

Questions

1. If camera (or eye) has a circular aperture, then the amount of light arriving at a point on the sensor plane is proportional to the area of the aperture. Note that we must consider the 2D area of the aperture, not just the 1D width of the aperture. With this in mind, how does the amount of light reaching any point in the image depend on the F-number? For example, if you wanted to double the amount of light in an image, how should you change the F-number?
2. Consider a fence consisting of cylindrical vertical posts that lie in a constant depth plane, the $Z = 10$ meter. That is, you are looking perpendicular to the depth plane containing the fence. Suppose the posts are spaced in the X direction every 10 cm, each post is $h = 2$ meters high, and the cylinder diameter of each post is $w = 1$ cm.

The fence posts appear as a sequence of vertical bars in the image. The posts are numbered $\dots, -2, -1, 0, 1, 2, \dots$ such that post 0 is at $X = 0$.

- (a) What is the visual angle of the height of post i ?
- (b) What is the visual angle of the width of post i ?
- (c) What is the visual angle between the centers of posts 0 and i ? That is, what is the angle from the center of the image to post i ? The angular distance from the center of the image is called the *eccentricity*.

Hint: The point of this exercise is not practice grade school arithmetic, doing mind numbing calculations. Rather, the point is for you to set up the calculation and to make sure you understand the definitions. You should also understand what a small angle approximation is, and when you can or cannot use it. For each question, you should ask yourself what is the small angle approximation and is it valid.

3. Suppose the left and right eyes are displaced by a distance $T_x = 6$ cm and the two eyes are parallel, i.e. both pointing in the Z direction.

Consider a scene consisting of a sphere whose center is at depth $Z = 10$ meters.

- (a) Suppose the visual angle subtended by the sphere is 0.5 degrees. What is the 3D diameter w of the sphere?
- (b) In order to use binocular disparities to perceive a depth difference between two points, your vision system needs to be able to detect a difference in the disparities of these two points. What is the maximum *difference* in disparities of two visible points on the sphere, measured in terms of visual angle? Hint: To answer this, you need to figure out which two points give the maximum depth difference.
- (c) Does the difference in disparity of two points on an object depend on where the eyes are looking, i.e. the vergence angle?
- (d) It is often claimed (incorrectly) that binocular vision only provides useful depth information for 3D points up to about 5 meters. How would you restate this claim in terms of binocular disparities?

4. (a) What is the *difference* in the binocular disparities of two scene points at depths 3 m and 6 m, respectively, *measured in degrees* ? Assume the eyes are 6 cm apart. State any other assumptions you make.
- (b) As I will discuss in lecture 2, since $\frac{1}{Z}$ effects come up so often in vision, it is common to have a special term of units of m^{-1} . The unit is called a *diopter*. Suppose two points are separated in depth by 0.4 diopters, i.e. $\frac{1}{Z_1} - \frac{1}{Z_2} = 0.4$. What is the difference in disparities of the two points? State your assumptions.

Answers

1. The cone of rays arriving at any point on the sensor has angular diameter A/f for both the X and Y directions, so the amount of light reaching a point on the sensor is proportional to $(\frac{A}{f})^2$, or the inverse square of the F-number.

Interestingly, the F-numbers that you can set on a typical camera have values

$$\sqrt{2}, \sqrt{4}, \sqrt{8}, \sqrt{16}, \sqrt{32}, \dots$$

which are usually written on the camera (or in the LCD menu) as

$$1.4, 2, 2.8, 4, 5.6, \dots$$

These settings step the amount of light in the image by a factor of 2, i.e. 2, 4, 8, 16, 32, ...

2. Note that the distance to the i th post in meters is $r = \sqrt{10^2 + (\frac{i}{10})^2}$ m.
 - (a) The angular height of post i is $\frac{h}{r}$ radians which decreases with i . Here we have a small angle since $h \ll r$, so we can use the small angle approximation.
 - (b) The angular width is $\frac{w}{r}$ radians which decreases with i . Again a small angle approximation is valid, since now $w \ll r$.
 - (c) Let e be the visual angle between the optical axis (Z axis) and post i . The letter e stands for *eccentricity*.

Because the posts are spaced every 0.1 m and are at a Z distance of 10 m, we get

$$\tan e = \frac{i/10}{10} = \frac{i}{100}. \quad (1)$$

For small i , we can use an small angle approximation

$$e = \frac{i}{100}$$

but as we get more into the periphery, this approximation gets worse, and we would need to take arctan to have an accurate expression.

$$e = \tan^{-1}\left(\frac{i}{100}\right).$$

3. (a)

$$\frac{\text{diameter}}{10} \cdot \frac{360}{2\pi} = 0.5^\circ$$

So the diameter is about 0.1 m.

- (b) The farthest point is at the side (rim) which is about at the same depth Z as the sphere center. The nearest point on a sphere is at the center of the sphere's projection ($Z - \Delta Z$). The two points have a depth difference which is approximately the radius of the sphere. You are being asked for the difference in disparity of these two points.

From the lecture, the disparity in radians is $\frac{T_x}{Z}$. The difference in disparity in radians thus:

$$T_x \left(\frac{1}{Z - \Delta Z} - \frac{1}{Z} \right)$$

But $\Delta Z = 0.05$ m from (a), i.e. half the diameter. Plugging in gives a difference of less than .002 degrees. This is very small, about 7 seconds of arc. (Recall that there are 60 minutes in 1 degree, and 60 seconds in 1 minute.)

- (c) No, it doesn't, at least if we use a simple approximation that rotating an eye by (θ_x, θ_y) shifts the visual directions of *all* points in the opposite direction, namely $(-\theta_x, -\theta_y)$. Look at the argument at the end of the lecture:

$$\text{disparity} = \left(\frac{x_l}{f} - \frac{x_r}{f} \right) - (\theta_l - \theta_r).$$

This holds for each point, so if we take the *difference* in disparities of two points, then the θ terms will cancel:

$$\text{disparity}_1 - \text{disparity}_2 = \left(\frac{x_{l,1}}{f} - \frac{x_{r,1}}{f} \right) - \left(\frac{x_{l,2}}{f} - \frac{x_{r,2}}{f} \right).$$

We sometimes say that the difference in disparity of two points is *invariant* to the positions of the eyes.

- (d) The claim basically says that all points beyond that distance (say 5 m) have roughly the same disparity, i.e. there may be small differences in disparities, but our visual systems cannot detect these differences.

Is this true? Absolutely not. For example, the difference in disparity between a point at 10 m and a point at infinity is $\frac{.06}{10} \cdot 57$ which is about $\frac{1}{3}$ degree. This is small but certainly not negligible!

4. (a) Assume the eyes are parallel, pointing straight ahead i.e. no vergence. As seen in the lecture, $x_l - x_r = \frac{T_x}{Z}$ radians. For the point at 3 m, $x_l - x_r = \frac{.06}{3} = \frac{1}{50}$ radians. For the point at 6 m, $x_l - x_r = \frac{.06}{6} = \frac{1}{100}$ radians. The difference is

$$\frac{1}{50} - \frac{1}{100} = \frac{1}{100}$$

or $\frac{57}{100}$ degrees, or about 34 arcmin.

- (b) Let the interocular distance T_X be 0.06 m. Multiplying by T_X gives $T_X \left(\frac{1}{Z_1} - \frac{1}{Z_2} \right) = 0.06 * 0.4 = .024$. This is the disparity difference in radians.

As we saw in Q3, it doesn't matter that we don't know the fixation (vergence) distance, since we are asking for the *difference* in disparities of the two points. Changing the vergence angle of the eyes would change the disparities of both points equally.