Questions

1. The Shi and Tomasi tracking method considers a deformation model:

$$\sum_{\mathbf{x} \in Ngd(\mathbf{x_0})} (I(\mathbf{x} + \mathbf{h} + \mathbf{D}(\mathbf{x} - \mathbf{x_0})) - J(\mathbf{x}))^2$$

where $\mathbf{x} = (x, y)$ and $\mathbf{x_0} = (x_0, y_0)$. Note that I have dropped the weighting function $W(\mathbf{x} - \mathbf{x_0})$ to simplify notation.

   (a) Rewrite this model by taking a first order Taylor series approximation. The notation will be easier if you let $(\Delta x, \Delta y) = (x - x_0, y - y_0)$.

   (b) Using (a), write out the least squares problem to be solved in terms of this first order model. The variables that we wish to solve for are the four $D_{i,j}$ and the h_x and h_y.

Answers

1. (a) The first order Taylor series approximation at $\mathbf{x}$ is:

$$I(\mathbf{x} + \mathbf{D}\Delta\mathbf{x} + \mathbf{h}) \approx I(\mathbf{x}) + (\frac{\partial I}{\partial x})(D_{11}\Delta x + D_{12}\Delta y + h_x) + (\frac{\partial I}{\partial y})(D_{21}\Delta x + D_{22}\Delta y + h_y)$$

where the partial derivatives of $I(\mathbf{x})$ are evaluated at $\mathbf{x} = (x, y)$.

(b) We organize the terms in (a). Let

$$\mathbf{u} = (D_{11}, D_{12}, D_{21}, D_{22}, h_x, h_y)$$

and let

$$\mathbf{a} = (\frac{\partial I}{\partial x}\Delta x, \frac{\partial I}{\partial x}\Delta y, \frac{\partial I}{\partial y}\Delta x, \frac{\partial I}{\partial y}\Delta y, \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y})$$

Substituting into the sum of squares, we get

$$\sum_{\mathbf{x} \in Ngd(\mathbf{x_0})} (I(\mathbf{x}) - J(\mathbf{x}) + \mathbf{a} \cdot \mathbf{u})^2$$

This expression can now be written in the form $\| \mathbf{Au} - \mathbf{b} \|^2$ where the rows of $\mathbf{A}$ are the $\mathbf{a}$ vectors corresponding to the different positions $\mathbf{x}$ in the neighborhood.

