

## lecture 24

review for final exam

## Tips for final exam

Technical things you do NOT need to review from lectures 1-8:

- blur (thin lens equation)
- slant & tilt
- radiometry definitions (radiance, irradiance, BRDF, camera response, color, vignetting, dynamic range)

## General Principle

- Course had 23 lectures  
 $\therefore \sim \frac{100}{23} \approx 4\frac{1}{2}$  point per lecture
- I have already used 70% (10 midterm + 60 assignment)
- I will try to use the last 30% to cover "other" material

## Image Processing (lectures 8-15)

- ~~edge detection (Canny)~~ A2
- ~~convolution~~ A2, A3
- ~~scale space constructions~~ A3
- ~~blob detection~~ A3
- image registration A3 + final
- SIFT final
- vanishing points final
- least squares final
- Hough and RANSAC final

## 3D Scene Inference (lectures 16-23)

- shape from shading final
- ~~shape from texture, depth from defocus~~ A3
- SVD final ~~non-linear least squares~~
- ~~camera calibration~~ A4
- homographies A4 (also final)
- structure from motion: factorization final
- stereo 1: epipolar geometry final
- stereo 2: correspondence final

Vision is "inverse optics"

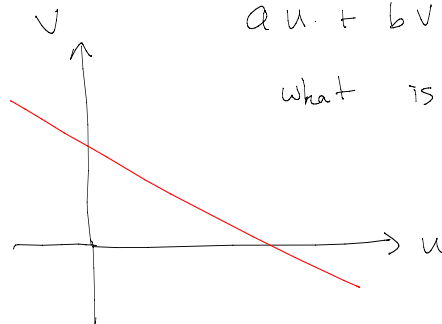
(given an image, infer the 3D scene that produced it)

Key difficulty: images often do not fully specify the 3D scene. (non-invertibility)

Given  $a, b, c$   
and a model

$$a_u u + b_v v + c = 0,$$

What is  $u, v$ ?

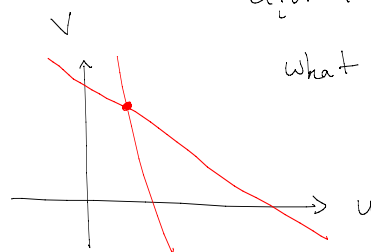


Solution: use more constraints / data

Given  $\{(a, b, c)_i\}$  and a model

$$a_{i,u} u + b_{i,v} v + c_i = 0,$$

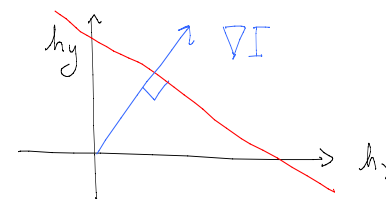
What is  $u, v$ ?



Example: Image Registration

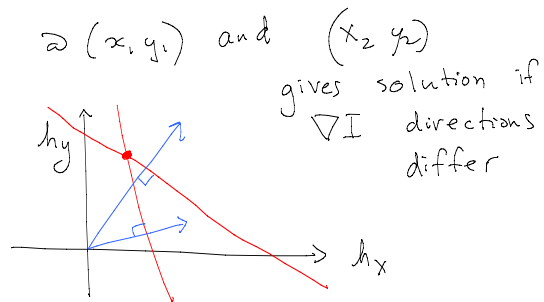
$$I(x, y) \quad J(x, y)$$

$$\frac{\partial I}{\partial x} h_x + \frac{\partial I}{\partial y} h_y + I - J = 0$$



To estimate  $(h_x, h_y)$  we need more constraints.

$$\frac{\partial I}{\partial x} h_x + \frac{\partial I}{\partial y} h_y + I - J = 0$$



Fitting a model with no noise

$$A\vec{x} = b \Rightarrow \text{solve for } \vec{x} \neq 0$$

$$A\vec{x} = 0 \Rightarrow \text{find null space (SVD)}$$

Other Examples

- vanishing points  $(x_v, y_v)$   $\left\{ \begin{array}{l} Ax=b \\ Ax=b \\ Ax=b \end{array} \right.$
- camera calibration (P)  $\left\{ \begin{array}{l} Ax=b \\ Ax=b \end{array} \right.$
- photometric stereo  $(p, \vec{n})$   $\left\{ \begin{array}{l} Ax=b \\ Ax=0 \end{array} \right.$
- structure from motion (RS)  $\left\{ \begin{array}{l} Ax=0 \\ Ax=0 \end{array} \right.$
- homographies (H)  $\left\{ \begin{array}{l} Ax=0 \\ Ax=0 \end{array} \right.$
- fundamental matrix (F)  $\left\{ \begin{array}{l} Ax=0 \\ Ax=0 \end{array} \right.$

Finding a null vector using SVD

$$\begin{matrix} n \\ m \end{matrix} \begin{bmatrix} A \\ \circ \end{bmatrix} \begin{matrix} x \\ \end{matrix} = \vec{0}$$

$$= \begin{matrix} m \times m \\ \end{matrix} \begin{bmatrix} U \\ \end{bmatrix} \begin{matrix} m \times n \\ \end{matrix} \begin{bmatrix} \Sigma \\ \circ \end{bmatrix} \begin{matrix} n \times n \\ \end{matrix} \begin{bmatrix} V^T \\ \text{row space of } A \\ \text{null space of } A \end{bmatrix} \begin{matrix} x \\ \end{matrix} = \vec{0}$$

MATLAB's SVD gives  $V_{n \times n}$ .

Vision is "inverse optics":

given an image, infer the parameters of a model that explains the image

Key difficulties:

- image data may not fully constrain the solution (use more data, constraints)
- images have noise  $\Rightarrow$  least squares
- not all data fits the model  $\Rightarrow$  ignore outliers (Hough, RANSAC)

Examples

- vanishing points  $(x_v, y_v)$
- camera calibration (P)
- photometric stereo  $(p, \vec{n})$
- structure from motion (RS)
- homographies (H)
- fundamental matrix (F)

$$A\vec{x} = 0$$

$$A\vec{x} = b$$

$$A = U \Sigma V^T$$

$$\begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

Typically a least squares model assumes certain singular values are 0. This gives the solution.

Sometimes, though, adding more data (and ignoring outliers) and setting singular values to 0 still doesn't fully constrain the solution.

Factorization Method for Structure from Motion

$$\begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}_{2F \times N} = \tilde{R}_{2F \times 3} S_{3 \times N} \begin{bmatrix} R \\ S \end{bmatrix}$$

$$= \left( \tilde{R}_{2F \times 3} Q_{3 \times 3} \right) \left( \tilde{Q}_{3 \times 3}^{-1} S_{3 \times N} \right)$$

## Projective Reconstruction Ambiguity

$$\begin{pmatrix} w_1 x_1 \\ w_1 y_1 \\ w_1 \end{pmatrix} = P_1 \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = P_1 M \bar{M}^{-1} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} w_2 x_2 \\ w_2 y_2 \\ w_2 \end{pmatrix} = P_2 \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = P_2 M \bar{M}^{-1} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

e.g.  $M$  could be translation, rotation, shear, or much more complex.

## "Photometric Stereo" (see shape from shading)

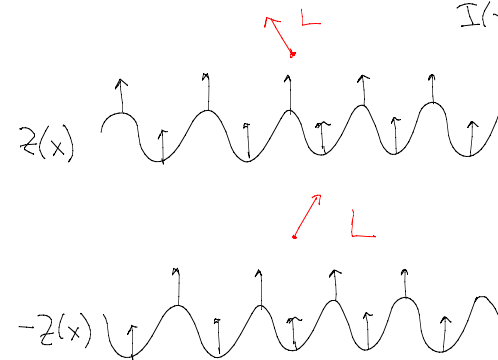
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -L_1 - \\ -L_2 - \\ -L_3 - \end{bmatrix}_{3 \times 3} \begin{bmatrix} \rho \vec{n} \end{bmatrix}_{3 \times 1}$$

$$= \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}_{3 \times 3} Q Q^{-1} \begin{bmatrix} \rho \vec{n} \end{bmatrix}_{3 \times 3}$$



## Shape from shading (see Exercises 2)

$$I(x) = \vec{n} \cdot \vec{L}$$



## Hollow Mask

### Illusion

(see youtube video)

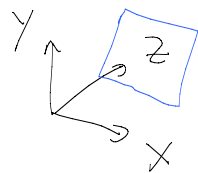
## Non-linear constraints

- shape from shading often assumes surface has constant reflectance
  - image registration  $I(x+h) \approx I(x)$  and stereo correspondence
  - orthogonal  $x, y$  camera axes in structure from motion (or skew  $s=0$  in camera calibration)
  - ...
- $\Rightarrow$  often one poses a non-linear, outlier-robust, least squares problem

Other similarities between cues / problems that arise from geometry

## Euclidean

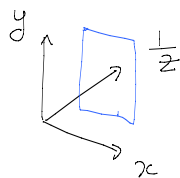
$$ax + by + cz = d$$



- shading on a sunny day
- structure from motion (factorization)

## Projective

$$ax + by + c = \frac{d}{z}$$



- vanishing points
- texture size & density
- depth from focus
- stereo disparity

## Shading and texture

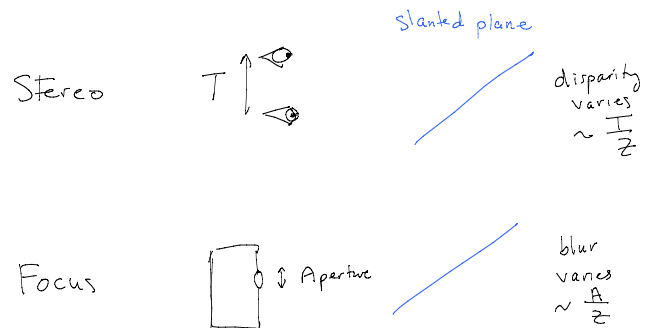
- surface normal & foreshortening
  - shading ( $\vec{n} \cdot \vec{L}$ )
  - texture ( $\vec{n} \cdot \vec{z}$ )
- distance
  - shading : vignetting,  $1/z^2$
  - texture : size, density

## flash camera & slanted plane



Think about shading (illumination) and texture information here.

Depth from focus and stereo



Homographies and Rectification

$$\begin{bmatrix} w \tilde{x} \\ w \tilde{y} \\ w \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

special cases

- $h_{31} = h_{32} = 0, h_{33} \neq 0$
- $h_{31}x + h_{32}y + h_{33} = 0$

Image Rectification

Suppose I want a homography that maps  $(x, y) = (0, 0)$  to the point at infinity in  $x$  axis direction

(Require the three rows are linearly independent)

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} * & * & 1 \\ * & * & 0 \\ a & b & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

More Computer Vision?

Winter 2011

- ECSE 626 Statistical Computer Vision
- COMP 766 Shape Analysis in Computer Vision

Fall 2011

- COMP 646 Computational Perception

COURSE  
EVALUATIONS

Deadline Sunday Dec. 5