

## Assignment 6

1. Implement *ray\_sphere\_intersect()* to return the closest hit of a sphere (if it exists) and render the pixel color of the sphere at that point

We have all the parts of the Ray-Sphere intersection:  $(\mathbf{o} + t\mathbf{d} - \mathbf{c}) \cdot \mathbf{R}^2 = 0$ , so solve for the equation:  $at^2 + bt + c = 0$

$\mathbf{a} = \mathbf{d} \cdot \mathbf{d}$ , where  $\mathbf{d}$  is the normalized *ray\_direction*

$\mathbf{b} = 2(\mathbf{o} - \mathbf{c}) \cdot \mathbf{d}$ , where  $\mathbf{o}$  is the *ray\_origin* and  $\mathbf{c}$  is the *sphere.position* (location of center)

$\mathbf{c} = (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - \mathbf{R}^2$ , where  $\mathbf{R}$  is the *sphere\_radius*

We can first calculate the *discriminant* of the quadratic equation. If it is less than 0 then we have no real solutions so return **false**.

Otherwise, calculate the two solutions for  $\mathbf{t}$ , with  $\mathbf{t1}$  being the smaller (closer) solution. If  $\mathbf{t1}$  is within the range of  $t\_near$  and  $t\_far$  then return  $\{\mathbf{t1}, \mathbf{true}\}$  to find our intersection point/color. If not, check the same conditions for  $\mathbf{t2}$ . If neither solution is within  $t\_near$  and  $t\_far$  then we have no valid hit on the screen and return **false**.

2. Implement *ray\_triangle\_intersect()* to return the closest hit of a triangle mesh (if it exists) and render the pixel color of the triangle at that point

To find the intersection most efficiently we use the **Möller Trumbore Algorithm**.

We can find  $\mathbf{E}_1 = \mathbf{P}_1 - \mathbf{P}_0$  and  $\mathbf{E}_2 = \mathbf{P}_2 - \mathbf{P}_0$  by using the vertices of our triangle where  $v_0, v_1, v_2$  correspond to the points  $P_0, P_1, P_2$

Next find

$\mathbf{S} = \mathbf{O} - \mathbf{P}_1$ , where  $\mathbf{O}$  is the *ray\_origin*

$\mathbf{S}_1 = \mathbf{D} \times \mathbf{E}_2$ , where  $\mathbf{D}$  is the *ray\_direction*

$\mathbf{S}_2 = \mathbf{S} \times \mathbf{E}_1$

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\mathbf{S}_1 \cdot \mathbf{E}_1} \begin{bmatrix} \mathbf{S}_2 \cdot \mathbf{E}_2 \\ \mathbf{S}_1 \cdot \mathbf{S} \\ \mathbf{S}_2 \cdot \mathbf{D} \end{bmatrix}$$

Then find the matrix value of  $\mathbf{t}$  using the formula:

Finally, the check that  $\mathbf{t}$  is inside the bounds of the triangle, using the barycentric coordinates  $\mathbf{b}_1$  and  $\mathbf{b}_2$  with the final coordinate  $= (1 - \mathbf{b}_1 - \mathbf{b}_2)$ . If all 3 coordinates are  $\geq 0$ , then  $\mathbf{t}$  lies inside (or on the edge) of the triangle and we return  $\{\mathbf{true}, \mathbf{t}\}$

Otherwise, return **false**.