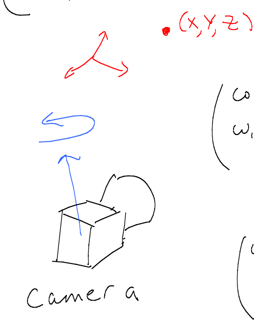
lecture 20

homographies (cont.)

Example 3: Camera Rotation

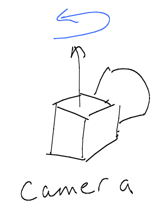
(Application - Image Stitching / Panoramas)



$$\begin{pmatrix} w_1 x_1 \\ w_1 y_1 \\ w \end{pmatrix} = K R_1 [I \mid -C] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} w_2 x_2 \\ w_2 y_2 \\ w_2 \end{pmatrix} = K R_2 [I \mid -C] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

If we assume $C_1 = C_2 = C$, then



$$(K R_1)^{-1} \begin{pmatrix} w_1 x_1 \\ w_1 y_1 \\ w \end{pmatrix} = [I \mid -C] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$(K R_2)^{-1} \begin{pmatrix} w_2 x_2 \\ w_2 y_2 \\ w_2 \end{pmatrix} = [I \mid -C] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

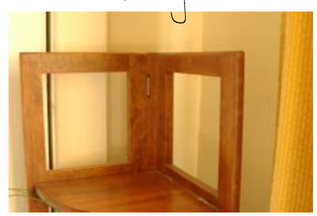
$$\therefore \begin{pmatrix} w_2 x_2 \\ w_2 y_2 \\ w_2 \end{pmatrix} = K R_2 (K R_1)^{-1} \begin{pmatrix} w_1 x_1 \\ w_1 y_1 \\ w \end{pmatrix} = K R_2 R_1^T K^{-1} \begin{pmatrix} w_1 x_1 \\ w_1 y_1 \\ w_1 \end{pmatrix}$$

$$H = K R_2 R_1^T K^{-1}$$

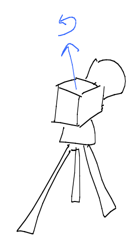
Notes:

- This is not merely a rotation since we are mapping pixel indices, not (mm) positions in the projection plane.
- In image stitching applications, one attempts to estimate H directly (see end of lecture)

Assignment 4



The mapping between pixels in these two images is approximately a homography (center of rotation $\neq$ center of projection i.e. $C_1 \neq C_2$)



Example 4: "image rectification"

for stereo in general (non-planar) 3D scenes

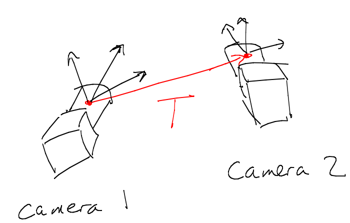
Assume we have estimated the camera internals and externals $K_1\ R_1\ C_1 \quad K_2\ R_2\ C_2$.

(called a "calibrated stereo rig")

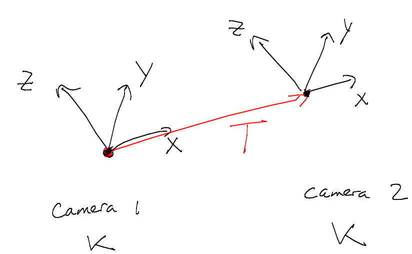
Let's transform the images using homographies such that:

- X Y Z axes of cameras become parallel
- camera internals become the same (K_1)

actual camera configuration



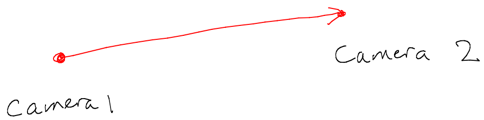
desired camera configuration



How to define "desired" configuration?

- new X axis is parallel to $\vec{T}$
- new Y axis is perpendicular to plane spanned by $\vec{T}$ and Z axis of camera 1
- new Z axis is perpendicular to new X and Y axis

$$\vec{T} = \vec{C}_2 - \vec{C}_1$$



Rotate camera 1 so that $\vec{T}$ vector is in the X axis direction of new camera 1 coordinate system

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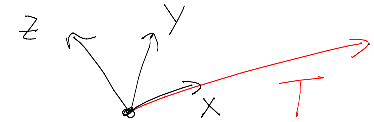
$$R_{rect} = \begin{bmatrix} \leftarrow & \vec{T}/\|\vec{T}\| & \rightarrow \\ \leftarrow & & \rightarrow \\ \leftarrow & & \rightarrow \end{bmatrix}$$

Here I am representing vector T in camera 1 coordinates (not scene coordinates)

Why? We want $\begin{bmatrix} |T| \\ 0 \\ 0 \end{bmatrix} = R_{rect} T$

$$R_{rect} = \begin{bmatrix} \leftarrow & \text{unit}(T) & \rightarrow \\ \leftarrow & \text{unit}(\hat{z} \times T) & \rightarrow \\ \leftarrow & \text{unit}((\hat{z} \times \vec{T}) \times T) & \rightarrow \end{bmatrix}$$



Rectifying camera 1's pixels

$$\begin{pmatrix} w\tilde{x}_1 \\ w\tilde{y}_1 \\ w \end{pmatrix} = K_1 \, R_{rect} \, K_1^{-1} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

Rectifying camera 2's pixels

$$\begin{pmatrix} w\tilde{x}_2 \\ w\tilde{y}_2 \\ w \end{pmatrix} = K_1 \, R_{rect} \, R_1 \, R_2^{-1} \, K_2^{-1} \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$$

The result is a pair of "rectified" images i.e. whose cameras have

- the same internal parameters (K_1)
- the same external rotation relative to world coordinates
- positions who differ only by a shift in X direction

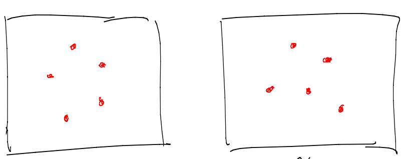
(Next week we say more about stereo geometry.)

NEXT

- Given 2 images, how do you find the homography between them?
- Given a homography H that maps one image domain to another, how do you remap the image intensities?
- An example that puts the two together

How to solve for the homography between two images?

Given $\{(x_i \, y_i \, \tilde{x}_i \, \tilde{y}_i)\}$



$I(x,y)$ $\qquad$ $\tilde{I}(\tilde{x},\tilde{y})$

Find homography $\begin{pmatrix} w\tilde{x} \\ w\tilde{y} \\ w \end{pmatrix} \approx H \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

$$\begin{pmatrix} w\tilde{x}_i \\ w\tilde{y}_i \\ w \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$(H_{31} x_i + H_{32} y_i + H_{33}) \, \tilde{x}_i = H_{11} x_i + H_{12} y_i + H_{13}$$

$$(H_{31} x_i + H_{32} y_i + H_{33}) \, \tilde{y}_i = H_{21} x_i + H_{22} y_i + H_{23}$$

$$(H_{31} x_i + H_{32} y_i + H_{33}) \, \tilde{x}_i = H_{11} x_i + H_{12} y_i + H_{13}$$

$$(H_{31} x_i + H_{32} y_i + H_{33}) \, \tilde{y}_i = H_{21} x_i + H_{22} y_i + H_{23}$$

$$\underbrace{\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i\tilde{x}_i & -y_i\tilde{x}_i & -\tilde{x}_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -x_i\tilde{y}_i & -y_i\tilde{y}_i & -\tilde{y}_i \\ & & & & \vdots & & & & \end{bmatrix}}_{2N \times 9 \text{ matrix}} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- $N = 4$ points $\therefore$ 8×9 matrix and compute the null space

- $N > 4$ points

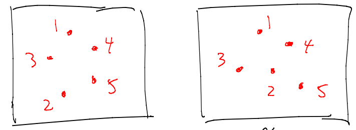
  Solve for H using least squares

  minimize $\| A \vec{h} \|$

  $2N \times 9$ $h \times 1$

  subject to $\| \vec{h} \| = 1$.

Give $I(x,y)$, $\tilde{I}(\tilde{x},\tilde{y})$, how do we find correspondences $\{(x_i\, y_i\, \tilde{x}_i\, \tilde{y}_i)\}$?



$I(x,y)$ $\tilde{I}(\tilde{x},\tilde{y})$

- by hand
- automatically eg. SIFT keypoint descriptors.

Using SIFT

For each keypoint $(x_i\, y_i)$ in I, make a set of $(\tilde{x}_i\, \tilde{y}_i)$ in $\tilde{I}$ that have similar SIFT descriptors

This gives a set of candidate matches $\{ (x_i\, y_i, \underbrace{\tilde{x}_i\, \tilde{y}_i}_{\text{there may be several of these for each } i}) \}$

RANSAC

→ repeat
- random sample 4 quadruples = 1 "trial" $(x_i\, y_i\, \tilde{x}_i\, \tilde{y}_i)$ i.e. 4 i values
- fit a model H for this trial
- go through all other candidate $x\, y\, \tilde{x}\, \tilde{y}$ quadruples and build the "consensus set" for this H
- increment counter

until (consensus set $> \tau_2$) or (counter == numTrials)

- refit H using largest consensus set and return

~~Use non-linear least squares to revise (See lecture notes.)~~

IGNORE THIS SLIDE.

~~e.g. Minimize~~

$$\cancel{\sum_{i=1}^{N} \| H \vec{x}_i - \tilde{\vec{x}}_i \|^2 + \| \vec{x}_i - H^{-1} \tilde{\vec{x}}_i \|^2}$$

~~subject to $\| H \| = 1$~~

~~Here the norm is defined in 2D pixel space, not the 3D homogeneous vector.~~

What about image intensities?

Given a homography H that maps one image domain to another, how do you remap the image intensities?

$H : \boxed{I} \rightarrow \boxed{\tilde{I}}$

given new

Forward mapping from I to $\tilde{I}$

for each x, y in $I(x,y)$

$$\begin{pmatrix} w\tilde{x} \\ w\tilde{y} \\ w \end{pmatrix} = H \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\underbrace{\tilde{I}(\tilde{x},\tilde{y})}_{\text{floats}} := \underbrace{I(x,y)}_{\text{integers}}$$

Inverse mapping from new $\tilde{I}$ to old I.

for each $(\tilde{x},\tilde{y})$ in $\tilde{I}$

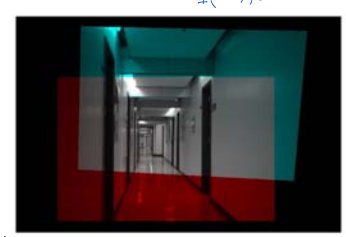
$$\begin{pmatrix} wx \\ wy \\ w \end{pmatrix} = H^{-1} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ 1 \end{pmatrix}$$

$$\tilde{I}(\tilde{x},\tilde{y}) := I(x, y)$$

H

H^{-1}

$\tilde{I}(\tilde{x},\tilde{y})$ $I(x,y)$



- Camera related by rotation.
- Keypoints selected and matched by hand.
- $\tilde{I}(\tilde{x},\tilde{y})$ inversed mapped to domain of (x,y) to obtain RGB values

$$H^{-1} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ 1 \end{pmatrix} = \begin{pmatrix} wx \\ wy \\ w \end{pmatrix}$$