

1. **Epipolar geometry**

Given an essential matrix \mathbf{E} that relates corresponding points in two image projection planes. Consider the SVD of \mathbf{E} , namely $\mathbf{E} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$. Assume singular values are ordered from largest to smallest.

Complete the sentence “The eigenvector of \mathbf{E} corresponding to the singular value 0 is ...”. by select all that apply.

- (a) ... the third column of \mathbf{V} . **answer**
- (b) ... the third row of \mathbf{V} .
- (c) ... the first row of \mathbf{V} .
- (d) ... the first column of \mathbf{V} .
- (e) ... the third row of \mathbf{U} .
- (f) ... the first row of \mathbf{U} .

Answer

Singular values are ordered from largest to smallest and so the third singular value is 0 (since \mathbf{E} is of rank 2). The SVD is $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$ and so the third column of \mathbf{V} or third row of \mathbf{V}^T would be the eigenvector with eigenvalue is 0.

If we wanted to be fancy and consider “left eigenvectors”, then we could have the third column of \mathbf{U} (call it \mathbf{U}_{*3}), since $\mathbf{U}_{*3}^T \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T = (0, 0, 1)^T \mathbf{\Sigma} \mathbf{V}^T = 0$. However, the third column of \mathbf{U} was not an option.

2. Consider two rectified cameras separated by a horizontal distance of 2. Suppose the projection planes for both cameras are $Z = 4$. What is the epipolar line in the right image that corresponds to a position $(x, y) = (1, 2)$ in the left image projection plane?

- (a) $y = 4$
- (b) $y = 2$ **answer**
- (c) $y = 1$
- (d) $y = \frac{1}{2}$
- (e) $x = 2$
- (f) $x = -2$

Answer

This one should be very easy. Answer is (b).

3. Suppose we estimate the fundamental matrix of a real stereo pair of images. We use least squares on a set of matching points and then impose the rank 2 constraint. Let the estimated matrix be \mathbf{F} .

Which of the following statements best describes the **actual** position \mathbf{p} of camera 1 in camera 2's image ?

- (a) $\mathbf{F}\mathbf{p} \approx 0$
- (b) $\mathbf{p}^T\mathbf{F} \approx 0$ **answer**
- (c) $\mathbf{F}\mathbf{p} = 0$
- (d) $\mathbf{p}^T\mathbf{F} = 0$

Answer Real images have noise, so the least squares estimates followed by imposing rank 2 constraint is extremely unlikely to give the exact correct solution. Rather one gets an approximation. The question is asking for the epipole in camera 2's image, which is defined by $\mathbf{e}_2^T\mathbf{F} = 0$.

4. Correspondence

A classical random dot stereogram consists of a foreground square in front of a background, such that (1) monocular regions exist on the left and right sides of the foreground square and (2) part of the background is visible to neither camera.

How could one change the foreground square so that all points on the background are visible to at least one camera? Select all that apply.

- (a) Decrease the width of the square. **answer**
- (b) Increase the width of the square.
- (c) Decrease the disparity of the square.
- (d) Increase the disparity of the square. **answer**

Answer: Decreasing the width of the square to say one pixel will surely do the job, as long as the disparity of the foreground square is at least 1 pixel greater than the disparity of the background. (This requires understanding the geometry of disparity space. If you don't understand, then you should review the slides.)

Similarly, keep the square size constant and bringing the square closer will also do the job. To see what's going on, if you put your finger in close to the screen, then parts of the screen below your finger will be hidden to both eyes. Now bring your finger closer to your eyes. Eventually you will get "double vision" and every point on the screen will be visible to at least one eye.

5. Photography

Suppose a photographer chooses the following camera settings: focal length $f = 80$ mm, shutter speed $\frac{1}{t} = 32$, f-number = 4.

Which of the following settings result in the same exposure and same field of view angle? Select all that apply.

- (a) $f = 80$ mm, $\frac{1}{t} = 128$, f-number = 2. **answer**
- (b) $f = 80$ mm, $\frac{1}{t} = 64$, f-number = 8.
- (c) $f = 40$ mm, $\frac{1}{t} = 128$, f-number = 4.

(d) $f = 40$ mm, $\frac{1}{t} = 32$, f-number = 8.

(e) $f = 160$ mm, $\frac{1}{t} = 16$, f-number = 4.

Note: the amount of light that passes through the aperture is proportional to the square of the aperture.

Answer

To get the same field of view angle, you need to keep the focal length. Thus, you change only the shutter speed and/or the aperture. So only (a) and (b) are possible answers. For (a), changing the shutter speed to 128 would mean a factor of 4 decrease and so the aperture would need to be doubled to exactly compensate (since doubling the aperture increases the exposure by a factor of 4). When we double the aperture, we halve the f-number (from 4 to 2). So (a) is correct. (b) is incorrect since increasing the shutter speed and f-number both decrease the exposure. So there is only one valid answer.