Questions

1. When a face is photographed from the front and from a small distance, the nose appears much larger than it should in comparison to the other parts of the face. Why?

2. Suppose we have a plane
$$aX + bY + cZ = d$$
that is observed by a laterally moving observer ($T_z = 0$). Give an expression for the image velocity field as a function of x, y, T_x, T_y, f and any other scene variables.

3. 3D points at infinity are not the same thing as vanishing points, but they are related. How?

Answers

1. Since the nose is the closest part of the face, the depth Z of the nose is less than for other parts of the face. For fixed ΔX, note that

$$\Delta x = f \frac{\Delta X}{Z}.$$

Thus for two neighboring points on the nose that are separated by ΔX and are at roughly at the same depth Z_{nose}, the Δx will be greater than the Δx value for two neighboring non-nose points with the same ΔX but greater Z.

2. The velocity field for lateral motion is

$$(v_x, v_y) = \frac{f}{Z} \, (-T_x, \ \ -T_y).$$

Multiplying the equation of a plane by $\frac{df}{Z}$ gives:

$$\frac{ax + by + cf}{d} = \frac{f}{Z}.$$

Substituting the right side into the velocity field expression above gives:

$$(v_x, v_y) = \frac{ax + by + cf}{d} \, (-T_x, \ \ -T_y).$$

The field has a constant direction but the magnitude is an **affine** transformation of $\mathbf{x} = (x, y)$, namely it is of the form $\mathbf{x} \rightarrow \mathbf{A}\mathbf{x} + \mathbf{b}$ where $\mathbf{A}$ is a 2×2 and $\mathbf{b}$ is 2×1.

3. Recall that vanishing points were defined in terms of parallel scene lines, namely lines that are all parallel to some vector (T_X, T_Y, T_Z), and that pass through different points X_0, Y_0, Z_0. All such lines define the same point at infinity, which is written in homogeneous coordinates $(T_X, T_Y, T_Z, 0)$.

