lecture 16

shape from shading

Given an image $I(x,y)$, compute a 3D model $\{(X,Y,Z)\}$ that explains the image intensities.

We need:

- a shading model
- some of the model's parameters

Typical Assumptions:

- weak perspective (see below).
- surface is smooth
- surface is Lambertian
- surface has uniform reflectance
- illumination from single direction

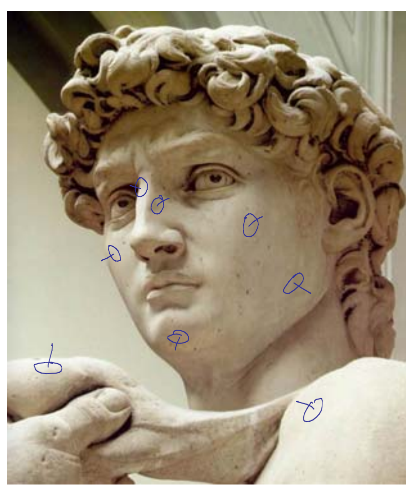
Shape-from-shading on a sunny day

Assumptions:

-
- ⋮
- illumination from single direction

$$\Rightarrow \quad E(x,y) = \vec{n}(x,y) \cdot \vec{l}_{src}$$

$E(x,y)$ units chosen so that $E = 1$ when $\vec{n} = \vec{l}_{src}$

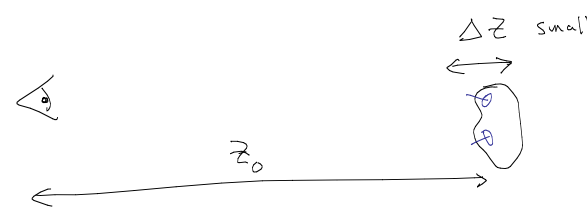


⌀ represents a

- disk lying tangent to surface, and
- normal vector $n(x,y)$

Goal is to compute $\vec{n}(x,y)$ at each pixel.

Weak Perspective ($Z \approx Z_0$)



Assume

$$x \approx f \frac{X}{Z_0}$$

$$y \approx f \frac{Y}{Z_0}$$

We are estimating the shape of a small surface, not the 3D geometry of the whole scene.

Weak Perspective

$$E(x,y) = E\left(\frac{f}{Z_0}X, \frac{f}{Z_0}Y\right)$$

pixels

Let surface depth map be $Z(X,Y)$.

$E(x,y)$ depends on $Z(X,Y)$ via a shading model.

[Notation: I will write $E(X,Y)$ from now on.]

Shape from shading on a sunny day

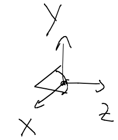
Recall lecture 6

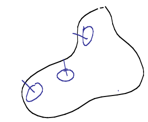
$$E(X,Y) = \vec{n}(X,Y) \cdot \vec{l}_{src}$$

surface normal at $(X, Y, Z(X,Y))$

How is $\vec{n}(X,Y)$ determined by $Z(X,Y)$?

Surface normal





$n(X,Y)$ is determined by $\left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}\right)$. How?

Consider two nearby points on the surface (X, Y, Z) and $(X+\Delta X, Y+\Delta Y, Z+\Delta Z)$.

$$Z(X+\Delta X, Y+\Delta Y) \approx Z(X,Y) + \frac{\partial Z}{\partial X}\Delta X + \frac{\partial Z}{\partial Y}\Delta Y$$

$$\Rightarrow \quad \Delta Z = \frac{\partial Z}{\partial X}\Delta X + \frac{\partial Z}{\partial Y}\Delta Y$$

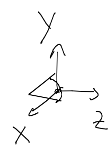
$$\Rightarrow \quad (\Delta X, \Delta Y, \Delta Z) \cdot \left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1\right) = 0$$

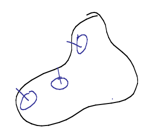
any step tangent to surface

$\therefore$ this is the surface normal direction

Unit Surface Normal

$$n(X,Y) = \frac{1}{\sqrt{\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 + 1}} \left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1\right)$$



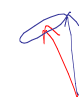


Shape from shading on a sunny day

$$E(X,Y) = \vec{n}(X,Y) \cdot \vec{l}_{src} = \frac{\left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1\right) \cdot \vec{l}_{src}}{\sqrt{\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 + 1}}$$

Note: $E(X_0,Y_0)$ and $\vec{l}_{src}$ do not uniquely determine $\vec{n}(X_0, Y_0)$.

l_{src}



Shape from shading on a sunny day

$$E(X,Y) = \frac{\left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1\right) \cdot \vec{l}_{src}}{\sqrt{\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 + 1}}$$

Given an image patch,

- assume or estimate $\vec{l}_{src}$
- estimate $\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}$ at each pixel

& integrate to estimate $Z(X,Y)$

There have been hundreds of attempts/methods to solve this problem.

☹ "Exact solutions" are not useful in practice since the assumptions are so strong.

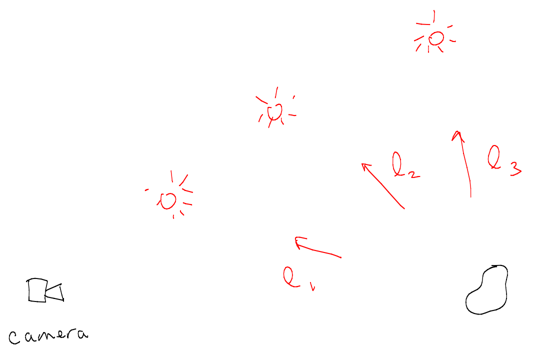
- Real surfaces are not exactly Lambertian.
- Single light source model l_{src} rarely holds
- Light source is not known

Surface reflectance rarely uniform.

SFS - Sunny Day is historically very important.

- Understanding why a problem is difficult led to many techiques for solving related problems
- Shading is still an important visual cue. Just don't expect an exact solution...

Related Problems: (1) Photometric Stereo



Idea is to use multiple light source directions. $\vec{\ell}_1 \; \vec{\ell}_2 \; \vec{\ell}_3$

$$\begin{bmatrix} E_1(x,y) \\ E_2(x,y) \\ E_3(x,y) \end{bmatrix} = \begin{bmatrix} \ell_x^1 & \ell_y^1 & \ell_z^1 \\ \ell_x^2 & \ell_y^2 & \ell_z^2 \\ \ell_x^3 & \ell_y^3 & \ell_z^3 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

If $\vec{\ell}$ vectors are known (lab setup) then 3×3 ℓ-matrix can be inverted and we can solve for $\vec{n}$.

$$\begin{bmatrix} E_1(x,y) \\ E_2(x,y) \\ E_3(x,y) \end{bmatrix} = \begin{bmatrix} \ell_x^1 & \ell_y^1 & \ell_z^1 \\ \ell_x^2 & \ell_y^2 & \ell_z^2 \\ \ell_x^3 & \ell_y^3 & \ell_z^3 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \cdot \underbrace{\rho(x,y)}_{\text{reflectance}}$$

Note: with three $\vec{\ell}$'s, we can in principle solve for a scalar reflectance too! (up to an unknown scale).

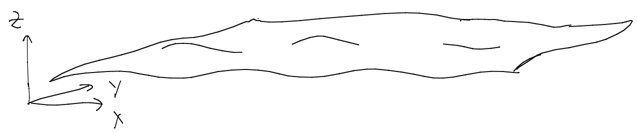
☹ still need to handle shadows, and non-Lambertian effects

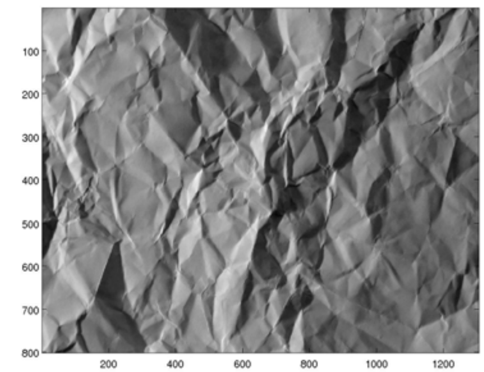
☺ Using more than 3 images helps!

Related Problems (2): linear SFS

$$n(x,y) = \frac{\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

Consider case: $\frac{\partial z}{\partial x}$ $\frac{\partial z}{\partial y}$ are small





$$n(x,y) = \frac{\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

Take Taylor series expansion of denominator

$$\frac{1}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}} \approx 1 - \frac{1}{2}\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] + \text{higher order terms}$$

$$\approx 1, \quad \text{if } \frac{\partial z}{\partial x} \; \frac{\partial z}{\partial y} \text{ are small}$$

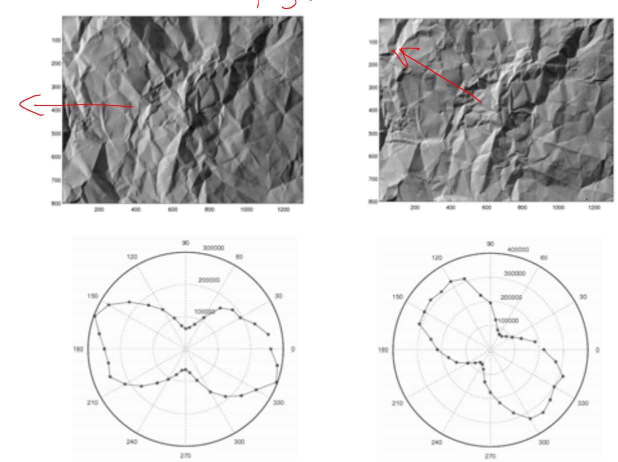
$$E(x,y) = \frac{\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right) \cdot \vec{\ell}_{src}}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

$$\approx \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right) \cdot (\ell_x, \ell_y, \ell_z)$$

linear shading

If $|l_z| \ll |(\ell_x \; \ell_y)|$, then we can get significant shading effects!

(ℓ_x, ℓ_y) direction



$$E(X,Y) = \left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1\right) \cdot (\ell_x, \ell_y, \ell_z)$$

Solution:

Integrate along lines in direction $(\ell_x\ \ell_y)$

e.g. if $\ell_y = 0$, then:

$$\int_{X_{min}}^{X_0} E(X,Y)\, dX = -\ell_z (X_0 - X_{min}) + \ell_x \int_{X_{min}}^{X_0} \frac{\partial Z}{\partial X}\, dX$$

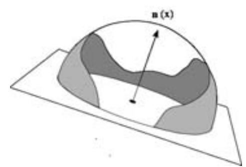
data

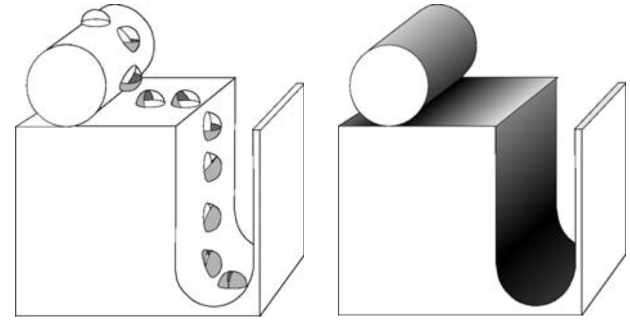
$$= -\ell_z (X_0 - X_{min}) + \ell_x \left(Z(X_0) - Z(X_{min}) \right)$$

Related Problems (3) Shape from shading on a cloudy day

$$E(X,Y) = \int_{\vec{\ell} \in V(X,Y)} \vec{n}(X,Y) \cdot \vec{\ell}\, d\Omega$$

$V(X,Y)$ is the set of directions in which the sky is visible from (X, Y, Z).





$$E(X,Y) = \frac{1}{\pi} \int_{\vec{\ell} \in V(X,Y)} \vec{n}(X,Y) \cdot \vec{\ell}\, d\Omega$$

$$\approx \frac{1}{2\pi} \int_{\vec{\ell} \in V(X,Y)} d\Omega$$

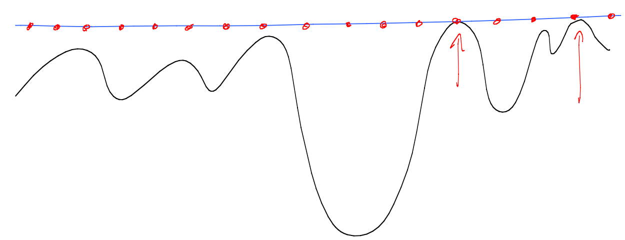
= % of sky that is visible from X, Y, Z

Shape from shading on a cloudy day

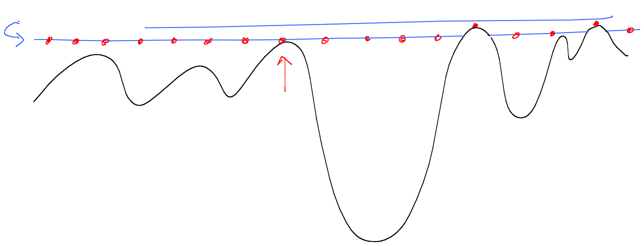
As always for SFS, assume weak perspective. Given an image $E(X,Y)$, find a surface depth map $Z(X,Y)$ that is consistent with

$$E(X,Y) = \frac{1}{2\pi} \int_{V(X,Y)} d\Omega$$

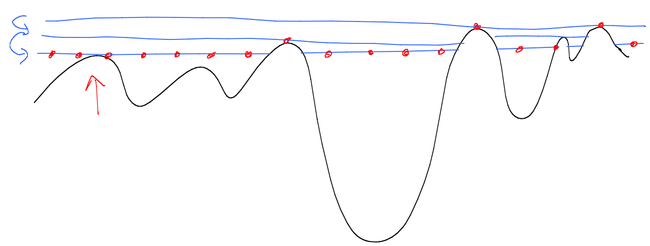
Water Spider Algorithm.



Water Spider Algorithm.

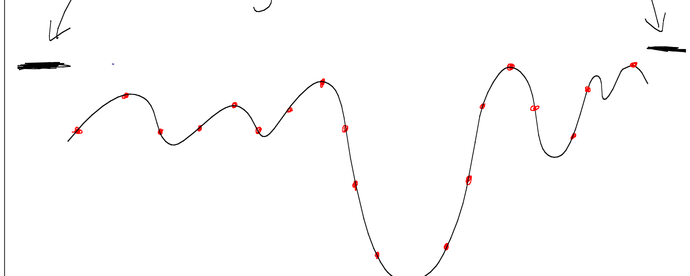


Water Spider Algorithm.

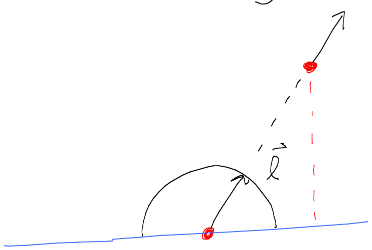


Water Spider Algorithm.

boundary conditions needed



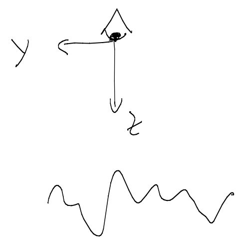
Local visibility constraints



Is the sky visible in direction $\vec{\ell}$?
Ask your neighbor.

Limitations

- $E(x,y) = \frac{1}{2\pi} \int_{V(x,y)} d\Omega$ is a very rough approximation
-



More general viewing geometry would be good.

Shape from shading is not a single problem

- sunny day (classical)
- photometric stereo
- linear
- cloudy day