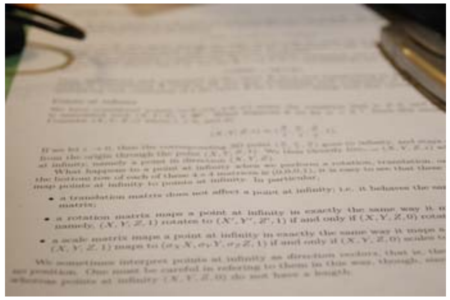
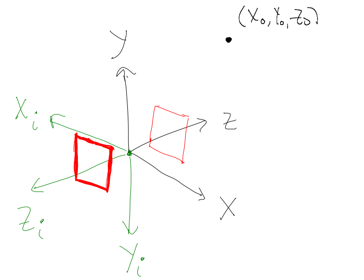
lecture 5

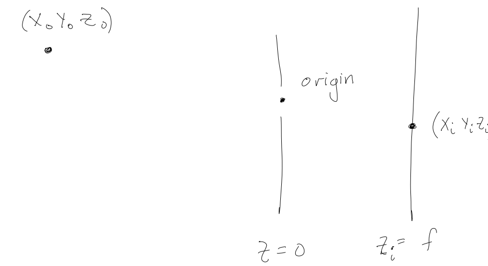
thin lens model, focus



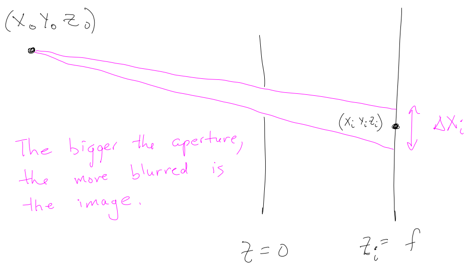
Real projection planes are behind the center of projection



Non-pinhole camera (finite aperture)

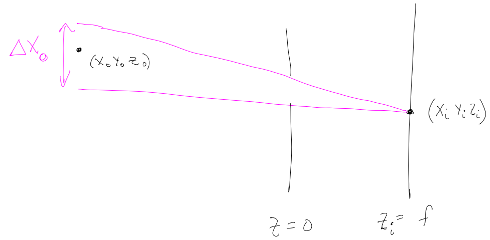


Forward projection

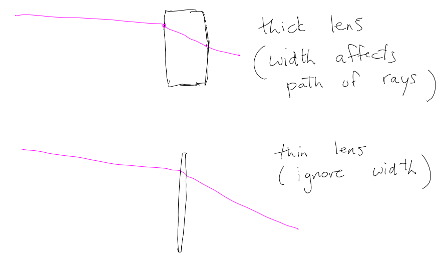


The bigger the aperture, the more blurred is the image.

Reverse Projection

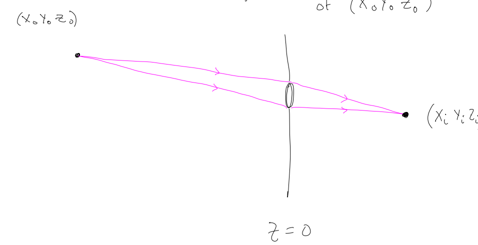


Thin lens



Conjugate Points

For all rays that leave ($X_0 Y_0 Z_0$) and pass through the lens, these rays all meet at some ($X_i Y_i Z_i$), called the conjugate point of ($X_0 Y_0 Z_0$)

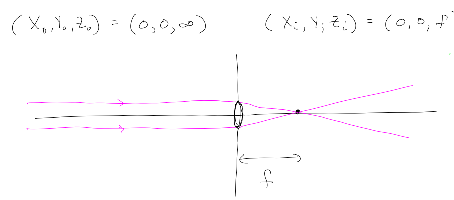


The path of a ray through a lens does not depend on the direction ← vs. →, so I will typically not specify the direction explicitly.

Our goal is to find a relationship between $X_0 Y_0 Z_0$ and $X_i Y_i Z_i$

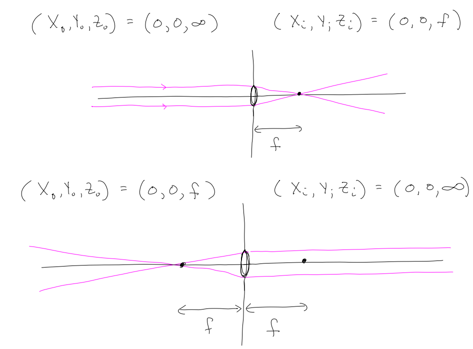
(thin lens equation $\frac{1}{f} = \frac{1}{Z_0} + \frac{1}{Z_i}$)

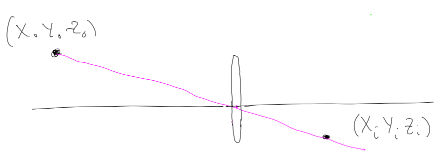
Focal length, f, of a lens



f is called the focal length.

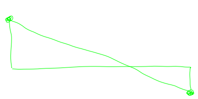
Note: if sensor plane is a distance f from the lens, then objects at ∞ will be "in focus" in the image.

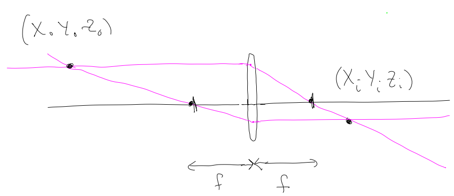




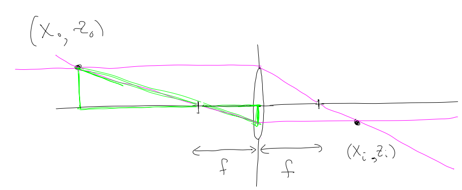
By similar triangles

$$\frac{X_i}{Z_i} = \frac{X_0}{Z_0}$$



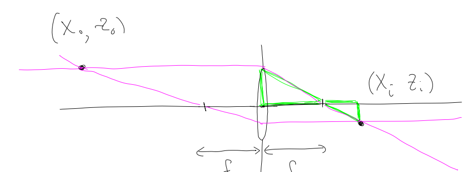


Note: this geometric argument only works for points very close to optical axis. A more complicated analysis is required for general points (omitted).



By similar triangles:

$$\frac{X_0}{Z_0 - f} = \frac{X_i}{f}$$



By similar triangles:

$$\frac{X_i}{Z_i - f} = \frac{X_0}{f}$$

$$\left.\begin{aligned}\frac{X_o}{z_o-f} &= \frac{X_i}{f}\\ \frac{X_i}{z_i-f} &= \frac{X_o}{f}\end{aligned}\right\}$$

$$\frac{X_i}{X_o} = \frac{f}{z_o-f} = \frac{z_i-f}{f}$$

$$\frac{1}{f} = \frac{1}{z_o} + \frac{1}{z_i}$$

THIN LENS EQUATION

$$\left.\begin{aligned}\frac{X_o}{z_o-f} &= \frac{X_i}{f}\\ \frac{Y_o}{z_o-f} &= \frac{Y_i}{f}\\ \frac{1}{f} &= \frac{1}{z_o}+\frac{1}{z_i}\end{aligned}\right\}$$

$$X_i = f\cdot\frac{X_o}{z_o-f}$$

$$Y_i = f\,\frac{Y_o}{z_o-f}$$

$$z_i = f\,\frac{z_o}{z_o-f}$$

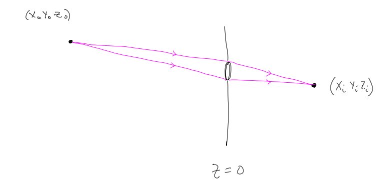
$$X_i = f\cdot\frac{X_o}{z_o-f}$$

$$Y_i = f\,\frac{Y_o}{z_o-f}$$

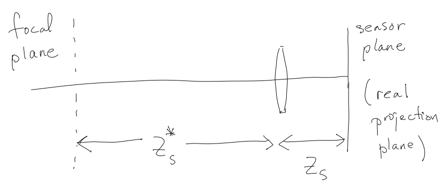
$$z_i = f\,\frac{z_o}{z_o-f}$$

$$\begin{bmatrix} X_i\\ Y_i\\ z_i\\ 1\end{bmatrix} \equiv \begin{bmatrix} fX_o\\ fY_o\\ fz_o\\ z_o-f\end{bmatrix} = \begin{bmatrix} f&0&0&0\\ 0&f&0&0\\ 0&0&f&0\\ 0&0&1&-f\end{bmatrix}\begin{bmatrix} X_o\\ Y_o\\ z_o\\ 1\end{bmatrix}$$

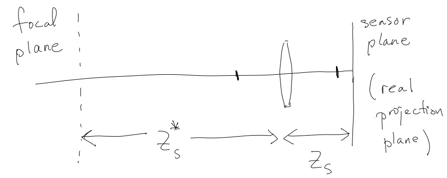
$$\begin{bmatrix} f&0&0&0\\ 0&f&0&0\\ 0&0&f&0\\ 0&0&1&-f\end{bmatrix}\begin{bmatrix} f&0&0&0\\ 0&f&0&0\\ 0&0&f&0\\ 0&0&1&-f\end{bmatrix} = \begin{bmatrix} f^2&0&0&0\\ 0&f^2&0&0\\ 0&0&f^2&0\\ 0&0&0&f^2\end{bmatrix}$$



$z = z_s^*$

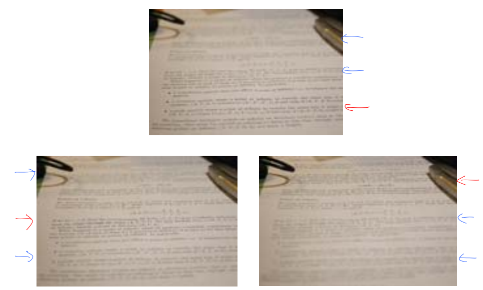


$$\frac{1}{f} = \frac{1}{z_s} + \frac{1}{z_s^*}$$

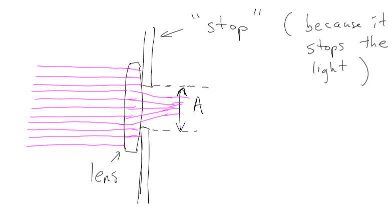


What if $z_s = f$?

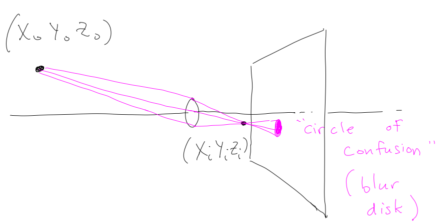
Moving the lens with respect to the sensor plane causes different parts of the scene to be "in" or "out" of focus.

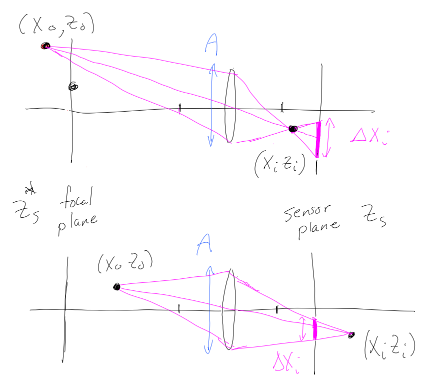


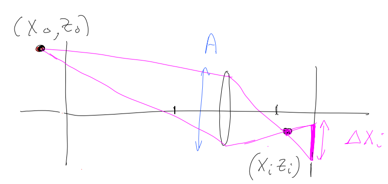
Aperture



In future diagrams, I will not draw the stop.

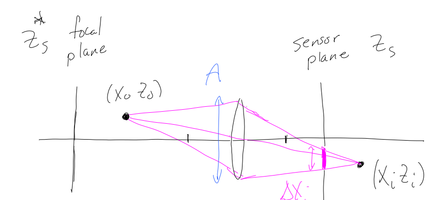






$$\frac{\Delta X_i}{z_s - z_i} = \frac{A}{z_i}$$

$$\Delta X_i = A\left(\frac{z_s}{z_i} - 1\right) = A\left(z_s\left(\frac{1}{f}-\frac{1}{z_o}\right) - 1\right)$$



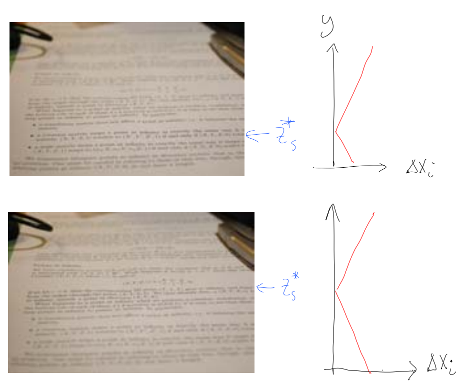
$$\frac{\Delta X_i}{z_i - z_s} = \frac{A}{z_i}$$

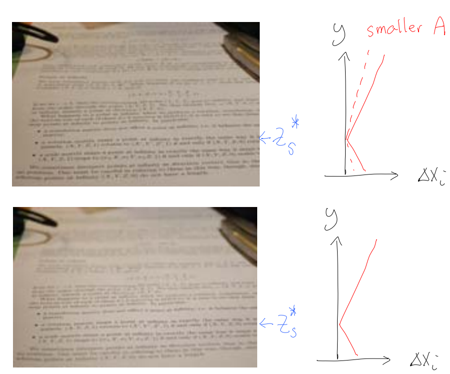
$$\Delta X_i = A\left(\frac{z_i - z_s}{z_i}\right) = A\left(1 - z_s\left(\frac{1}{f}-\frac{1}{z_o}\right)\right)$$

$$\Delta X_i = A\left|z_s\left(\frac{1}{f}-\frac{1}{z_o}\right) - 1\right|$$

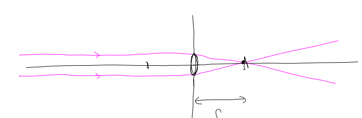
Recall that if you have a plane $aX + bY + cZ = 1$ in the scene, then $ax + by + cf = \frac{f}{Z}$.

So the blur width ΔX_i varies linearly with x, y.





"f-number" $N \equiv \frac{f}{A}$



- Aperture A often written "f/#"
  eg. $f = 80$ mm, $N = 5.6$, $f/\# = \frac{80}{5.6}$ mm
- f-number is (angle subtended by lens)$^{-1}$