1. Consider a Hough transform method for identifying the rectangles in a noisy image, based on the outputs (x, y, θ) from an edge detector.

   What would be the dimension of the Hough voting space for this problem? Do not assume that the sides of each rectangle are parallel to the x and y axes.

   - 2
   - 3
   - 4
   - 5 (**answer**)
   - 9

   Solution: A rectangle with general orientation and position can be parameterized by 5 values, for example, a center point, a long axis length, a short axis length, and a rotation angle for the long axis length away from horizontal.

   A Hough transform method could create a coarse binning of this 5D space. For each edge detected, it could iterate over all bins and vote for whichever parameters are consistent with that edge.

   Comments: This question was not handled well (only 56% correct). This incorrect answers were spread over all choices.

2. Consider an image that contains a uniform gray disk on a white background, and consider a point $\mathbf{x}$ within this gray disk.

   Which of the following are possible values of the eigenvalues (λ_1, λ_2) of the second moment matrix $\mathbf{M}$ that is defined at $\mathbf{x}$?

   Select all that apply.

   - (0, 0) (**answer**)
   - (3, 1) (**answer**)
   - (4, 0)
   - (2, -1)

   Added as announcement during quiz: the gray disk is exactly 20 pixels wide, and the second moment matrix calculation only involves pixels from a window that is 10 pixels wide.

   Solution: If $\mathbf{x}$ is at the center of the gray disk, then all gradient vectors that contribute to $\mathbf{M}$ will be 0. In this case, both eigenvalues will be 0. If $\mathbf{x}$ sits near the boundary of the gray disk, then the boundary will be curved in the neighborhood of $\mathbf{x}$ and so multiple gradient directions will occur, which can lead to two positive eigenvalues.

   The only way that just one eigenvalue can be positive is if the gradient vectors that contribute to $\mathbf{M}$ are all parallel. But this cannot occur, since the boundary of the gray disk is curved.

   A few important notes about this case:

- The problem with the original formulation of the question is that a student could argue that if the disk was very large and the number of pixels in the image is very large, then the disk boundary would appear to be a straight line when viewed only a very small window.)
- As one student pointed out in office hours, if you approximate a gray disk with a pixellated image (so it has jagged edges) then there will be points on the disk boundary that form a straight edge over a local neighborhood. The student claimed that this sampling artifact can lead to a 0 eigenvalue. (I did not independently verify this.) If that is true, it is an interesting observation. But it is not enough for me to regrade the question, since this would require that I penalize students that did *not* select that choice.

The last choice is not possible since $\mathbf{M}$ cannot have a negative eigenvalue.

Comments: This question was handled ok. The most common mistake (40 students) was to select $(4, 0)$ as a choice. I have no way of evaluating how many students wrote up Matlab code and made the discovery above. BTW, since the question doesn't ask you to answer it by running Matlab, students should assume that it is meant to be answered by reasoning alone.

3. The Lucas-Kanade method assumes that the two images have approximately constant gradient over the local windows over which regions in each image *are* [1] compared, and that this condition is achieved by smoothing.

   - true
   - false **(answer)**

   Solution: This statement is false. The L-K method is based on intensity gradients, but it does not assume that the images have constant gradient in the local windows. Indeed, when the gradient *it* is constant, within the local window, it is impossible to estimate the 2D displacement (h_x, h_y) since the $\mathbf{M}$ matrix is not invertible.

   Comments: 60 (out of 102) students got this question wrong.

4. Consider an image containing a vertical white stripe on a gray background. The stripe is at x positions $x_0 < x < x_1$. Suppose we convolve this image with a normalized Laplacian of a Gaussian filter. Where would the global minima occur in scale space (x, y, σ)?

   - on a vertical line in the middle of the stripe and at scale $\sigma = x_1 - x_0$
   - on a vertical line in the middle of the stripe and at scale $\sigma = (x_1 - x_0)/2$ **(answer)**
   - on a vertical line in the middle of the stripe and at scale $\sigma = (x_1 + x_0)/2$
   - on the vertical lines at the edge of the stripe and at scale $\sigma = x_1 - x_0$
   - on the vertical lines at the edge of the stripe and at scale $\sigma = (x_1 - x_0)/2$

   Solution: This is just the same as the 1D problem presented in the slides. The scale where the minimum occurs is the half width of the box.

   Comments: Only 5 students go this one wrong.

[1] The word was missing. This typo was corrected during the exam.