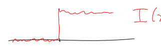
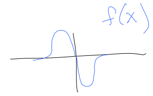
lecture 10

- 2D edge detection
- "corner" detection

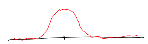
REVIEW Edge detection (Canny)

$I(x) = a\, u(x) + n(x)$





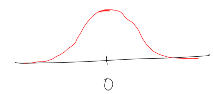
Compute $f(x) * I(x)$ and look for peak in response.



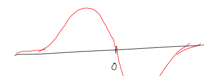
$f(x) * I(x)$

Choosing f(x)

$$g(x) = e^{-\frac{x^2}{2}}$$



$$\frac{d}{dx} g(x) = -x\, e^{-\frac{x^2}{2}}$$



$$\equiv f(x)$$

Gaussian function ("Normal" distribution)

$$G(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$G(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}, \quad g(x) = e^{-\frac{x^2}{2}}$$

How are the two related?

$$g_s(x) = e^{-\frac{s^2 x^2}{2}}$$

$$\frac{s}{\sqrt{2\pi}}\, g_s(x) = \frac{s}{\sqrt{2\pi}}\, e^{-\frac{s^2 x^2}{2}}$$

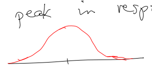
$$= G(x) \quad \text{where } \sigma = \frac{1}{s}$$

Edge detection (Canny)

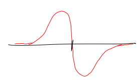
$I(x) = a\, u(x) + n(x)$

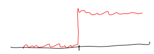
Compute $\frac{d}{dx}\left(g(x) * I(x) \right)$ and look for peak in response.

$g(x)$

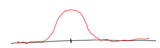


$\frac{d}{dx} g(x)$

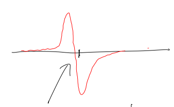




$I(x)$



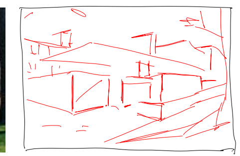
$I(x) * \frac{d\,g(x)}{dx}$



$\frac{d}{dx}\left(I(x) * \frac{d\,g(x)}{dx} \right)$

estimated location of edge ("zero crossing")

2D Edge Detection



Edges can occur at any orientation.

2D convolution

$$I(x,y) * f(x,y)$$

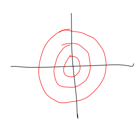
$$\equiv \sum_{x'} \sum_{y'} I(x',y')\, f(x-x', y-y') \quad \text{(discrete)}$$

$$\equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x',y')\, f(x-x', y-y')\, dx'\, dy' \quad \text{(continuous)}$$

2D blur

$$g(x) = e^{-\frac{x^2}{2}}$$

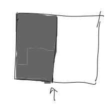
$$g(x,y) = e^{-\frac{x^2}{2}}\, e^{-\frac{y^2}{2}} = e^{-\frac{1}{2}(x^2+y^2)}$$



Vertical Edge detection

$I(x,y) = a\, u(x-x_0) + n(x,y)$

Compute $\frac{\partial}{\partial x}\left(g(x,y) * I(x,y) \right)$



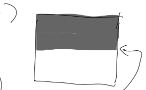
edge location $x = x_0$

Look for pixels (x,y) where there is a peak in response in the x direction.

Horizontal Edge detection

$I(x,y) = a\, u(y-y_0) + n(x,y)$

Compute $\frac{\partial}{\partial y}\left(g(x,y) * I(x,y) \right)$



edge location $y = y_0$

Look for pixels (x,y) where there is a peak in response in the y direction.

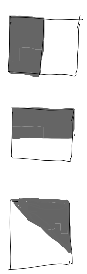
Edge detection at any orientation

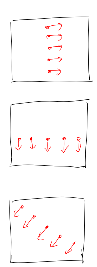
$$\left(\frac{\partial}{\partial x} g(x,y) * I(x,y),\ \frac{\partial}{\partial y} g(x,y) * I(x,y) \right)$$

$$= \nabla \left(g(x,y) * I(x,y) \right)$$

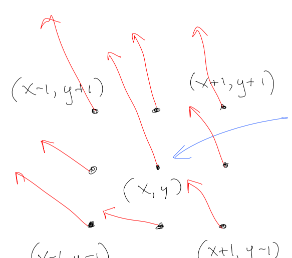
gradient operator

$I(x,y)$ | $\nabla \left(I(x,y) * g(x,y) \right)$





$\nabla g(x,y) * I(x,y)$



How to decide if there is an edge at (x,y)?

2D Edge Detection

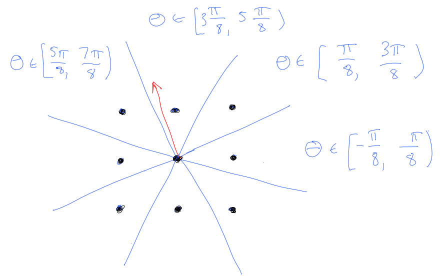
- compute $\nabla \left(g(x,y) * I(x,y) \right)$
- to decide if there is an edge at (x,y), check for a local maxima of $|\nabla (g * I)|$ in the direction of $\nabla (g * I)(x,y)$

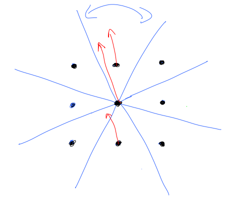
$$\nabla\left(g(x,y) * I(x,y)\right)$$

$$= (\cos\theta, \sin\theta)\ \left|\nabla\left(g(x,y) * I(x,y)\right)\right|$$

To decide if there is an edge at (x,y)
- compute $\theta(x,y)$ and look for maxima of $|\nabla g * I|$ in direction θ.

Example Approach (Trucco & Verri textbook)

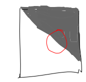




In case $\theta \in [3\frac{\pi}{8}, 5\frac{\pi}{8})$, check that three conditions are met:

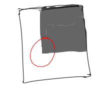
- $|\nabla g * I(x, y+1)| < |\nabla g * I(x,y)|$
- $|\nabla g * I(x, y-1)| < |\nabla g * I(x,y)|$
- $|\nabla g * I(x,y)| >$ threshold

edge

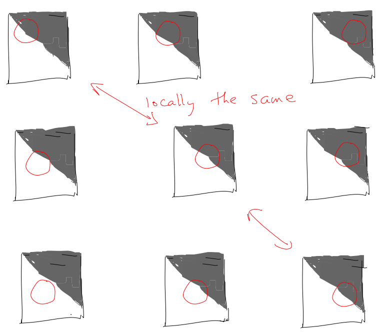


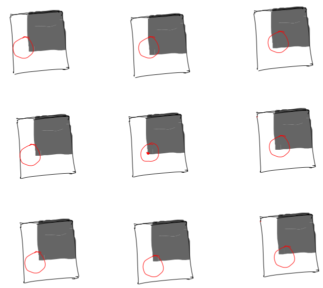
Pixels along an edge may be difficult to distinguish from each other.

Corner



Position is well defined.



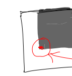


Notation: write $I(x,y)$ instead of $g(x,y) * I(x,y)$.

e.g. $\nabla I(x,y)$ instead of $\nabla g(x,y) * I(x,y)$

Is there a "corner" at (x_0, y_0)?

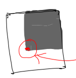
Formulation # 1:



How does:

$$\sum_{(x,y) \in Ngd(x_0 y_0)} \left(I(x,y) - I(x+\Delta x, y+\Delta y)\right)^2$$

vary with $(\Delta x, \Delta y) \in \{-1,0,1\} \times \{-1,0,1\}$?



Example:

Ngd(x_0, y_0) could be 5×5 pixels or a disk of diameter 5 pixels.

Formulation # 2

$$I(x+\Delta x, y+\Delta y)$$

$$\approx I(x,y) + \frac{\partial I(x,y)}{\partial x}\Delta x + \frac{\partial I(x,y)}{\partial y}\Delta y$$

$$\sum_{(x,y) \in Ngd(x_0 y_0)} \left(I(x,y) - I(x+\Delta x, y+\Delta y)\right)^2$$

$$\approx \sum_{(x,y) \in Ngd(x_0 y_0)} \left(\frac{\partial I(x,y)}{\partial x}\Delta x + \frac{\partial I(x,y)}{\partial y}\Delta y\right)^2$$

$$\sum_{(x,y) \in Ngd(x_0 y_0)} \left(\frac{\partial I(x,y)}{\partial x}\Delta x + \frac{\partial I(x,y)}{\partial y}\Delta y\right)^2$$

$$= \sum_{(x,y) \in Ngd(x_0 y_0)} (\Delta x, \Delta y)\ \nabla I(x,y)^T\ \nabla I(x,y) \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix}$$

$$= (\Delta x, \Delta y) \sum_{(x,y) \in Ngd(x_0 y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix} \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix}$$

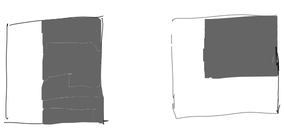
$$\geq 0$$

Second Moment Matrix

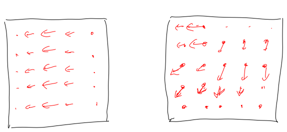
$$M = \sum_{(x,y) \in Ngd(x_0 y_0)} \begin{pmatrix}\frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y}\end{pmatrix} \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right)$$

$$= \sum_{(x,y) \in Ngd(x_0 y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

$I(x)$



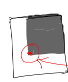
$\nabla I(x)$



$$M = \sum_{Ngd} \nabla I^T \nabla I \qquad \begin{bmatrix} 23 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 8 & 7 \\ 7 & 8 \end{bmatrix}$$

Is there a "corner" at (x_0, y_0)?

Formulation # 2:



How does quadratic form

$$(\Delta x\ \Delta y)\ M \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix}$$

vary with $(\Delta x, \Delta y)$ on unit circle?