

# COMP 558 Assignment 2

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Due: Sunday Nov. 1, 2020 (by midnight, 11:59pm)

This assignment focusses on the material of lectures 7, 9, and 11, namely corners, scale space, and SIFT features. It also reinforces some of the earlier lectures on filtering.

## Instructions

Use the mycourses discussion board for any assignment related questions. This will help us reach the entire class when we respond. However, follow appropriate protocol, e.g., do not reveal the answer to a particular question or ask if your proposed solution is correct.

You are also free to discuss the questions with each other. *However, the solutions that you submit must reflect your own work and must be written by you.* You are not permitted to submit code or text that you have copied from one another or from any third party sources including internet sites.

Submit a single zipped file **A2.zip** to your mycourses Assignment 2 folder containing:

- A single PDF file with figures and text for each question. Please do not spend time with fancy typesetting. Just make sure that your explanations are clear and that the figures and their captions are easy to interpret and that you answer each part of each question.
- The Matlab code that you wrote so that we can run it if necessary. You may submit a single script. If you submit multiple files, then please a README to explain which is which.
- The images that you used to test your code.

In order to receive full points, your solution code must be properly commented, and you must provide a *clear and concise* description of your computed results in the PDF. Note that the TAs have limited time and will spend at most 20-30 minutes grading each submission.

**Late assignment policy:** Late assignments will be accepted up to only 3 days late and will be penalized by 10% of the total marks, per day late. For example, an assignment that is submitted 2 days late and that receives a grade of 90/100 would then receive an 18 point penalty. Submitting one minute after midnight on the due date means it is 1 day late (not a fraction of a day), and same for second day.

## Tip

- To examine the RGB values in an image, use the figure tab *Tools* → *Datatips*. This will allow you to click on pixels and see their values.

## Instructions for all questions:

- For plots that have only positive values, use the `gray` colormap. For plots that have positive and negative values, use the `jet` colormap. Include a `colorbar` to indicate the range of values in the plot.
- **For each question, include results on the same two test images.** One image must be synthetic and contain a variety of shapes, including squares, rectangles, disks, ellipsoids and at least one more complex shape which can be a combination (overlap) of the regular shapes. The second image should be natural one of your choice. If you wish to use an RGB image, then you'll need to convert it to gray.

Use `imagesc` so that the size of your image is indicated in your plot. This is especially important for the last question.

It is sufficient for your images to have height and width between 300 and 500 pixels, so resize your images (use `imresize`). Don't waste your time computing on larger images than that.

## Question 1 Gaussian scale space [10 points]

*Compute a Gaussian scale space.* The scale must go from  $\sigma = \sigma_0$  to  $\sigma = 16\sigma_0$ , and have *uniform spacing on a log scale* as described in the lectures, namely four slices for each doubling of  $\sigma$ . This will give you 17 discrete slices of the scale space. (You are free to choose  $\sigma_0$ . It should be at least 1.)

Do **not** construct a pyramid. Rather, at all scales, the scale space must be the same size as the original image. The same holds for all questions below – no pyramids!

*Use `subplot` to create a 4 x 4 grid showing slices 2 to 17.*

## Question 2 Harris-Stevens [10 points]

Write a script that applies the Harris-Stevens operator to each level of the Gaussian scale space. **[reworded Oct. 20:** The neighborhood for the second moment matrix should be defined by a Gaussian whose standard deviation (call it  $\sigma_{\text{window}}$ ) is twice the scale  $\sigma$  defined at that level.]

*Use `subplot` to create a 4 x 4 grid showing slices 2 to 17. Briefly discuss your plots. Does the operator highlight the locally distinctive points? Does the position of locally distinctive points change across scale?*

### Question 3 Difference of Gaussians [10 points]

Compute the Difference of Gaussians scale space, as defined in the SIFT lecture, namely: compute the difference between Gaussian scale space layers  $j$  and  $j+1$ . [Added Oct 21: Note that the DOG scale space only has 16 layers.]

*Use subplot to create a 4 x 4 grid showing the 16 DOG responses at different scales. Briefly discuss how the zero-crossings vary across scale. (Do not compute zero crossings. You did that already in Assignment 1.)*

### Question 4 SIFT Keypoint detection [20 points]

Do a brute force search through the Difference of Gaussian scale space and find the local extrema (maximum and minima) as described in the lecture. Examine 3x3x3 neighborhoods and check if the center point is larger than all its 26 neighbors (maximum) or smaller than all its 26 neighbors (minimum). These  $(x, y, \sigma)$  minima and maxima are candidate keypoint locations. Only consider keypoints whose position is at least  $2 \cdot \sigma$  away from the x and y image boundaries, where  $\sigma$  is the scale of the keypoint. [Updated Oct 21] Also, only search layers 2 to 15 in the DOG scale space, since points in layers 1 and 16 don't have a 3x3x3 neighborhood. Finally, you may find it better to choose a threshold of the magnitude of the Difference of Gaussian function at the keypoint.

Create an array containing these keypoints. The array should also include a column for the dominant orientation which you will compute later.

*Show the keypoints by displaying a red circle superimposed on the original gray level image. Use the Matlab `viscircles` command.*

In the next question, you will replace some of these red circles with blue ones. (It is fine if you just submit the figure for the next question.)

### Question 5 Hessian constraint [20 points]

Lowe's (2004) SIFT paper suggests using a Hessian matrix to decide whether a keypoint is well localized, namely compute the Hessian of the Difference of Gaussians function at the keypoint, and verify that the eigenvalues of the Hessian matrix are sufficiently close to each other. See the paper for more detail.<sup>1</sup>

*You will need to come up with a discrete approximation of the Hessian, using the ideas of local differences that we discussed earlier in the course. State your approximation for the Hessian.*

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<sup>1</sup> Lowe refers to the *principal curvatures* of the surface. We are not expecting that you know what that term means in general. You only need to know what it means in the present context, namely it refers to the two eigenvalues of the Hessian at the keypoint.

*If a candidate keypoint does not meet Lowe's condition, then do not include it in your list of keypoints. Replace the figure in the previous question by drawing a blue circle instead of a red one.*

### **Question 6 SIFT feature dominant orientation [20 points]**

For each keypoint, compute an orientation histogram as follows. Compute the intensity gradient at each point in scale space by filtering the pixels with local differences, and weight the magnitudes of the gradient vectors by a Gaussian with standard deviation  $1.5 \sigma$ . Define 36 bins of 10 degrees each, centered on 0, 10, 20, ..., 350 degrees. (This will avoid the instability of horizontal and vertical edges, namely the bin boundaries will be at -5 deg, 5 deg, 15 deg, etc.)

Estimate the dominant orientation of these weighted gradient vectors, namely find the histogram bin that has the largest value (sum of weighted gradient magnitudes). This bin defines the direction  $\theta$  of the keypoint. If any other orientation bin(s) have a value that is over 80% of the largest value, then also define a keypoint for each of these orientations.

*Further annotate the keypoints from earlier, by adding a red line at the location of the keypoint. The line's endpoints should be the keypoint and a point that is  $2\sigma$  away, and the direction of the line should be in the dominant orientation defined by the keypoint. If there are multiple orientations at a keypoint, then multiple lines should be drawn.*

In addition, *plot the histograms for at least two keypoints and identify these keypoints in the image.* The orientation at the peak of the histograms should correspond to the orientation shown in the image. Note that the image y axis points down, so we suggest making the angle go clockwise.

### **Question 7 Scale Invariance [10 points]**

Use `imresize` to create a new image that is rescaled by some factor between 0.5 and 2 (but different from 1, obviously) e.g. 0.8 or 1.2. Detect keypoints and annotate them using the red circle and line as before.

[Oct 21: requirements simplified]

Compare the keypoints (locations, scales, orientations) for this rescaled image to the keypoints found for the original image. Are the keypoints scale invariant, that is, do the keypoints in the newly rescaled images correspond in the expected way with the keypoints in the original images?

Be sure to use `imagesc` so that the size of your image is indicated in each figure.