# Questions

1. Many animals such as birds, snakes, fish and deer have white bellies. It has been claimed that such surface pigmentation *decreases* the visibility of the animal (i.e. "camouflage") when it is viewed *from the side* under natural lighting.

   Briefly justify this claim. Hint: Think shading.

2. Consider the linear shading model which assumes a low relief surface.

$$I(X, Y) \;=\; (\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1) \cdot (L_X, L_Y, L_Z)$$

   Suppose we have a depth function

$$Z(X, Y) \;=\; Z_0 + a \sin(k_0 Y)$$

   and a light source direction "from above" $(0, L_Y, L_Z)$. Write out the linear shading model in this case.

3. Consider the case of a surface with depth map

$$Z(X, Y) = a \sin(k_X X)$$

   and suppose $\mathbf{L} = (0, 0, -1)$.

   (a) Does the linear shading model apply?

   (b) What is the resulting intensity $I(X, Y)$, assuming the $\mathbf{N} \cdot \mathbf{L}$ model ? Give a formula and describe qualitatively.

4. Suppose we invert the depth of the surface $Z(X, Y) = -Z(X, Y)$ so that hills become valleys and valleys become hills.

   (a) How do the surface normals change?

   (b) According to the $\mathbf{N} \cdot \mathbf{L}$ model, the intensities will change when we invert the surface. Is it possible to change the light source direction so that the intensities do not change? That is, if you invert the surface *and* you change the light source direction in a certain way, then can you get the same image intensities again? Explain.

5. (a) Consider the uniform colored illumination case on slide 39. Compare the grey world method with the 'max-RGB' adaptation method. (See lecture notes: the grey world method divides each RGB channel by the mean in that channel, and the maxRGB method divides each channel by the largest value in that channel.) How do the results of the two methods differ? Specifically consider two cases:

   - case 1: the scene contains a white surface patch
   - case 2: the scene does not contain a white surface patch

   (b) Consider the example showed in slide 40: a colored checkerboard surface illuminated by bright yellowish light on one half and dimmer blueish light on the other half. In the lecture notes I described a "grey world" assumption that could be used to compensate for color of the illuminance. How could the grey world method be *generalized* to handle the case with the shadow shown in the slide ?

# Solutions

1. If an animal has a uniform color, then under natural lighting the top part of the animal will have the highest luminance and the bottom part will have the lowest luminance. This is because light tends to be from above (sunlight, cloudy day). The white colored belly partially cancels out this shadow/shading effect, since the illumination and reflectance gradients go in opposite directions. This phenomena is called *countershading*. It is a well known principle of camouflage.

2. Since $\frac{\partial Z}{\partial X} = 0$, the linear shading model reduces to:

$$I(X,Y) \; = \; L_Y \frac{\partial Z(X,Y)}{\partial Y} - L_Z .$$

3. (a) It applies, but it doesn't give any shading. Since $L_X$ and $L_Y$ are both 0, and the model says that $I(X,Y) = L_Z$, i.e. a constant. This would be approximately correct if $\mid \frac{\partial Z}{\partial X} \mid$ and $\mid \frac{\partial Z}{\partial Y} \mid$ were near 0. But if these slopes were not neglible, then some intensity variations (shading) would indeed occur, which would not be captured by the linear model.

   (b) Plugging into the $I = \mathbf{N} \cdot \mathbf{L}$ formula gives:

$$I(X,Y) = \frac{1}{\sqrt{1 + (ak_X)^2 \cos^2(k_X X)}}$$

   By inspection, it has a maximum value of 1 when the cosine has value 0. The minimum value depends on $ak_X$ and would be close to 0 if $ak_X$ is large. Notice that the maxima and minima occur at twice the frequency of the depth map. The maxima occur at both the hills and valleys, where the surface normal points to the light source. The minima occur on the steepest part of the slope.

   This is known as the *frequency doubling* effect.

4. (a) When we flip the surface in depth, the sign of the partial derivatives change, so the surface normal

$$\frac{1}{\sqrt{1 + (\frac{\partial Z}{\partial X})^2 + (\frac{\partial Z}{\partial Y})^2}} (\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1)$$

   becomes

$$\frac{1}{\sqrt{1 + (\frac{\partial Z}{\partial X})^2 + (\frac{\partial Z}{\partial Y})^2}} ( \; - \frac{\partial Z}{\partial X}, \;\; - \frac{\partial Z}{\partial Y}, \;\; - 1)$$

   (b) If we move the light source $(L_X, L_Y, L_Z) \rightarrow (-L_X, -L_Y, L_Z)$ by flipping the signs of both $L_X$ and $L_Z$, then $I(X,Y) = \mathbf{N}(X,Y) \cdot \mathbf{L}$ will remain the same.

   There is an issue with shadows, however. You can have cast shadows in the original surface but not when you flip the surface, or vice-versa. In the absence of cast shadows, though, there is a two-fold ambiguity. Two different depth functions and light source directrions can give exactly the same image. This is known as the *depth reversal ambiguity*. I'll come back to this in lecture 14.

5. (a) If the scene contains a white patch, then the maxRGB method would scale the RGB values by $(1/max(R), 1/max(G), 1/max(B)$ which would bring the RGB value in the white patch back to (1,1,1) i.e. white reflectance. The grey world assumption method would scale by the mean intensity in each image channel, i..e $(1/mean(R), 1/mean(G), 1/mean(B)$. If indeed the scene was grey on average then a white patch in that scene would be scaled to a neutral value. Specifically, the white patch has $I_{RGB}$ equal to the illumiance, and so it would scale to

$$(\frac{illuminance_R}{mean(R)}, \frac{illuminance_G}{mean(G)}, \frac{illuminance_B}{mean(B)}) = (1/g, 1/g, 1/g)$$

where $g < 1$ is the mean reflectance value in the scene in each channel, i.e. $g$ for 'grey' world. Note that $1/g > 1$.

If the scene does not contain a white patch, however, then the situation is more subtle. The RGB image values again will be scaled by $(1/max(R), 1/max(G), 1/max(B)$. If the patch that has the highest value in the red channel indeed has reflectance 1 in that channel, then the scaling will indeed remove the illumination effect for that channel, and the same with the green and blue channels. However, if the patch that has the red channel does not have reflectance one in that channel, then it will be rescaled to value 1.

Note that the maxRGB method is based on a different assumption than the grey world method. The maxRGB method is assuming that there exists at least one patch in the scene that has reflectance 1 in the R channel, and similarly for G and B channels in possibly different patches. The methods will do the right thing *if their underlying assumption is true.* In particular the maxRGB method will do the right rescaling if there is a white patch in the scene (independent of whether the average reflectance in the scene is neutral i.e. grey) and also if the illuminance is constance over position $(x, y)$.

(b) The grey world assumption could be applied separately in the left and right halves of the image, since the two halves have very different averages intensities.[1] The average value in the left half is yellowish and so the visual system would divide the RGB channels in the left half by a yellowish average value, bringing the values in the left to neutral, and it would divide the RGB values in the right half by a bluesh average value, bring the values in the right half to neutral. Note in particular that the average values in the left and right half would both be (1,1,1) and so not just the color (hue and saturation) effect would be compensated, but also the intensity would be compensated.

[1] How the visual system could do that is not described here. I note only that this would not happen in the retina, which only contains very local operations.