


Sample space



Events

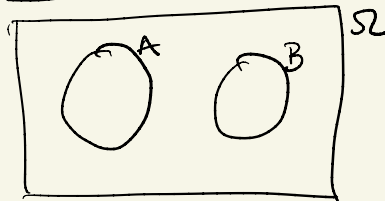
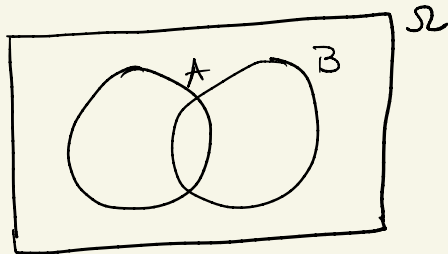
$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$

$$A^c, \bar{A} = \{1, 3, 5\}$$

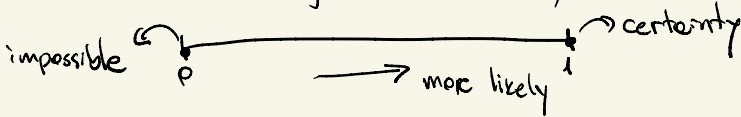
$$A \cup B$$

$$A \cap B$$



$$A \cap B = \emptyset$$

Probability = measure of the likelihood of an event



Naive definition

$$P(A) = \frac{\# \text{ outcomes favorable to } A}{\# \text{ outcomes in } \Omega}$$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(\boxed{\begin{smallmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{smallmatrix}}) = \frac{1}{6}$$

Probability as relative frequency:

$$P(A) = \lim_{n \rightarrow \infty} \frac{\#(A)}{n}$$

Axiomatic definition of probability

$$P: \Omega \rightarrow [0, 1]$$

$$1) A \subset \Omega, P(A) \geq 0$$

$$2) P(\Omega) = 1$$

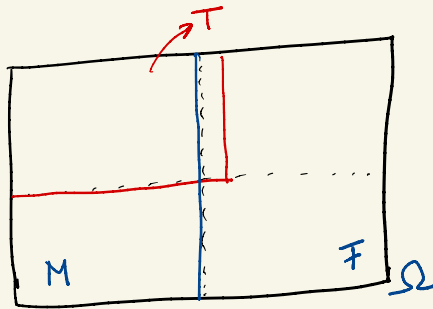
$$3) A \text{ and } B \text{ are disjoint events, } P(A \cup B) = P(A) + P(B)$$

Corollaries:

$$P(A^c) = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$$P(T) = 0.30$$

$$P(T|M) = 0.50$$

$$P(T|F) = 0.10$$

$$P(T \cap M)$$

$$P(T|M) = \frac{P(T \cap M)}{P(M)}$$

$$P(A|B) P(B) = P(A \cap B) \rightarrow \text{Multiplicative Rule}$$

$$P(B|A) P(A) = P(A \cap B)$$

- 0.8% Women 40-50 have cancer
- If woman has cancer, the mammogram will be positive with probability 90% $P(+|Cancer)$
- If woman does not have cancer, the mammogram will still be positive with probability 7%

$$P(Cancer | +) = \frac{P(Cancer \text{ and } +)}{P(+)} = \frac{P(+|Cancer) P(Cancer)}{P(+)}$$

$$= \frac{P(+|Cancer) P(Cancer)}{P(+|Cancer) P(Cancer) + P(+|\overline{Cancer}) \underbrace{P(\overline{Cancer})}_{1 - 0.008}}$$

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