

1. (a) Define the term mobility as used in spatial mechanisms
(b) With an aid of neat diagram, classify the kinematic pairs used in spatial mechanisms according to the number of degrees of freedom they have
2. Points A , B , and C on a rigid body, rotate by 30° about an axis that passes through the origin pointing toward point D . Find the new coordinates (A' , B' , and C') if the coordinates of point D are $(2, 2, 20)$.
3. frame $\{B\}$ is located initially coincident with a frame $\{A\}$. Frame $\{B\}$ is rotated 30 degrees about Z_B , and then the resulting frame is rotated about X_B by 45 degrees. Give the rotation matrix that will change the description of vectors from B_p to A_p .
4. A point $p = (7, 3, 2)^T$ is attached to uvw frame and is subjected to the transformations described next. Find the coordinates of the point relative to reference frame
 - (i) Rotation of 90° about z -axis, followed by a translation of $(4, -3, 7)$ along the xyz frame and finally a rotation of 90° about the y -axis.
 - (ii) Rotation of 90° about w -axis, followed by translation of $(4, -3, 7)$ along uvw frame and finally a rotation of 90° about the v -axis

EMT 2542: Industrial Robotics

1. Figure 1 shows a cylindrical robot with spherical wrist. For this configuration
 - (a) Assign the coordinate frame based on the DH-representation.
 - (b) Fill out the link parameter table
 - (c) Write the ${}^k_{k-1}H$ for $k = 1, 2, 3$.
 - (d) Derive the forward kinematic equation for the manipulator.

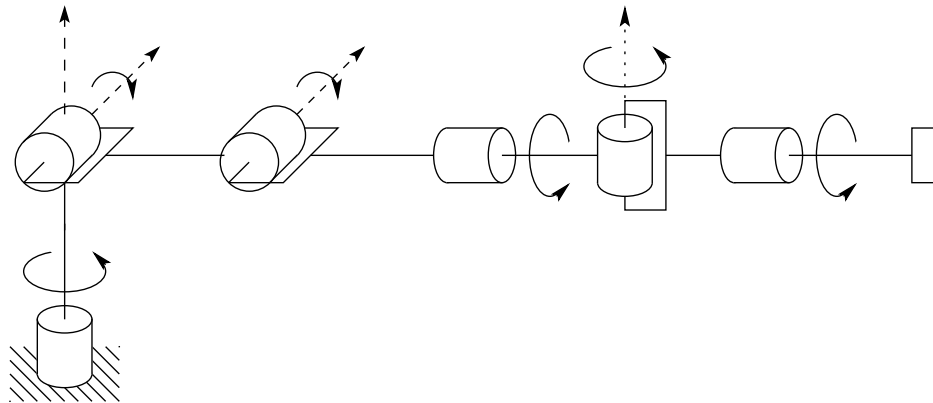


Figure 1: Q1

2. For the stanford manipulator shown in Fig. 2, derive the forward kinematic equations using the DH-convention
3. Assign link frames to the RPR planar robot shown in Fig. 3 , and give the linkage parameters

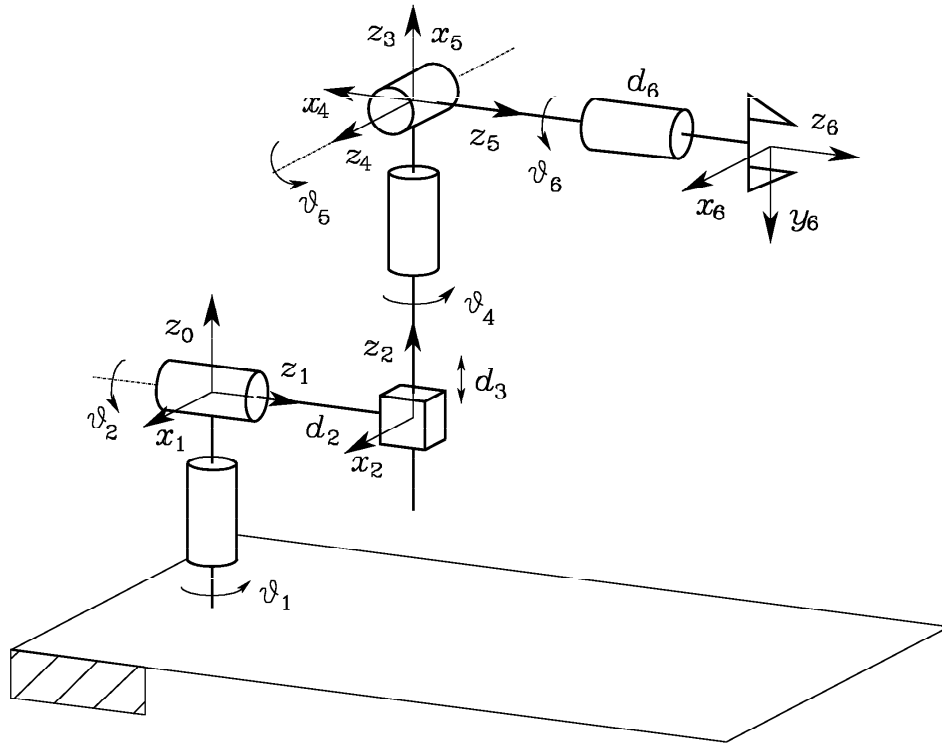


Figure 2: Q2

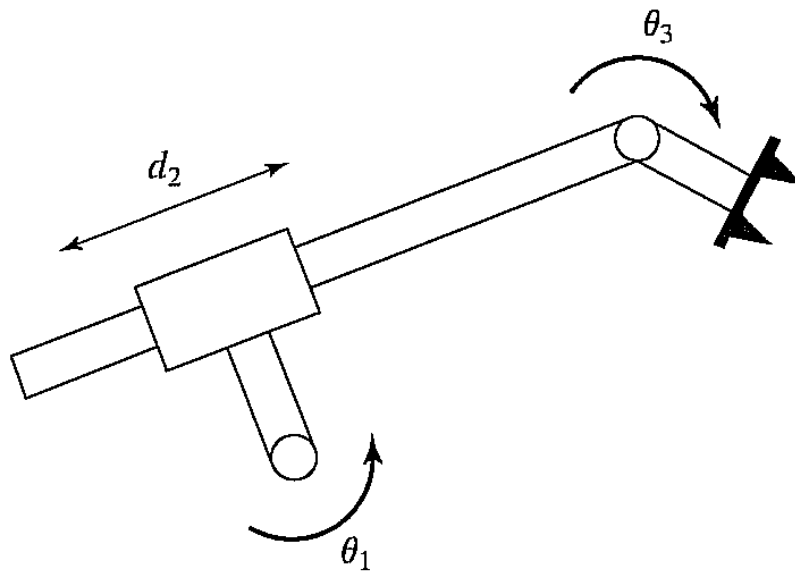


Figure 3: Q3

BACHELOR OF SCIENCE IN MECHATRONIC ENGINEERING

EMT 2542: ROBOT KINEMATICS AND DYNAMICS, ASSIGNMENT ONE

DATE: 1ST JULY 2021

TIME: 1 HOUR

INSTRUCTIONS

1. Answer ALL questions and show your working steps where applicable.

(1) With the aid of neat diagrams, classify robots based on their configurations and explain the applications of each configuration. [5 Marks]

(2) Points A , B , and C on a rigid body, rotate by 30° about an axis that passes through the origin pointing toward point D . Find the new coordinates (A' , B' , and C') if the coordinates of point D are $(2, 2, 20)$. [4 Marks]

(3) In line with efforts to combat COVID-19, you have been tasked with designing a robot that will assist health care providers in drug and food delivery to patients so as to minimize physical interactions.

(a) Explain your design process and state your main design considerations.

(b) State and justify the type of robot to be developed.

(c) Detail, the type of sensors, actuators, payload and the control mechanism to be incorporated for the robot to effectively execute its tasks. Give reasons for your selection.

(d) Draw a sketch of the robot with the different components highlighted. [8 Marks]

(4) $P = (2, 1)$ is a point in the xy -plane. Point P is moved along the diagonal of 30° for a distance of 8 units to a new point P' . Get the coordinates of point $P' = (x_2, y_2)$ [3 Marks]

(5) A point $p = (7, 3, 2)^T$ is attached to uvw frame and is subjected to the following transformations. Find the coordinates of the point relative to reference frame.

(i) Rotation of 90° about z -axis, followed by a translation of $(4, -3, 7)$ along the xyz frame and finally a rotation of 90° about the y -axis.

(ii) Rotation of 90° about w -axis, followed by translation of $(4, -3, 7)$ along uvw frame and finally a rotation of 90° about the v -axis [6 Marks]

(6) A frame $\{B\}$ is located initially coincident with a frame $\{A\}$. Frame $\{B\}$ is rotated 30 degrees about Z_B , and then the resulting frame is rotated about X_B by 45 degrees. Give the rotation matrix that will change the description of vectors from B_p to A_p . [4 Marks]

INSTRUCTIONS

Attempt ALL the questions.

QUESTIONS ONE

Coordinate system B is initially aligned with coordinate system A . It is translated to the point $[15, 4, 10]^T$ and then rotated 30 degrees about its x-axis. Lastly, the coordinate system is rotated 60 degrees about an axis that passes through the point $[12, 0, 8]^T$, measured in the current coordinate system, which is parallel to the y-axis. Find the position of frame B relative to frame A , ${}^B_A T$.

[15 Mks]

QUESTIONS TWO

A special three-degree-of freedom spraying robot has been designed as shown in Fig. Q2.

- (a) Assign the coordinate frame based on the $D - H$ representation.
- (b) Fill out the link parameter table
- (c) Write the ${}^k_{k-1}H$ for $k = 1, 2, 3$.
- (d) For fixed l_2 (2 degree of freedom robot) and

$${}^0_{tool}H = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 0 & -1 & y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find θ_i for $i = 1, 2$. We assume that point $(x, y, 0)$ is within the workspace of the robot.

[20 Mks]

QUESTIONS THREE

Find the composite rotation matrix representing the following

- (a) A rotation of α about u-axis, a rotation of θ about w-axis and rotation of ϕ about y-axis
 \times z
- (b) Rotation of ϕ about y-axis, a rotation of θ about w-axis and a rotation of α about the u-axis.

[15 Mks]

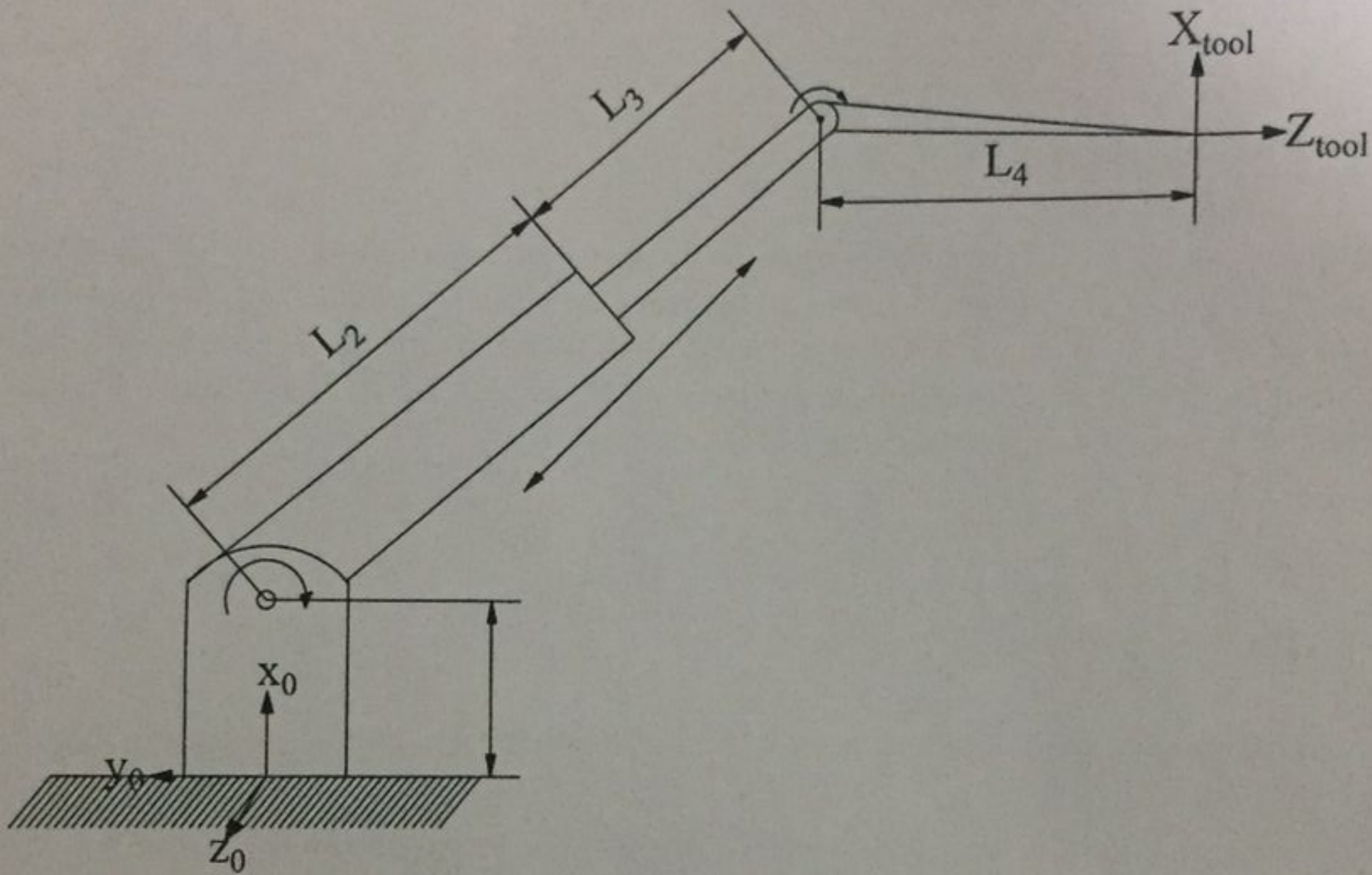


Fig. Q.2

QUESTION FOUR

- (a) For a 3-DOF articulated arm, the joint-link transformation matrices, with joint variables $\theta_1, \theta_2, \theta_3, L_2, L_3$ are

$$T_o^1(\theta_1) = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1^2(\theta_2) = \begin{bmatrix} c_2 & -s_2 & 0 & L_2 c_2 \\ s_2 & c_2 & 0 & L_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3(\theta_3) = \begin{bmatrix} c_3 & -s_3 & 0 & L_3 c_3 \\ s_3 & c_3 & 0 & L_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (i) Determine the simplified transformation matrix for the end point of the arm.
- (ii) For the 3 DOF articulated arm, whose direct kinematic model has been obtained in a(i) above, determine the joint displacements if the total configuration matrix with $L_2 = 0.75$ m, $L_3 = 0.3$ m is

$$T_0^3 = \begin{bmatrix} -0.2241 & 0.8365 & 0.5 & 0.1777 \\ -0.1292 & -0.4830 & -0.866 & 0.1026 \\ 0.9654 & -0.2588 & 0 & 0.5776 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

QUESTION FIVE

- (a) Explain why the inverse kinematic problem is more difficult to solve than forward kinematic problem
- (b) Why is closed-form solution preferred in inverse kinematic problem?
- (c) For the two link planar manipulator RR , find the joint variables given that the position of the end-effector is described by (X, Y)
- (d) It is desired to place the origin of the hand frame of a cylindrical robot shown in Fig. 1 at $[6 \ 4 \ 10]^T$. Calculate the joint variables of the robot.

EMT 2542: Industrial Robotics

1. Figure 1 shows a cylindrical robot with spherical wrist. For this configuration

- Assign the coordinate frame based on the DH-representation.
- Fill out the link parameter table
- Write the ${}^k_{k-1}H$ for $k = 1, 2, 3$.
- Derive the forward kinematic equation for the manipulator.

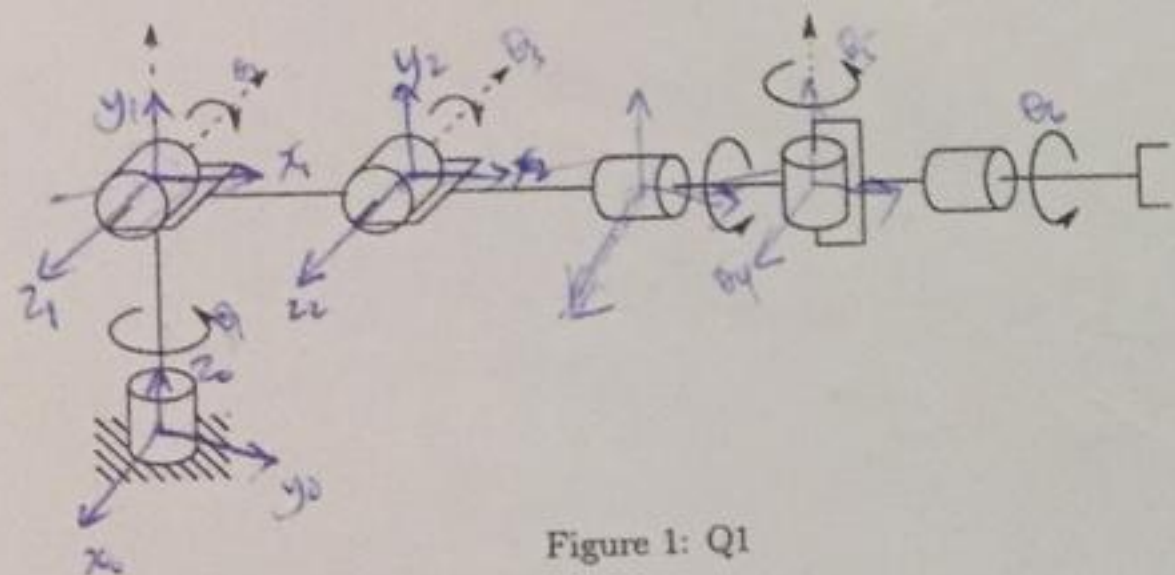


Figure 1: Q1

- For the stanford manipulator shown in Fig. 2, derive the forward kinematic equations using the DH-convention
- Assign link frames to the RPR planar robot shown in Fig. 3, and give the linkage parameters

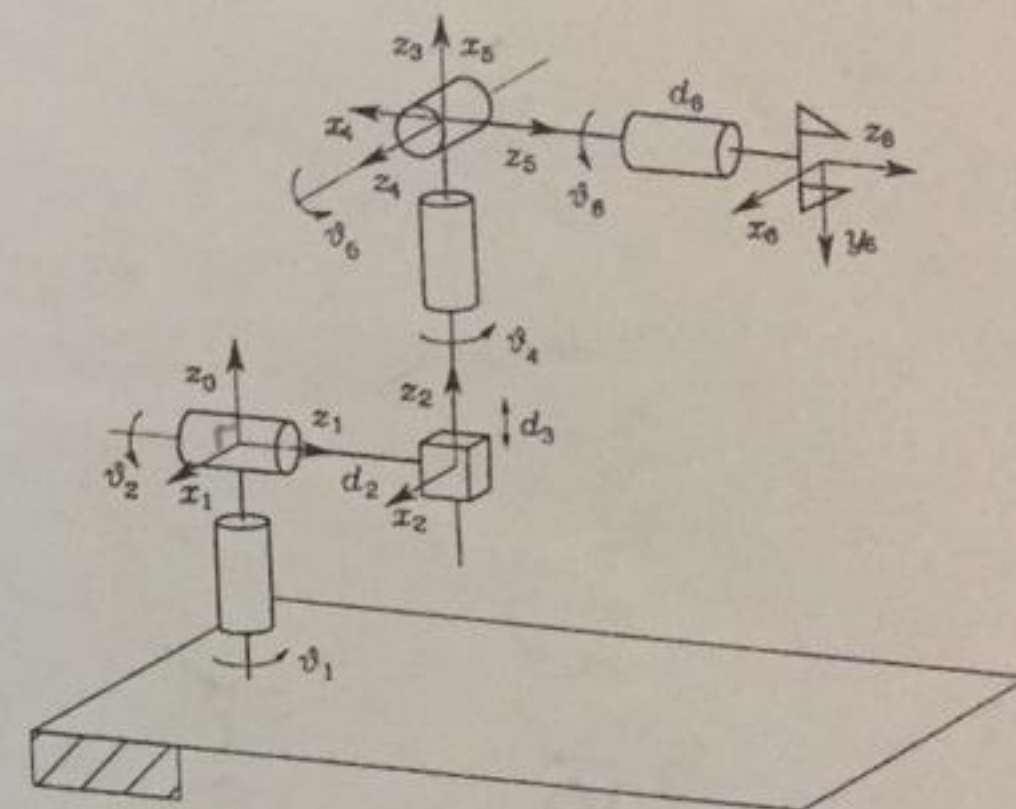


Figure 2: Q2

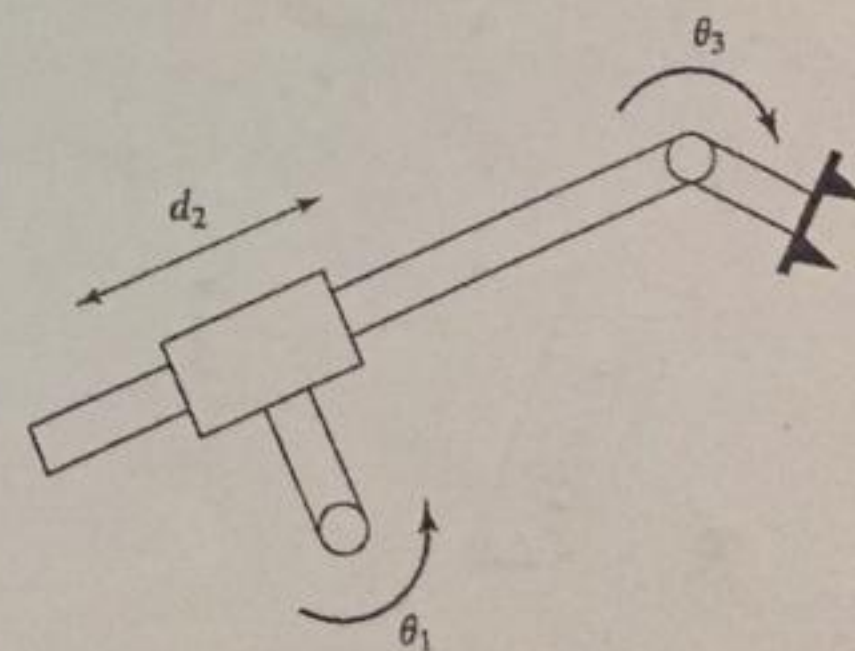


Figure 3: Q3

	a_i	α_i	d_i	θ_i
1	0	90	d_1	
2	a_2	0	0	
3	a_3	0	0	
4	0	-90	0	
5	0	0	0	
6	0	0	d_6	



0.02 kg/m

10 cm

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UNIVERSITY EXAMINATION 2020/2021

FIFTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF
BACHELOR OF SCIENCE IN MECHATRONIC ENGINEERING

EMT 2542: INDUSTRIAL ROBOTICS

DATE: AUGUST 2021

TIME: 2 HOURS

INSTRUCTIONS

1. This paper contains **FIVE (5)** questions.
2. Each question carries 20 marks..
3. Answer **ANY THREE** questions.

QUESTION ONE

[20 MARKS]

- (a) Differentiate between Newton-Euler and Lagrange-Euler dynamic modeling methods as used in robotics [3 Marks]
- (b) Describe degeneracy and dexterity as used in robots [2 Marks]
- (c) Fig. Q1(c) shows a two-link planar elbow robotic arm, where the mass of link k is denoted by m_k . The joint variable and generalized force vector are given as $q = [\theta_1 \ \theta_2]^T$, $\tau = [\tau_1 \ \tau_2]^T$, respectively. For this robotic arm configuration, determine its dynamics using Lagrange-Euler method. [12 Marks]
- (d) With the aid of a diagram, describe the vectors that represent the orientation of end-effector. [3 Marks]

QUESTION TWO

[20 MARKS]

- (a) Describe the three types of joints that are commonly found in robots. [3 Marks]
- (b) Explain the two main categories of sensors used in industrial robotics giving a few examples of each. [4 Marks]
- (c) A frame $\{B\}$ is rotated relative to frame $\{A\}$ about the Z-axis by 30 degrees and then translated by $[4, 3, 0]^T$. Find ${}^B_A T$. [4 Marks]

- (d) Coordinate system B is initially aligned with coordinate system A . It is translated to the point $[5, 4, 1]^T$ and then rotated 30 degrees about its x-axis. Lastly, the coordinate system is rotated 60 degrees about an axis that passes through the point $[2, 0, 2]^T$, measured in the current coordinate system, which is parallel to the y-axis. Find ${}^B_A T$. [9 Marks]

[20 MARKS]

QUESTION THREE

Fig. Q3 shows a cylindrical robot with spherical wrist. For the given configuration

- (a) Assign the coordinate frame based on the $D-H$ representation. [5 Marks]
 (b) Fill out the link parameter table [5 Marks]
 (c) Write the ${}^k_{k-1}H$ for $k = 1, 2, 3$. [2 Marks]
 (d) Derive the forward kinematic equation for the manipulator. [8 Marks]

[20 MARKS]

QUESTION FOUR

- (a) With the aid of diagrams, explain the classification of robots based on their basic configuration. [5 Marks]
 (b) A point $P = [7 \ 3 \ 2]^T$ is attached to uvw frame and is subjected to rotation of 90° about z-axis followed by rotation of 90° about y-axis and finally a translation of $(4, -3, 7)$. Find the coordinates of the point relative to the reference frame at the conclusion of transformation. [7 Marks]
 (c) A frame $\{B\}$ is described as initially coincident with $\{A\}$. We then rotate $\{B\}$ about the vector $\theta = [0.707 \ 0.707 \ 0.0]^T$ (passing through the point ${}^A P = [1.0 \ 2.0 \ 3.0]$) by an amount $\theta = 30$ degrees. Give the frame description of $\{B\}$. [8 Marks]

QUESTION FIVE

[20 MARKS]

- (a) Differentiate forward kinematic problem from inverse kinematic problem. Also, explain why the inverse kinematic problem is more difficult to solve. [5 Marks]
 (b) Describe workspace-boundary and workspace-interior singularities [2 Marks]
 (c) Describe the approaches used to solve the inverse kinematic problem. [3 Marks]
 (d) For the two link planar manipulator shown in Fig. Q5(d), find the joint variables using one of the approach described in (c) above given that the position of the end-effector is described by (P_x, P_y) [10 Marks]

Figures

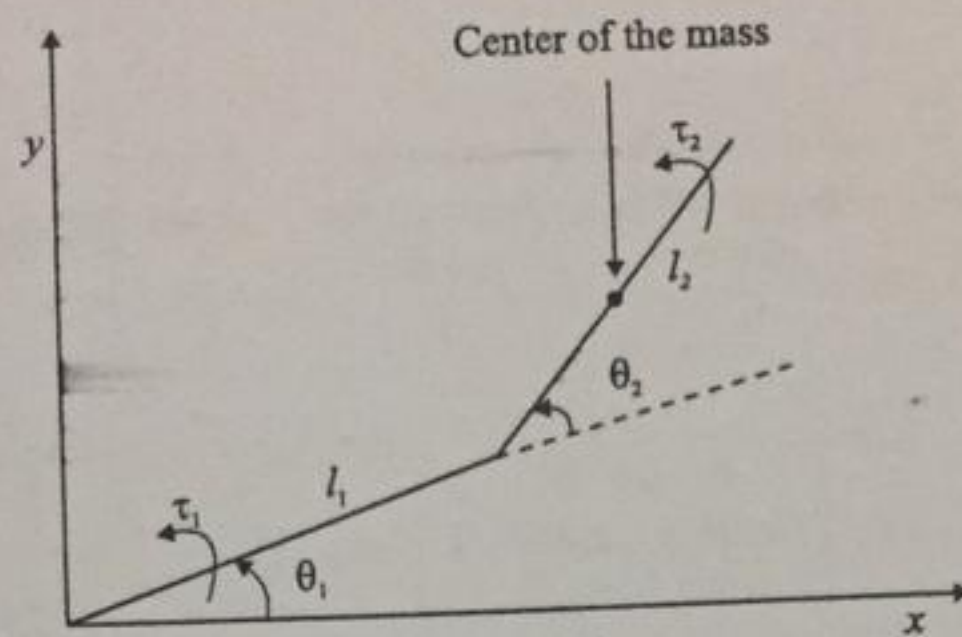


Fig. Q1(c)

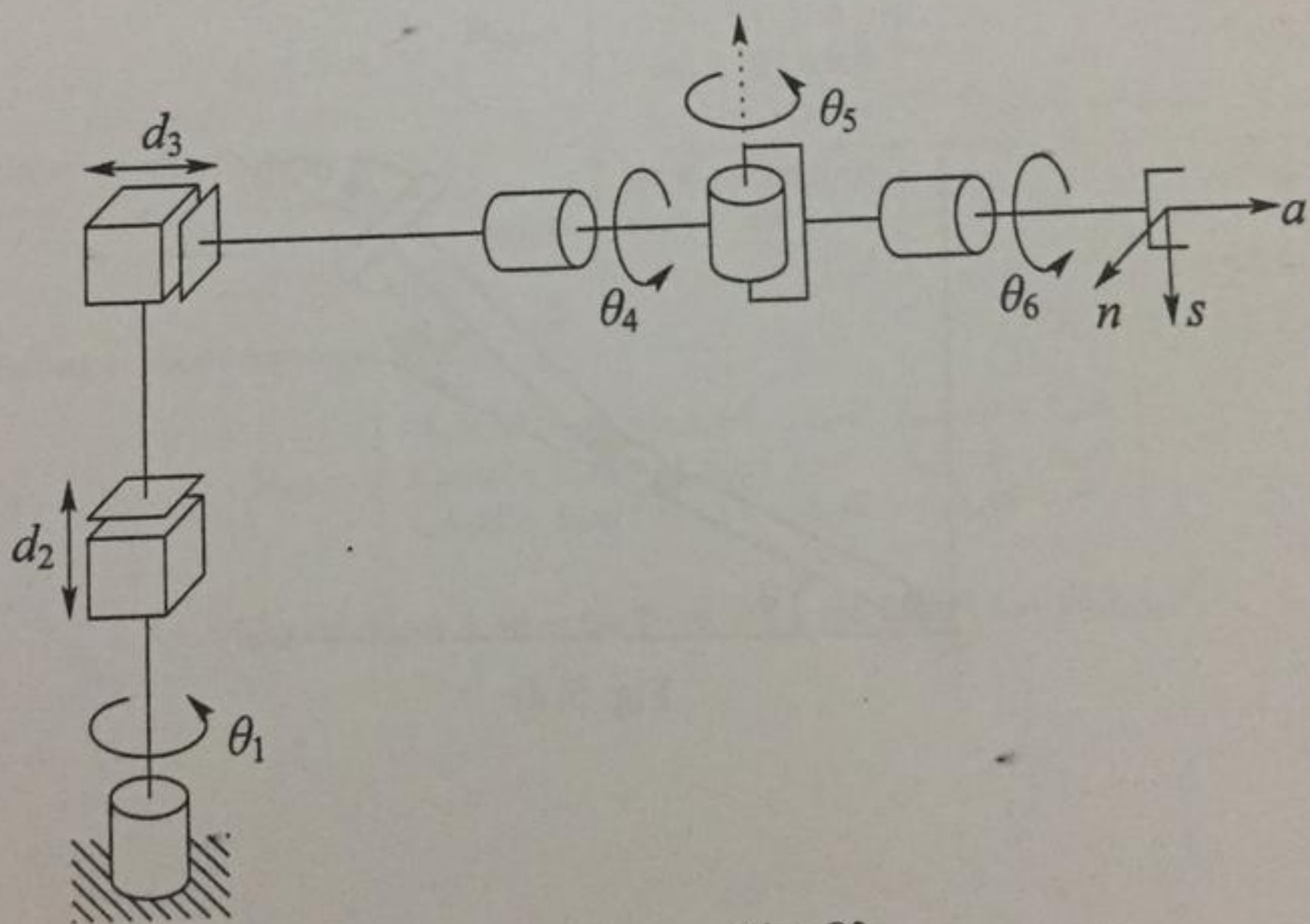


Fig.: Q3

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0.366 & -0.933 & 4 \\ 0 & 0.5 & 0.866 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0.366 & -0.933 & 3 \\ 0 & 0.5 & 0.866 & 2.732 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0 & 0.866 & 7 \\ 0.933 & 0.366 & -0.75 & 3 \\ 0.75 & 0.5 & 0.933 & 2.732 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0 & 0.866 & 9.262 \\ 0.933 & 0.366 & -0.75 & 2.674 \\ 0.75 & 0.5 & 0.933 & 0.566 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0\mathbf{p}_2 = {}^0\mathbf{p}_1 + l_1 \cos \theta_1 \mathbf{e}_1 + l_1 \sin \theta_1 \mathbf{e}_2$$

$$l_1 \cos \theta_1 + l_2 \cos \theta_2$$

$$\cos$$

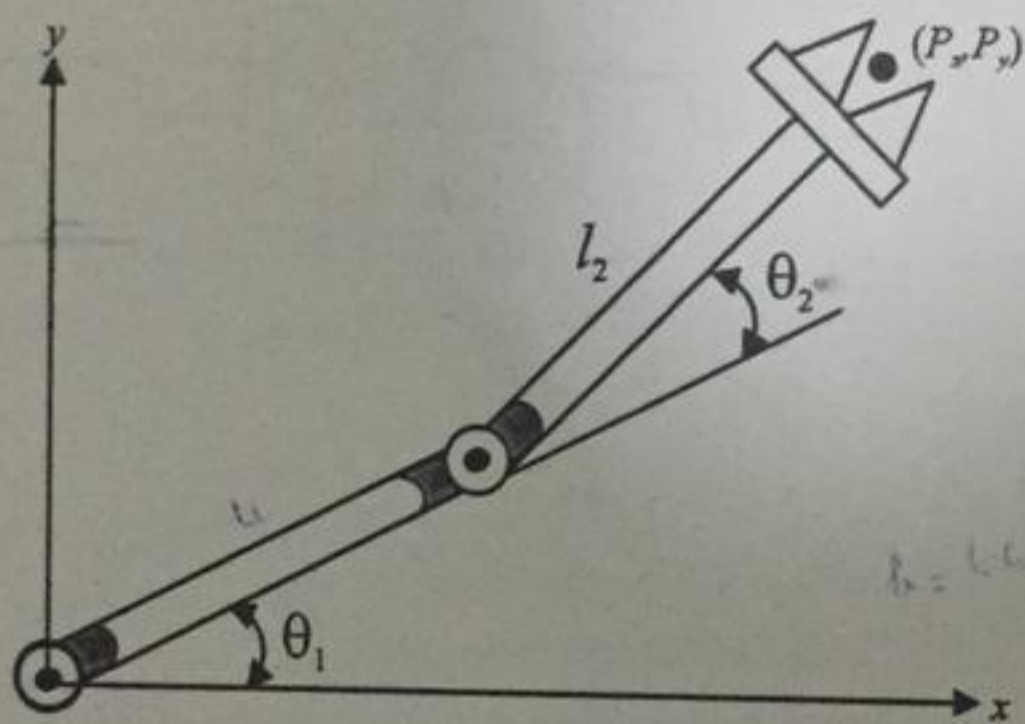


Fig. 5(d)

Appendix: Standard Transformations

- D-H representation

$${}^k_{k-1}H = \begin{bmatrix} \cos \theta_k & -\cos \alpha_k \sin \theta_k & \sin \alpha_k \sin \theta_k & a_k \cos \theta_k \\ \sin \theta_k & \cos \alpha_k \cos \theta_k & -\sin \alpha_k \cos \theta_k & a_k \sin \theta_k \\ 0 & \sin \alpha_k & \cos \alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

- Rotation about x-axis

$$R_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \quad (2)$$

- Rotation about y-axis

$$R_{y,\beta} = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \quad (3)$$

- Rotation about z-axis

$$R_{z,\gamma} = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

- Rotation about a general axis

$$R_{k,\theta} = \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix} \quad (5)$$

where $c\theta = \cos \theta$, $s\theta = \sin \theta$, $v\theta = 1 - \cos \theta$, and $\theta = [k_x k_y k_z]^T$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s\theta & 0 \\ s\theta & 0 & c\theta & 0 \\ 0 & 1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta & 0 & -s\theta & -d_3 s\theta \\ s\theta & 0 & c\theta & d_3 c\theta \\ 0 & 1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$