COMP 546

Lecture 14

Maximum likelihood models

Tues. Feb. 27, 2018

Overview of today

- Informal notion of likelihood

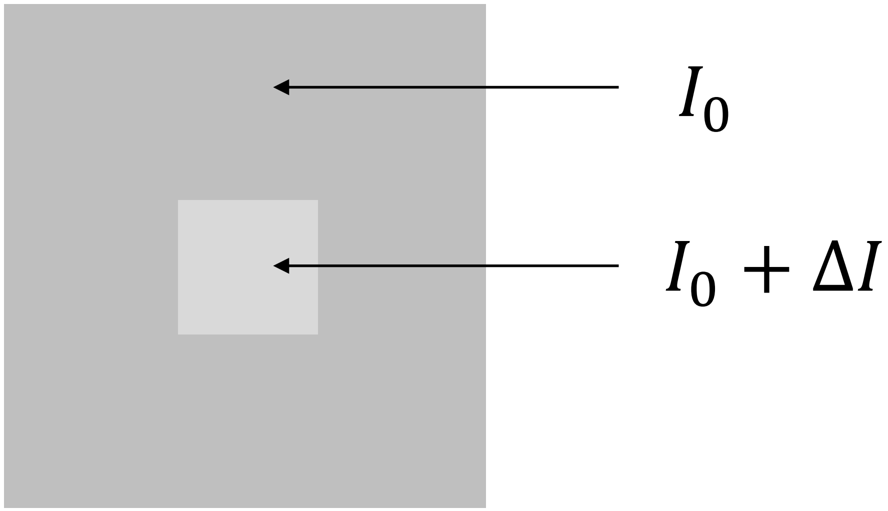
- Formal definition of likelihood as conditional probability

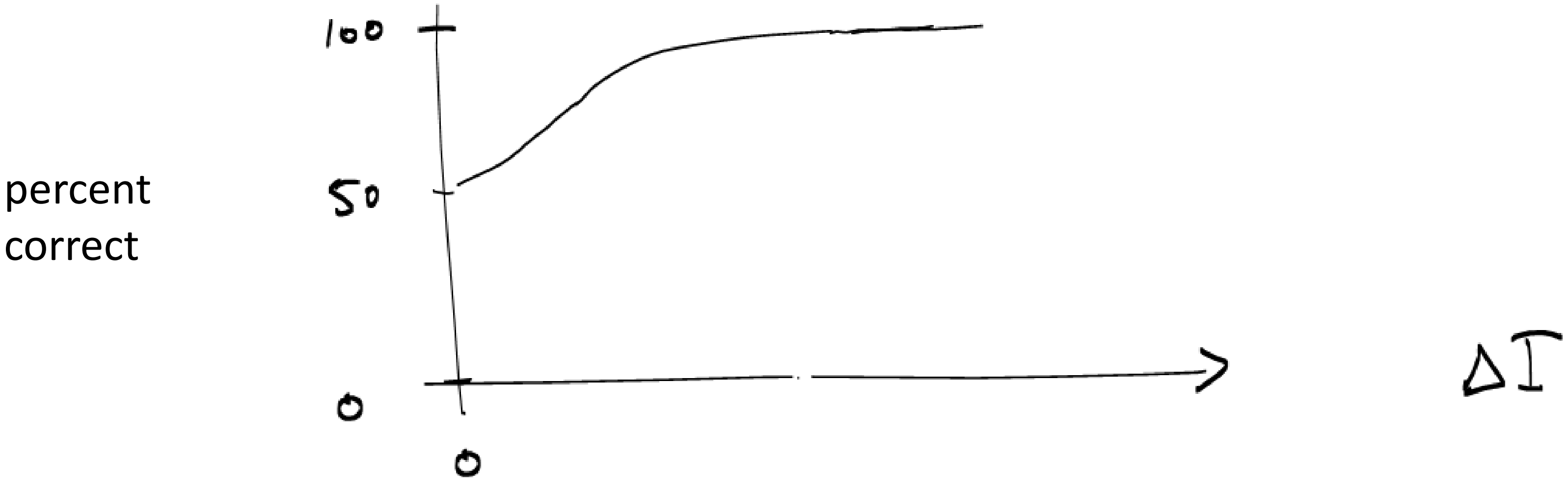
- Maximum likelihood problems (sketch)

$$\text{Scene} \xrightarrow[\text{image formation}]{} \text{Image} \xrightarrow[\text{vision}]{} \text{Estimated Scene}$$

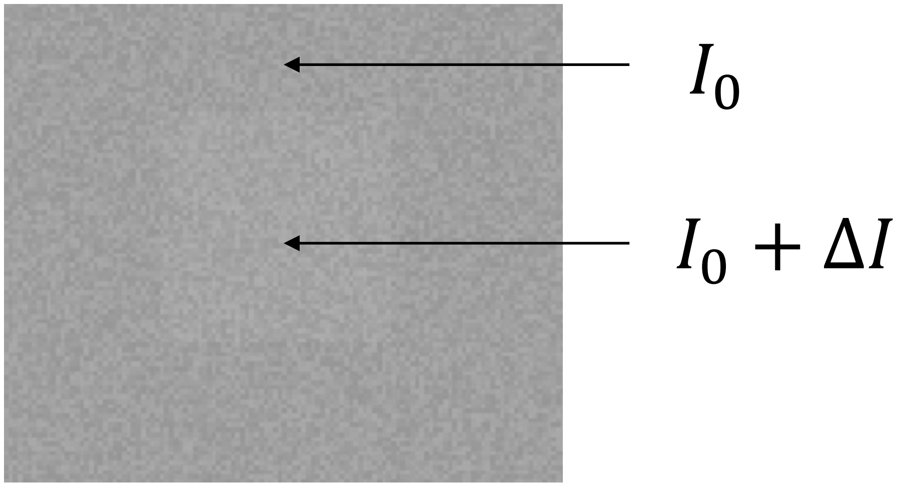
$S = s$	$I = i$	$S = \hat{s}$
luminance	image intensity	luminance
orientation	filter responses	orientation
disparity		disparity
motion		motion
surface slant, tilt		surface slant, tilt
…		…

Task: detecting an intensity increment

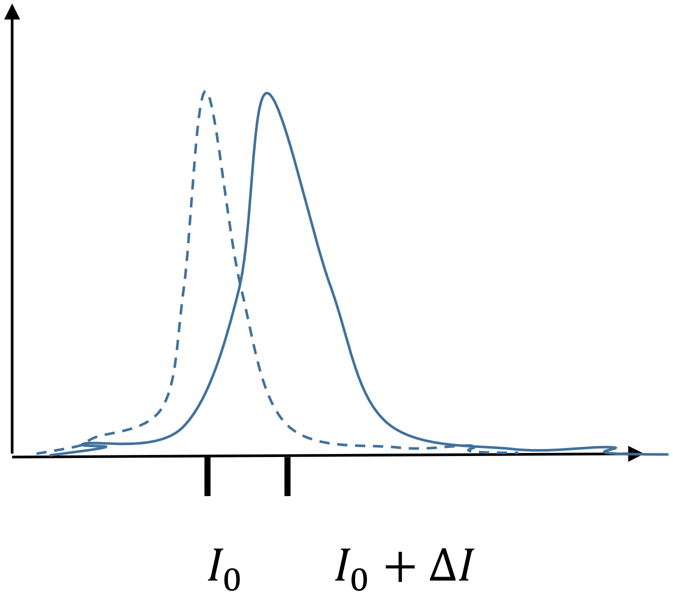




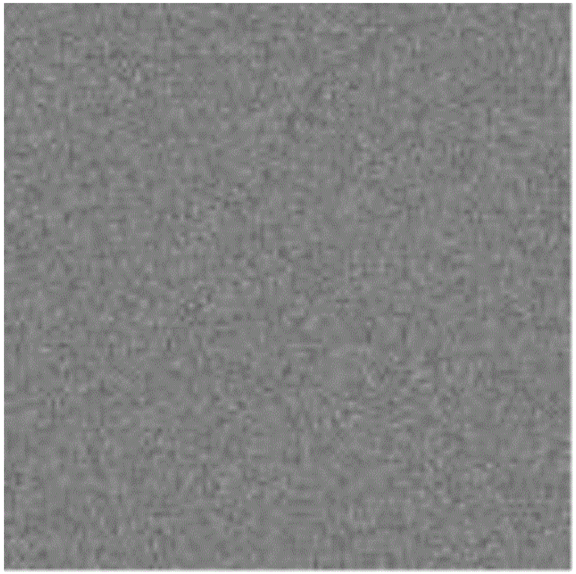
If ΔI is small and noise is big, then the task becomes more difficult.

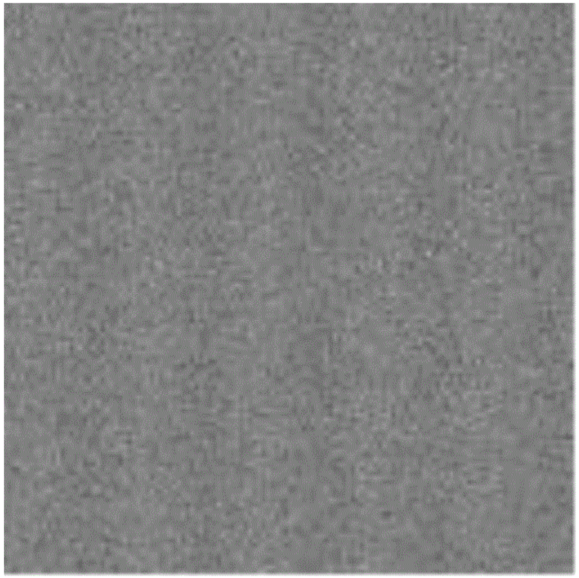


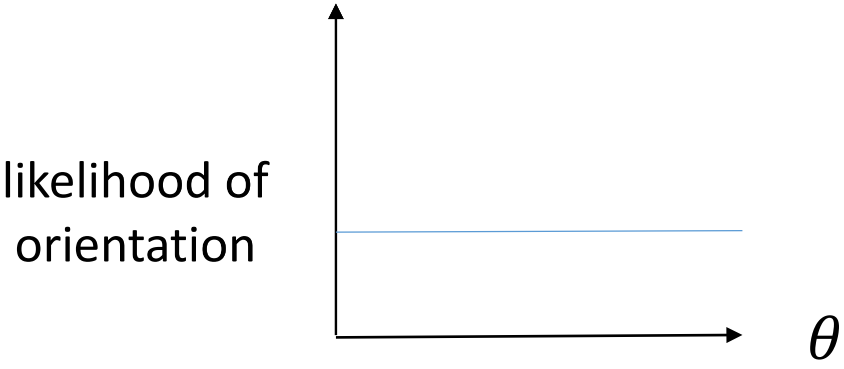
likelihood of intensity in center (solid) and background (dashed)

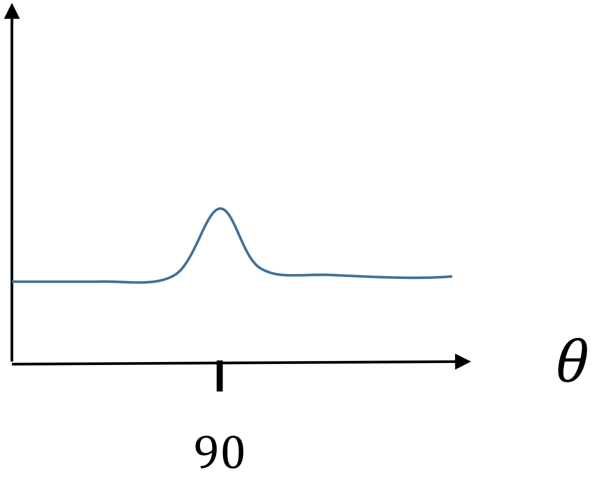


Task: estimate orientation

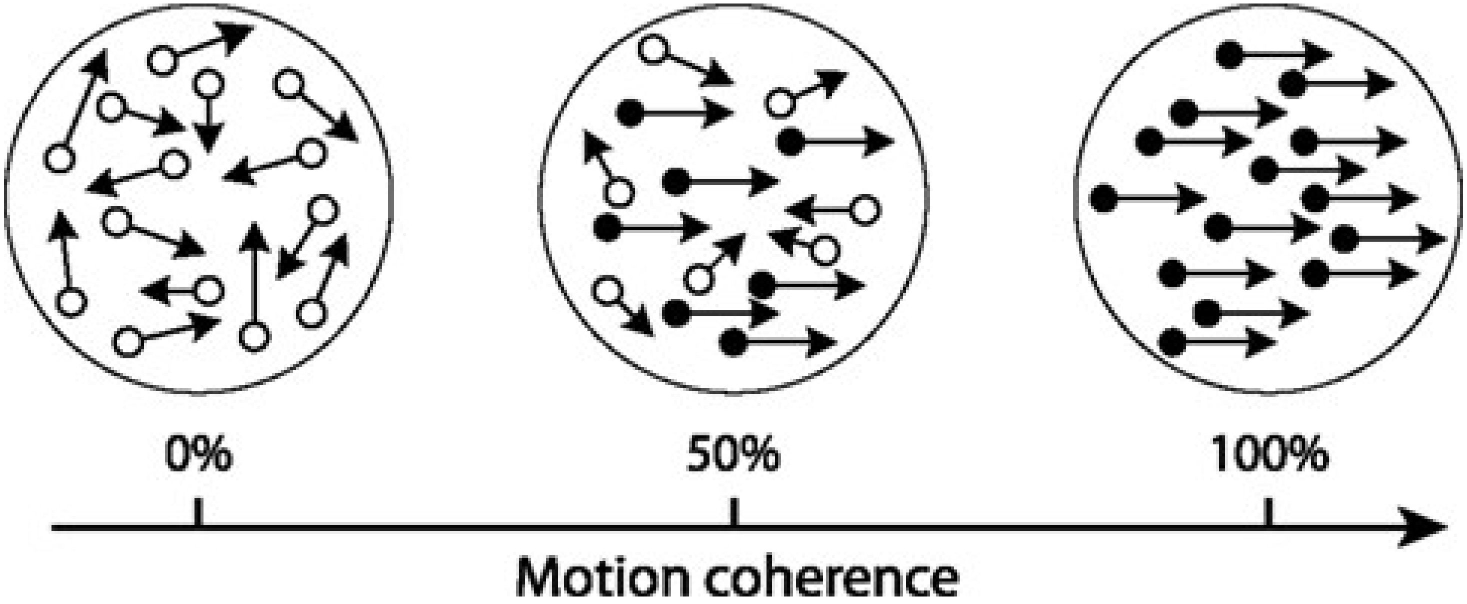


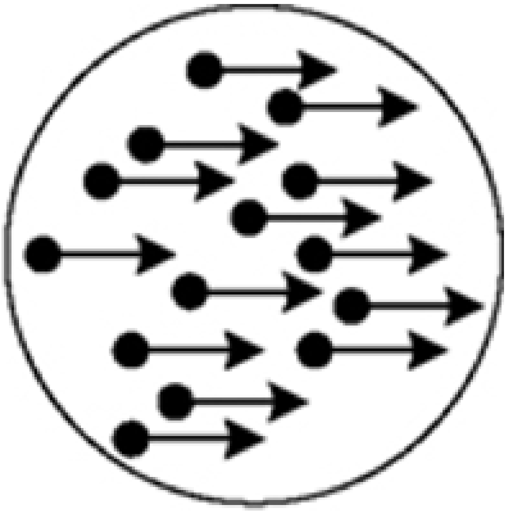






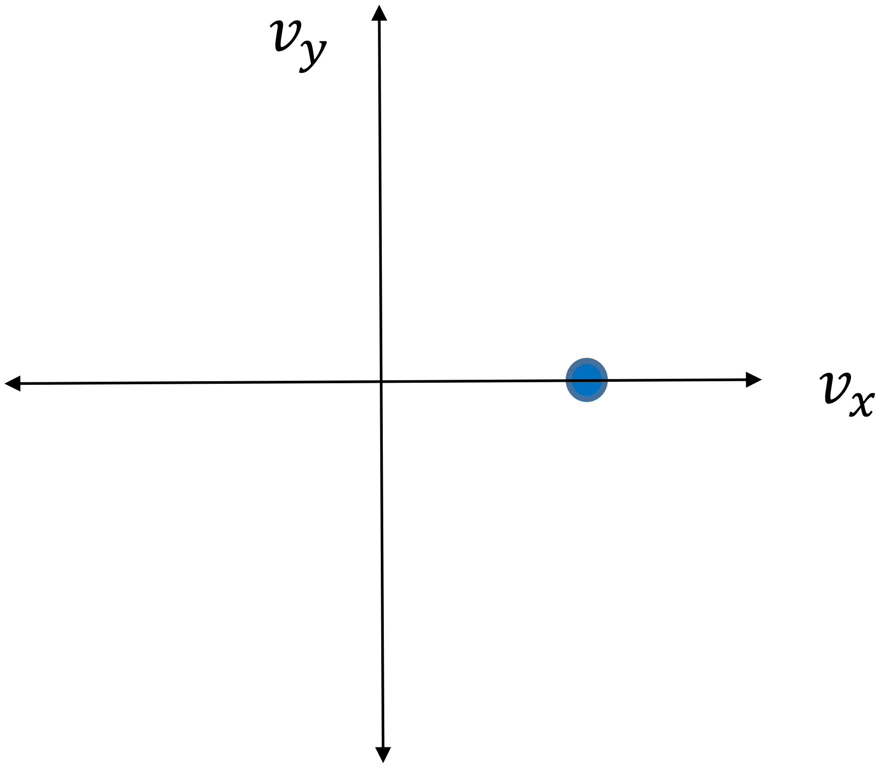
Task: estimate velocity of black dots

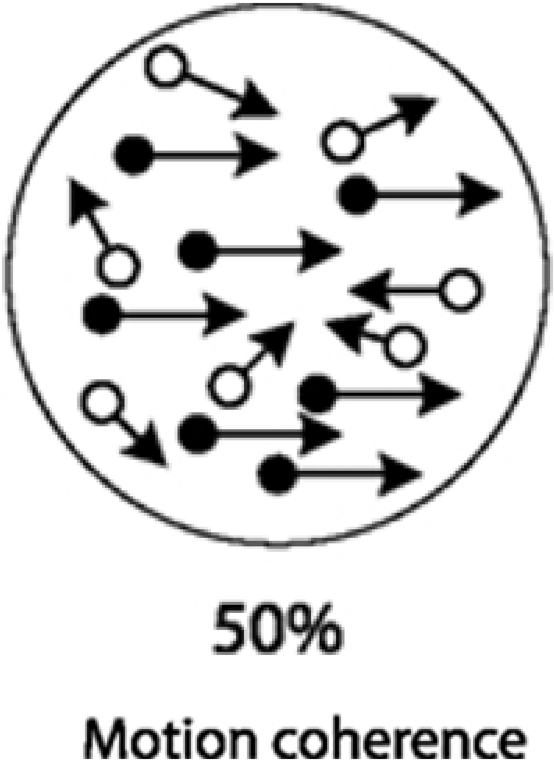


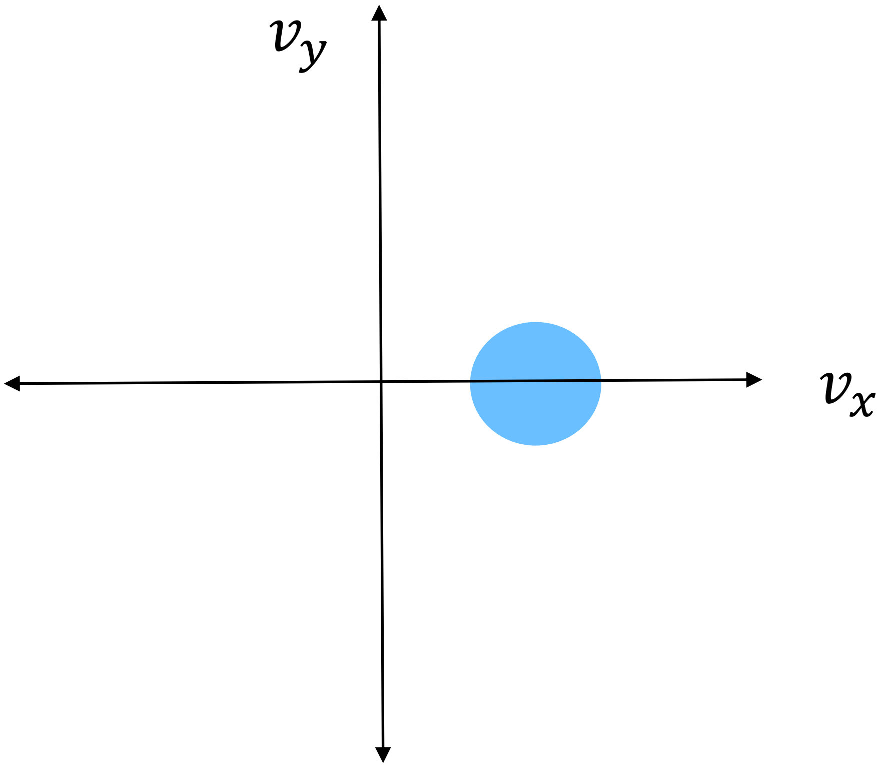


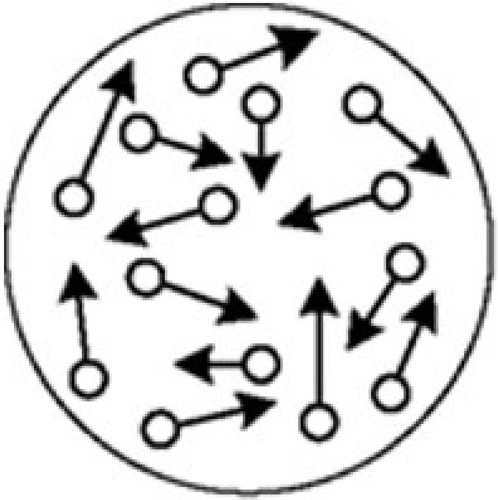
100%

Motion coherence



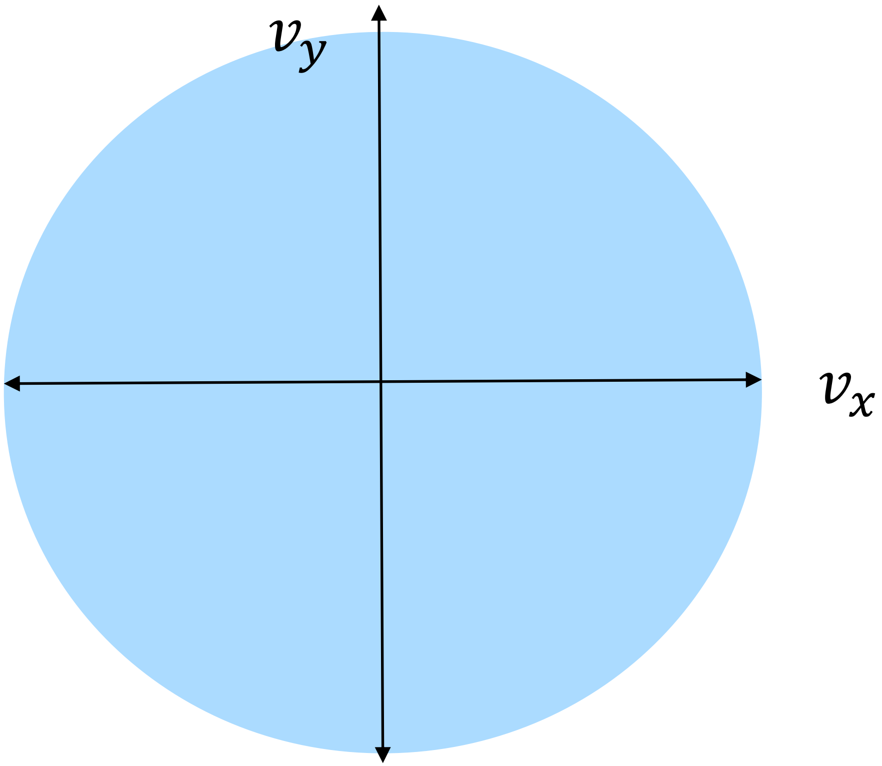

50%
Motion coherence


v_y
v_x

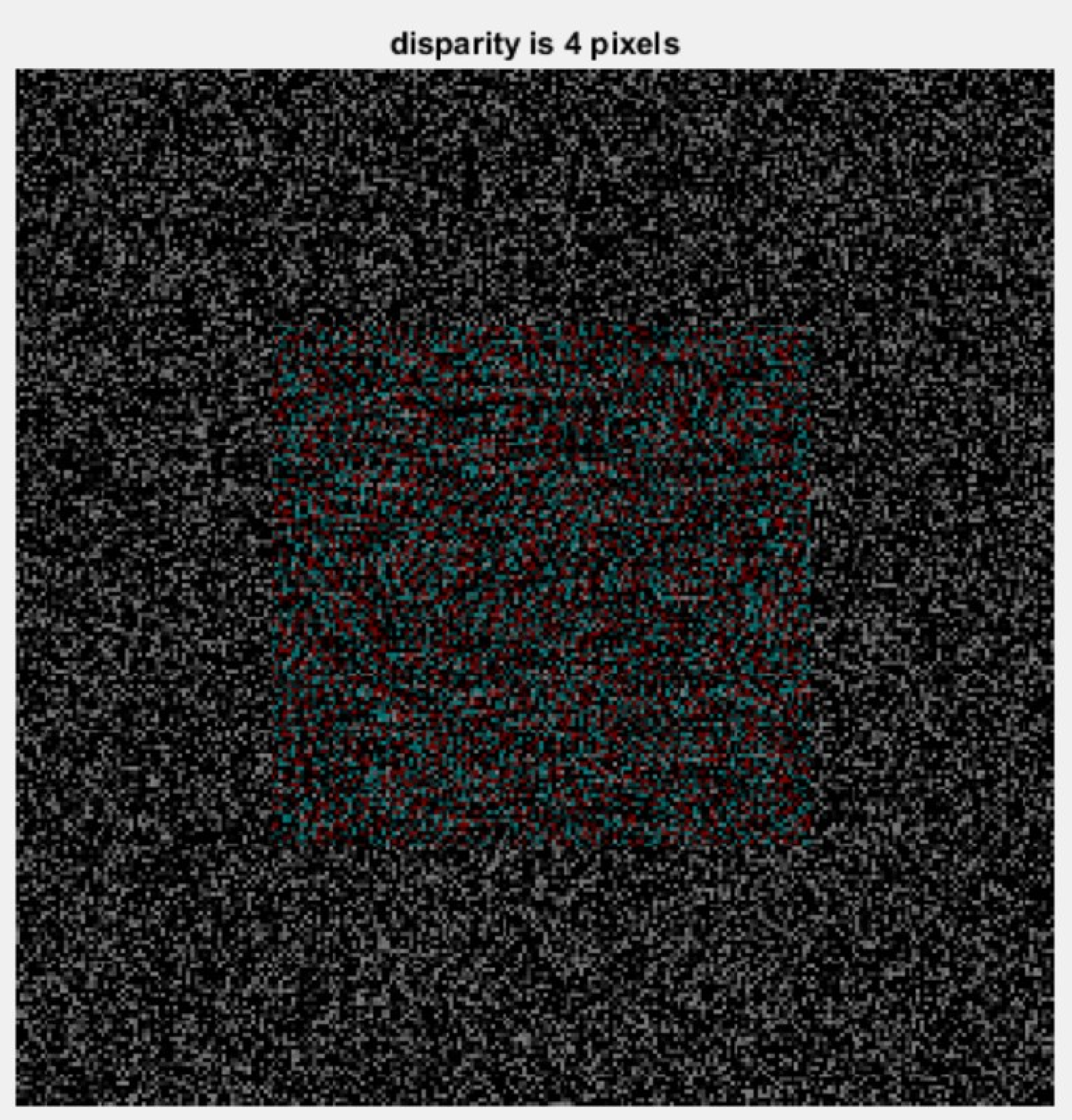


0%

Motion coherence



Task: estimate disparity of patch

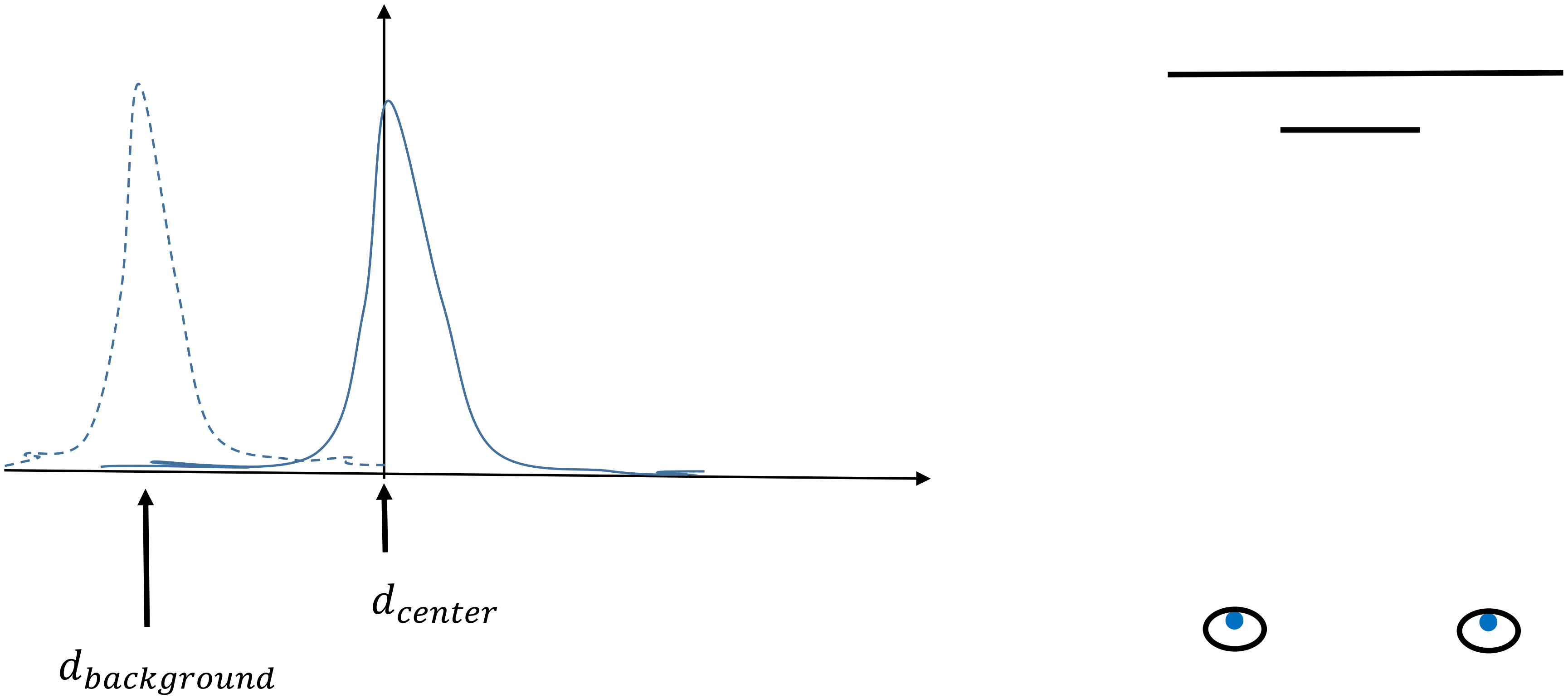


left eye

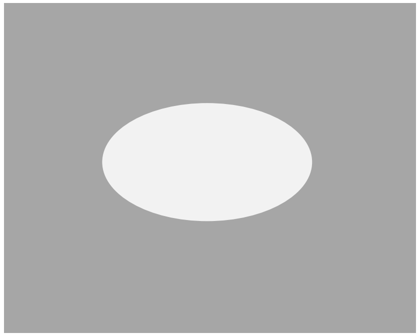
right eye

We could add noise by independently randomizing B&W value of bits in left and right images.

likelihood of disparities of background (dashed) and central (solid) square

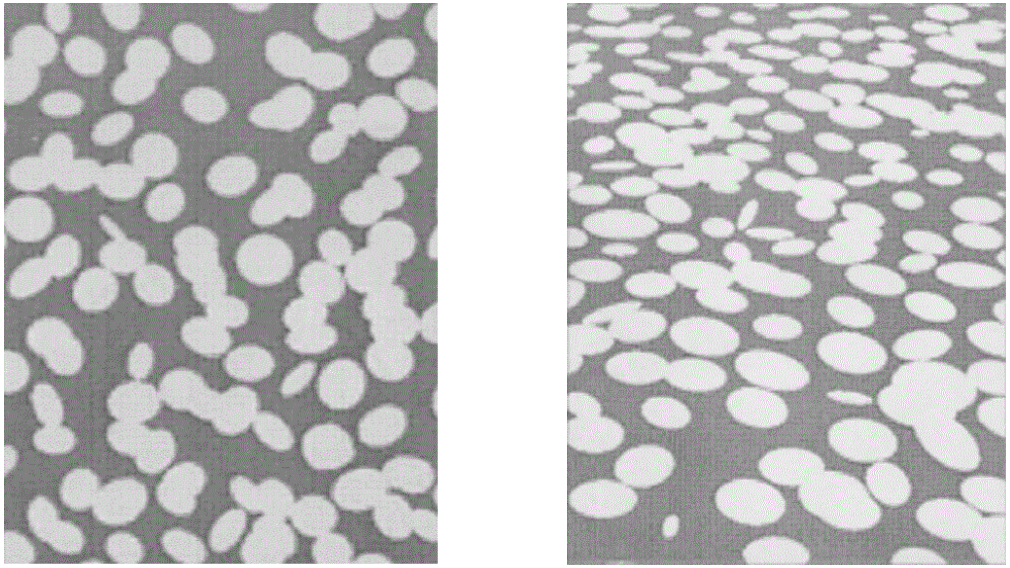


Task: estimate surface slant



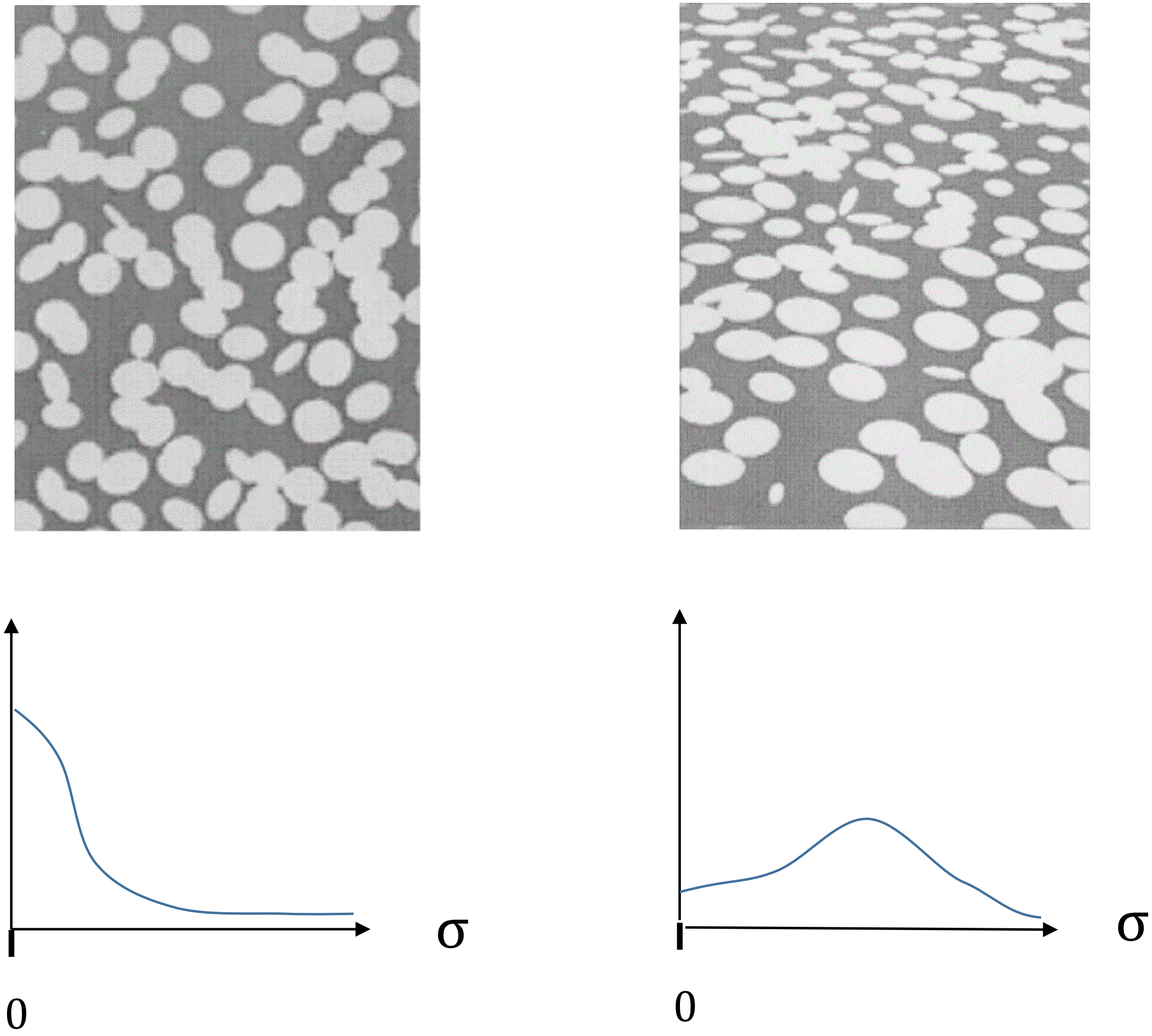
Is this an ellipse on a frontoparallel plane,
or a disk on a slanted plane?

Task: estimate the slant from texture



Random distribution of disk shapes and sizes (rather than pixel noise).

[Knill, 1998]



likelihood
of slant σ

What is the formal definition of “likelihood” ?

Review of Probability (Sketch)

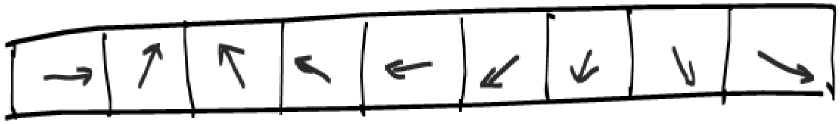
$$\left.\begin{array}{l} p(I) \\ \\ p(S) \end{array}\right\} \text{marginal probabilities}$$

$p(I, S)$ – joint probability

$$\left.\begin{array}{l} p(I|S) \equiv \dfrac{p(I, S)}{p(S)} \\ \\ p(S|I) = \dfrac{p(I, S)}{p(I)} \end{array}\right\} \text{Conditional probabilities}$$

scene variables (not directly measured)

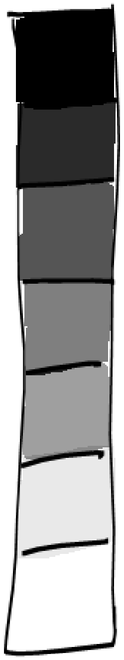
$S = s$



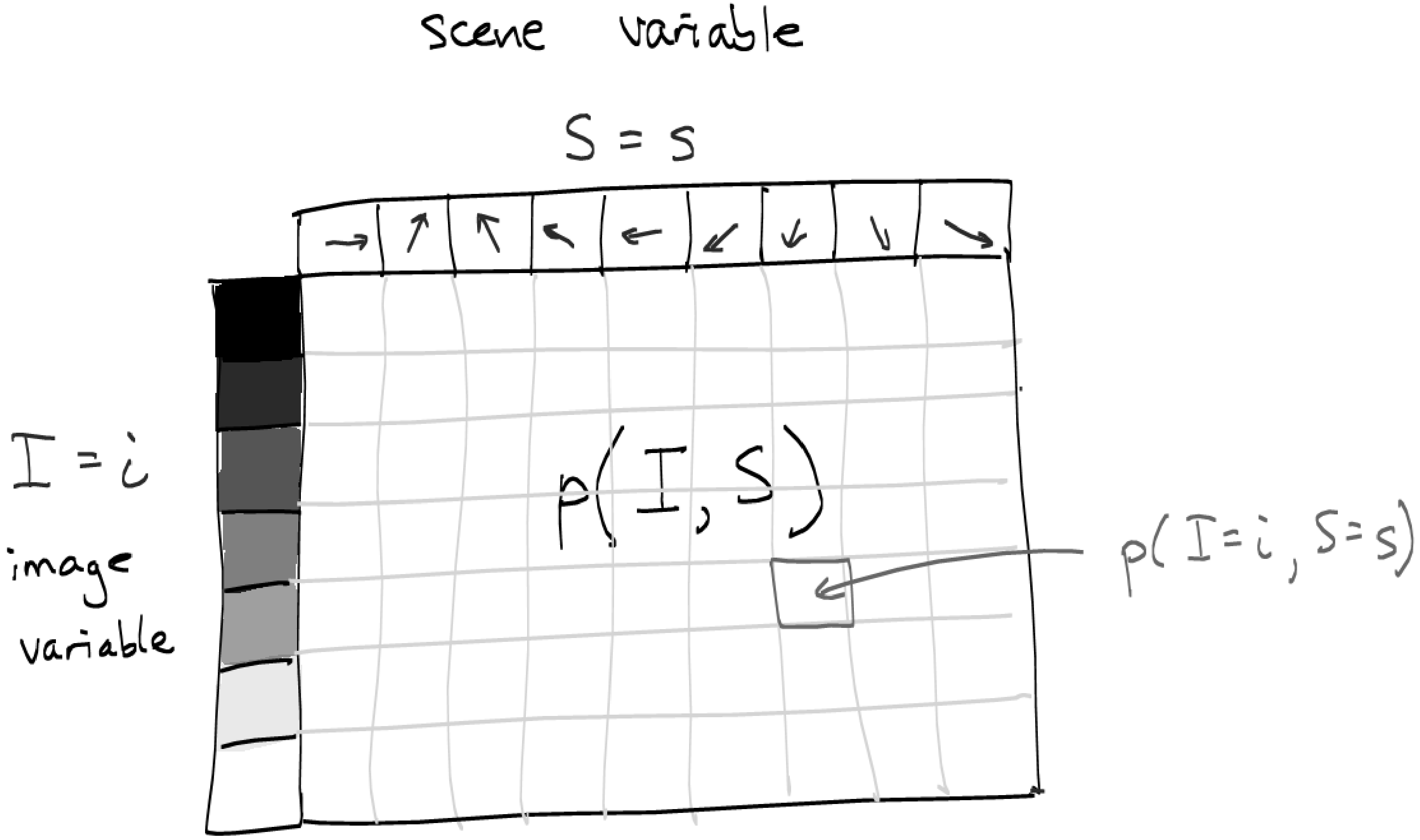
$p(S = s)$

$I = i$

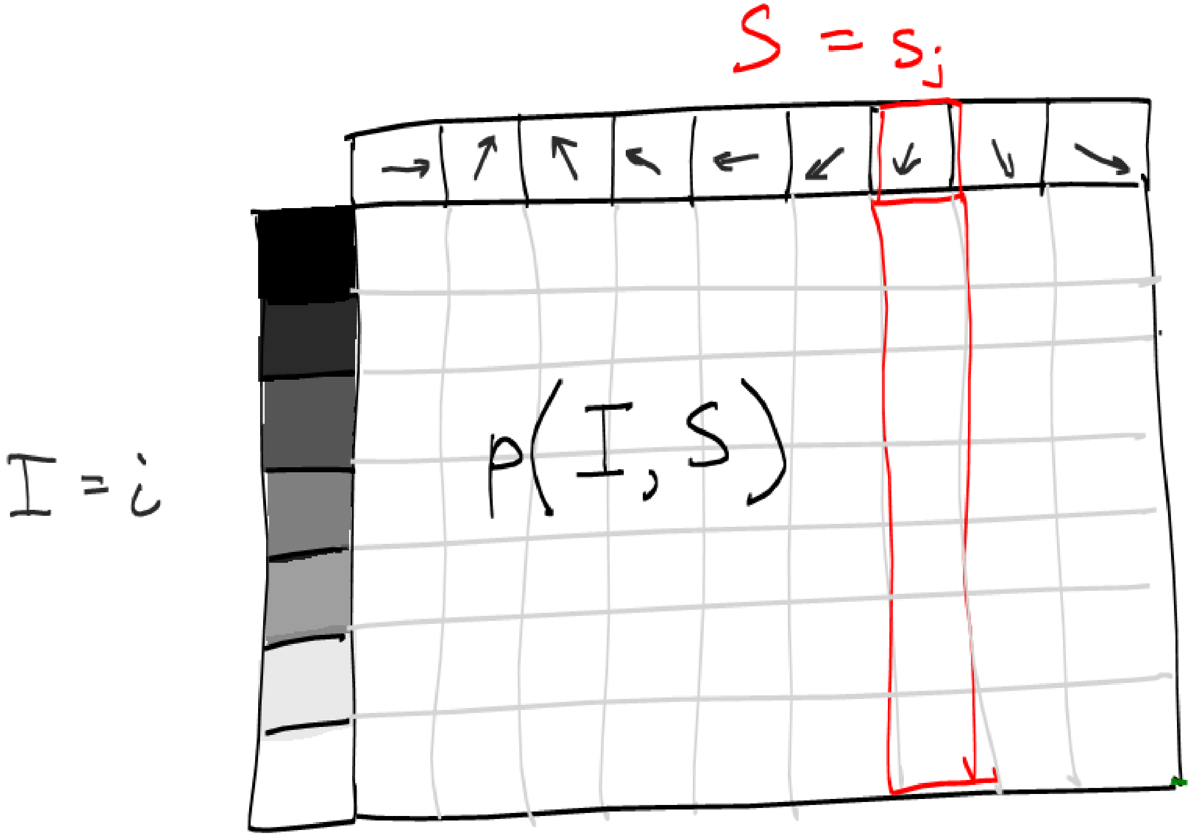
image intensities or Gabor outputs (directly measured)



$p(I = i)$

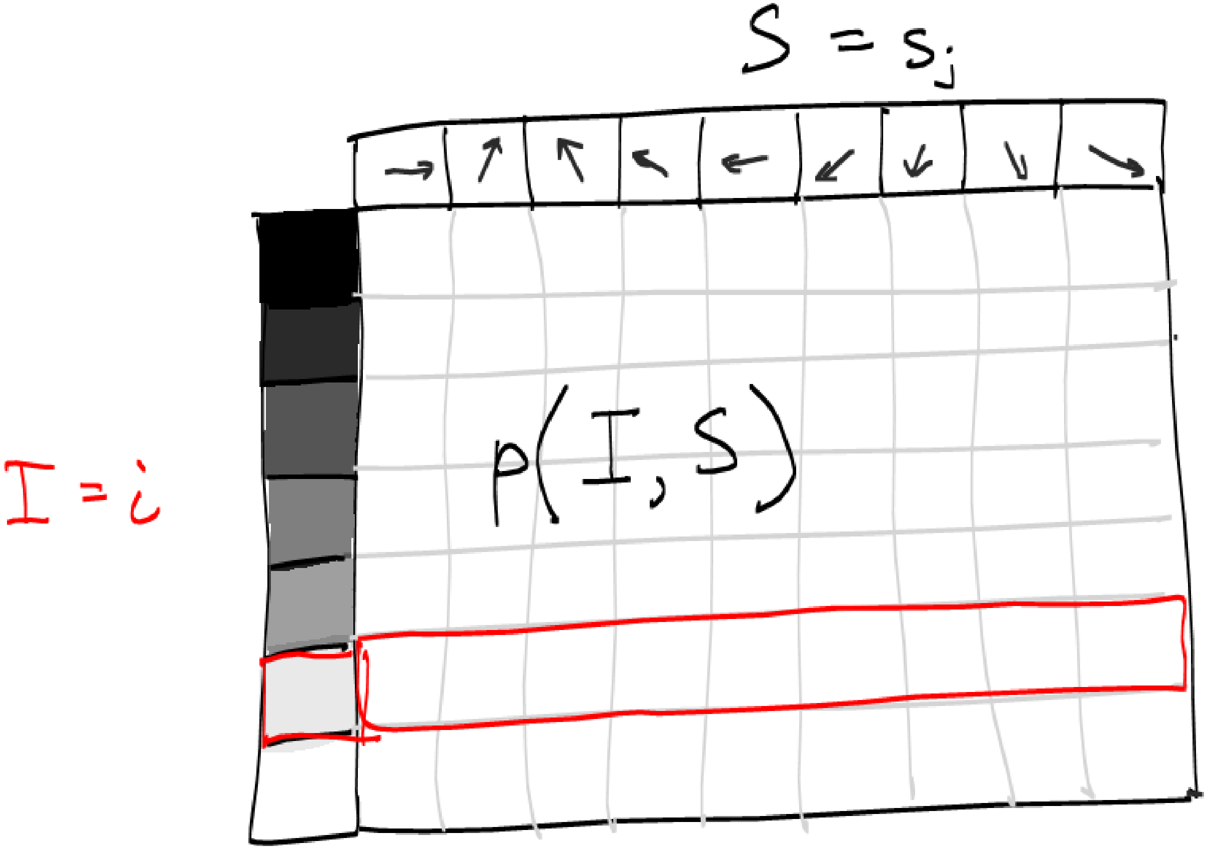


$p(I,S)$ is a "joint" probability function.



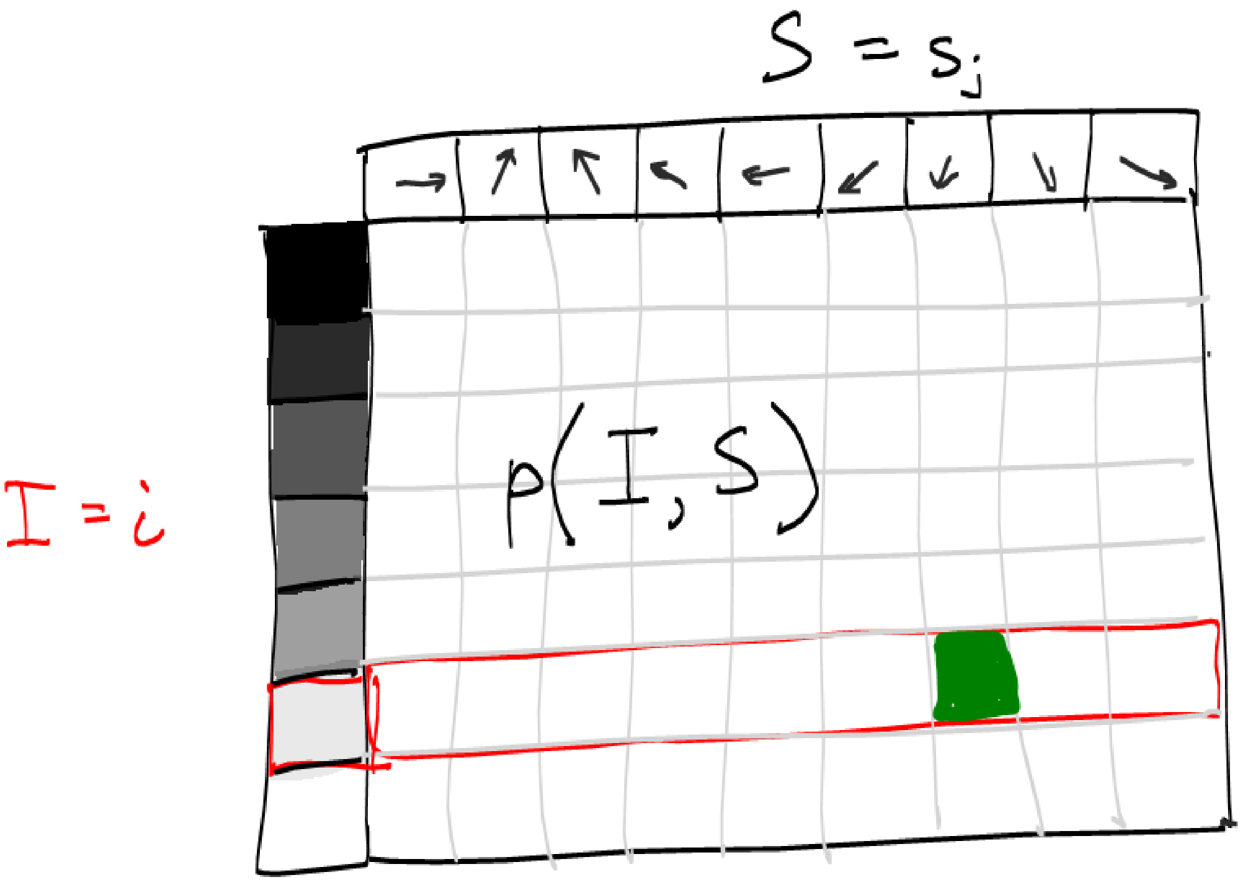
$p(S)$ is a "marginal" probability function

$$p(S=s_j) = \sum_{i \in I} p(I=i, S=s_j)$$



$p(I)$ is a "marginal" probability function

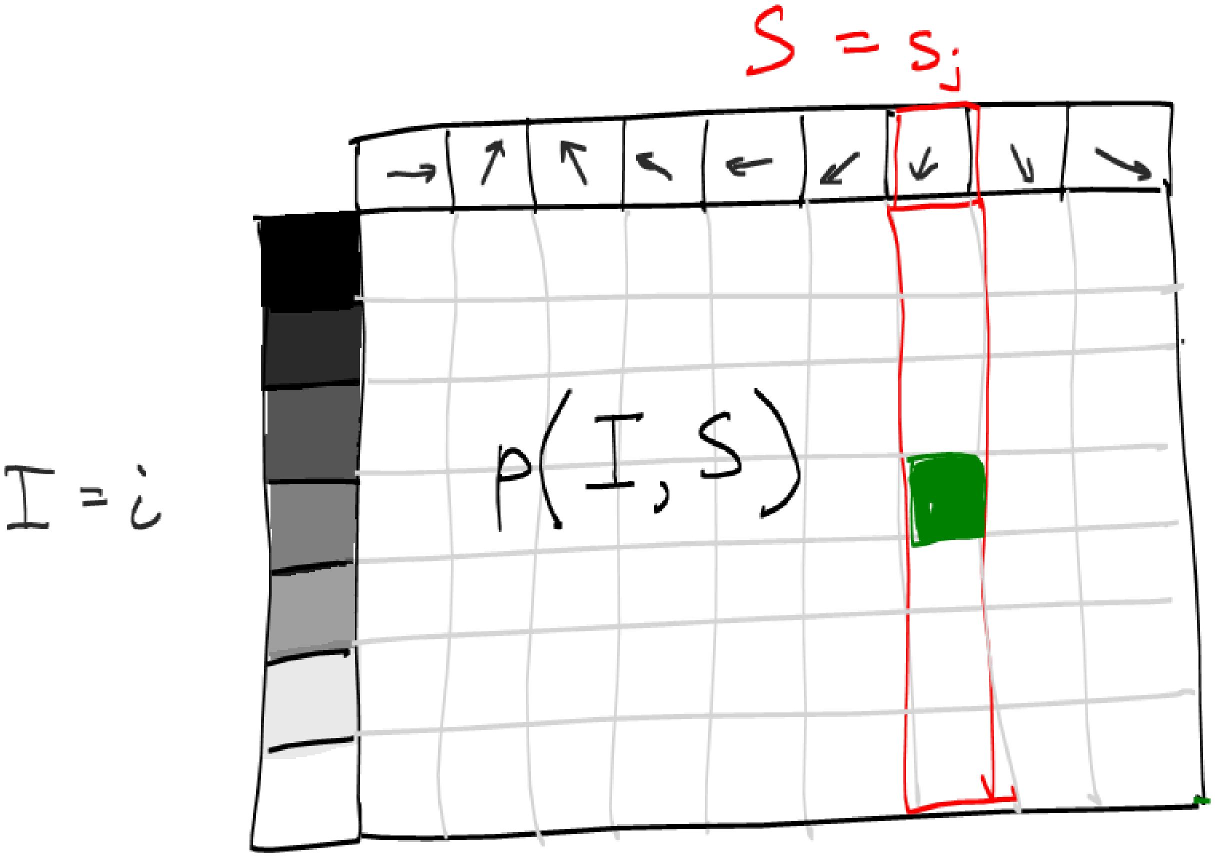
$$p(I=i) = \sum_{s_j \in S} p(I=i, S=s_j)$$



$$p(S \mid I) = \frac{p(S, I)}{p(I)}$$

is a conditional probability function.

$$p(S=s_j \mid I=i) = \frac{p(I=i, S=s_j)}{\sum_{s_j \in S} p(I=i, S=s_j)}$$



$$p(I \mid S) = \frac{p(I,S)}{p(S)}$$

is a conditional probability function.

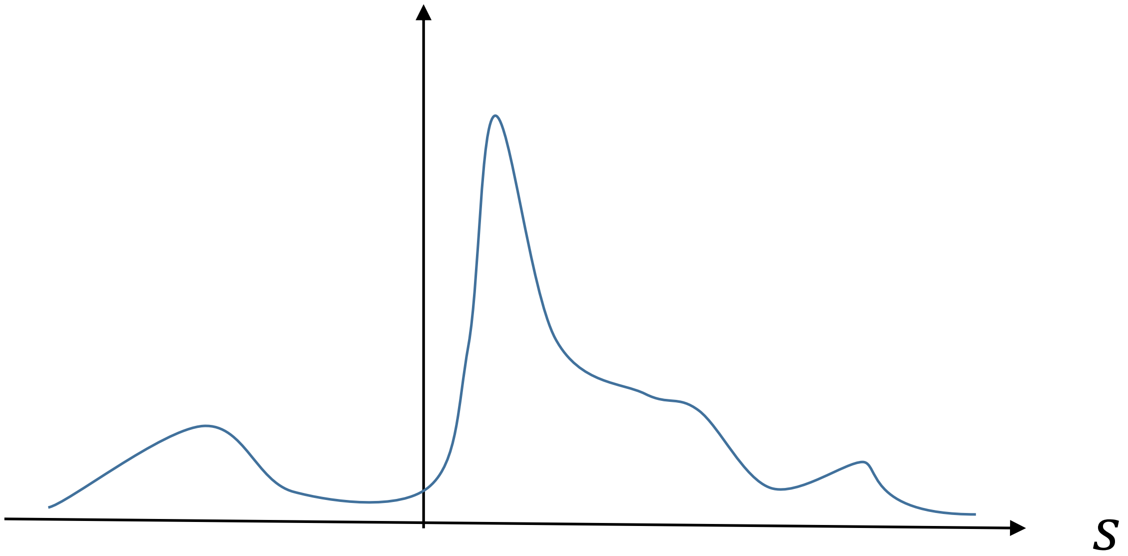
$$p(I=i \mid S=s_j) \equiv \frac{p(I=i,\, S=s_j)}{\sum_{i \in I} p(I=i,\, S=s_j)}$$

Likelihood

The conditional probability

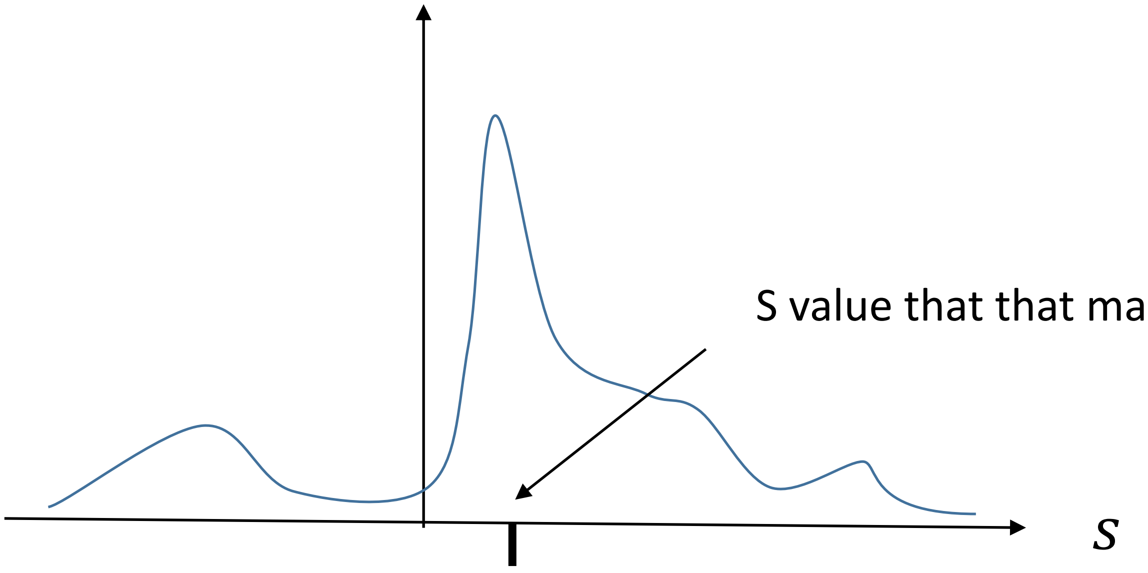
$$p(\, I = i \mid S = s \,)$$

is known as the “likelihood” of $S = s$, for a given image $I = i$.



Maximum likelihood estimation:

Given an image $I = i,$ choose the scene $S = s$ that maximizes $p(I = i \mid S = s)$.



S value that that maximizes likelihood

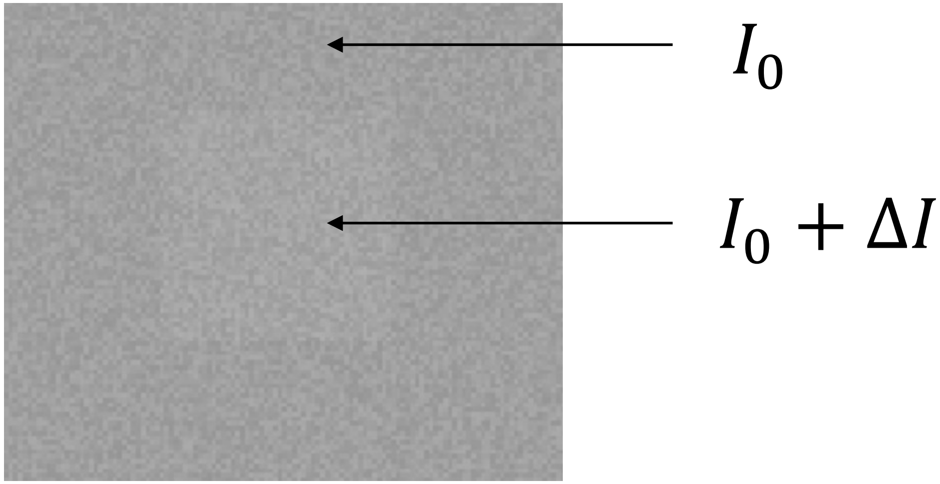
Maximum likelihood estimation in vision

Image *I* = i $\longrightarrow$ Estimated S = $\hat{s}$

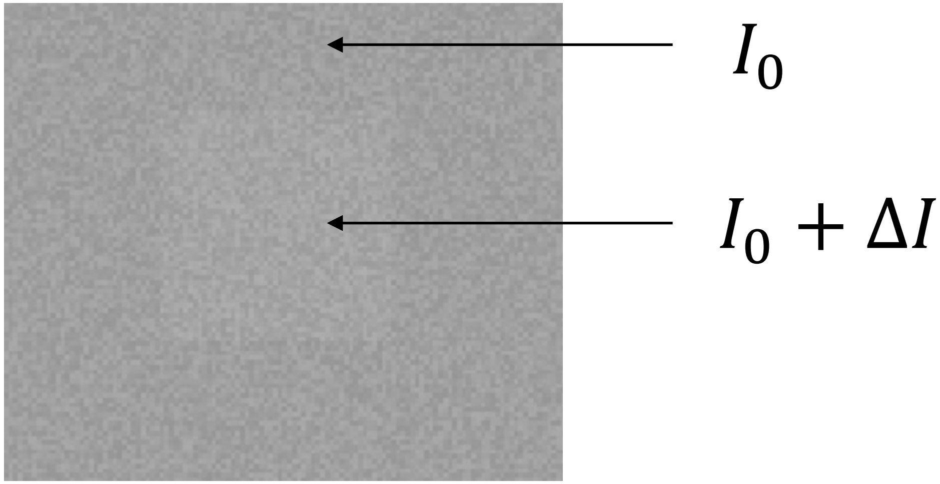
image intensity
filter responses

luminance
orientation
disparity
motion
surface slant, tilt
…

Task: estimate $I_{0,} I_0 + \Delta I$ in presence of noise



Task: estimate $I_{0,} I_0 + \Delta I$ in presence of noise



Additive Gaussian noise n : mean 0 and variance ${\sigma_n}^2$.

$$I_{center}(x, y) = I_0 + \Delta I + n(x, y)$$

$$I_{surround}(x, y) = I_0 + n(x, y)$$

$$I_{surround}(x, y) = I_0 + n(x, y)$$

Let’s define a likelihood function for I_0 :

$$p(\, I_{surround}(x, y) \mid I_0 \,) = p\big(\, n(x, y)\big)$$

$$I_{surround}(x,y) = I_0 + n(x,y)$$

$$p(\, I_{surround}(x,y) \mid I_0 \,) = p\Big(\overbrace{n(x,y)}^{I_{surround}(x,y) \;-\; I_0}\Big)$$

$$I_{surround}(x,y) = I_0 + n(x,y)$$

$$p(\, I_{surround}(x,y) \mid I_0 \,) = \underbrace{p\Big(\overbrace{n(x,y)}^{I_{surround}(x,y) - I_0} \Big)}_{\uparrow}$$

Gaussian pixel noise $$\frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{n(x,y)^2}{2\,{\sigma_n}^2}}$$

Independent Random Variables

Two random variables X_1 and X_1 are independent if, for all values x_1 and x_2,

$$p(\, X_1 = x_1, X_2 = x_2) = p(\, X_1 = x_1) \; p(X_2 = x_2)$$

The same definition holds for many random variables.

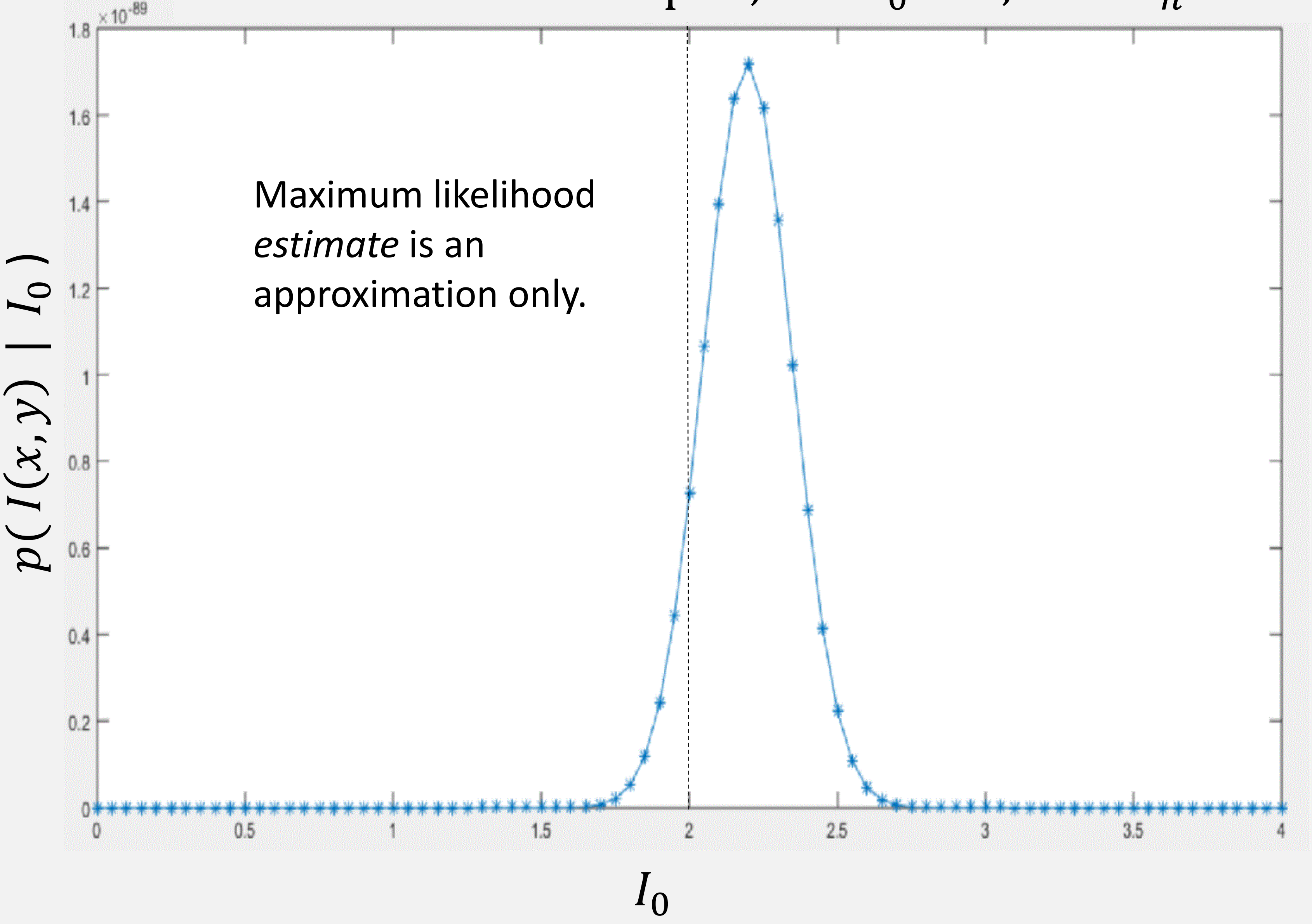
The example here is pixel noise.

$$I_{surround}(x, y) = I_0 + n(x, y)$$

Likelihood for I_0 for all pixels (x, y) in the surround:

$$p(I_{surround} \mid I_0) = \prod_{(x,y)} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(I_{surround}(x,y) - I_0)^2}{2 \sigma_n{}^2}}$$

$N = 400$ noise samples, $I_0 = 2$, $\sigma_n = 3$



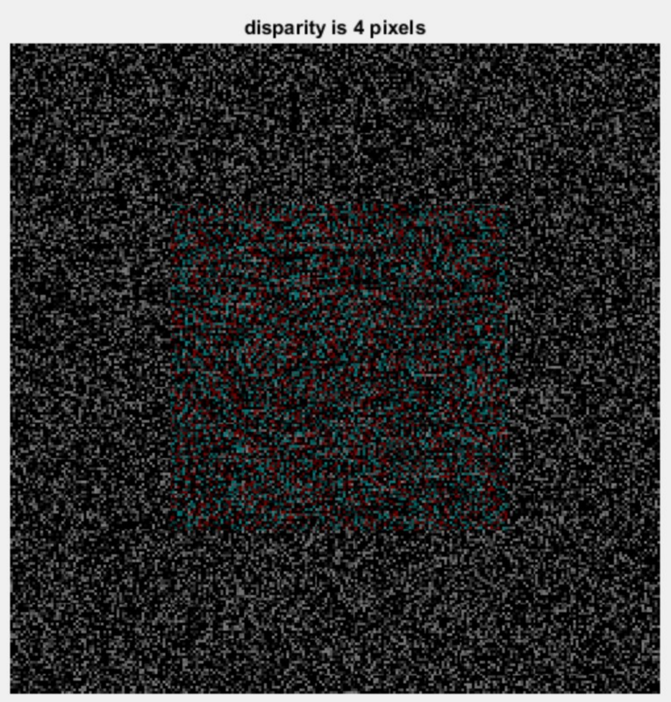
Maximum likelihood *estimate* is an approximation only.

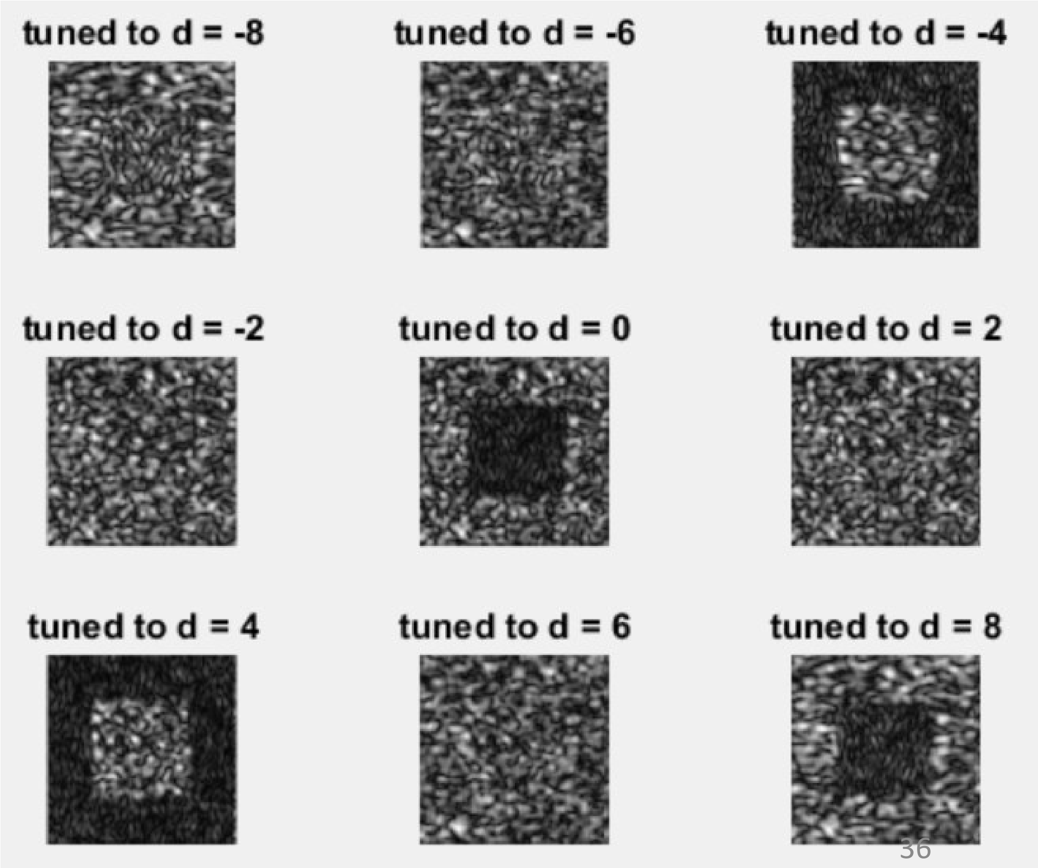
See exercises. http://www.cim.mcgill.ca/~langer/546/MATLAB/likelihood.m

Task: estimate disparity of patch

$$\mathrm{p}(responses(x,y,d_{tuned}) \; = \; \vec{r} \mid disparity = d\ )$$

It not obvious how to write down such a function.

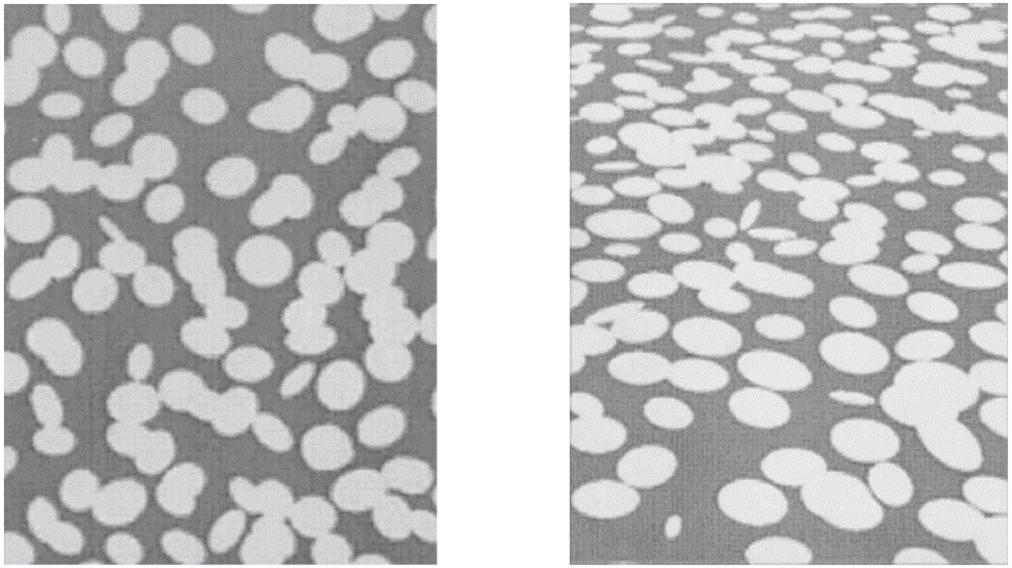




Task: estimate slant of surface

$$\mathrm{p}(\, I = i \mid S = s \,)$$

Given a set of image ellipses, $I = i$, and assuming some probability distribution of disk shapes *on the surface*, define the likelihood of different surface slants $S = s$. (For details, see papers by David Knill in 1990's.)

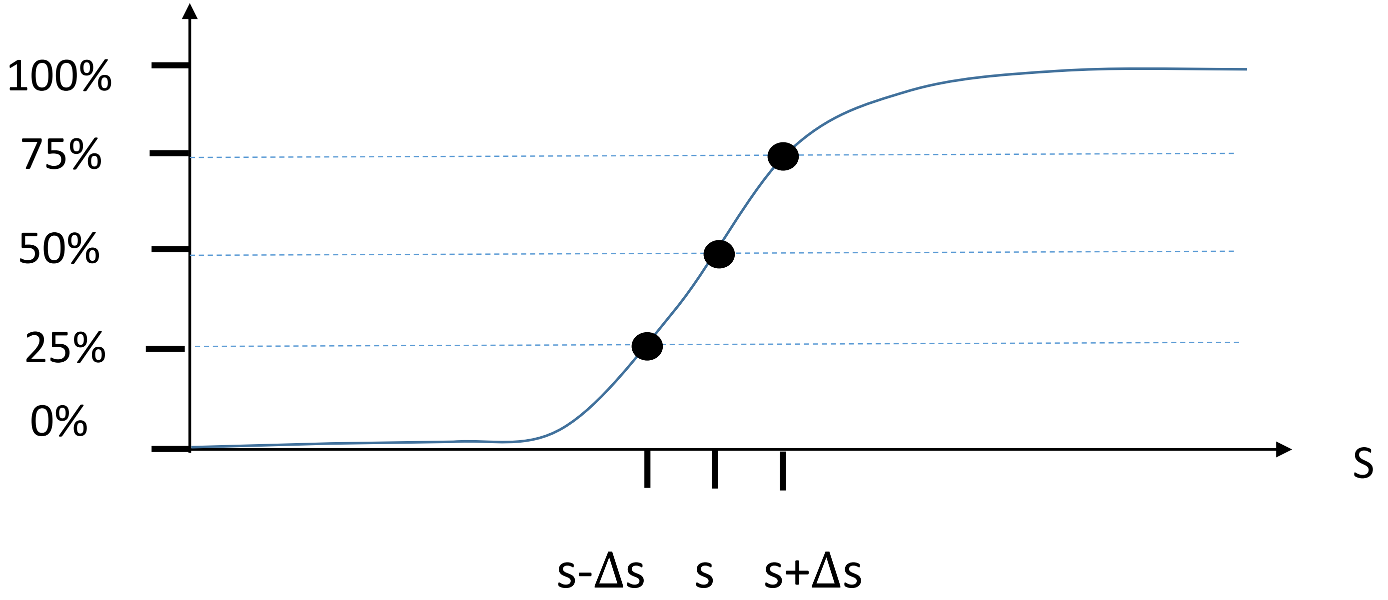


[Knill, 1998]

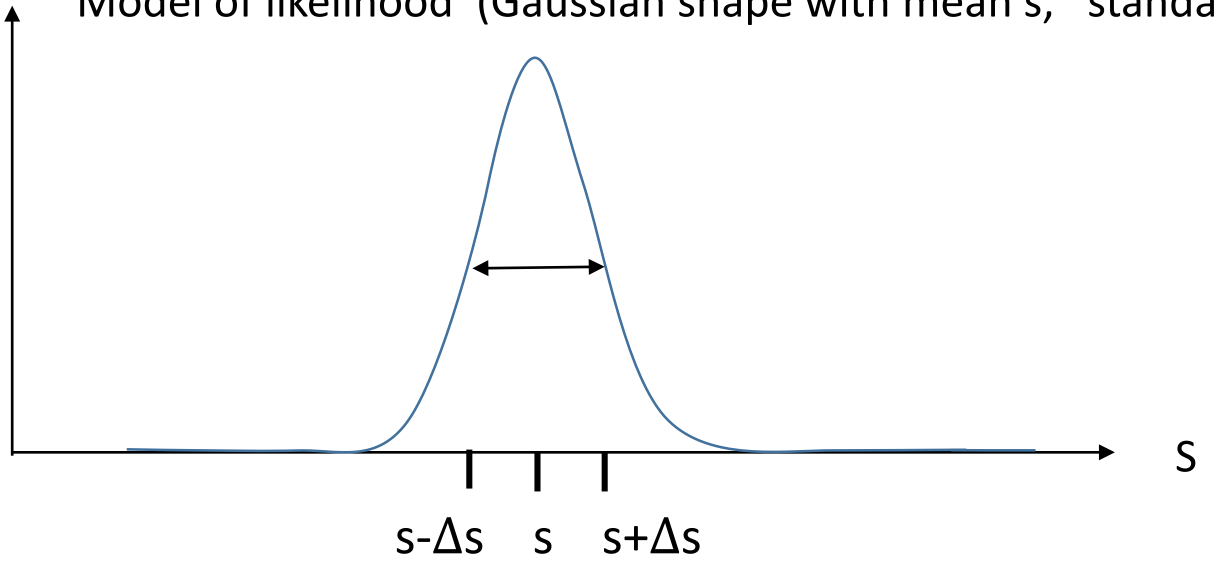
The above examples take an
“ideal observer” approach.

Can we model a human observer’s
uncertainty, using a likelihood function ?

Psychometric function (fit with cumulative Gaussian i.e. blurred step edge)



Model of likelihood (Gaussian shape with mean s, standard deviation Δs)



Q: How can a such a likelihood model *explain or predict* how a vision system estimates a scene parameter?

A: It can tell us how people combine different cues.
(Next lecture)