lecture 13

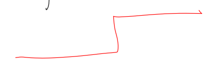
more scale space

(edges, corners, ridges, boxes)

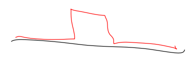
Summary of lecture 12

$I(x) * G(x,\sigma)$ ~ blur an image

$I(x) * \frac{d}{dx} g_\sigma(x)$ ~ edge detection



$I(x) * \sigma \frac{d^2}{dx^2} g_\sigma(x)$ ~ box detection



Gradient filters

$$\nabla g_\sigma(x,y) = \left(\frac{\partial}{\partial x} g_\sigma(x,y), \frac{\partial}{\partial y} g_\sigma(x,y) \right)$$

$$\nabla G(x,y,\sigma) = \text{etc.}$$

Notation :

$$\text{“} \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \text{”} \equiv \left(\frac{\partial I(x,y)}{\partial x} * g_\sigma(x,y), \frac{\partial I(x,y)}{\partial y} * g_\sigma(x,y) \right)$$

$I(x,y) * G(x,y,\sigma)$

368 x 550 pixels

$\sigma = 2$

$\sigma = 4$

$\sigma = 8$

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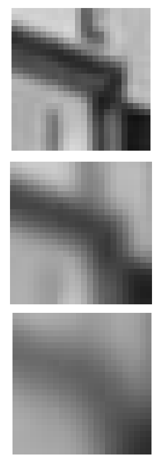
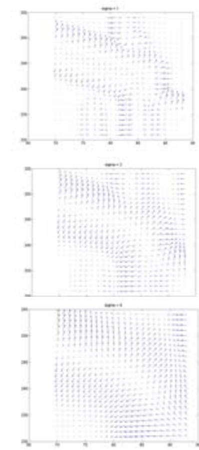
eg. look at region 24 x 24 pixels

$\sigma = 1$

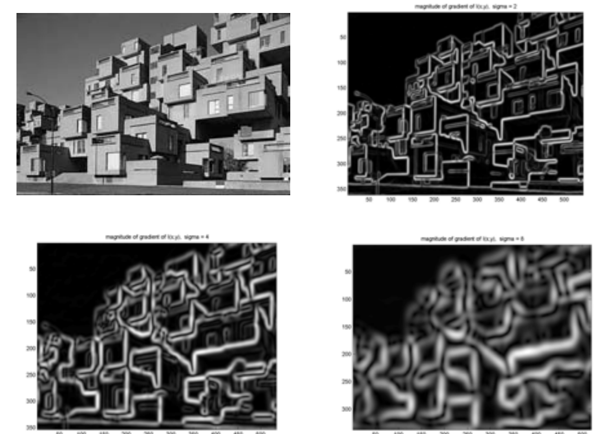
$\sigma = 2$

$\sigma = 4$





$I * \nabla g_\sigma$



Second Moment Matrix

$$\sum_{(x,y) \in Ngd(x_0,y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}_{\sigma_D}$$

derivative scale

$$= \sum_{(x,y) \in Ngd(x_0,y_0)} \begin{bmatrix} \left(I(x,y) * \frac{\partial g_\sigma(x,y)}{\partial x} \right)^2 & \left(I(x,y) * \frac{\partial g_\sigma(x,y)}{\partial x} \right)\left(I(x,y) * \frac{\partial g_\sigma(x,y)}{\partial y} \right) \\ \left(I(x,y) * \frac{\partial g_\sigma(x,y)}{\partial x} \right)\left(I(x,y) * \frac{\partial g_\sigma(x,y)}{\partial y} \right) & \left(I(x,y) * \frac{\partial g_\sigma(x,y)}{\partial y} \right)^2 \end{bmatrix}$$

Second Moment Matrix

$$M(x,y) = \sum_{(x',y') \in Ngd(x,y)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}_{\sigma_D}$$

$$\sum_{(x',y')} g_{\sigma_I}(x-x', y-y') \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}_{\sigma_D}$$

integration scale

eval @ (x',y')

$$= g_{\sigma_I}(x,y) * \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}_{\sigma_D}$$

If we fix ratio $\sigma_I : \sigma_D$ (e.g. 3:1) then we get a scale space

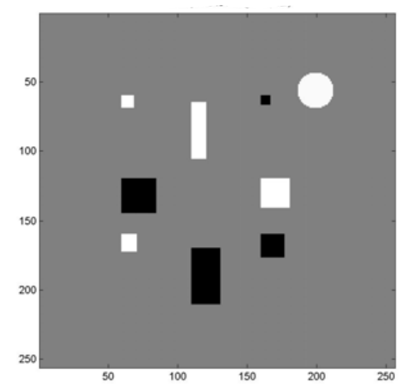
$$M(x,y,\sigma_D)$$

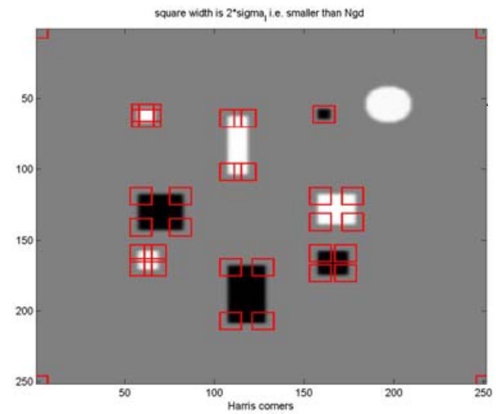
$$= g_{\sigma_I}(x,y) * \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}_{\sigma_D}$$

Example : Harris corners

$$M(x,y,\sigma_D) = g_{\sigma_I}(x,y) * \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}_{\sigma_D}$$

- We would like to know points where both eigenvalues are large.
- $\text{Harris}(M) = \det M - 0.1\,(\text{tr}\,M)^2$
- Find local maxima.

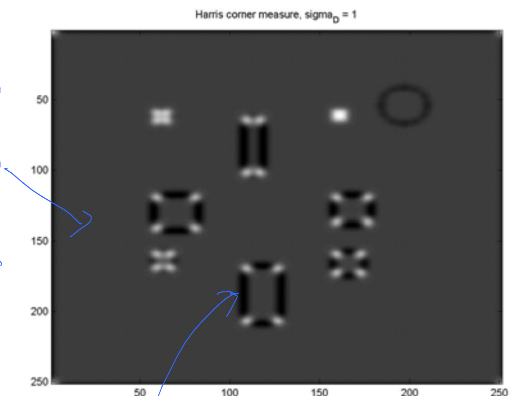




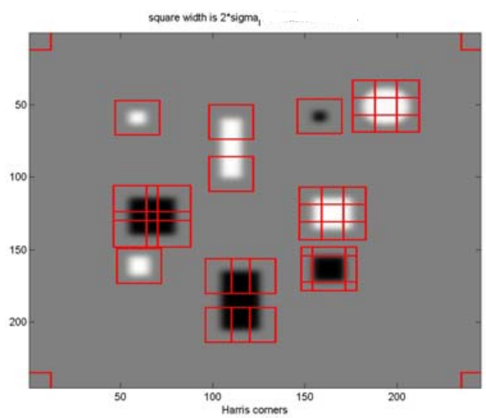
$\text{tr}(M) = 0$

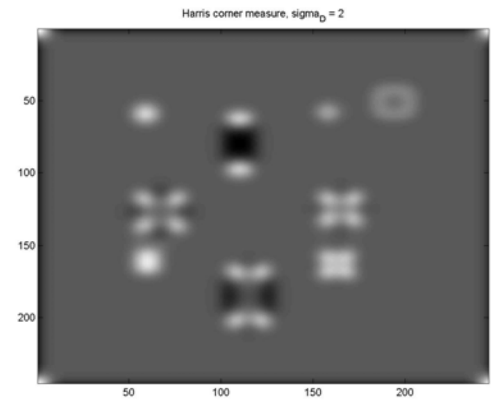
$\det(M) = 0$

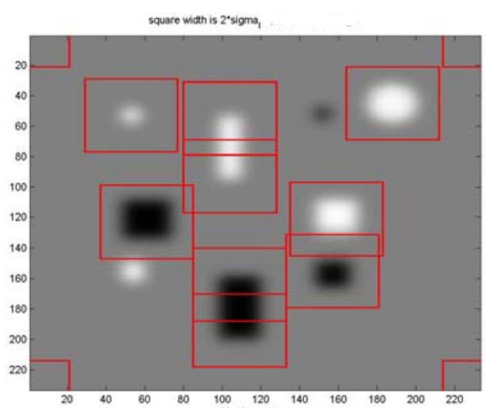
$\Rightarrow$ Harris $= 0$

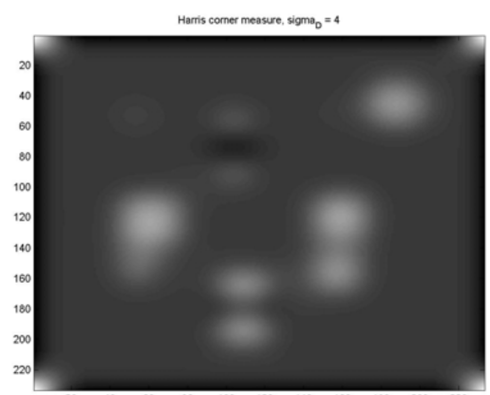


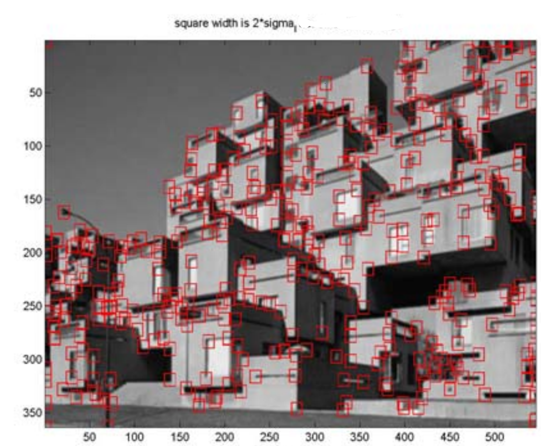
$\text{tr}(M) > 0 \quad \det(M) = 0 \Rightarrow \text{Harris} < 0$

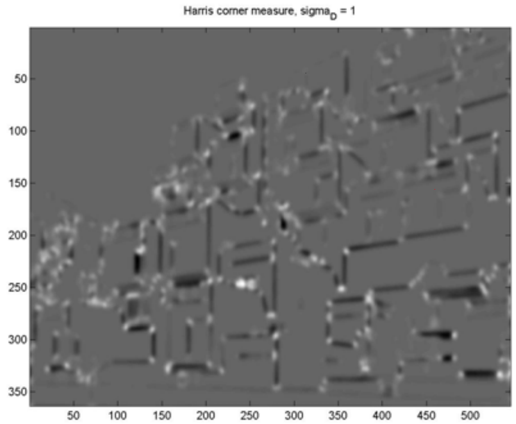


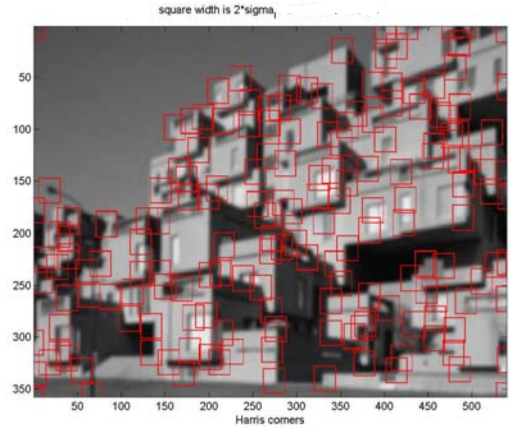


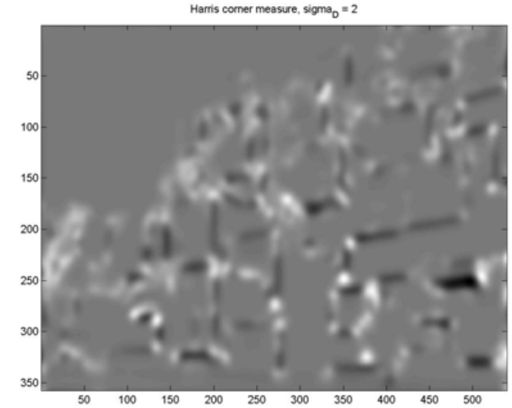


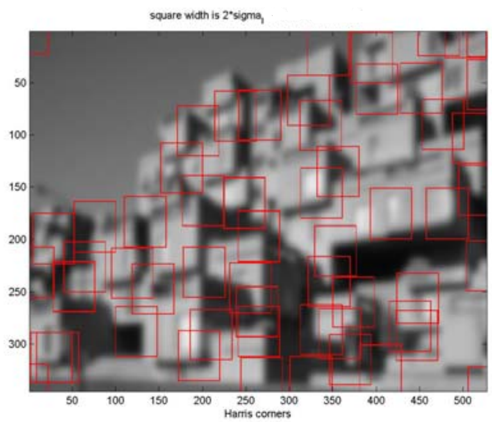


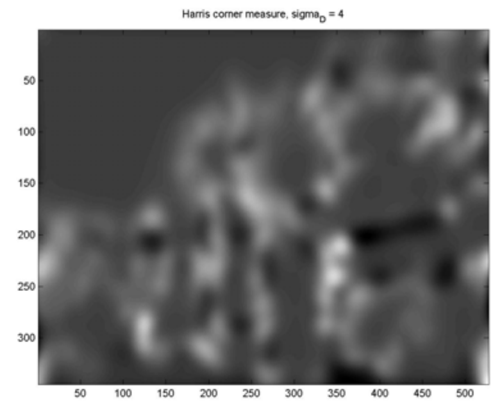




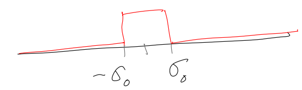




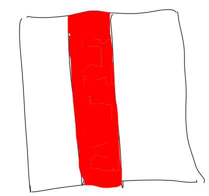




Bar / Ridge detection



1D case (last class)



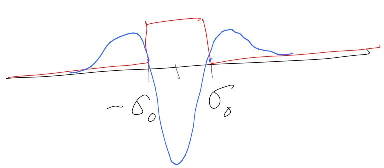
2D case

$$I(x,y) = u(x+\sigma_0) - u(x-\sigma_0)$$

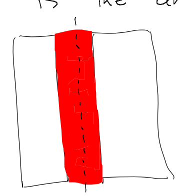
Recall (last lecture)

$$I(x) * \sigma \frac{\partial^2 g_\sigma(x)}{\partial x^2}$$

produces a negative minimum at $(x, \sigma) = (0, \sigma_0)$.



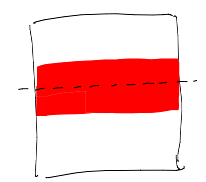
Q: What is the analogous 2D result?



2D case

$$I(x,y) = u(x+\sigma_0) - u(x-\sigma_0)$$

A: Convolve with $\frac{\partial^2}{\partial x^2} g_\sigma(x,y)$ gives a minimum at $(x,y,\sigma) = (0, y, \sigma_0)$



$$I(x,y) = u(y+\sigma_0) - u(y-\sigma_0)$$

A: Convolve with $\frac{\partial^2}{\partial y^2} g_\sigma(x,y)$ gives a minimum at $(x,y,\sigma) = (x, 0, \sigma_0)$

Proof:

It is straightforward to show

$$u(x-\sigma_0) * G(x,y,\sigma) = u(x-\sigma_0) * G(x,\sigma)$$

Thus,

$$\sigma^2 \frac{\partial^2}{\partial x^2} u(x-\sigma_0) * G(x,y,\sigma) = \sigma^2 u(x-\sigma_0) * \frac{d^2}{dx^2} G(x,\sigma)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\frac{\partial^2}{\partial x^2} u(x-\sigma_0) * g_\sigma(x,y) = u(x-\sigma_0) * \frac{d^2}{dx^2} \sigma g_\sigma(x)$$

How to generalize to a bar at an arbitrary orientation?

Use $\nabla^2 g_\sigma(x,y)$

$$\equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) g_\sigma(x,y)$$

"Laplacian" of a (normalized) Gaussian

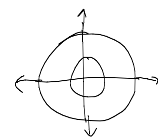
Use $I(x,y) * \nabla^2 g_\sigma(x,y)$

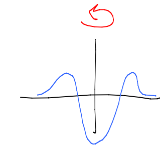
$$\nabla^2 g_\sigma(x,y) \equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) g_\sigma(x,y)$$

modified Nov. 23, 2010

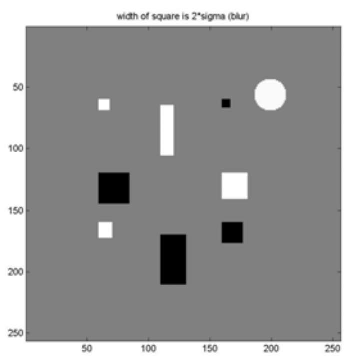
$$= \frac{2}{\sigma^2}\left(\frac{x^2+y^2}{2\sigma^2} - 1\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

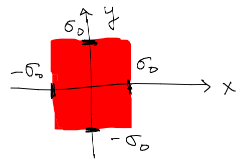
paraboloid





2D Box Detection





$$I(x,y) = \left(u(x+\sigma_0) - u(x-\sigma_0)\right) \cdot \left(u(y+\sigma_0) - u(y+\sigma_0)\right)$$

Easy to show

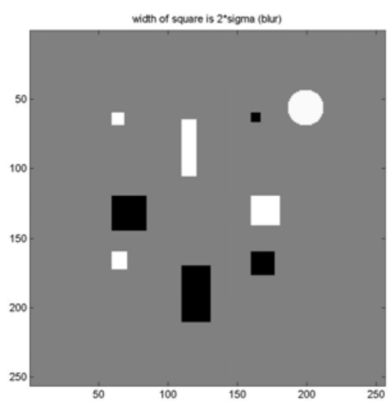
$I(x,y) * \nabla^2 g_\sigma(x,y,\sigma)$ has a minimum at $(0,0,\sigma_0)$

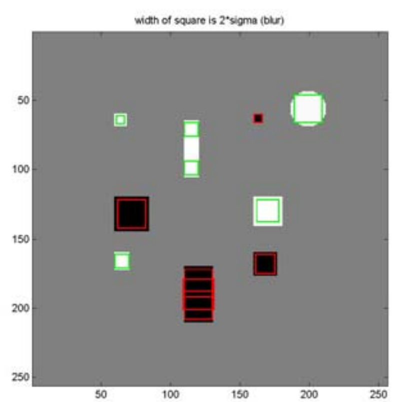
Examples

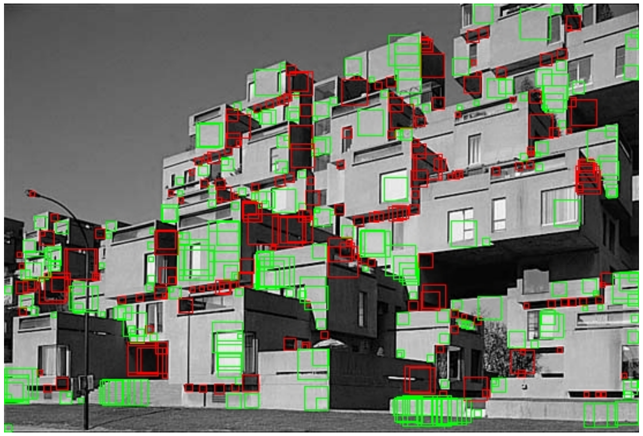
Given $I(x,y)$,

- compute $I(x,y) * \nabla^2 g_\sigma(x,y)$
- find local maxima / minima in (x,y,σ)

such that $|I * \nabla g_\sigma| >$ threshold







Recall Summary of lecture 12

$I(x) * G(x,\sigma) \sim$ blur an image

$I(x) * \frac{d}{dx} g_\sigma(x) \sim$ edge detection

$\frac{d}{dx}\, \sigma G(x,\sigma)$

$I(x) * \sigma \frac{d^2}{dx^2} g_\sigma(x) \sim$ box detection

$\frac{d^2}{dx^2}\, \sigma^2 G(x,\sigma)$

Summary Today

$I(x,y) * G(x,y,\sigma) \sim$ blur an image

$I(x,y) * \nabla g_\sigma(x) \sim$ edge detection

$\nabla\, \sigma G(x,\sigma)$

$I(x,y) * \nabla^2 g_\sigma(x) \sim \left.\begin{matrix}\text{ridge}\\ \text{box}\end{matrix}\right\}$ detection

$\nabla^2 \sigma^2 G(x,y,\sigma)$