Lecture 3 Class Activities

Image Filtering

Mon. Sept 14, 2020

Outline for today

- Review main concepts & do several exercises (need pen and paper, etc) → 1 hour

- Poll and Breakout rooms to discuss course/zoom → 20 min

Concepts and Terminology

Image Filtering (Math)

- Local difference
- Local average
- Convolution
- Filter
- Impulse Function
- Impulse Response Function
- Boundary issues : zero padding
- Algebraic properties of convolution
- Cross-correlation
- Gaussian function
- 2D extension of all the above

Matlab code

- conv(I, D)
- conv(I, B)
- conv(I, D, ‘same’)
- conv(I, D, ‘valid’)

Local difference

$$I_{diff}(x) = \frac{1}{2}I(x+1) - \frac{1}{2}I(x-1)$$

Approximates a derivative.
Central difference (versus forward difference versus backwards difference)

Local average

$$I_{smooth}(x) = \frac{1}{4}I(x+1) + \frac{1}{2}I(x) + \frac{1}{4}I(x-1)$$

Smoothing = blurring
Weights sum to 1 (taking an *average*)

Convolution

$$I(x) * f(x) \equiv \sum_{x'} I(x')\, f(x - x')$$

Note the opposite signs on the x'.
How to interpret this operation?

Filtering

$$I(x) * f(x) \equiv \sum_{x'} I(x') \, f(x - x')$$

For the difference and smoothing operations, what is the *filter* function $f(x)$?
(See lecture notes.)

$$I_{diff}(x) = \frac{1}{2} I(x+1) - \frac{1}{2} I(x-1)$$

$$I_{smooth}(x) = \frac{1}{4} I(x+1) + \frac{1}{2} I(x) + \frac{1}{4} I(x-1)$$

How to visualize convolution?

$$I(x) * f(x) \equiv \sum_{x'} I(x')\, f(x - x')$$

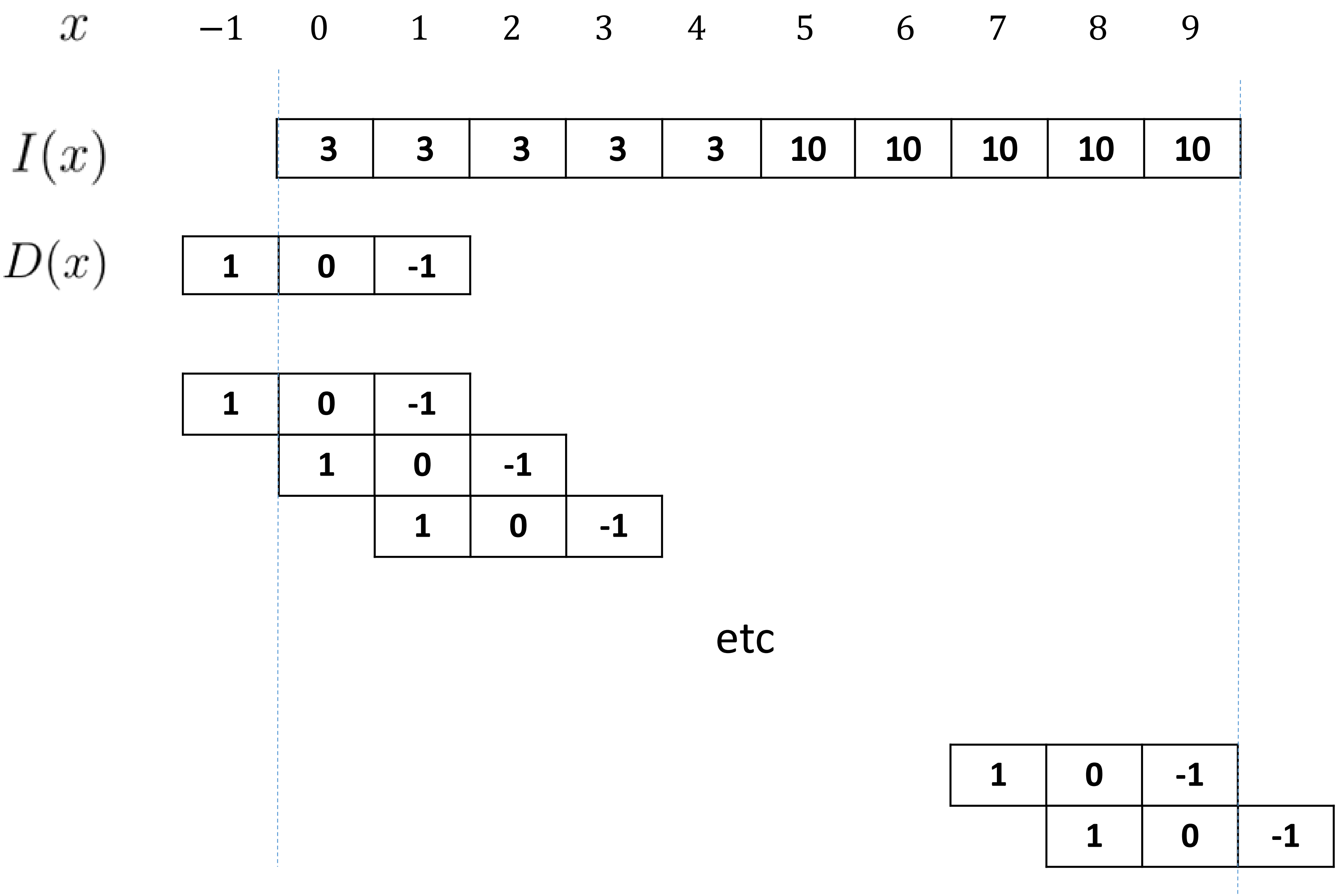
$I(x)$

3	**3**	**3**	**3**	**3**	**10**	**10**	**10**	**10**	**10**

$f(x)$

f(-1)	*f(0)*	*f(1)*

We need to specify what are the x values for $I(x)$ and $f(x)$.



One can show that the length of $I * f$ is len(I) + len(f) – 1 in general.

Impulse function

$$\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(x - x_0) = \begin{cases} 1, & x = x_0 \\ 0, & \text{otherwise} \end{cases}$$

Exercise

$$D(x) = \begin{cases} \frac{1}{2}, & x = -1 \\ -\frac{1}{2}, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

Exercise: Write this filter as a sum of impulse functions.

Exercise

$$D(x) = \begin{cases} \frac{1}{2}, & x = -1 \\ -\frac{1}{2}, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

Exercise: Write this filter as a sum of impulse functions.

Answer:

$$D(x) = -\frac{1}{2}\delta(x-1) + \frac{1}{2}\delta(x+1)$$

Exercise

$$B(x) = \begin{cases} \frac{1}{4}, & x = -1 \\ \frac{1}{2}, & x = 0 \\ \frac{1}{4}, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

Exercise: Write this filter as a sum of impulse functions.

Exercise

$$B(x) = \begin{cases} \frac{1}{4}, & x = -1 \\ \frac{1}{2}, & x = 0 \\ \frac{1}{4}, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

Exercise: Write this filter as a sum of impulse functions.

Answer: $$B(x) = \frac{1}{4}\delta(x-1) + \frac{1}{2}\delta(x) + \frac{1}{2}\delta(x+1)$$

"Impulse Response" Function

x	-2	-1	0	1	2	3	4	5	6	7
$\delta(x)$	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$f(x)$		*f(-1)*	*f(0)*	*f(1)*						

Exercise: What is $\delta(x) * f(x)$?

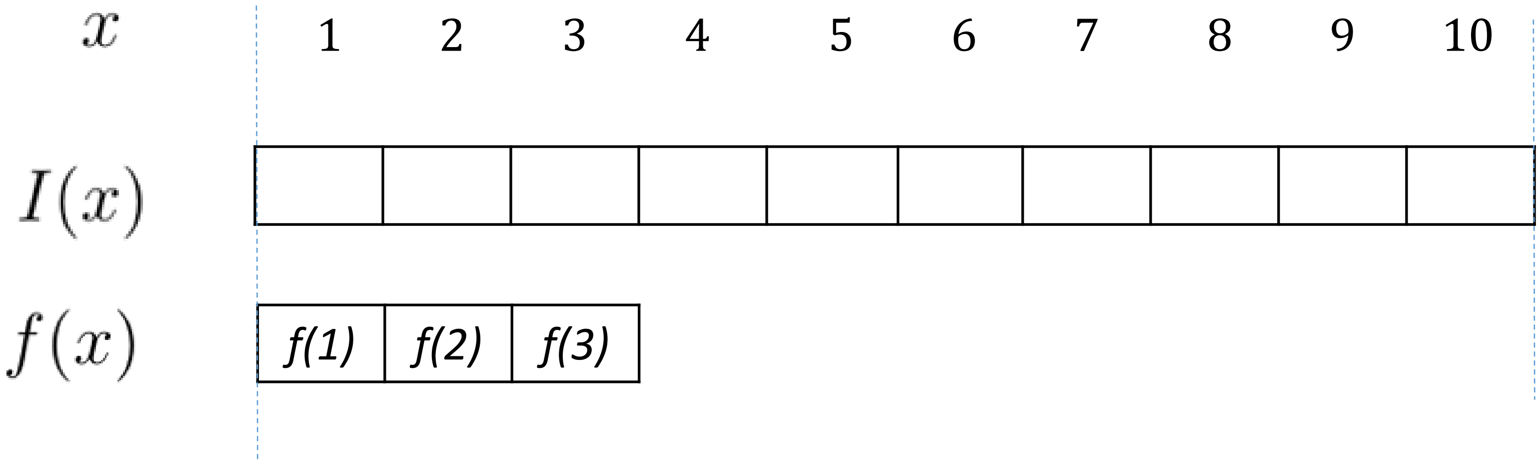
"Impulse Response" Function

x	-2	-1	0	1	2	3	4	5	6	7
$\delta(x)$	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$f(x)$		*f(-1)*	*f(0)*	*f(1)*						
$\delta(x) * f(x)$	**0**	*f(-1)*	*f(0)*	*f(1)*	**0**	**0**	**0**	**0**	**0**	**0**

Exercise: What is $\delta(x) * f(x)$?

Answer: $f(x)$.

This holds for general $f(x)$, thus the name impulse response function



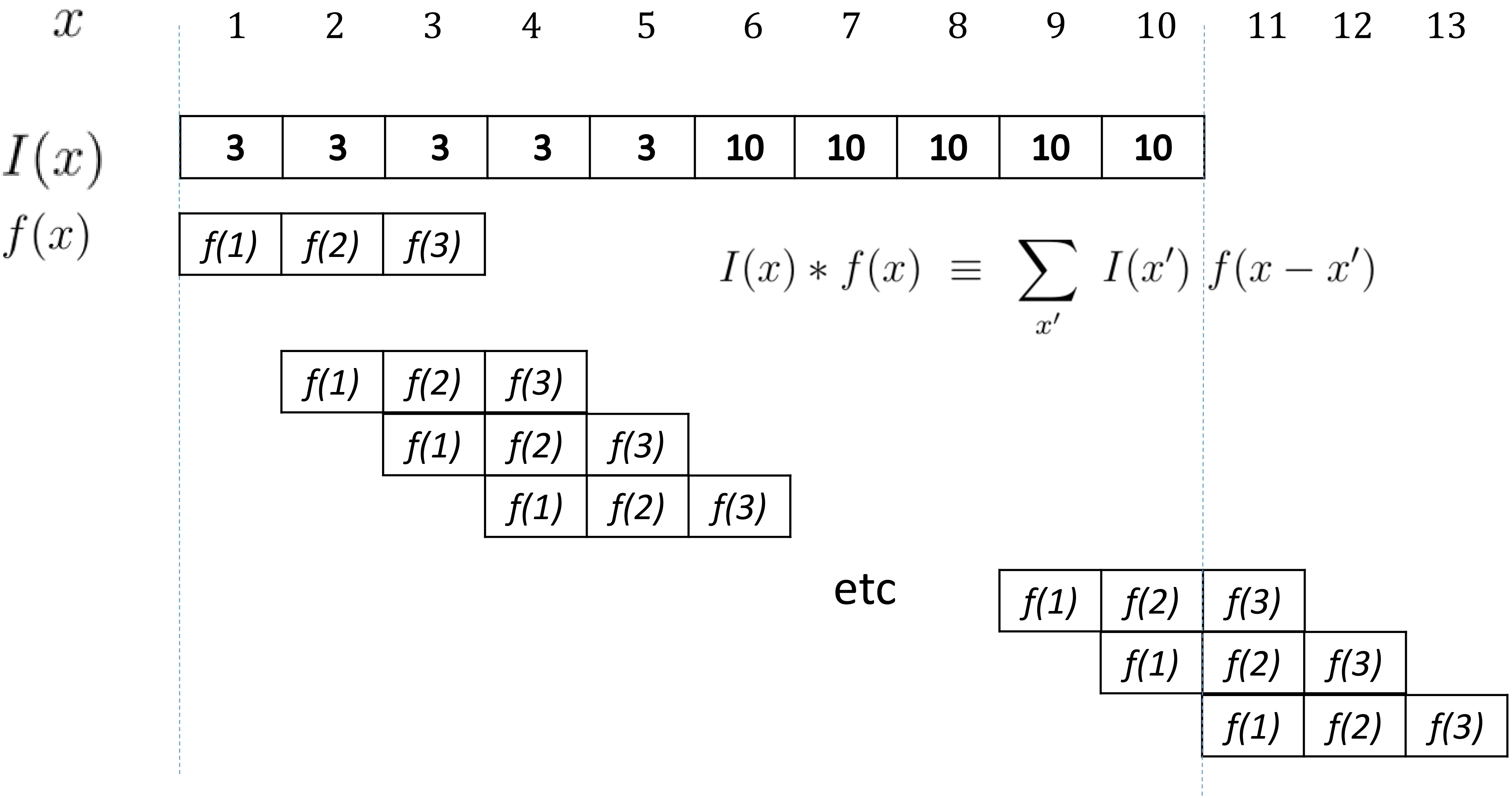
$$I(x) * f(x) \;\equiv\; \sum_{x'} I(x')\; f(x - x')$$

Exercise : for this example, what will be the range of indices of $I(x) * f(x)$?

i.e. For which values of x could $I(x) * f(x)$ be non-zero ?

(Draw picture above, and discuss in breakout rooms.)

Break/Breakout room (7 minutes)



$$I(x) * f(x) \equiv \sum_{x'} I(x')\, f(x - x')$$

Exercise : what will be the range of indices of $I(x) * f(x)$?

Answer: for this example, x would range from 2 to 13.

Algebraic properties of convolution

Commutative: $$I * f = f * I$$

Associative: $$I * (f_1 * f_2) = (I * f_1) * f_2$$

Distributive: $$(I_1 + I_2) * f = I_1 * f + I_2 * f$$

Exercise :

In Matlab, one computes $I(x) * f(x)$ using `conv(I, f)`.
How would you write the above expressions in Matlab ?
To test for equality, use `isequal( , )`.

Commutative: $$I(x) * f(x) = f(x) * I(x)$$

isequal(conv(I,f), conv(f,I))

Associative: $$I * (f_1 * f_2) = (I * f_1) * f_2$$

Distributive: $$(I_1 + I_2) * f = I_1 * f + I_2 * f$$

Commutative: $$I(x) * f(x) = f(x) * I(x)$$

isequal(conv(I,f), conv(f,I))

Associative: $$I * (f_1 * f_2) = (I * f_1) * f_2$$

isequal(conv(I, conv(f1, f2)), conv(conv(I, f1), f2))

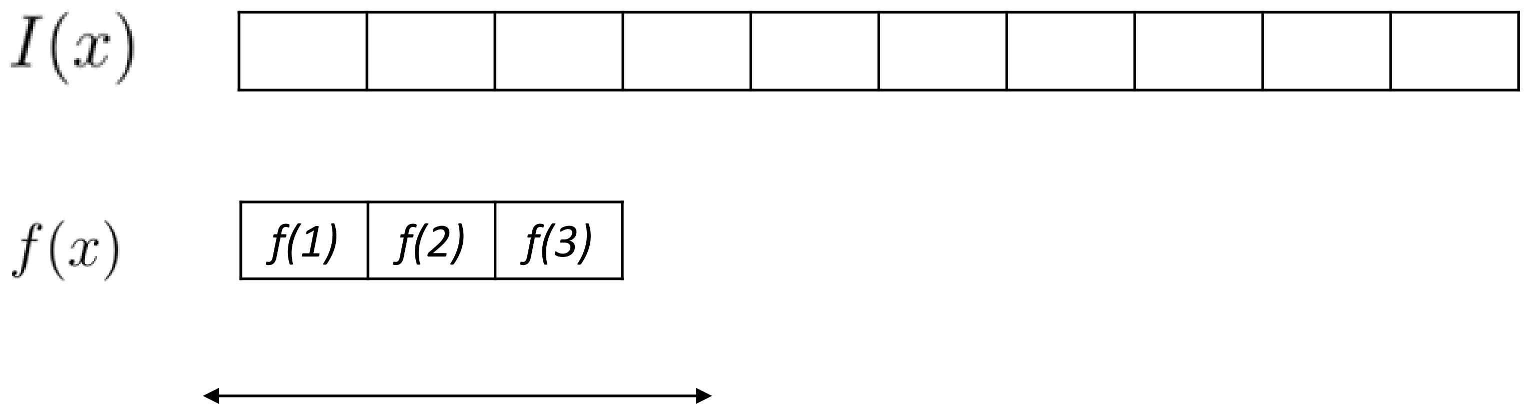
Distributive: $$(I_1 + I_2) * f = I_1 * f + I_2 * f$$

isequal(conv(I1 + I2,f), conv(I1, f) + conv(I2, f))

The + operator gives an error if the vectors have different length.

Cross-correlation

$$f(x) \otimes I(x) = \sum_{x'} f(x' - x)\ I(x')$$



Notice that the argument of $f(x)$ is flipped, relative to the definition of convolution.

This gives a different geometric interpretation. More on this later in course…

Gaussian function

$$G(x;\ \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Discrete approximation to Gaussian

$$G(x;\ \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Exercise:

When we sample the function at integer values of x,
what problems arise ?

(answer in chat please)

Discrete approximation to Gaussian

$$G(x;\ \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Exercise:

When we sample the function at integer values of x,
what problems arise ?

Answer:

1) The Gaussian has infinite tails, but we use only finite samples.
2) The Gaussian is supposed to sum to 1, but this is not guaranteed if we use finite samples.

Poll

- How much time did you spend preparing for today's 558 class ?
- Which *study* strategies did you use (or do you expect to you) most ?

(Turn off screen sharing, go to gallery view)

[Unfortunately zoom did not save poll results.]

Breakout Rooms (max 20 minutes)

Please introduce yourselves, turn on cameras, and go to gallery view.

Could one person in each group please take notes and submit them to Discussion board for this lecture ? Others can comment on postings.

Suggested topics for discussion (academic only please):

- How is the course going for you so far? (material, format) Why?

- Which Zoom teaching methods in other large courses seem to work better/worse ? Why?