

Questions

1. The Apple iPhone 4 specifications say that the screen is 3.5-inch (diagonal) with 960×640 pixel resolution. Holding the phone at arm's length (say 21 inches), how many pixels are there per degree of visual angle?

BTW, the iPhone 6 has similar resolution <http://www.apple.com/ca/iphone-6/specs/>

2. High quality printers generate output with 600 dots per inch (dpi). How many dots are there per degree of visual angle, assuming a reading distance of 12 inches.
3. Suppose you buy a high definition television whose screen has 1080 pixel rows and is 54 cm high. As we will see a few lectures from now, the region of highest resolution of eye has about 120 cells per degree of vision angle. How far should you sit from the screen so that your peak eye resolution matches the screen resolution?
4. Consider a digital camera that has 3000×2000 pixels over a sensor area $30\text{mm} \times 20\text{mm}$. Let's pretend it is a pinhole camera. Suppose the distance from the pinhole to the sensor plane is 50 mm. What is the angular resolution in pixels per degree of visual angle?
5. Suppose we have a camera lens with focal length $f = 25$ mm (or 40 diopters). Suppose the distance from the center of the lens to the sensor plane is $Z_s = 30$ mm, and the lens aperture has diameter $A = 5$ mm.
 - (a) What is the depth of scene points that would be exactly in focus on the sensor plane?
 - (b) For points at a greater distance than in (a), what is the maximum blur width w on the sensor plane (measured in degrees of visual angle)?
6. (a) Suppose you accommodate to (focus at) some depth $Z_{focalplane}$. Show that the blur width in radians of a point at any other depth Z_o is

$$A \left| \frac{1}{Z_{focalplane}} - \frac{1}{Z_o} \right|.$$

The absolute value is needed to account for the fact that the focussed point can be in front or behind the sensor plane, i.e. you cannot have 'negative blur.'

Hint: Answering this just requires writing out the geometry and doing some basic algebraic manipulations. The derivation is several steps and you need to get a bit lucky since it is not obvious which way to proceed. But give it a shot, or at least have a look.

Also note that the answer to Questions 5(a,b) are consistent with this expression.

- (b) Plot this relationship (blur width in radians as a function $\frac{1}{Z_o}$). This is easy, except that it is weird to think of a plot in which the variable on the horizontal axis is $\frac{1}{Z_o}$.
- (c) How does depth of field vary with pupil size? In particular, if you double the pupil diameter, what is the effect on the depth of field?

7. Suppose an eye is focussed on an object at depth 2 m.
- (a) What would be the blur width of a distant point (at $Z = \infty$), measured in degrees of visual angle? Assume the distance from the center of the lens to the retina is 2 cm and the pupil diameter is 2 mm.
 - (b) Suppose the person's "depth of field" is about $\frac{1}{3}$ diopter. What is the corresponding range of depths that would appear to be in focus? State your assumptions.
8. Consider a person who is myopic and can read without blur when the page is at a distance between 10 and 12.5 cm. If this person puts on glasses with lenses that are -5 diopters, what is the new range of depths at which this person can read without blur?

Answers

1. Assuming the pixels are square, the diagonal has $\sqrt{960^2 + 640^2} \approx 1140$ pixels. The visual angle is $\frac{3.5}{21} \approx \frac{1}{6}$ radians, or about 10 degrees. So the phone has about 110 pixels per degree. As we will see, this is well matched to the density at the center of the fovea, which is why its called a *retinal display*.

2.

$$\begin{aligned} 600 \text{ dots per } \frac{1}{12} \text{ radian} &= 7200 \text{ dots per radian} \\ &= 7200/180 \times \pi \text{ dots per degree} \\ &= 125 \text{ dots per degree} \end{aligned}$$

3. 1080 rows per 54 cm implies 20 rows of pixels per cm on the screen, or 120 rows per 6 cm on the screen. For a peak eye resolution of about 120 sensors (cones) per degree, we need to sit at a distance such that 6 cm on the screen is about 1 degree of visual angle, or about $\frac{1}{57}$ radians. We would need to sit at Z such that $6/Z = 1/57$ and so $Z = 6 \cdot 57$ cm or about 3.4 m from the screen.
4. 100 pixels per mm on the sensor, so distance between pixel samples is .01 mm. The angular distance between pixels is .01 mm / 50 mm or $1/5000$ radians. Equivalently, the resolution is 5000 pixels per radian or $5000 \cdot \pi/180 \approx 87$ pixels per degree.
5. (a) Since $\frac{1}{f} = \frac{1}{Z_o} + \frac{1}{Z_s}$, we have $\frac{1}{Z_o} = 40 - \frac{100}{3}$ and so $Z_o = \frac{3}{20}m = 15$ cm which is rather close.
(b) For an object point on the optical axis ($X = Y = 0$), rays will be focussed at a point between the lens and the sensor surface i.e. in front of the sensor surface. This defines two triangles.

$$\frac{A}{Z_i} = \frac{w}{|Z_i - Z_s|}.$$

Note in the lecture I presented a detailed example in which the focus point is behind the sensor surface. Here the focus point is in front of the sensor surface.

For scene points that are farther away ($Z_o \rightarrow \infty$), the focussed image obeys $Z_i \rightarrow f$ and the blur is largest at points at infinity. The width of the blur on the sensor plane is thus:

$$w = \frac{A|f - Z_s|}{f} = \frac{0.005m \cdot |.03m - .025m|}{.025m} = .001m$$

which has visual angle of $\frac{1}{30}$ radians, or about 2 degrees. This is a large amount of blur, which is not surprising since from (a) we saw that the camera is focussed on a point that is quite near.

6. (a) Let Z_o, Z_i, f satisfies the thin lens law. Take the case $Z_i < Z_s$, i.e. Z_o is further than the focus distance. By similar triangles,

$$\frac{A}{Z_i} = \frac{w}{|Z_s - Z_i|}$$

and so

$$w = A \frac{|Z_s - Z_i|}{Z_i} = AZ_s \left| \left(\frac{1}{Z_i} - \frac{1}{Z_s} \right) \right|.$$

Applying the thin lens law

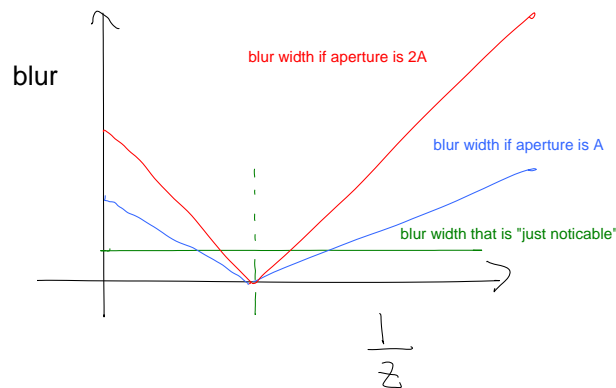
$$w = AZ_s \left| \left(\left(\frac{1}{Z_o} - \frac{1}{f} \right) - \left(\frac{1}{Z_{focalplane}} - \frac{1}{f} \right) \right) \right|$$

Since the blur width in radians is $\frac{w}{Z_s}$, we have

$$\frac{w}{Z_s} = A \left| \frac{1}{Z_o} - \frac{1}{Z_{focalplane}} \right|$$

Similar reasoning applies for $Z_i > Z_s$.

- (b) Verify from the formula that the blur width is zero when Z_0 is in focus, namely $\frac{1}{f} = \frac{1}{Z_0} + \frac{1}{Z_s}$. Next, note that when blur width is **plotted on a $\frac{1}{Z_0}$ axis**, it increases linearly away *in both directions* away from the zero blur depth. Finally, note that the slope of this linear function is A , the pupil diameter.



- (c) If you change pupil diameter A , you are just changing the slope of the linear function (see the formula in the question). So, if you double the slope A , then you halve (in diopters) the distance between the focus depth and the depth where “just noticeable” blur occurs. In the figure above, consider the range of depths (in diopters) of the green line segment between the two blue sloped lines versus the range of depths between the two red sloped curves.

7. (a) We can blindly use the formula from the previous question to get the answer immediately: $.002|\frac{1}{2} - \frac{1}{\infty}|$ or .001 radians.

Alternatively, we can pretend we don't know that nice formula and instead go through a calculation. This at least shows that we know what's going on.

First calculate the focal length. From the thin lens law,

$$\frac{1}{f} = \frac{1}{2} + \frac{1}{.02} = 50.5, \quad f \approx .0198.$$

Then using similar triangles, calculate the *blur width* in meters using

$$\frac{A}{f} = \frac{\text{blurwidth}}{.02 - f}$$

So $\text{blurwidth} = .002 \times 50.5 \times (.02 - .0198) \approx .00002m = .02mm$. Then, visual angle of $\text{blurwidth} \approx .02/20 = .001$ radians, or $\approx .057$ degrees.

- (b) The question says the eye is focussed at a distance 2 meters or 0.5 D. Let the depth of field be defined as the range of depths that are in focus. If the depth of field is $\frac{1}{3}$ diopter, then from the plot above that blur width increases linearly with dioptric distance from the focal plane, we can conclude that the limits of apparent focus are $\frac{1}{6}$ D in front of and $\frac{1}{6}$ D beyond the focal depth. So the range of depths that are in focus (in diopters) is about $[1/2 + 1/6, 1/2 - 1/6]$, or $[2/3, 1/3]$. So the range of depths that appear to be in focus is about 1.5 m to 3 m.
8. The range of depths over which one can read without blur is determined by two factors: the depth of field (which might vary between observers and between lighting conditions), and the range of accommodation. For both of these factors, putting on glasses just shifts the range of depths that have any given focus, and the shift is by diopters.

Use the relation

$$\frac{1}{f} = \frac{1}{Z_i} + \frac{1}{Z_0}.$$

We are given two values of Z_0 , 10 and 12.5 cm, and for these values $\frac{1}{Z_0}$ is $10D$ and $8D$, respectively (where D is diopters). If we add a lens with power $-5D$, then we replace $\frac{1}{f}$ with $\frac{1}{f} - 5$ to get the new power. To keep the same amount of blur in the image, we keep $\frac{1}{Z_i}$ fixed. This keeps the geometry of rays the same between the eye lens and the retina. Thus, we need to subtract 5 from each $\frac{1}{Z_0}$ to keep the left and right sides of the above equation the same. So the two new values of $\frac{1}{Z_0}$ will be $10-5 = 5 D$ and $8-5 = 3 D$, which correspond to distances $Z_0 = 20$ and $33.3cm$ which is a more typical distance for holding the book.