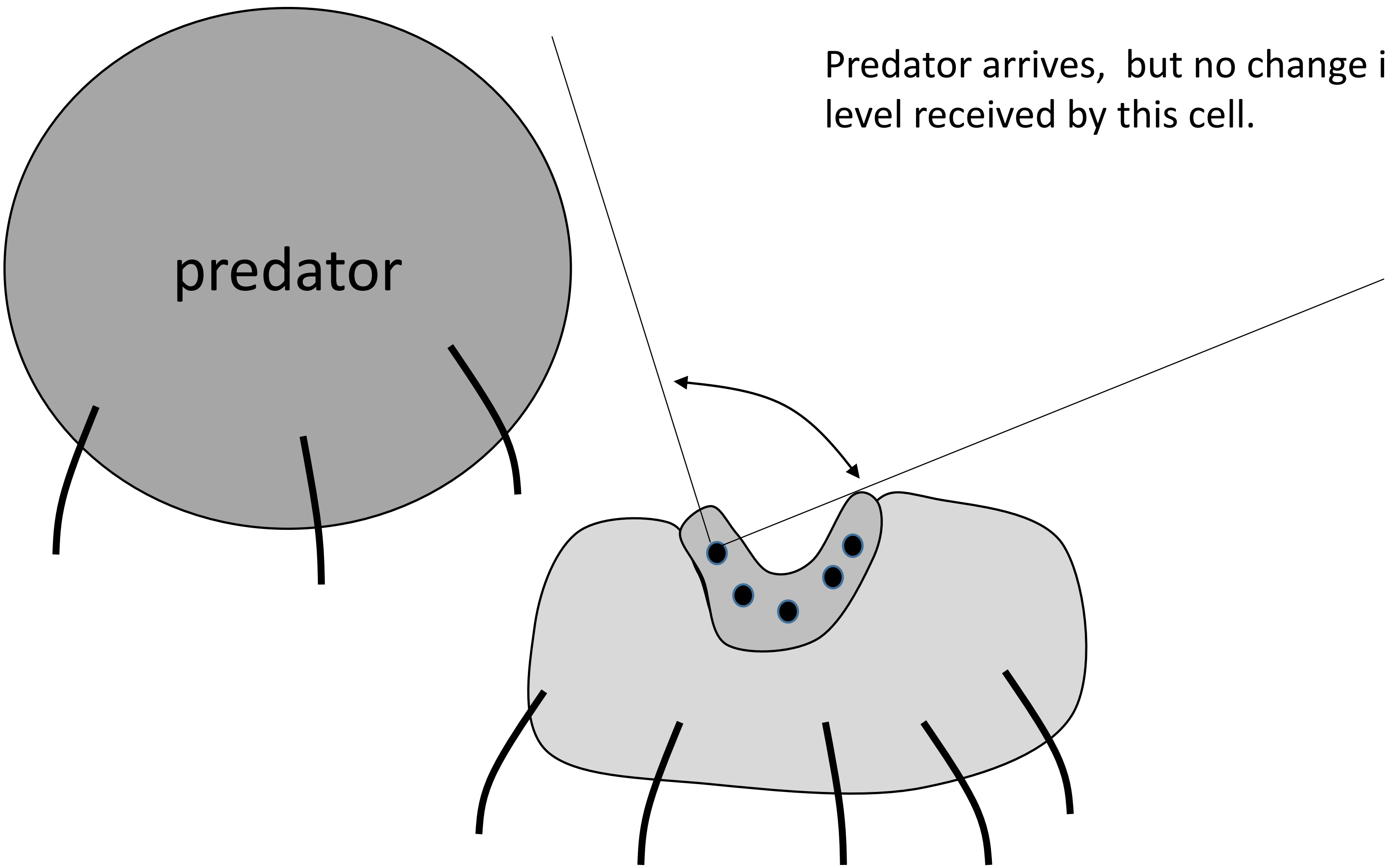


Origins of spatial vision

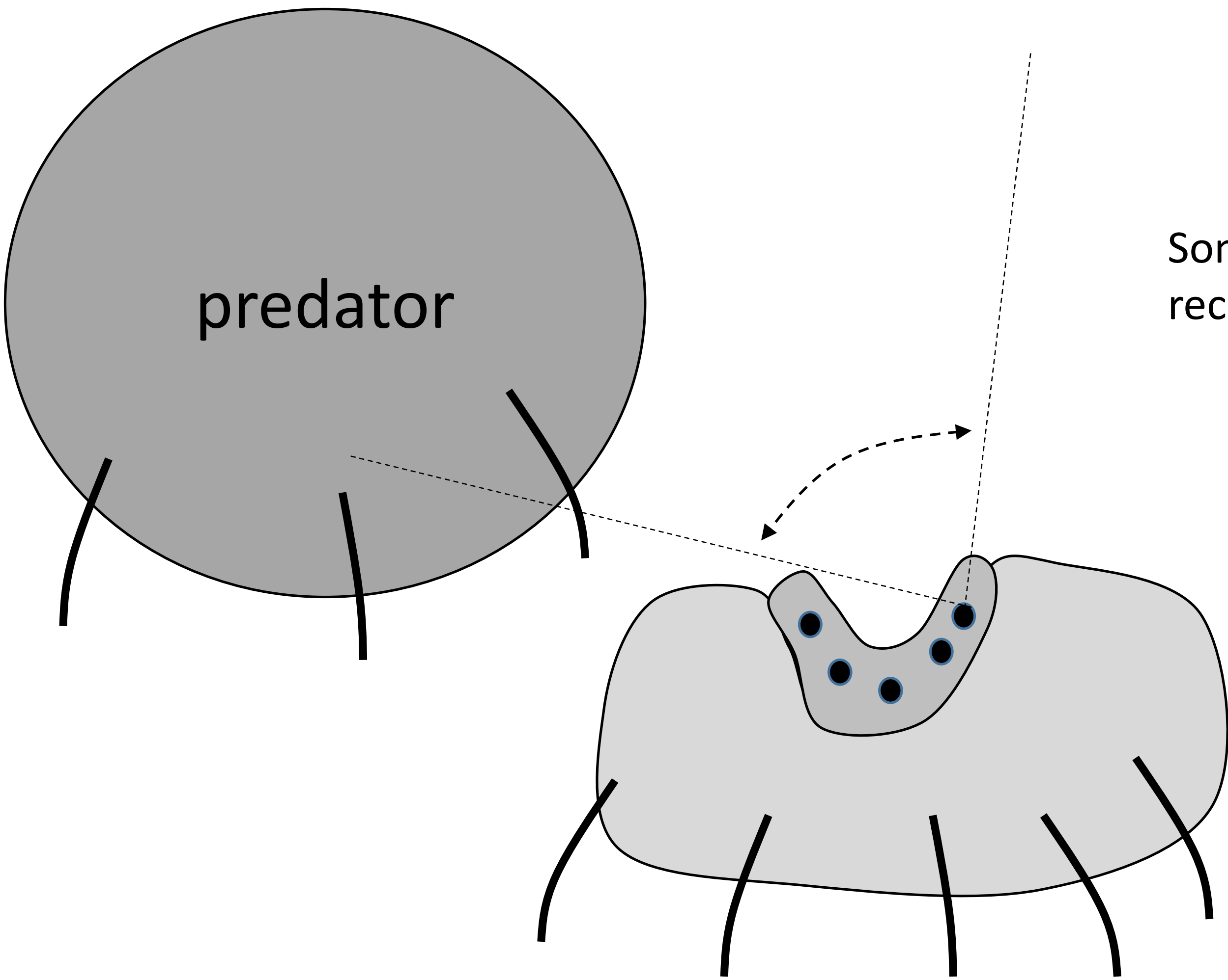


Origins of spatial vision

Predator arrives, but no change in light level received by this cell.

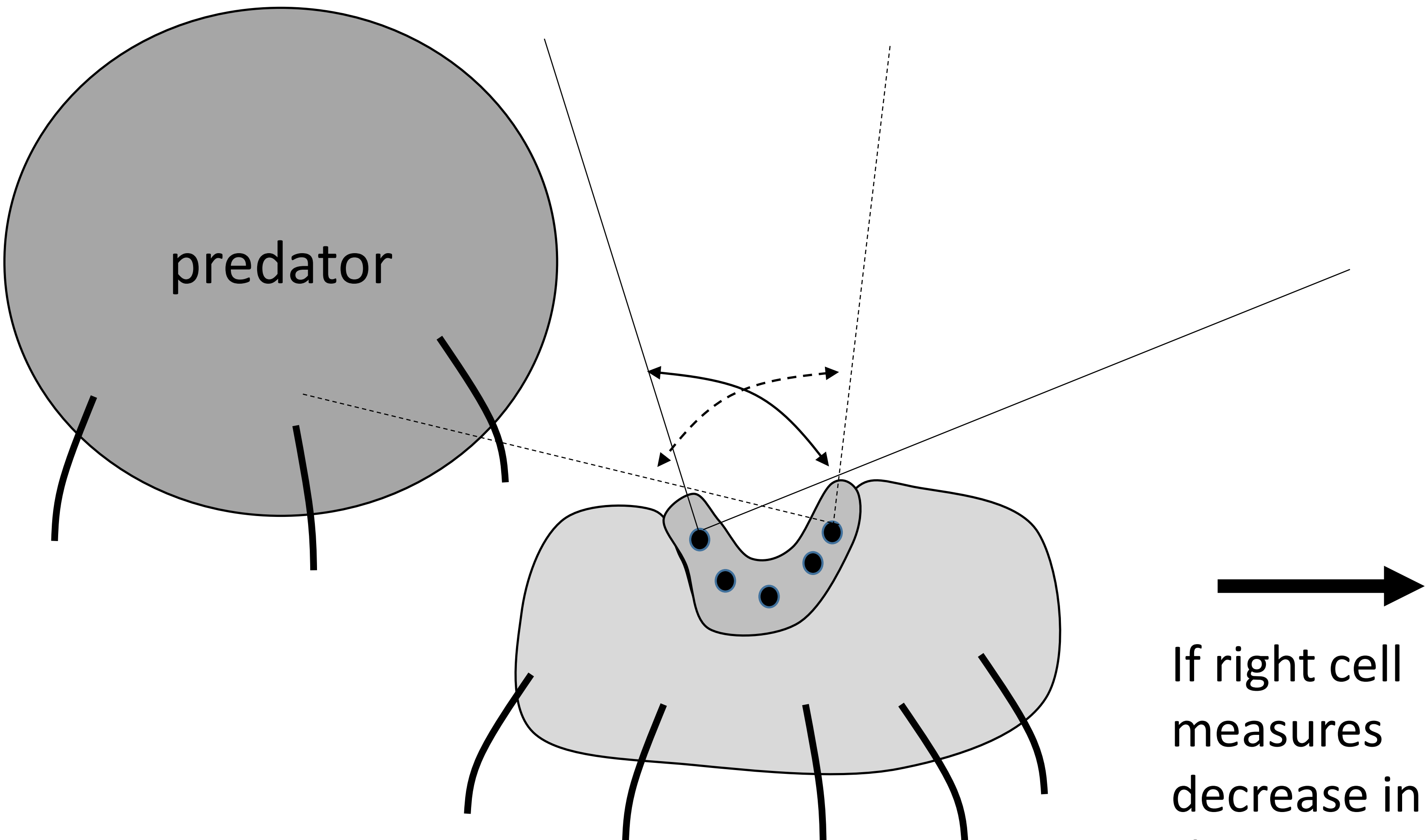


Origins of spatial vision



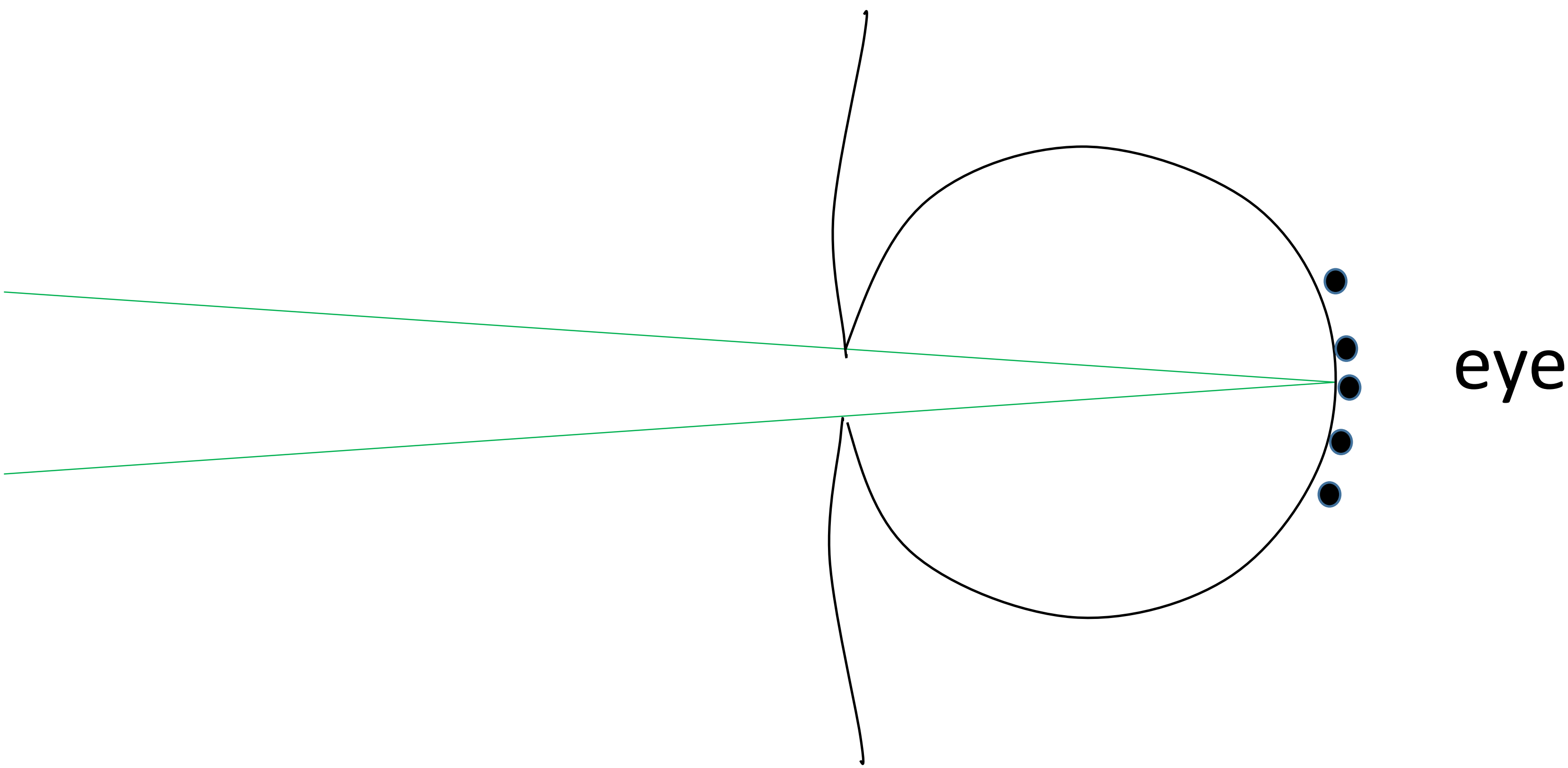
Some change in light level received by this cell.

Origins of spatial vision

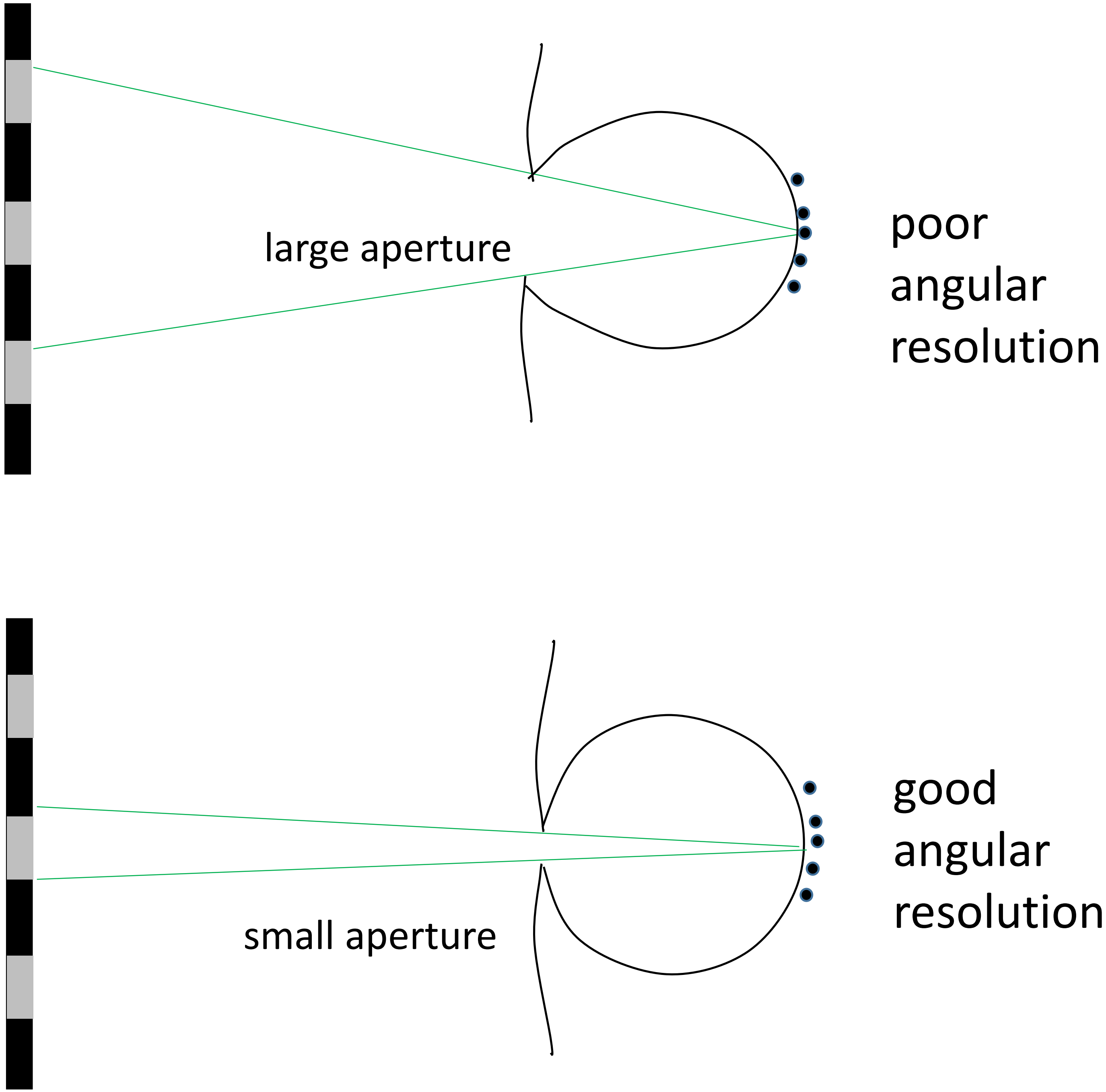


If right cell measures decrease in light, then move right.

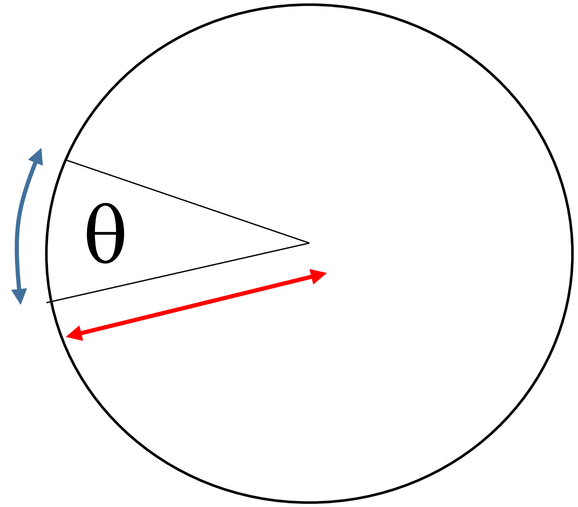
Evolution of eyes



As pit becomes more concave, angular resolution improves
(but amount of light decreases)

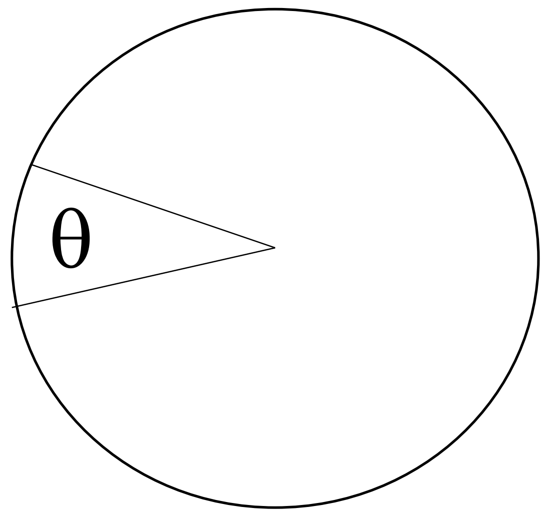

large aperture
poor angular resolution
small aperture
good angular resolution

Radians



$$\theta \ \ radians \ = \ \frac{arclength \ on \ circle}{radius \ of \ circle}$$

Radians vs. degrees

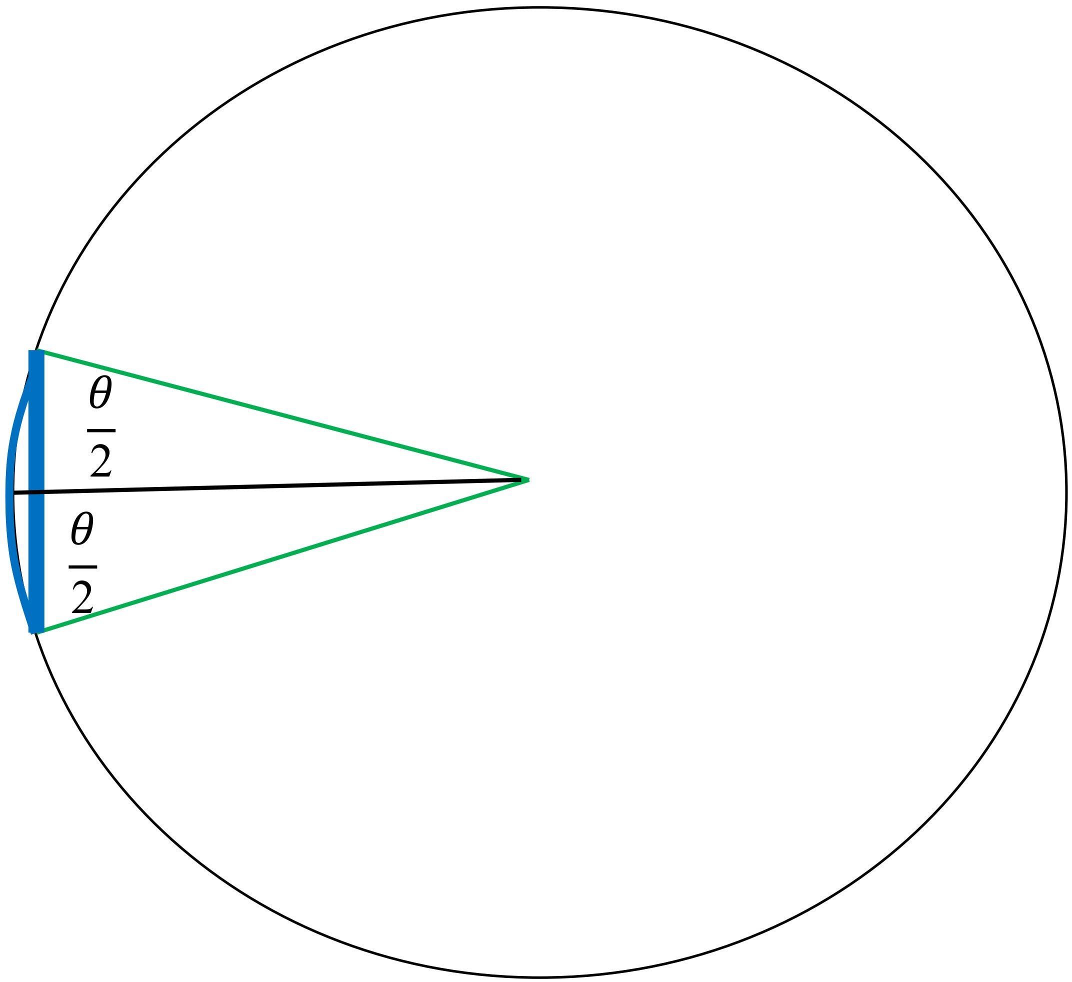


$$\theta\ \cancel{radians}\ *\ \frac{180\ degrees}{\pi\ \cancel{radians}} = \theta * \frac{180}{\pi}\ degrees$$

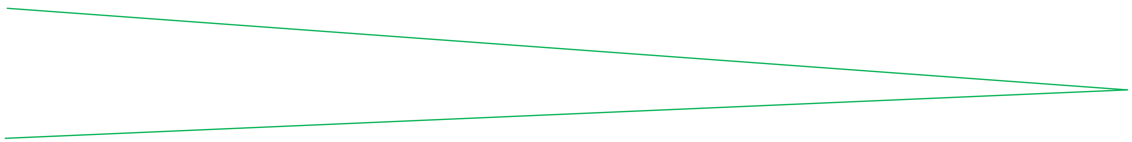
$$1\ radian \approx 57\ deg$$

Small angle approximation

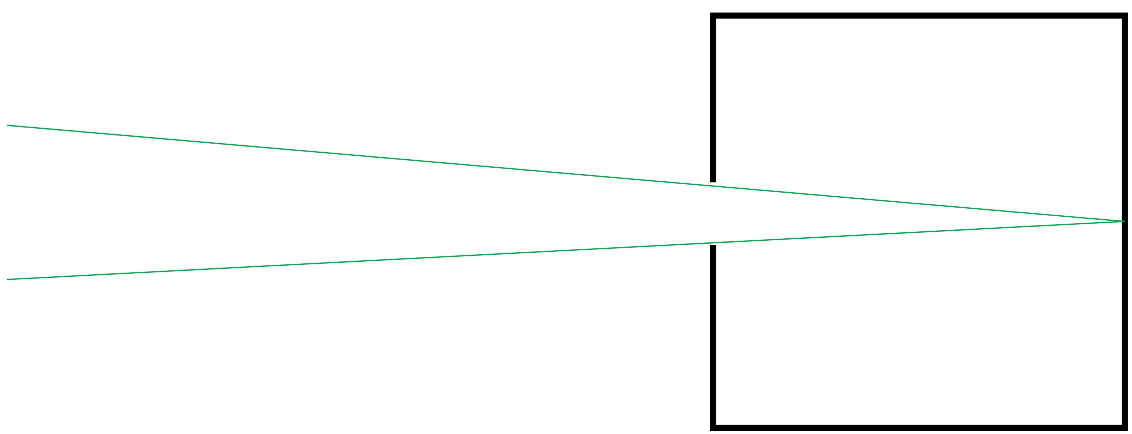
$$\theta \approx 2\,tan\frac{\theta}{2}$$



Aperture angle from a few slides ago….

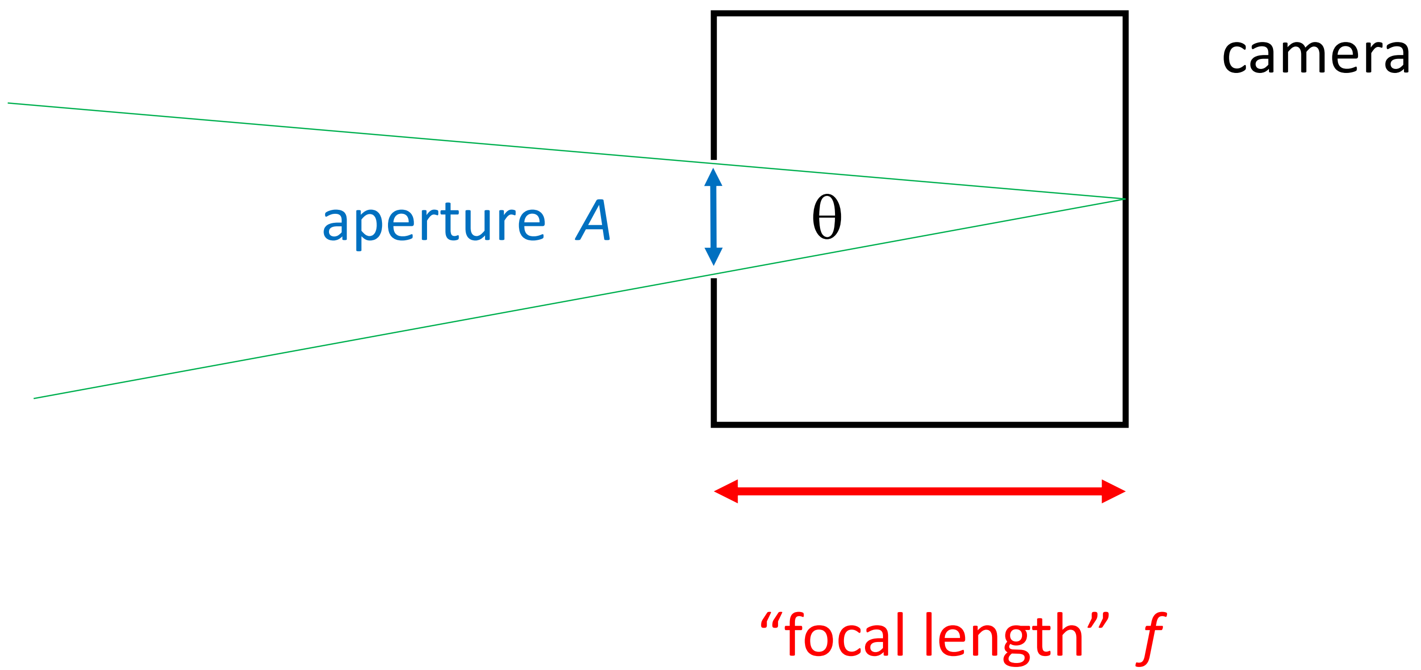


eye



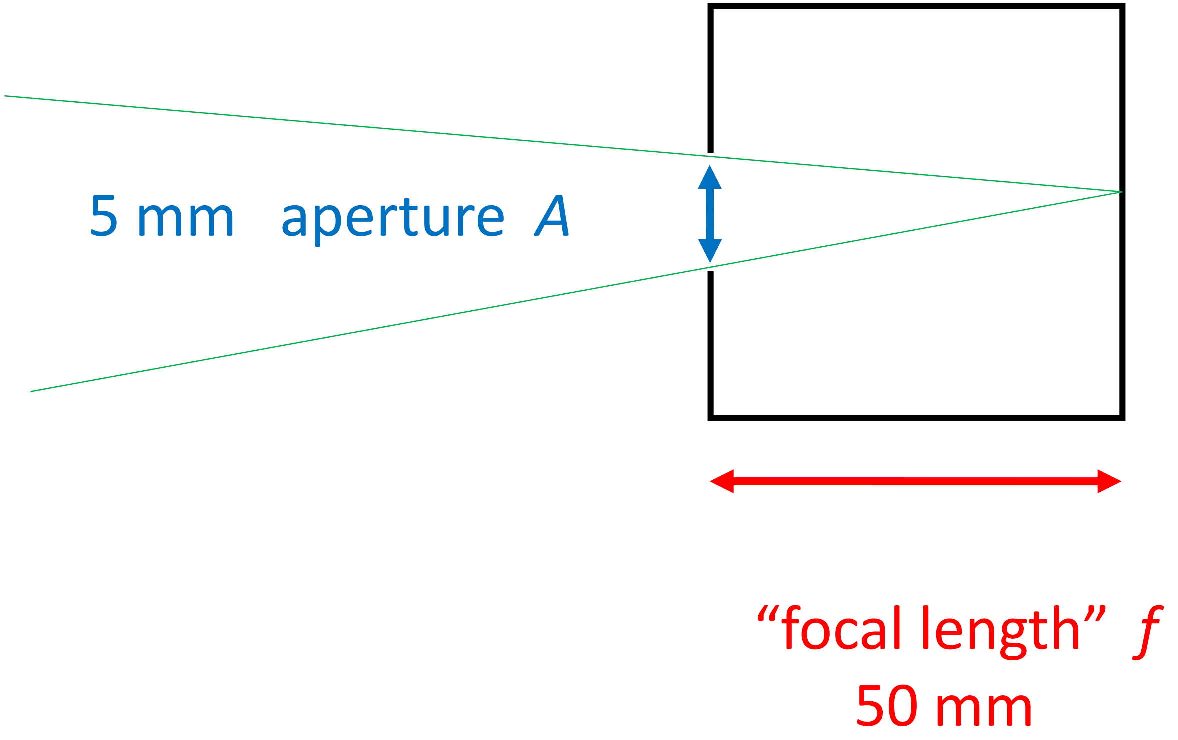
camera

“F number“ (photography)



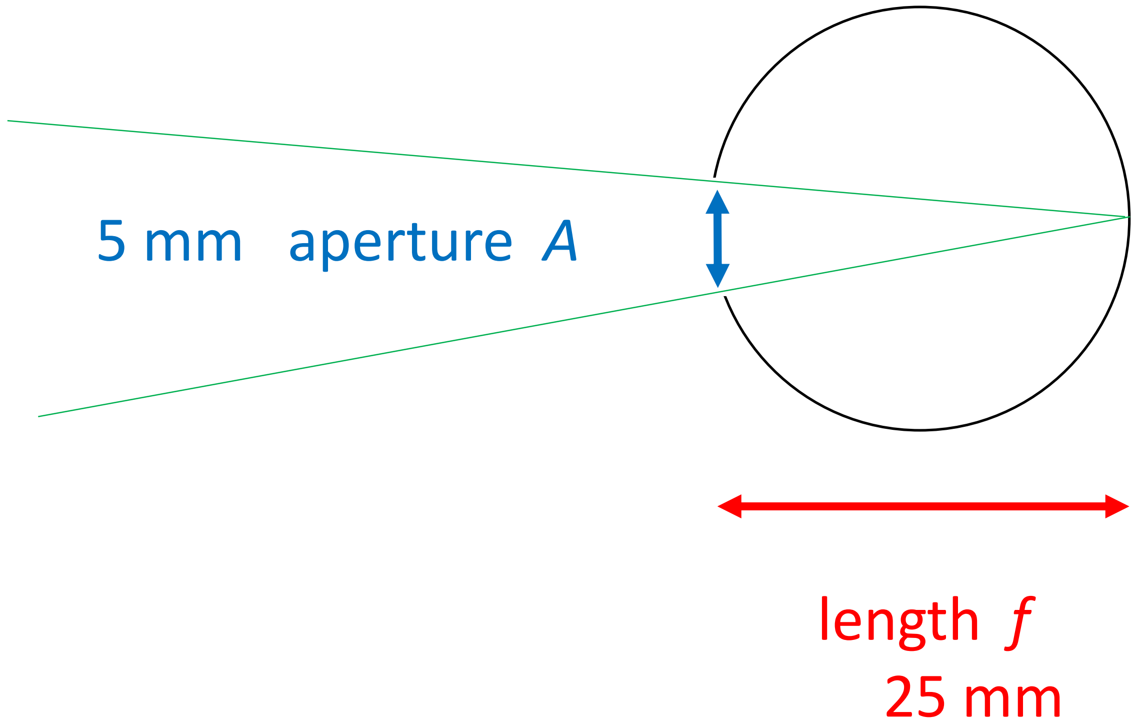
$$F\ number \equiv \frac{f}{A} \approx \frac{1}{\theta}$$

ASIDE: camera



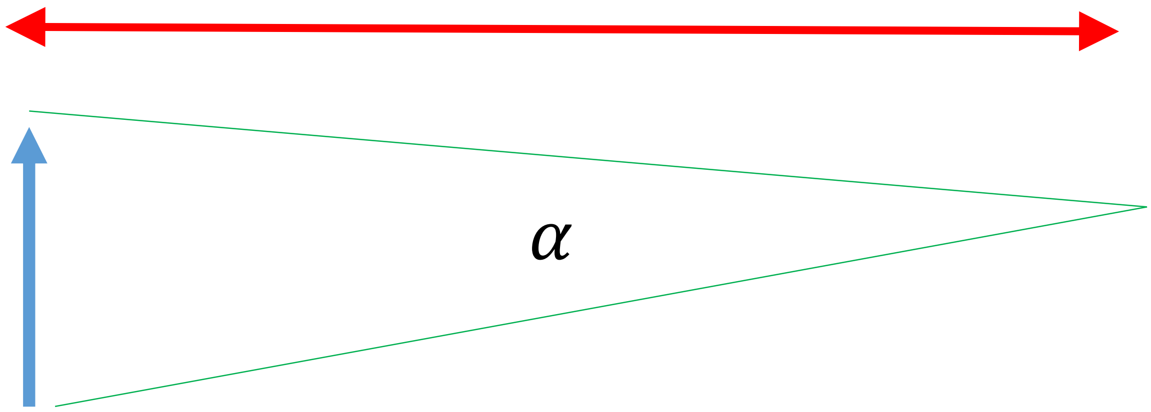
$$F\ number \equiv \frac{f}{A} = \frac{50}{5} = 10$$

eye (ignore lens)



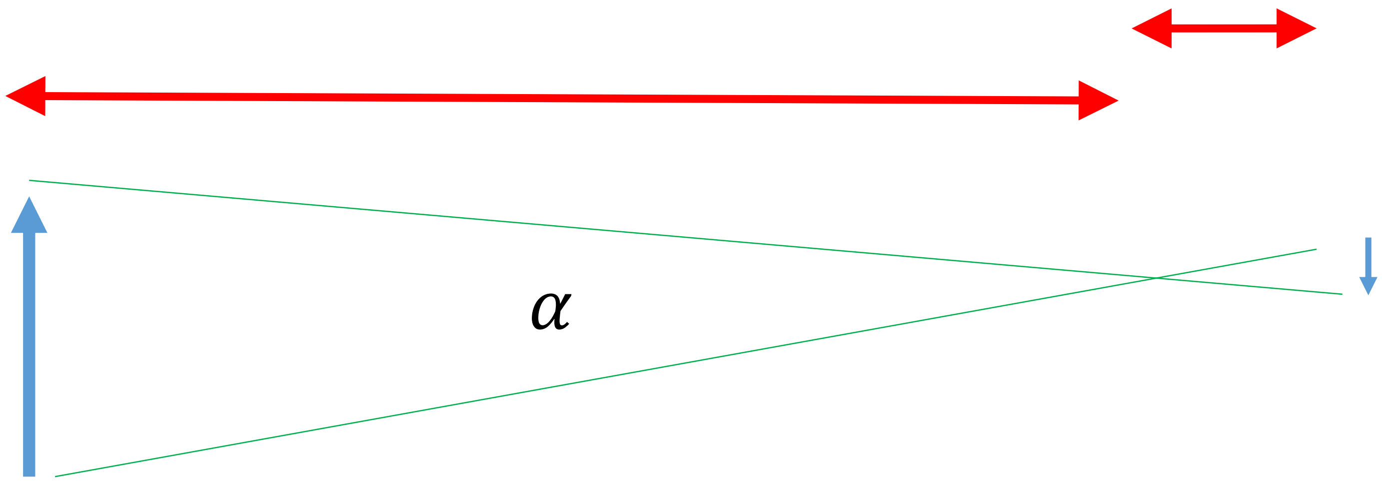
$$F\ number \equiv \frac{f}{A} = \frac{25}{5} = 5$$

Visual Angle



$$\alpha \approx \frac{object\ height}{distance}$$

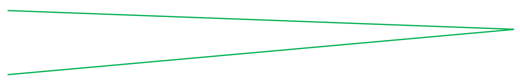
Visual Angle



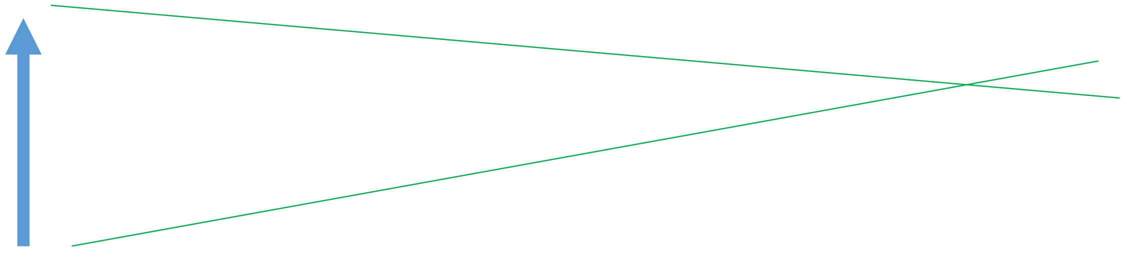
$$\alpha \approx \frac{image\ size\ of\ object}{diameter\ of\ eyeball}$$

Two different concepts

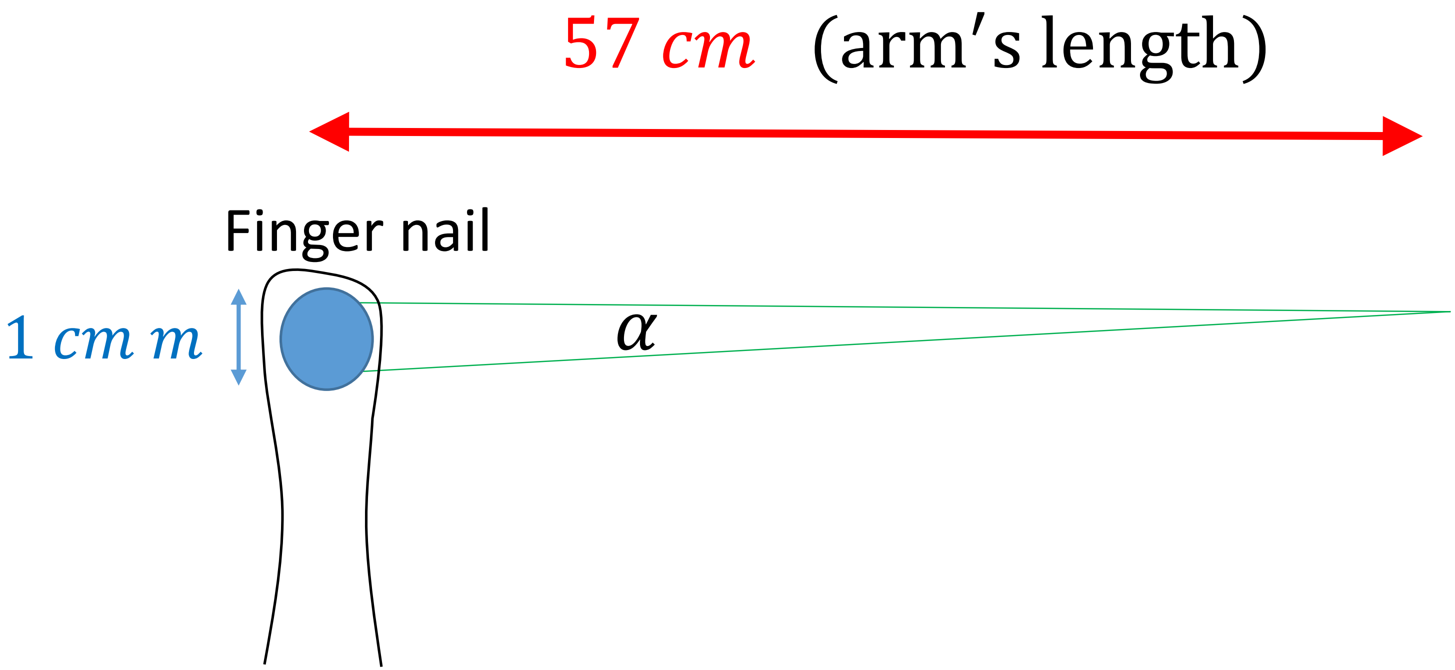
Aperture angle



Visual angle

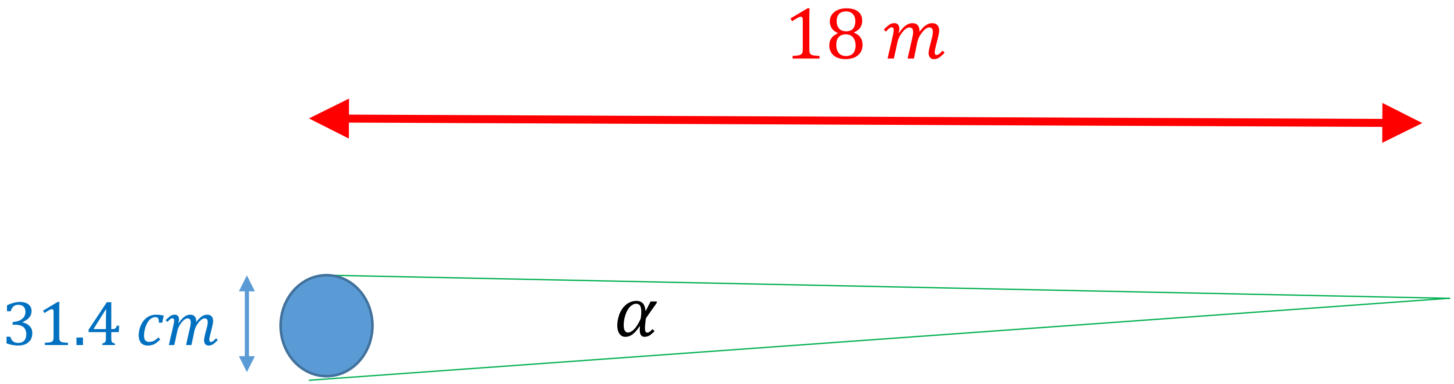


Visual Angle Example 1



$$\alpha \approx \frac{object\ height}{distance} = \frac{1\ cm}{\frac{180}{\pi}\ cm} = 1 \text{ degree}$$

Visual Angle Example 2



$$\alpha \approx \frac{object\ height}{distance} = \frac{\frac{\pi}{10}\ m}{18\ m} = \frac{\pi}{180}\ radians = 1 \text{ degree}$$

Example 3:   moon

Visual angle of moon is about $\frac{1}{2}\ deg$.

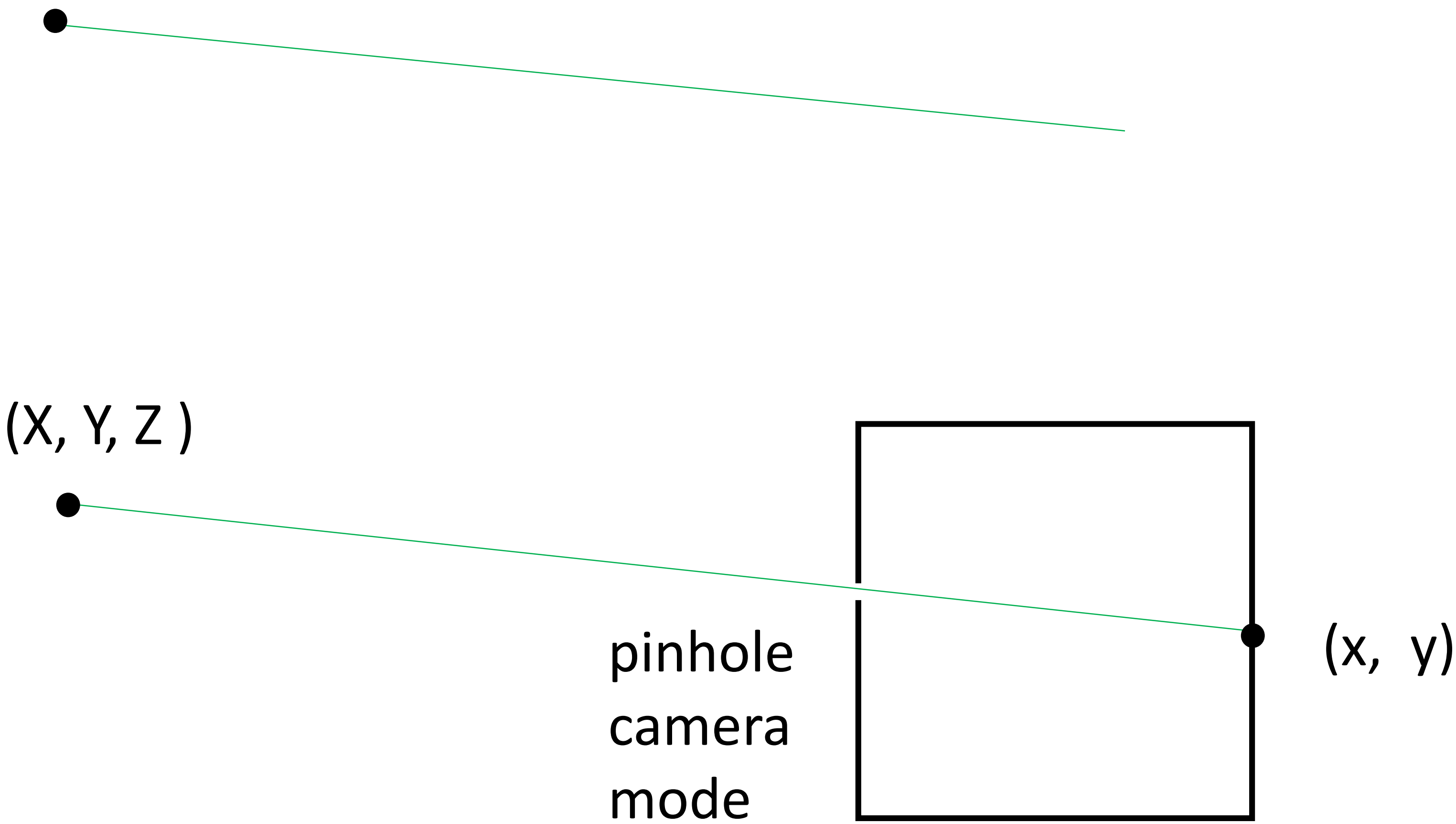
Units of visual angle

1 radian = $\frac{180}{\pi}$ deg

1 deg = 60 minutes (or “arcmin”)

1 minute = 60 seconds (or “arcsec”)

Image position



Pinhole camera

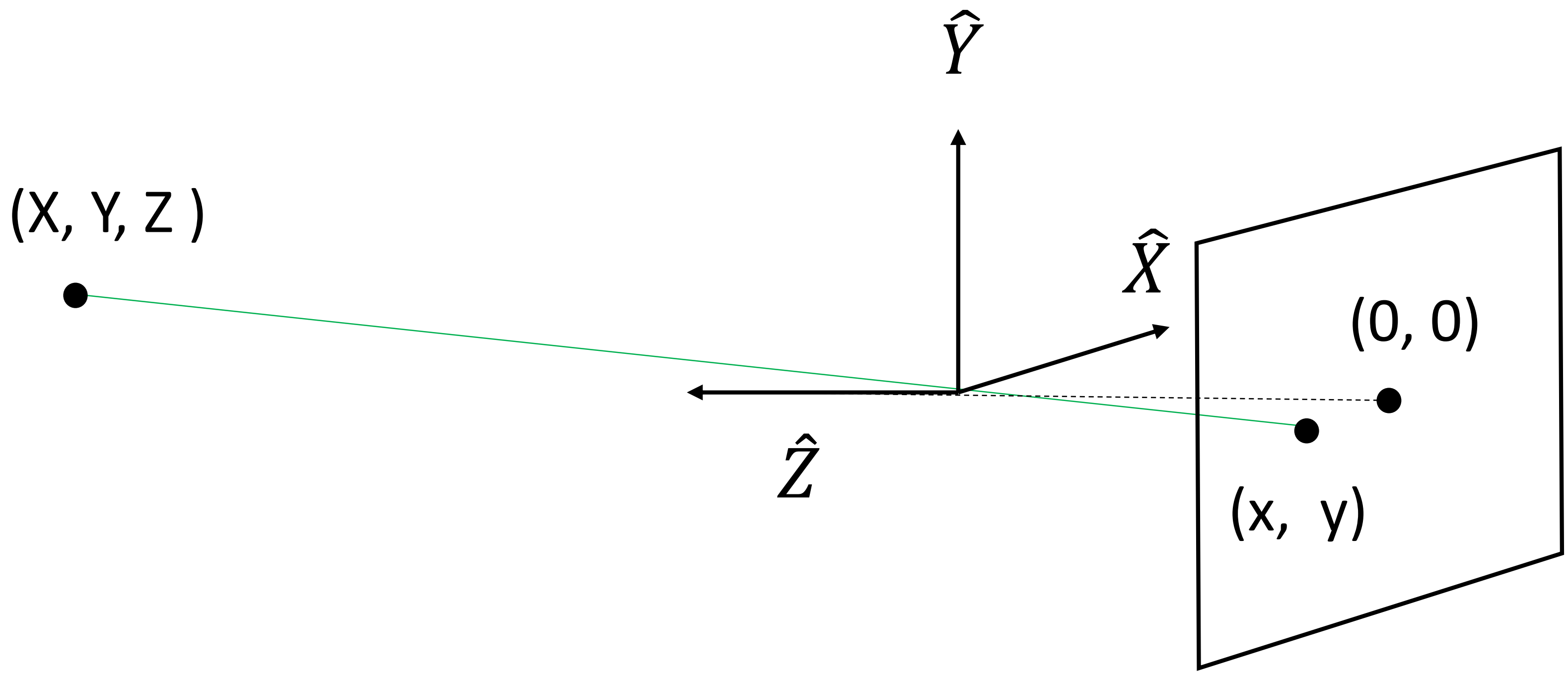
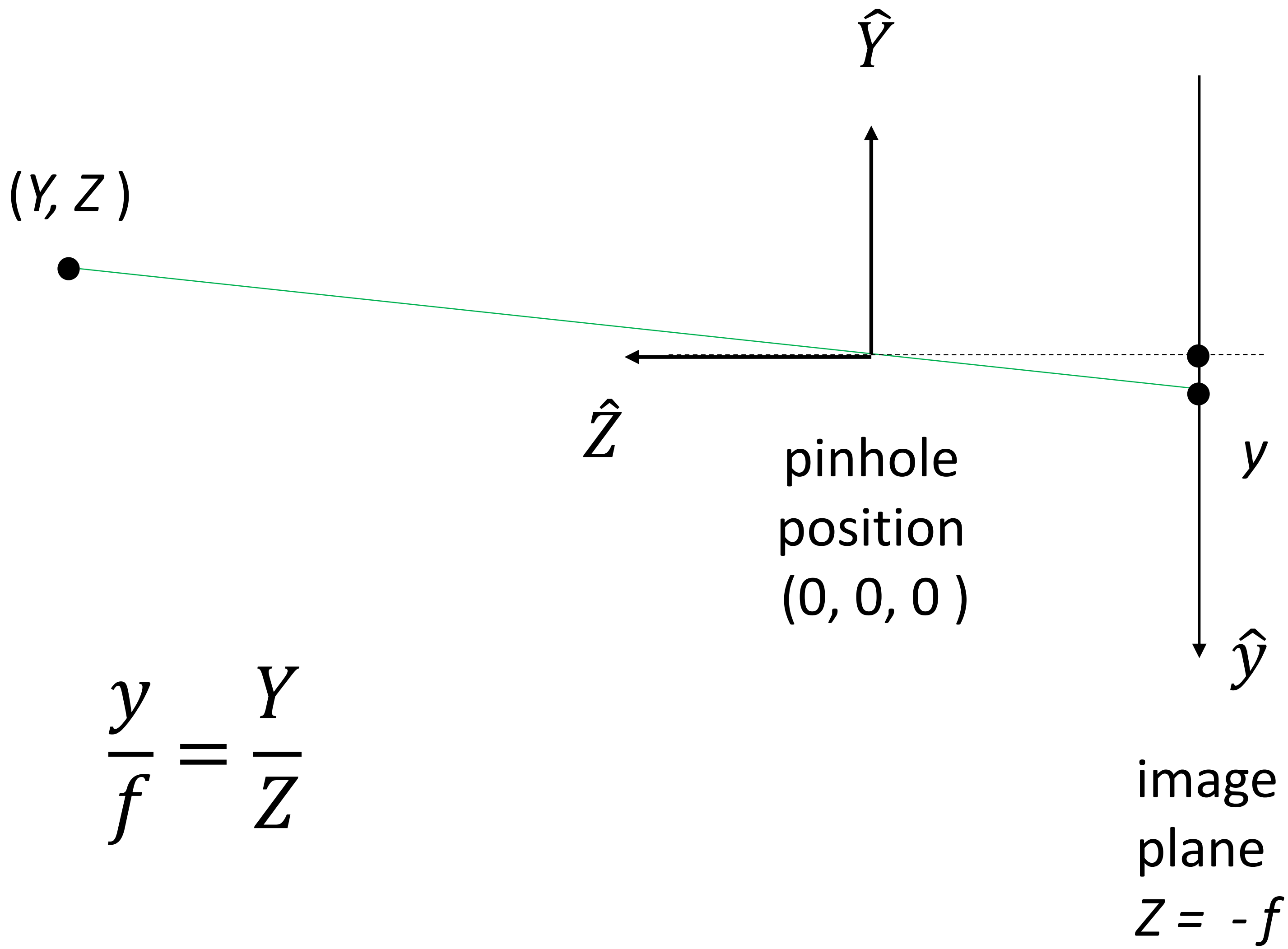


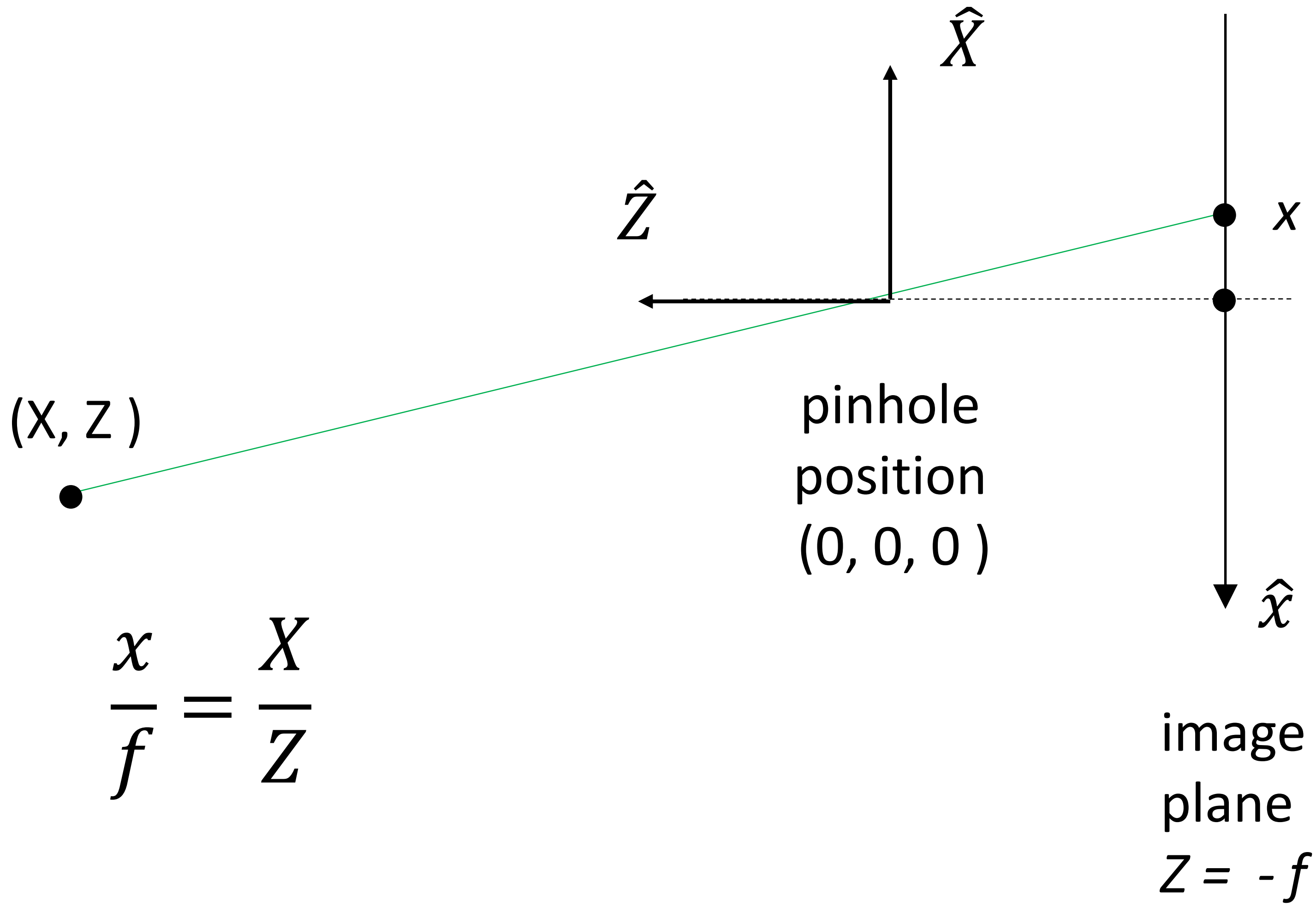
image
plane
behind
pinhole

View from side (YZ)



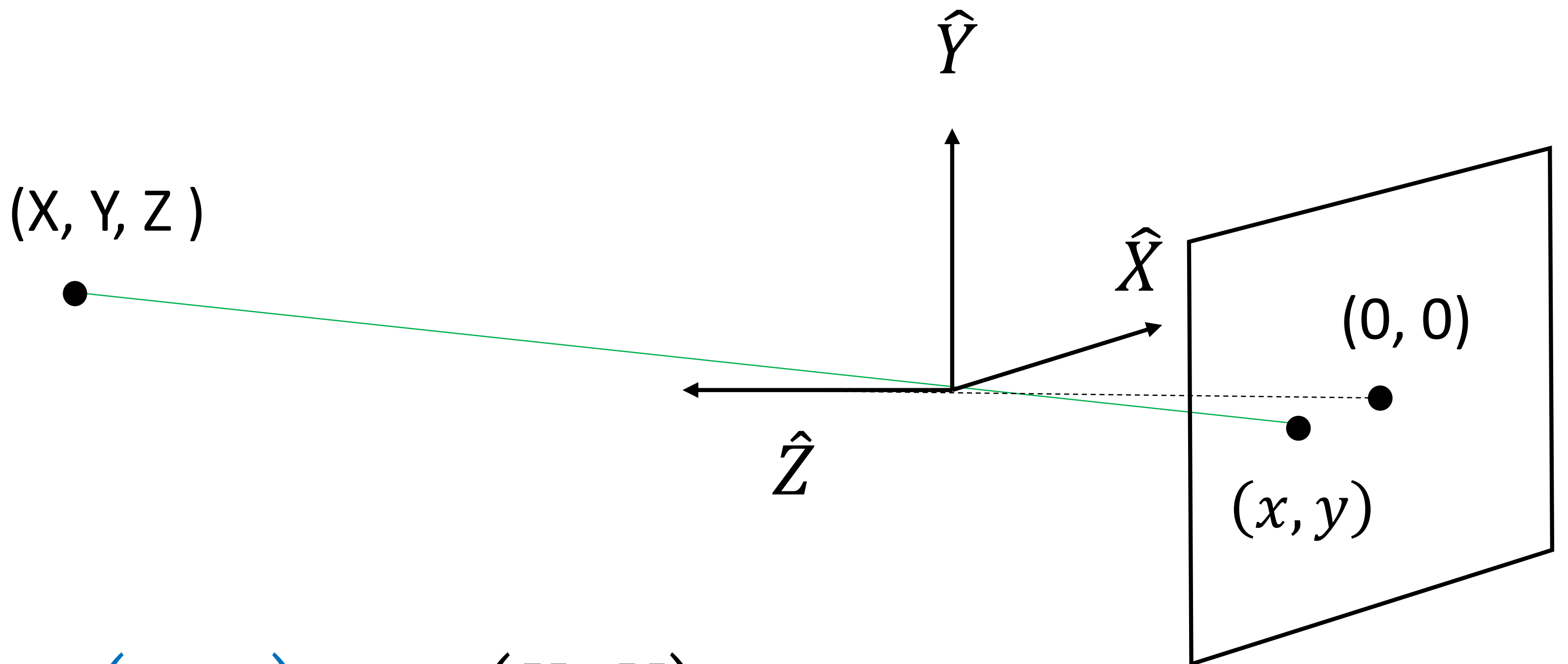
$$\frac{y}{f} = \frac{Y}{Z}$$

View from above (XZ)



$$\frac{x}{f} = \frac{X}{Z}$$

Image position *in radians**

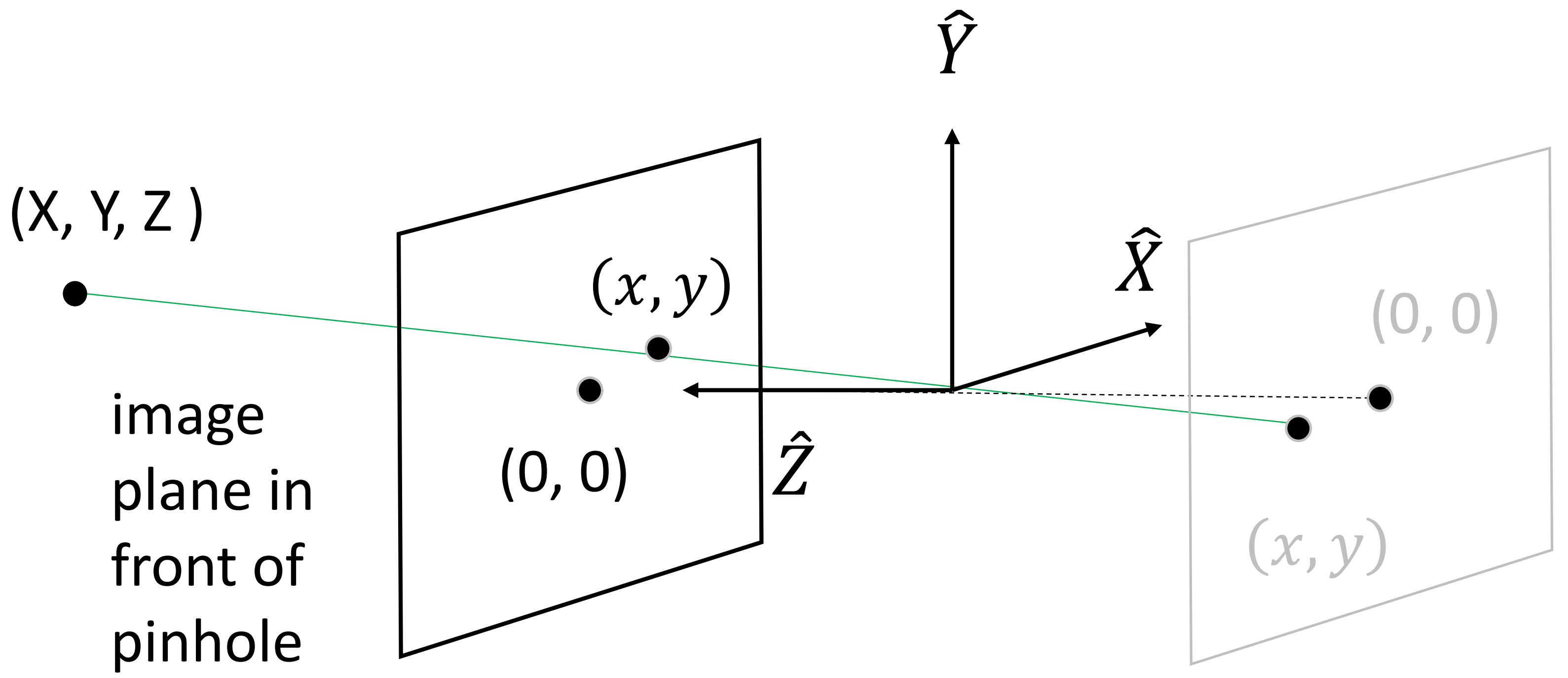


$$\left(\frac{x}{f}, \frac{y}{f}\right) = \left(\frac{X}{Z}, \frac{Y}{Z}\right)$$

image plane Z=-f behind pinhole

*assuming small angle approximation

Visual direction *in radians**



$$\left(\frac{x}{f}, \frac{y}{f}\right) = \left(\frac{X}{Z}, \frac{Y}{Z}\right)$$

Example (ground and horizon)

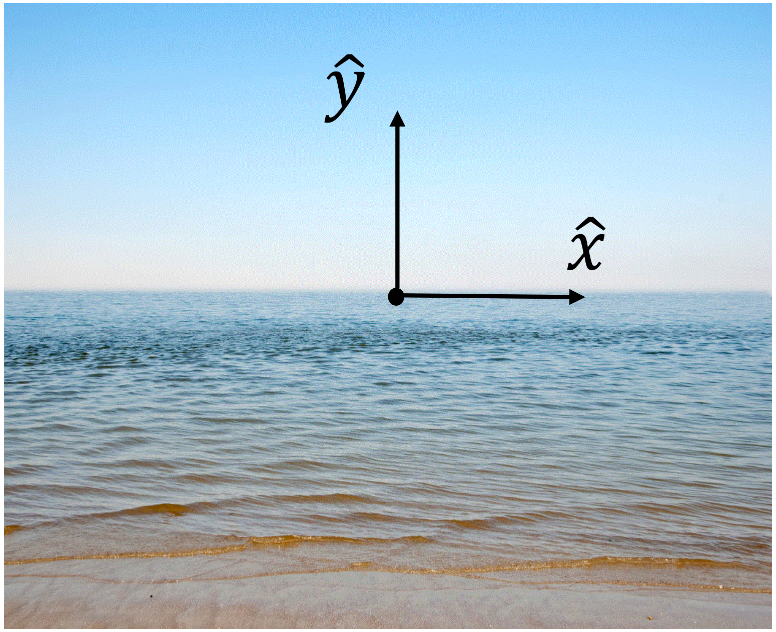
Image projection
(upside down and backwards)

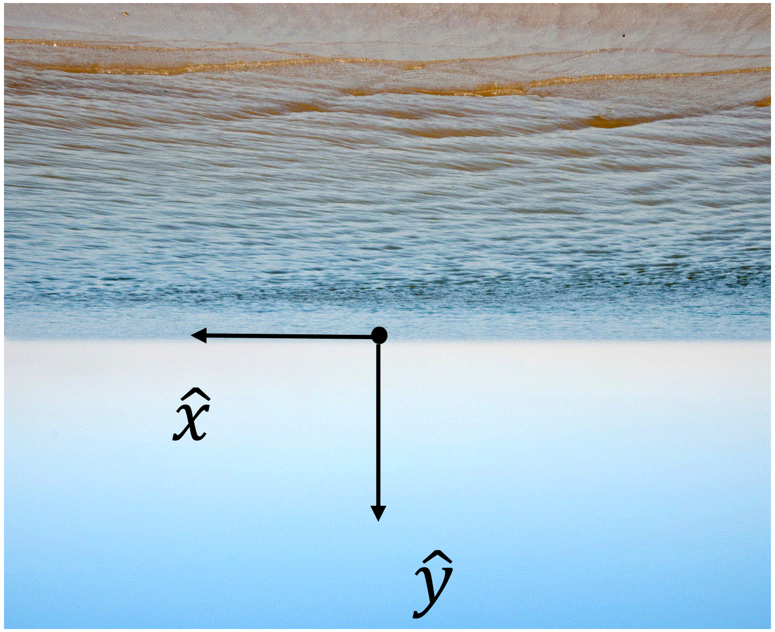
Visual direction

(image plane in front of pinhole)

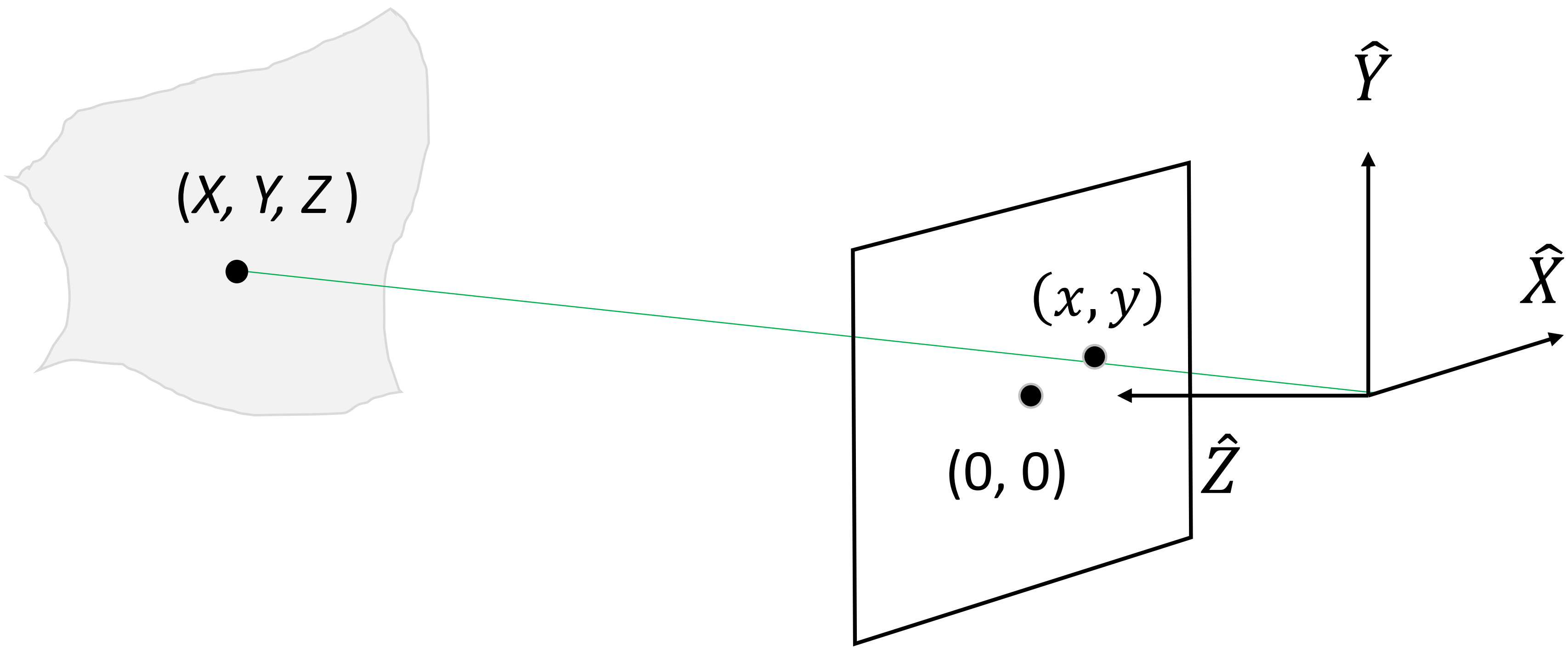
Image projection

(image plane behind pinhole)



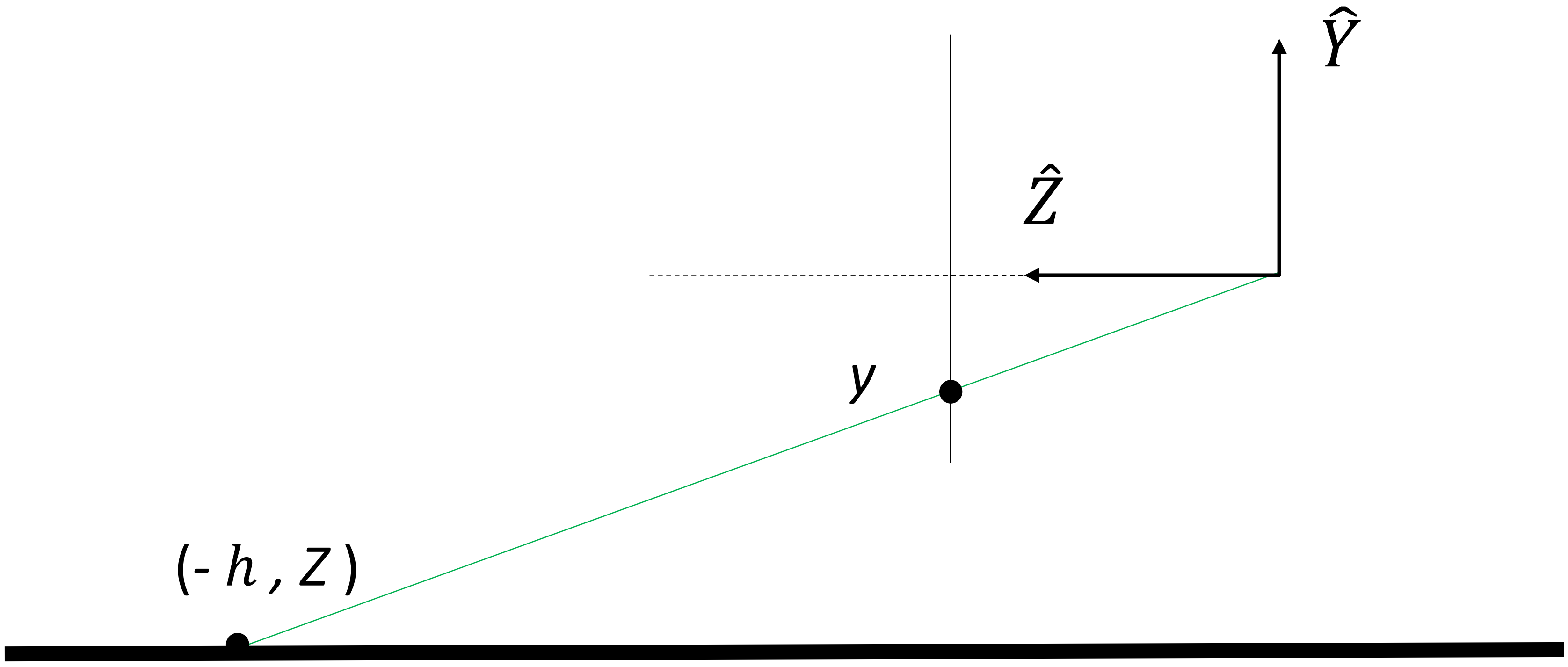


Depth Map



The mapping $Z(x, y)$ from image positions (x, y)
to depth Z values on a 3D surface is called a "depth map".

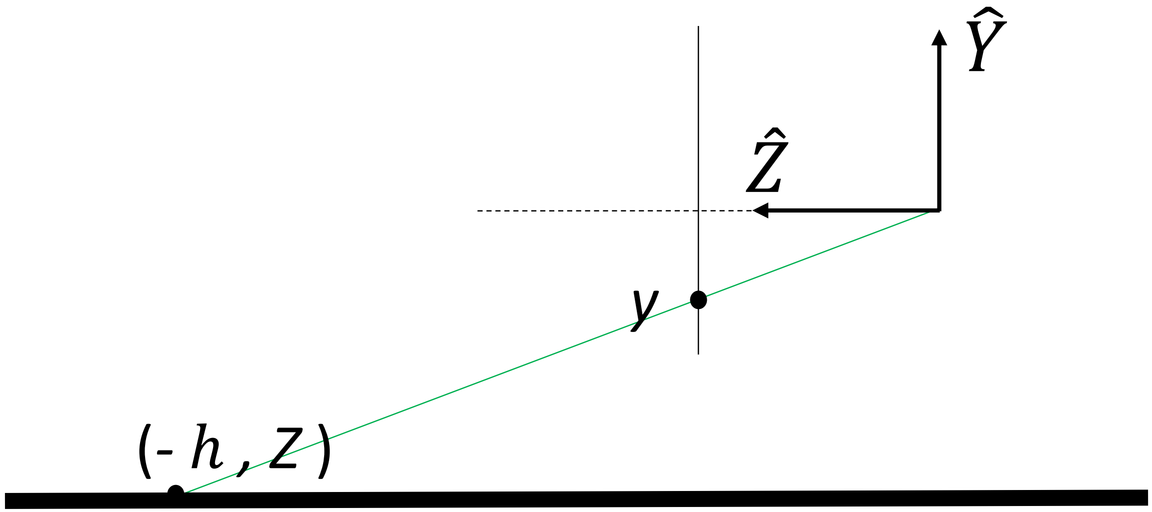
What is the depth map of a ground plane ?



Ground plane

$$Y = -h$$

What is the depth map of a ground plane ?



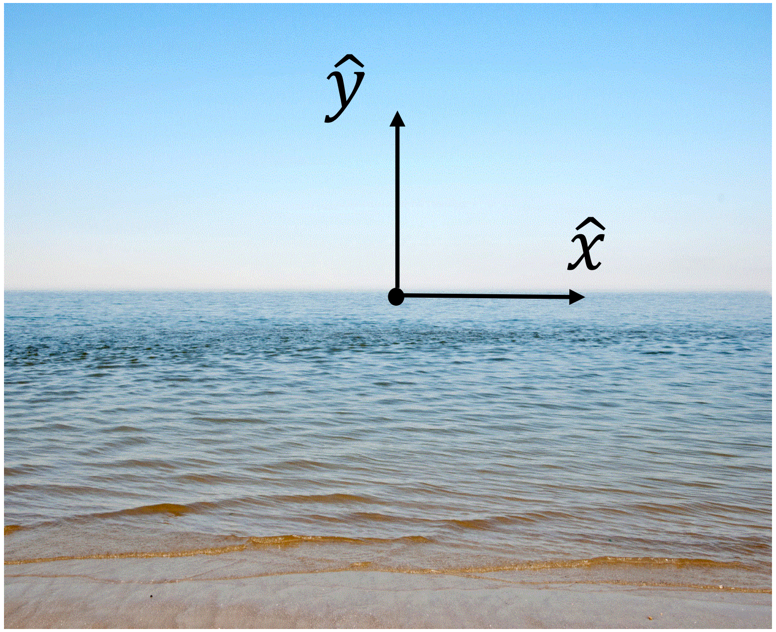
Ground plane $\quad Y = -h$

$$\frac{y}{f} = \frac{Y}{Z}$$

Thus, $\quad Z = \dfrac{-hf}{y}$

Visual direction

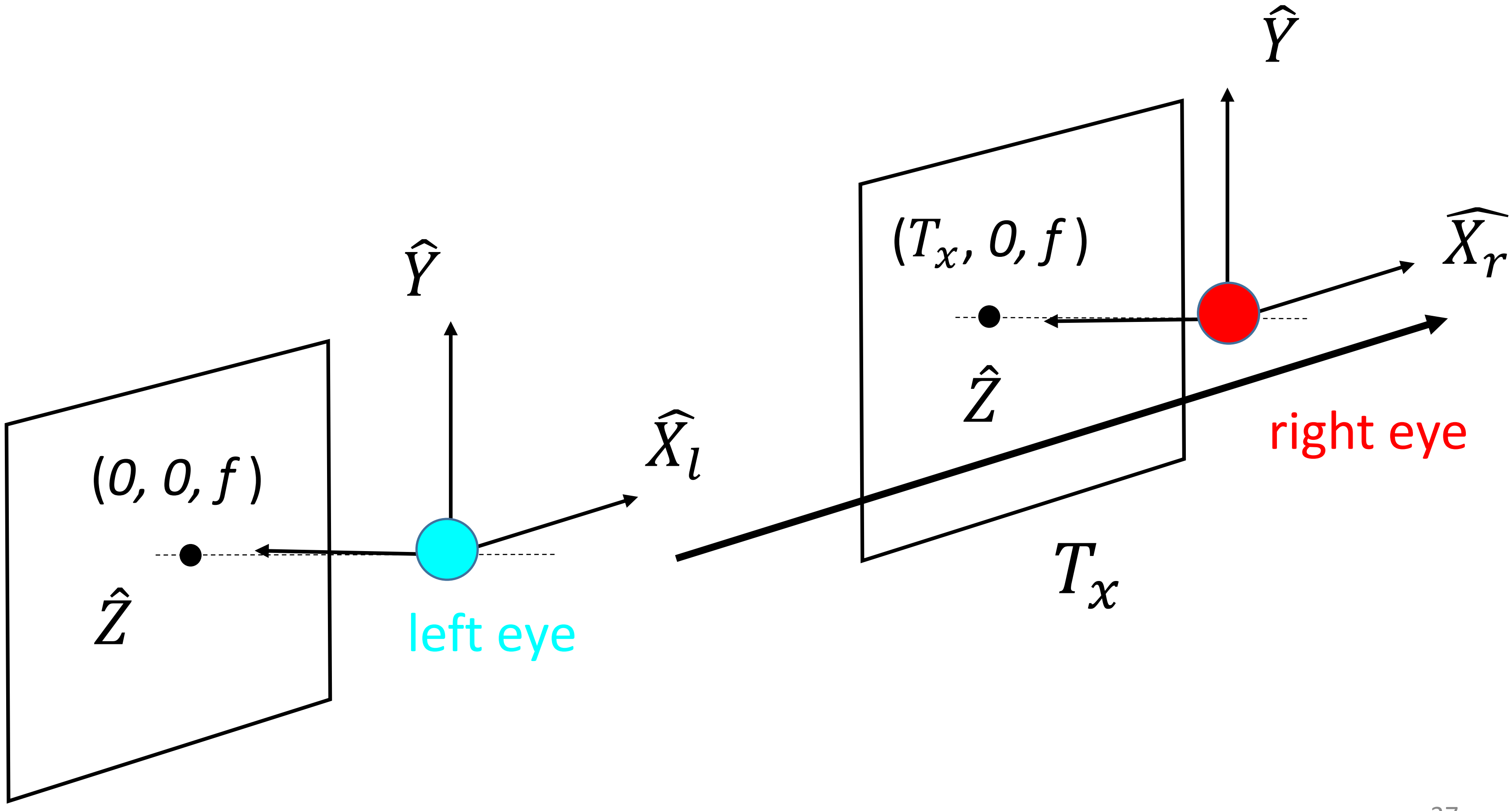
(image plane in front of pinhole)



$$Z = -h\left(\frac{f}{y}\right)$$

Binocular Vision

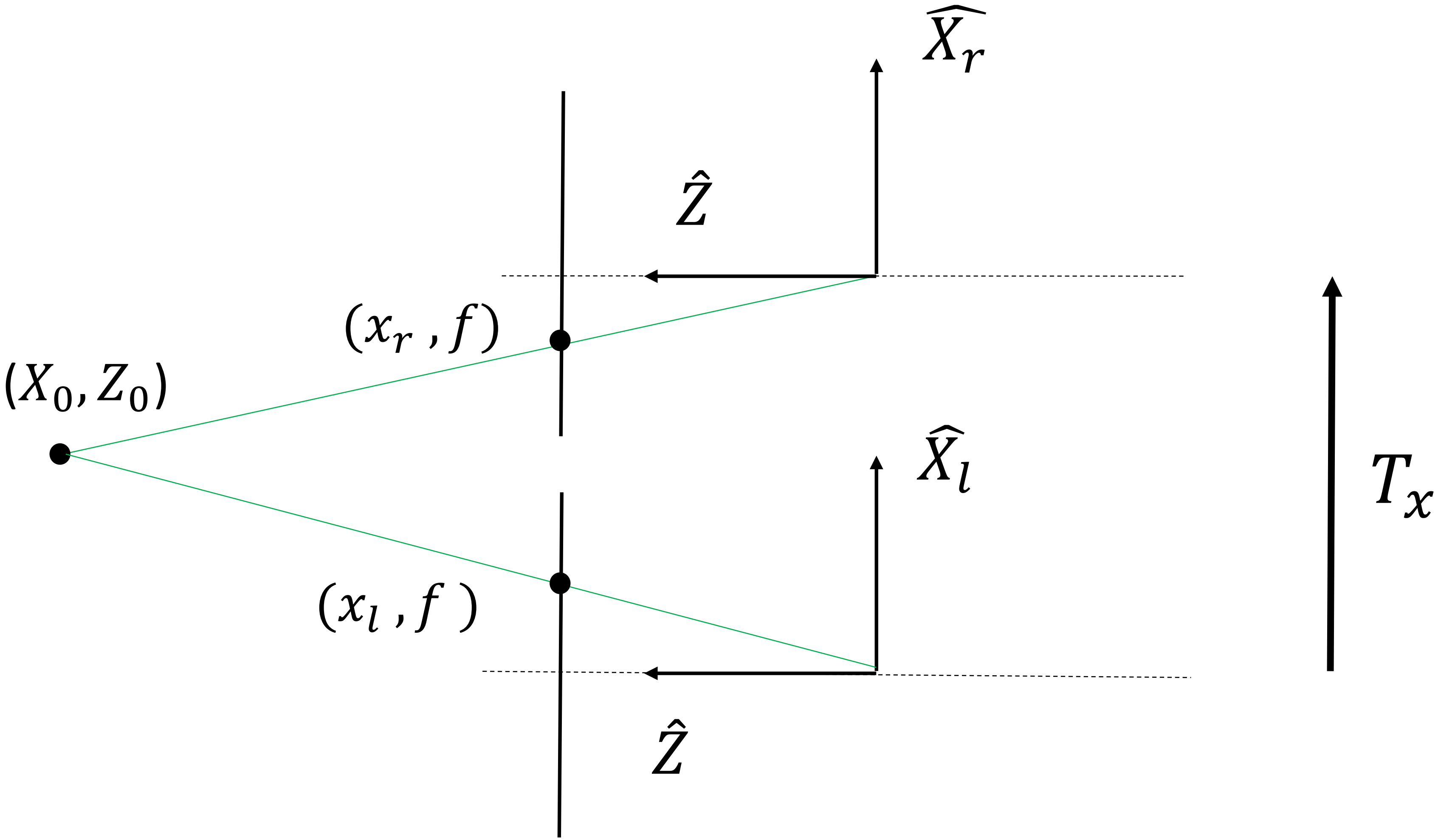
Assume eyes are separated by T_X in the X direction.
T_X is the *interocular distance*.



What is the *difference* in or visual direction (or image position) of each 3D object in the left and right images?

How does this difference depend on depth ?

View from above (XZ)



$$\text{Binocular disparity} \equiv \frac{x_l}{f} - \frac{x_r}{f}$$

is the difference in visual direction of a 3D point as seen by two eye.

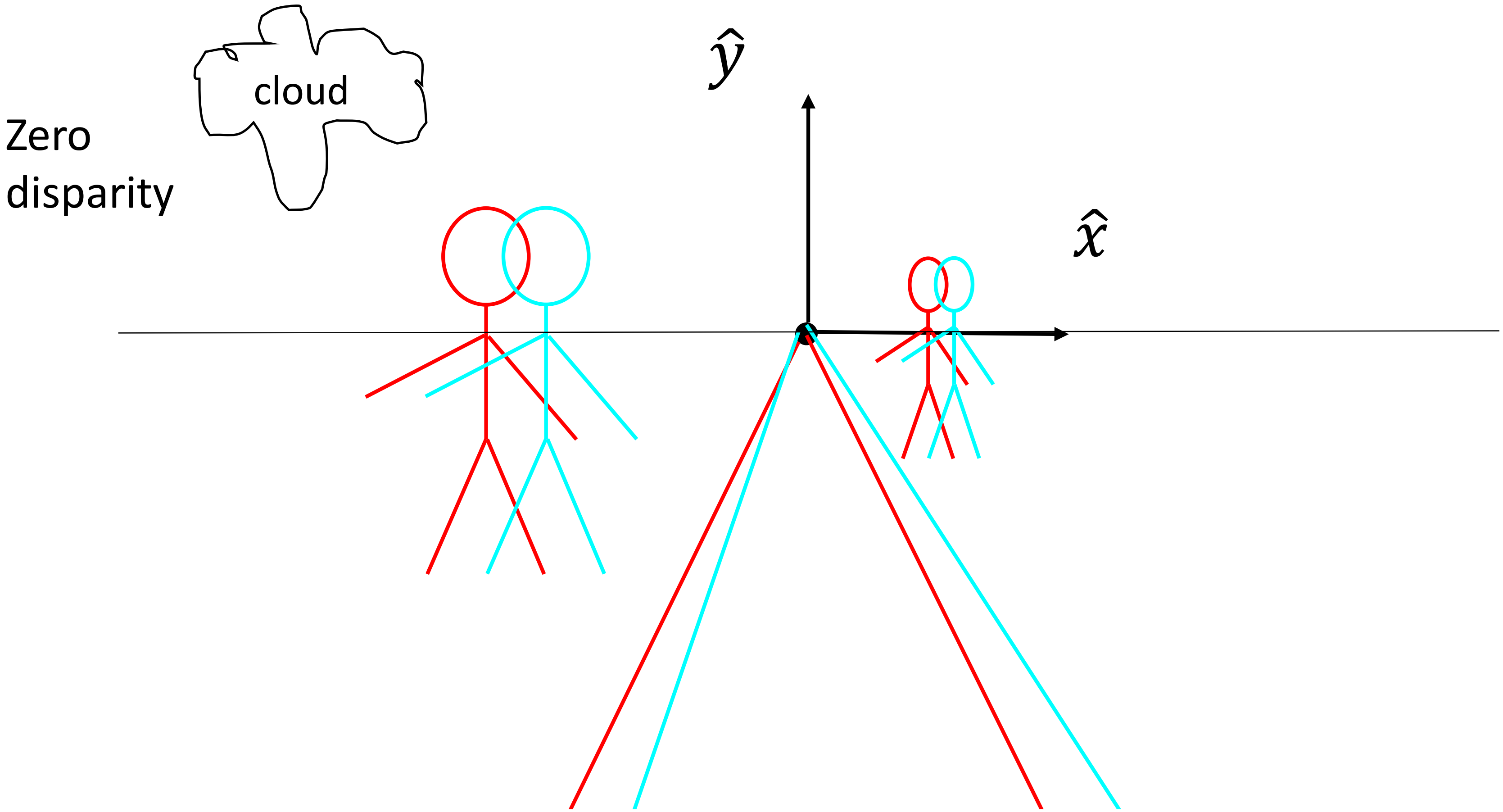
$$\text{Binocular disparity} \equiv \frac{x_l}{f} - \frac{x_r}{f}$$

$$\frac{x_l}{f} = \frac{X_0}{Z_0}$$

$$\frac{x_r}{f} = \frac{X_0 - T_x}{Z_0}$$

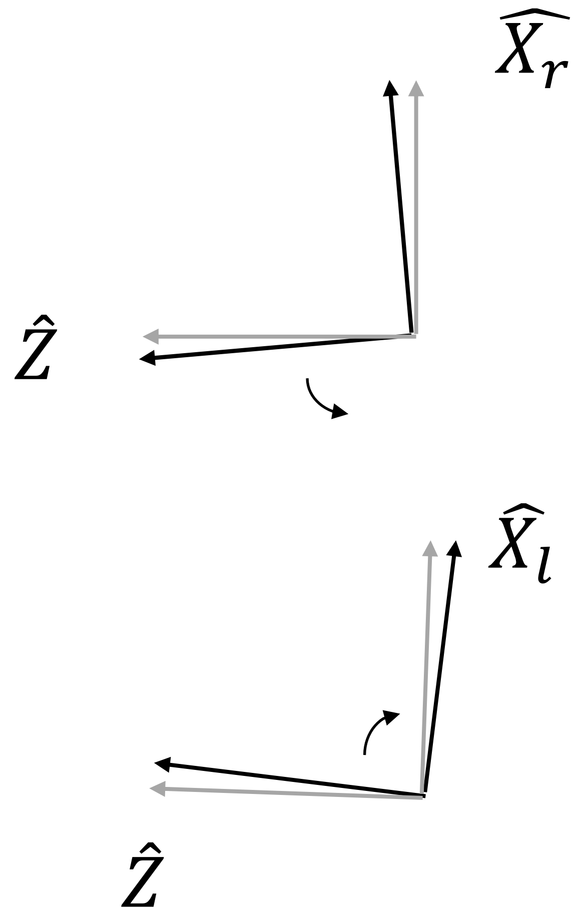
$$\text{Thus, binocular disparity} = \frac{T_x}{Z_0}$$

Superimposing left and right eye images



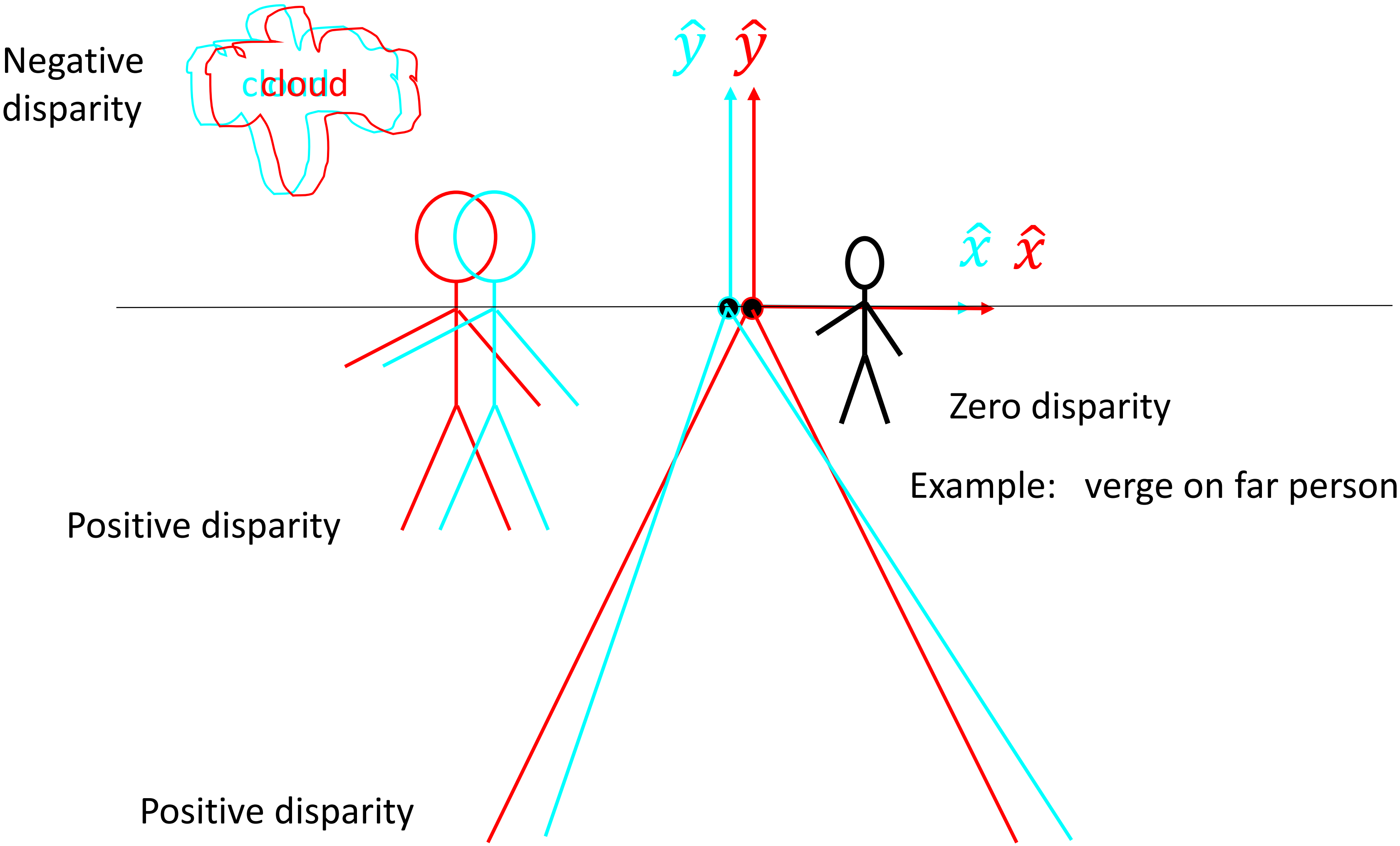
binocular disparity $= \dfrac{T_x}{Z_0} = \dfrac{T_x}{-h}\left(\dfrac{y}{f}\right)$

Vergence (rotating the eyes)



Here we assume horizontal rotation only ("pan").

Vergence


Negative disparity
cloud
cloud
$\hat{y}$
$\hat{y}$
$\hat{x}$
$\hat{x}$
Zero disparity
Example: verge on far person
Positive disparity
Positive disparity

Let θ_l and θ_r be the rotations of the left and right eyes due to vergence.

The rotations can be *approximated* by a shift in image position.

$$\text{Binocular disparity} \equiv \left(\frac{x_l}{f} - \theta_l\right) - \left(\frac{x_r}{f} - \theta_r\right)$$

$$= \left(\frac{x_l}{f} - \frac{x_r}{f}\right) - (\theta_l - \theta_r)$$