NORTHWESTERN UNIVERSITY | SCHOOL OF PROFESSIONAL STUDIES

Fall 2019 Midterm Exam

MSDS 400: Math for Data Scientists

Points possible: 100

Description: The midterm exam will cover topics from sessions 1-4.

Resources: The exam is completely open book. You may use course textbooks, materials provided on Canvas, graphing calculators (such as TI 83 or 84); *but any more advanced calculators, Excel Solver, Web calculators, Web-graphic calculators, or simplex method calculators are not allowed. Programming languages other than Python are also not permitted*.

For questions that require calculations, all calculations should be shown, not just the final answer. This will allow for partial credit for those answers that might be set up correctly but have calculation errors. For questions that specifically require Python, the code and output should be included with your answer. For questions that require graphs, only use Python.

Restrictions: All answers are to be your work only. You are not to receive assistance from any other person.

To complete the exam:

1. Answer all questions on the exam thoroughly. Create a Microsoft Word document, including the question number, the question, your typed answer, and graphs if required. You may use Word's equation editor to complete your answers.

2. Once you have completed your exam, return to the exam item where you downloaded the exam PDF, click View/Complete Assignment, and submit your document.

1. Reebok is designing a new type of Crossfit shoe, the Nano XIX. The fixed cost for the production will be $24,000. The variable cost will be $18 per pair of shoes. The shoes will sell for $120 for each pair. Graph the cost and revenue functions and determine how many pairs of sneakers will have to be sold for the company to break even on this new line of shoes.

`Problem 1`

Reebok is designing a new type of Crossfit shoe, the Nano XIX. The fixed cost for the production will be $24,000. The variable cost will be $18 per pair of shoes. The shoes will sell for $120 for each pair. Graph the cost and revenue functions and determine how many pairs of sneakers will have to be sold for the company to break even on this new line of shoes.

C(x) = mx + b; b = fixed cost, m = marginal/variable cost

b = $24,000 m = $18 per pair of shoes

R(x) = px; p = price per unit

p = $120 per pair of shoes

C(x) = 18x + 24,000

R(x) = 120x

Graph of C(x) and R(x) is shown below.

Break even quantity of sneakers is found when `R(x) = C(x)`

By this equation, the break-even quantity can be found graphically or algebraically.

Via Algebra:

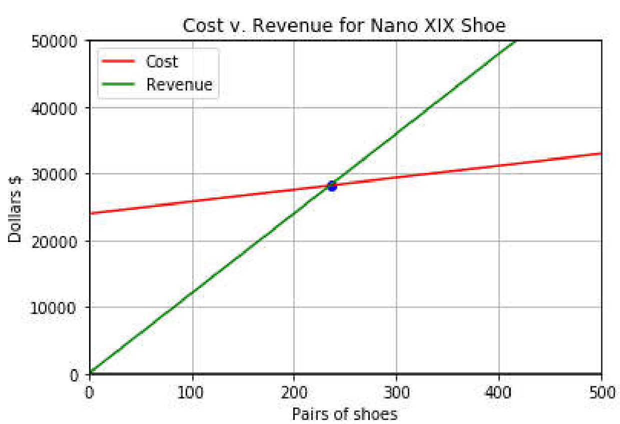
120x = 18x +24,000

102x = 24,000

x = 235.294 pairs of shoes. X must be a whole number. Cannot sell fraction of a pair of shoes. Therefore, will round up to the next whole pair of shoes as the break-even quantity.

`Break even quantity is 236 pairs of shoes`

```
import matplotlib.pyplot
from matplotlib.pyplot import *
import numpy
from numpy import arange

#Set-up Parameters

x_max_lim = 500
y_max_lim =50000
x= arange(0,x_max_lim+0.1,0.1) #Set up x-range
b = 24000  #fixed cost
m = 18     #variable cost
p = 120    #price per unit

#Set-up Linear functions

C = (m*x)+b
R = p*x
P = R - C

#Create Graph

figure()
xlim(0,x_max_lim)
ylim(0,y_max_lim)
xlabel('Pairs of shoes')
ylabel('Dollars $')
hlines(0,0,x_max_lim,color='k')
vlines(0,0,y_max_lim,color='k')
grid(True)
plot(x,C,'r-') #Graph cost function
plot(x,R,'g-') #Graph revenue function
title('Cost v. Revenue for Nano XIX Shoe')
legend(['Cost','Revenue'])
x = [236]
y = [28320]
scatter(x,y,color='b')
show()

print('The break-even ordered pair is (235.3,28235.29).')
print('''Becuase there cannot be a partial number for pairs of shoes,
I will round up to the next whole number as the break even quantity.''')
print('\nThe company will have to sell 236 pairs of sneakers in order to break even on this new line of shoes.')
```




```
The break-even ordered pair is (235.3,28235.29).
Becuase there cannot be a partial number for pairs of shoes,
I will round up to the next whole number as the break even quantity.

The company will have to sell 236 pairs of sneakers in order to break even on this new line of shoes.
```

#1.)

$C(x) = mx + b$; b = fixed cost

$b = \$24,000$; m = Marginal / variable cost

$m = \$18$/pair of shoes.

$R(x) = px$; $p = \$120$/pair

$\underline{C(x) = 18x + 24,000}$ $\underline{R(x) = 120x}$

$P(x) = 120x - (18x + 24,000) = 102x - 24,000$

$\underline{P(x) = 102x - 24,000}$

Break even qty: $R(x) = C(x)$

$120x = 18x + 24,000$

$102x = 24,000$

$x = 235.294$ shoes

round

$\boxed{x = 236 \text{ shoes}} \longrightarrow R(x) = \$28,320$

2. Arielle invests a total of $17,500 in three products. She invests one part in a mutual fund which has an annual return of 11%. She invests the second part in government bonds at 7% per year. The third part she puts in CDs at 5% per year. She invests twice as much in the mutual fund as in the CDs. In the first year Arielle's investments bring a total return of $1495. How much did she invest in each product?

Problem 2

Arielle invests a total of $17,500 in three products. She invests one part in a mutual fund which has an annual return of 11 percent. She invests the second part in government bonds at 7 percent per year. The third part she puts in CDs at 5 percent per year. She invests twice as much in the mutual fund as in the CDs. In the first year Arielle's investments bring a total return of $1495. How much did she invest in each product?

```
import numpy
from numpy import *
from numpy.linalg import *
```

```
A = matrix([[1,1,1],[0.11,0.07,0.05],[1,0,-2]]) #set up matrix from system of equations
rhs = matrix([17500,1495,0])
rhs = transpose(rhs) #set up right hand side of augmented matrix
result = linalg.solve(A,rhs)
print(result)
print('\nArielle invested $9,000 in Mutual Funds, $4,000 in Government Bonds, and $4,500 in CDs.')
```

```
[[9000.]
 [4000.]
 [4500.]]

Arielle invested $9,000 in Mutual Funds, $4,000 in Government Bonds, and $4,500 in CDs.
```

#2.)

Total = \$17,500

x_1 = mutal funds ; x_2 = gov bonds ; x_3 = CDs

mutual funds 2x CDs

$$x_1 + x_2 + x_3 = \$17,500$$

$$0.11x_1 + 0.07x_2 + 0.05x_3 = \$1495$$

$$x_1 = 2x_3$$

1

$$\left.\begin{aligned} x_1 + x_2 + x_3 &= 17500 \\ 0.11x_1 + 0.07x_2 + 0.05x_3 &= 1495 \\ x_1 \qquad\quad - 2x_3 &= 0 \end{aligned}\right\} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 17500 \\ 0.11 & 0.07 & 0.05 & 1495 \\ 1 & 0 & -2 & 0 \end{array}\right]$$

1b Use Gauss-Jordan Method to find solution

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 17500 \\ 0.11 & 0.07 & 0.05 & 1495 \\ 1 & 0 & -2 & 0 \end{array}\right] -R_1 + R_3 \to R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 17500 \\ 0.11 & 0.07 & 0.05 & 1495 \\ 0 & -1 & -3 & -17500 \end{array}\right] -0.11R_1 + R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 17500 \\ 0 & -0.04 & -0.06 & -430 \\ 0 & -1 & -3 & -17500 \end{array}\right] 0.04R_1 + R_2 \to R_1 \left[\begin{array}{ccc|c} 0.04 & 0 & -0.02 & 270 \\ 0 & -0.04 & -0.06 & -430 \\ 0 & -1 & -3 & -17500 \end{array}\right] -0.04R_3 + R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 0.04 & 0 & -0.02 & 270 \\ 0 & -0.04 & -0.06 & -430 \\ 0 & 0 & 0.06 & 270 \end{array}\right] R_3 + R_2 \to R_2 \left[\begin{array}{ccc|c} 0.04 & 0 & -0.02 & 270 \\ 0 & -0.04 & 0 & -160 \\ 0 & 0 & 0.06 & 270 \end{array}\right]$$

#2.) cont.

$$\left[\begin{array}{ccc|c} 0.04 & 0 & -0.02 & 270 \\ 0 & -0.04 & 0 & -160 \\ 0 & 0 & 0.06 & 270 \end{array}\right] \quad 3R_1 + R_3 \to R_1 \quad \left[\begin{array}{ccc|c} 0.12 & 0 & 0 & 1080 \\ 0 & -0.04 & 0 & -160 \\ 0 & 0 & 0.06 & 270 \end{array}\right] \begin{array}{l} R_1/0.12 \to R_1 \\ R_2/-0.04 \to R_2 \\ R_3/0.06 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 9000 \\ 0 & 1 & 0 & 4000 \\ 0 & 0 & 1 & 4500 \end{array}\right]$$

$$\boxed{x_1 = \$9,000 \qquad x_2 = \$4,000 \qquad x_3 = \$4,500}$$

3. Vandelay Industries has 252 sales reps, each to be assigned to one of four marketing teams. If the first team is to have three times as many members as the second team and the third team is to have twice as many members as the fourth team, how can the members be distributed among the teams?

`Problem 3`

Vandelay Industries has 252 sales reps, each to be assigned to one of four marketing teams. If the first team is to have three times as many members as the second team and the third team is to have twice as many members as the fourth team, how can the members be distributed among the teams?

First, I defined my variables x1, x2, x3, and x4 as follows:

`x1 = Number of members on team 1`

`x2 = Number of members on team 2`

`x3 = Number of members on team 3`

`x4 = Number of members on team 4`

There is not enough information to solve this system of equations outright. Therfore, using the Echelon Method, I was able to solve the system of equations in terms of x4. The coordinates for the solution are as follows:

`(189 - (9/4)x4, 63 - (3/4)x4, 2x4, x4)`

Next, I considered what the possible values were for all of my variables. Since the variables represent people, I first noticed that all variables must be integers greater than zero.

Then, I solved for what value of x4 would lead x1 and x2 to result in zero. The value of x4 to cause x1 and x2 to be zero was found to be: `x4 = 84`.

Therefore, I was able to create the table of values below for the possible distribution of sales reps among marketing teams 1, 2, 3, and 4 provided the given constraints in the problem and that `0 <= x4 <= 84`.

```
print('{:>10}{:>10}{:>10}{:>10}{:>10}'.format('x1','x2','x3','x4','Total'))

for i in range(0,85,1):
    x4 = i
    x3 = 2*x4
    x2 = 63-((3/4)*x4)
    x1 = 189-((9/4)*x4)
    total = x1 + x2 + x3 + x4
    print('{:10.0f}{:10.0f}{:10.0f}{:10.0f}{:10.0f}'.format(x1,x2,x3,x4,total))
```

Below is the table for all possible solutions for how to distribute the sales reps amongst the four marketing teams:

x1	x2	x3	x4	Total
189	63	0	0	252
187	62	2	1	252
184	62	4	2	252
182	61	6	3	252
180	60	8	4	252
178	59	10	5	252
176	58	12	6	252
173	58	14	7	252
171	57	16	8	252
169	56	18	9	252
166	56	20	10	252
164	55	22	11	252
162	54	24	12	252
160	53	26	13	252
158	52	28	14	252
155	52	30	15	252
153	51	32	16	252
151	50	34	17	252
148	50	36	18	252
146	49	38	19	252
144	48	40	20	252
142	47	42	21	252
140	46	44	22	252
137	46	46	23	252
135	45	48	24	252
133	44	50	25	252
130	44	52	26	252
128	43	54	27	252
126	42	56	28	252
124	41	58	29	252
122	40	60	30	252
119	40	62	31	252
117	39	64	32	252
115	38	66	33	252
112	38	68	34	252
110	37	70	35	252
108	36	72	36	252
106	35	74	37	252
104	34	76	38	252
101	34	78	39	252
99	33	80	40	252
97	32	82	41	252
94	32	84	42	252
92	31	86	43	252
90	30	88	44	252
88	29	90	45	252
86	28	92	46	252
83	28	94	47	252
81	27	96	48	252
79	26	98	49	252
76	26	100	50	252
74	25	102	51	252
72	24	104	52	252
70	23	106	53	252
68	22	108	54	252
65	22	110	55	252
63	21	112	56	252
61	20	114	57	252

58	20	116	58	252
56	19	118	59	252
54	18	120	60	252
52	17	122	61	252
50	16	124	62	252
47	16	126	63	252
45	15	128	64	252
43	14	130	65	252
40	14	132	66	252
38	13	134	67	252
36	12	136	68	252
34	11	138	69	252
32	10	140	70	252
29	10	142	71	252
27	9	144	72	252
25	8	146	73	252
22	8	148	74	252
20	7	150	75	252
18	6	152	76	252
16	5	154	77	252
14	4	156	78	252
11	4	158	79	252
9	3	160	80	252
7	2	162	81	252
4	2	164	82	252
2	1	166	83	252
0	0	168	84	252

#3.)

$t = 252$ sales reps , 4 marketing teams

team #1 = x_1 ; team #2 = x_2, team #3 = x_3, team #4 = x_4 } members on the team

$x_1 = 3x_2$ $\qquad$ $x_1 + x_2 + x_3 + x_4 = 252$

$x_3 = 2x_4$

} # teams > # of equations, cannot solve outright.

1. $x_1 + x_2 + x_3 + x_4 = 252$
2. $x_1 - 3x_2 = 0$
3. $x_3 - 2x_4 = 0$

} Solve w/ Echelon Method.

$$R_1 - R_2 = \cancel{x_1} + x_2 + x_3 + x_4 - (\cancel{x_1} - 3x_2) + 252 - 0 \rightarrow R_2 = 4x_2 + x_3 + x_4 = 252$$

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 252 \\ 4x_2 + x_3 + x_4 &= 252 \\ x_3 - 2x_4 &= 0 \end{aligned}$$

} now use back substitution

R_3: $x_3 = 2x_4 \rightarrow (?, ?, 2x_4, x_4)$

R_2: $4x_2 + 2x_4 + x_4 = 252$

$4x_2 + 3x_4 = 252 \rightarrow 4x_2 = 252 - 3x_4$

$x_2 = 63 - \frac{3x_4}{4}$ } $(?, 63 - \frac{3x_4}{4}, 2x_4, x_4)$

R_1: $x_1 + (63 - \frac{3x_4}{4}) + 2x_4 + x_4 = 252$

#3.) cont.

R_1: $\left(x_1 + \left(63 - \frac{3x_4}{4}\right) + 2x_4 + x_4 = 252\right) 4$

$4x_1 + 252 - 3x_4 + 8x_4 + 4x_4 = 1008$

$4x_1 - 3x_4 + 8x_4 + 4x_4 = 756$

$4x_1 + 9x_4 = 756$

$4x_1 = 756 - 9x_4$

$x_1 = 189 - \frac{9}{4}x_4$

Solution (x_1, x_2, x_3, x_4)

$\boxed{\left(189 - \frac{9}{4}x_4,\ 63 - \frac{3}{4}x_4,\ 2x_4,\ x_4\right)}$

Find range of values for x_4:

$\boxed{x_4 \geq 0,\quad x_3 \geq 0,\quad x_2 \geq 0,\quad x_1 \geq 0}$ These are the constraints

$189 - \frac{9}{4}x_4 = 0$

$189 = \frac{9}{4}x_4$

~~$x_1 = 0$ @ 84~~

$x_1 = 0$ @ $x_4 = 84$

$63 - \frac{3}{4}x_4 = 0$

$63 = \frac{3}{4}x_4$

$x_2 = 0$ @ $x_4 = 84$

$0 \leq x_4 \leq 84$

4. Phil's Candy makes three types of artisanal chocolate bars: cherry, almond, and raisin. Matrix A gives the amount of ingredients in one batch. Matrix B gives the costs of ingredients from suppliers J and K. Calculate the cost of 100 batches of each candy using ingredients from supplier K.

$$A = \begin{array}{cc} \begin{matrix}\text{sugar} & \text{choc} & \text{milk}\end{matrix} & \\ \begin{bmatrix} 6 & 8 & 1 \\ 6 & 4 & 1 \\ 5 & 7 & 1 \end{bmatrix} & \begin{matrix}\text{cherry} \\ \text{almond} \\ \text{raisin}\end{matrix} \end{array}$$

$$B = \begin{array}{cc} \begin{matrix} J & K \end{matrix} & \\ \begin{bmatrix} 4 & 3 \\ 4 & 5 \\ 2 & 2 \end{bmatrix} & \begin{matrix}\text{sugar} \\ \text{choc} \\ \text{milk}\end{matrix} \end{array}$$

`Problem 4`

Phil's Candy makes three types of artisanal chocolate bars: cherry, almond, and raisin. Matrix A gives the amount of ingredients in one batch. Matrix B gives the costs of ingredients from suppliers J and K. Calculate the cost of 100 batches of each candy using ingredients from supplier K.

Multiplying matrices A & B by A*B results in matrix C which holds data indicating the cost per batch for each artisanal chocolate bar by the supplier of ingredients. Then using scalar multiplication, D = 100xC, where D tells the cost of 100 batches of each candy per supplier. Supplier J cost information is in column 1 while supplier K cost information is in column 2.

```
import numpy
from numpy import *

A = matrix([[6,8,1],[6,4,1],[5,7,1]])
B = matrix([[4,3],[4,5],[2,2]])
C = A*B
print('Matrix C \n', C)
D = 100*C
print('\nMatrix D\n', D)
```

```
Matrix C
 [[58 60]
 [42 40]
 [50 52]]

Matrix D
 [[5800 6000]
 [4200 4000]
 [5000 5200]]
```

Therefore, we can conclude that the cost of 100 batches of each candy bar using supplier K is 6,000 , 4,000 , and 5,200 units of currency for cherry, almond, and raisin artisanal chocolate bars, respectively.

The total cost of 100 batches of each candy using ingredients from supplier K is 15,200 units of currency.

#4.) say: ingredient unit = #lb cost unit = $

$$A = \begin{array}{c|c} & \begin{array}{ccc} \text{sugar} & \text{choc} & \text{milk} \end{array} \\ \hline & \left[\begin{array}{ccc} 6\ \text{lb/batch} & 8\ \text{lb/batch} & 1\ \text{lb/b} \\ 6\ \text{lb/batch} & 4\ \text{lb/batch} & 1 \\ 5\ \text{lb/batch} & 7\ \text{lb/batch} & 1 \end{array}\right] \begin{array}{l} \text{cherry} \\ \text{almond} \\ \text{raisin} \end{array} \end{array} = (3 \times 3)$$

(rows x columns)

ingredients in one batch

$$B = \begin{array}{c} \begin{array}{cc} J & K \end{array} \\ \left[\begin{array}{cc} 4 & 3 \\ 4 & 5 \\ 2 & 2 \end{array}\right] \end{array} \begin{array}{l} \text{sugar} \\ \text{choc} \\ \text{milk} \end{array} \quad (3 \times 2)$$

cost = $/lb

cost of ingredients from supplier J + K

$A * B \Rightarrow (3 \times 3) * (3 \times 2) \longrightarrow (3 \times 2)$

$$\begin{array}{cc} J & K \end{array} \\ \left[\begin{array}{cc} 4 & 3 \\ 4 & 5 \\ 2 & 2 \end{array}\right]$$

$$\begin{array}{c|ccc} & s & c & m \\ \hline c & 6 & 8 & 1 \\ a & 6 & 4 & 1 \\ r & 5 & 7 & 1 \end{array} \quad \cancel{\left[\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right]}$$

①

$$\begin{array}{cc} J & K \end{array} \\ \left[\begin{array}{cc} 4 & 3 \\ 4 & 5 \\ 2 & 2 \end{array}\right] \begin{array}{l} \text{sugar} \\ \text{choc} \\ \text{milk} \end{array}$$

$$\begin{array}{c|ccc} & s & c & m \\ \hline c & 6 & 8 & 1 \\ a & 6 & 4 & 1 \\ r & 5 & 7 & 1 \end{array} \quad \left[\begin{array}{cc} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{array}\right]$$

① $24 + 32 + 2 = 58 = \underset{\text{sugar}}{(6*4)} + \underset{\text{choc}}{(8*4)} + \underset{\text{milk}}{(1*2)} =$ 58 $/batch

$6\ \text{lb/batch} * 4\ \$/\text{lb} = 24\ \$/\text{batch}$

② $(6*3) + (8*5) + (1*2) =$ $60 /batch

③ $(6*4) + (4*4) + (1*2) =$ $42/batch

④ $(6*3) + (4*5) + (1*2) =$ $40/batch

⑤ $(5*4) + (7*4) + (1*2) =$ $50/batch

⑥ $(5*3) + (7*5) + (1*2) =$ $52/batch

#4 (cont.)

$$C = A * B = \begin{array}{cc} J & k \\ \left[\begin{array}{cc} 58 & 60 \\ 42 & 40 \\ 50 & 52 \end{array}\right] \end{array} \begin{array}{l} \text{cherry} \\ \text{almond} \\ \text{raisin} \end{array} \qquad [=] \ \$/\text{batch}$$

4a $$D = 100 * C = \begin{array}{cc} J & k \\ \left[\begin{array}{cc} 5{,}800 & 6{,}000 \\ 4{,}200 & 4{,}000 \\ 5{,}000 & 5{,}200 \end{array}\right] \end{array} \left.\begin{array}{l} \text{cherry} \\ \text{almond} \\ \text{raisin} \end{array}\right\} \text{cost per 100 batches}$$

5. Welsh-Ryan Arena seats 15,000 people. Courtside seats cost \$8, first level seats cost \$6, and upper deck seats cost \$4. The total revenue for a sellout is \$76,000. If half the courtside seats, half the upper deck seats, and all the first level seats are sold, then the total revenue is \$44,000. How many of each type of seat are there?

```
Problem 5
```

Welsh-Ryan Arena seats 15,000 people. Courtside seats cost 8 dollars, first level seats cost 6 dollars, and upper deck seats cost 4 dollars. The total revenue for a sellout is 76,000 dollars. If half the courtside seats, half the upper deck seats, and all the first level seats are sold, then the total revenue is 44,000 dollars. How many of each type of seat are there?

```
import numpy
from numpy import *
from numpy.linalg import *
```

```
A = matrix([[1,1,1],[8,6,4],[4,6,2]])
rhs = matrix([15000,76000,44000])
rhs = transpose(rhs)
result = linalg.solve(A,rhs)
result
```

```
matrix([[ 3000.],
        [ 2000.],
        [10000.]])
```

Based on the code above, the conclusion is that there are 3,000 courtside seats, 2,000 first level seats, and 10,000 upper deck seats in Welsh-Ryan Arena.

Additionally, the answer can be found using the Gauss-Jordan method as shown by hand. This method yields the same result of 3,000 courtside seats, 2,000 first level seats, and 10,000 upper deck seats within Welsh-Ryan Arena.

#5.)

total seats = 15,000   type: x_1 = courtside ; \$8

x_2 = first level ; \$6

sell at = \$76,000 x_3 = upper deck ; \$4

$$x_1 + x_2 + x_3 = 15000$$
$$8x_1 + 6x_2 + 4x_3 = 76,000$$
$$8\left(\frac{x_1}{2}\right) + 6(x_2) + 4\left(\frac{x_3}{2}\right) = 44,000$$

$$\left.\right\} \quad \begin{aligned} x_1 + x_2 + x_3 &= 15,000 \\ 8x_1 + 6x_2 + 4x_3 &= 76,000 \\ 4x_1 + 6x_2 + 2x_3 &= 44,000 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 15,000 \\ 8 & 6 & 4 & 76,000 \\ 4 & 6 & 2 & 44,000 \end{array}\right] \; -8R_1 + R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15,000 \\ 0 & -2 & -4 & -44,000 \\ 4 & 6 & 2 & 44,000 \end{array}\right] \; -4R_1 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 15,000 \\ 0 & -2 & -4 & -44,000 \\ 0 & 2 & -2 & -16,000 \end{array}\right] \; R_2 + R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 15,000 \\ 0 & -2 & -4 & -44,000 \\ 0 & 0 & -6 & -60,000 \end{array}\right] \; 2R_1 + R_2 \to R_1$$

$$\left[\begin{array}{ccc|c} 2 & 0 & -2 & -14,000 \\ 0 & -2 & -4 & -44,000 \\ 0 & 0 & -6 & -60,000 \end{array}\right] \; -3R_1 + R_3 \to R_1 \quad \left[\begin{array}{ccc|c} -6 & 0 & 0 & -18,000 \\ 0 & -2 & -4 & -44,000 \\ 0 & 0 & -6 & -60,000 \end{array}\right] \; -2R_3 + 3R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} -6 & 0 & 0 & -18,000 \\ 0 & -6 & 0 & -12,000 \\ 0 & 0 & -6 & -60,000 \end{array}\right] \begin{array}{l} R_1/-6 \\ R_2/-6 \to \\ R_3/-6 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3,000 \\ 0 & 1 & 0 & 2,000 \\ 0 & 0 & 1 & 10,000 \end{array}\right]$$

$$\boxed{\begin{aligned} x_1 &= 3,000 \\ x_2 &= 2,000 \\ x_3 &= 10,000 \end{aligned}}$$

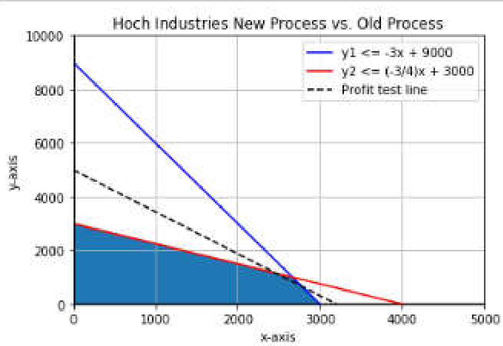
6. Due to new environmental restrictions, Hoch Industries must use a new process to reduce pollution. The old process emits 6 g of Sulphur and 3 g of lead per liter of chemical made. The new process emits 2 g of Sulphur and 4 g of lead per liter of chemical made. The company makes a profit of 25¢ per liter under the old process and 16¢ per liter under the new process. No more than 18,000 g of Sulphur and no more than 12,000 g of lead can be emitted daily. How many liters of chemicals should be made daily under each process to maximize profits? What is the maximum profit?

First will solve by graphing the feasible region and profit lines to test corner points:

`Problem 6`

Due to new environmental restrictions, Hoch Industries must use a new process to reduce pollution. The old process emits 6 g of Sulphur and 3 g of lead per liter of chemical made. The new process emits 2 g of Sulphur and 4 g of lead per liter of chemical made. The company makes a profit of 25¢ per liter under the old process and 16¢ per liter under the new process. No more than 18,000 g of Sulphur and no more than 12,000 g of lead can be emitted daily. How many liters of chemicals should be made daily under each process to maximize profits? What is the maximum profit?

```python
import matplotlib.pyplot
from matplotlib.pyplot import *
import numpy
from numpy import arange

#Set up parameters and lines

x_max_lim = 5000
y_max_lim =10000
x= arange(0,x_max_lim+0.1,.1)
z = 800                       #Units are in dollars, Objective Function value to check corner points of feasible region

#Set up linear equations for the inequalities

y1 = -3*x + 9000              #Sulfur inequality
y2 = (-3/4)*x + 3000          #Lead inequality
y3 = (-0.25/0.16)*x +(z/0.16)

figure()
xlim(0,x_max_lim)
ylim(0,y_max_lim)
hlines(0,0,x_max_lim,color='k')        # x-axis
vlines(0,0,y_max_lim,color='k')        # y-axis
grid(True)

xlabel('x-axis')
ylabel('y-axis')
title ('Hoch Industries New Process vs. Old Process')

plot(x,y1,'b')               #Sulfur inequality
plot(x,y2,'r')               #Lead inequality
plot(x,y3,'k--')             #Objective Function Test Line

legend(['y1 <= -3x + 9000','y2 <= (-3/4)x + 3000','Profit test line'])

x = [0,0,(8000/3),3000]
y = [0,3000,1000,0]
fill(x,y)

show()
```



Graphing the feasible region indicates that the corner point (8000/3 , 1000) is where the profit is maximized. This is consistent with Hoch Inudstries making 8000/3, or 2,666.67, liters of chemical via the old process and 1,000 liters of chemical via the new process. This will result in the maximum profit of 826.67 dollars.

This problem can also be solved using Linear Programming techniques in Python:

```
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize

# declare your variables
x1 = LpVariable("x1", 0, None) # x1>=0
x2 = LpVariable("x2", 0, None) # x2>=0

# defines the problem
prob = LpProblem("problem", LpMaximize)

# defines the constraints
prob += 6*x1 + 2*x2 <= 18000
prob += 3*x1 + 4*x2 <= 12000

# defines the objective function to maximize
prob += 0.25*x1 + 0.16*x2

# solve the problem
status = prob.solve()
LpStatus[status]

#Calculate value of objective function

OF = 0.25*value(x1) + 0.16*value(x2)

# print the results
print("Pulp solutions for liters of old process and liters of new process are: ")
print('Liters of old process:', round(value(x1),2))
print('Liters of new process:', value(x2))
print('The maximum profit is: $',round(OF,2))
```

```
Pulp solutions for liters of old process and liters of new process are:
Liters of old process: 2666.67
Liters of new process: 1000.0
The maximum profit is: $ 826.67
```

Both techniques conclude that a maximum profit of $826.67 is achieved when Hock Industries produces 2,666.67 liters of the old process and 1,000 liters of the new process.

#6.) x_2 x_1

<u>New (y)</u>

- 2g sulfur / liter
- 4g lead / liter
- \$0.16 / liter - profit

<u>old (x)</u>

- 6g sulphur / liter
- 3g lead / liter
- 25¢ = \$0.25 / liter - profit

sulphur $\leq 18{,}000g$ lead $\leq 12{,}000 g$; x = liters of old process

y = liters of new process

① $6x + 2y \leq 18{,}000$ sulfur

② $3x + 4y \leq 12{,}000$ lead

O.F: profit = $\$0.25x + \$0.16y \rightarrow \boxed{z = \$0.25x + \$0.16y}$ $x \geq 0$; $y \geq 0$

① $6x + 2y \leq 18{,}000$

$2y \leq -6x + 18{,}000$

$y_1 \leq -3x + 9{,}000$

② $3x + 4y \leq 12{,}000$

$4y \leq -3x + 12{,}000$

$y_2 \leq \frac{-3}{4}x + 3{,}000$

<u>Find corner points</u>

$C_1 = (0,0) \rightarrow$ axis

$C_2 = -3/4x + 3{,}000 \rightarrow x = 0, \; y = 3{,}000 \rightarrow (0, 3{,}000)$

$C_3 = -3x + 9000 = \frac{-3}{4}x + 3000$

$-3x + 6000 = \frac{-3}{4}x$

$6000 = \frac{9}{4}x \rightarrow x = \frac{8000}{3}, \; y = 1000 \quad \left(\frac{8000}{3}, 1000\right)$

$C_4 = 0 = -3x + 9{,}000 \rightarrow x = 3000, \; y = 0 \quad (3000, 0)$

$z = 0.25x + 0.16y \rightarrow 0.16y = -0.25x + z$

$y_3 = \frac{-0.25x}{0.16} + \frac{z}{0.16}$

#6.) cont.

max corner point in feasible region = $\left(\frac{8000}{3}, 1000\right)$

- max profit comes when Hoth makes $\frac{8000}{3}$ or 2666.67 liters of the old process and 1,000 liters of the new process.

- max profit is =

$$z = \$0.25(8000/3) + \$0.16(1000) =$$

$$\underbrace{\boxed{z = \$826.67}}_{\text{max profit}}$$

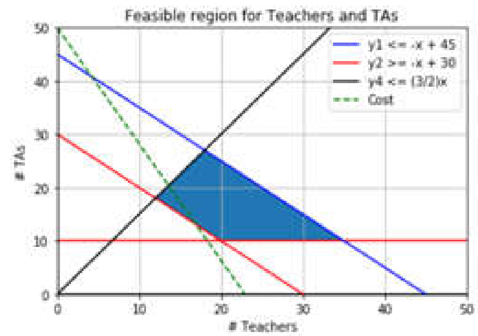
7. Northwestern is looking to hire teachers and TA's to fill its staffing needs for its summer program at minimum cost. The average monthly salary of a teacher is \$2400 and the average monthly salary of a TA is \$1100. The program can accommodate up to 45 staff members and needs at least 30 to run properly. They must have at least 10 TA's and may have up to 3 TA's for every 2 teachers. How many teachers and TA's the should the program hire to minimize costs. What is the minimum cost?

   First approach to this problem is to set up constraints and graph the feasible region:

   `Problem 7`

   Northwestern is looking to hire teachers and TA's to fill its staffing needs for its summer program at minimum cost. The average monthly salary of a teacher is 2400 dollars and the average monthly salary of a TA is 1100 dollars. The program can accommodate up to 45 staff members and needs at least 30 to run properly. They must have at least 10 TA's and may have up to 3 TA's for every 2 teachers. How many teachers and TA's the should the program hire to minimize costs. What is the minimum cost?

```
import matplotlib.pyplot
from matplotlib.pyplot import *
import numpy
from numpy import arange

#Set up parameters and lines

x_max_lim = 50
y_max_lim =50
x= arange(0,x_max_lim+0.1,.1)
w = 55000                       #Cost of staff; units are in dollars

#Set up linear equations for the inequalities

y1 = -x + 45                 #Max total staff members
y2 = -x + 30                 #Min total staff members
y3 = (3/2)*x                 #Ratio of TA's to teachers
y4 = (-24/11)*x +(w/1100)

figure()
xlim(0,x_max_lim)
ylim(0,y_max_lim)
hlines(0,0,x_max_lim,color='k')          # x-axis
vlines(0,0,y_max_lim,color='k')          # y-axis
hlines(10,0,x_max_lim,color='r')         # Minimum number of TA's allowed
grid(True)

xlabel('# Teachers')
ylabel('# TAs')
title ('Feasible region for Teachers and TAs')

plot(x,y1,'b')               #Max total staff members
plot(x,y2,'r')               #Min total staff members
plot(x,y3,'k')               #Ratio of TA:teacher
plot(x,y4,'g--')             #Test Objective Function

legend(['y1 <= -x + 45','y2 >= -x + 30','y4 <= (3/2)x','Cost'])

x = [12,18,35,20]
y = [18,27,10,10]
fill(x,y)

show()
```



After graphing the feasible region for the problem at hand, the Corner Point Theorem is utilized. The corner points were tested and evaluated by examining the movement in a Cost test line. This revealed the corner point (12,18) as the minimun solution. This solution indicates that Northwestern should hire 12 teachers and 18 TAs at a minimum summer program cost of $48,600 per month.

The graphical approach to this problem yields a result indicating that Northwestern should hire 12 teachers and 18 TA's for their summer program to minimize their expenses to $48,600 per month.

This problem can also be approached as a Linear Programming problem using the pulp module in Python:

```
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize

# declare your variables
y1 = LpVariable("y1", 0, None) # y1>=0
y2 = LpVariable("y2", 0, None) # y2>=0

# defines the problem
prob = LpProblem("problem", LpMinimize)

# defines the constraints
prob += y1 + y2 <= 45
prob += y1 + y2 >= 30
prob += y2 >= 10
prob += y2 - y1*(3/2) <= 0

# defines the objective function to optimize
prob += 2400*y1 + 1100*y2

# solve the problem
status = prob.solve()
LpStatus[status]

#Calculate value of objective function

OF = 2400*value(y1) + 1100*value(y2)

# print the results
print("Pulp solutions for numbers of teachers and TAs needed for the summer program are: ")
print('Teachers: ', value(y1))
print('TAs: ', value(y2))
print('The minimum program cost: $',round(OF,2))
```

```
Pulp solutions for numbers of teachers and TAs needed for the summer program are:
Teachers:  12.0
TAs:  18.0
The minimum program cost: $ 48600.0
```

Both methods give the same solution!

#7.)

Goal = minimize cost!

teacher = \$2,400 / month. ; TA = \$1,100

x = # of teachers $= x_1 = y_1$

y = # of TA's $= x_2 = y_2$

\- OF: cost = \$2,400 * x + \$1,100 y

$$\begin{cases} (1)\ x + y \le 45 \\ (2)\ x + y \ge 30 \\ (3)\ y \ge 10 \\ (4)\ y \le \frac{3}{2}x \end{cases}$$

(y) (x)

3 TA's for every 2 teachers

$$\frac{y}{x} \le \frac{3}{2}$$

$$\frac{y}{x} = \frac{TA}{teacher} \le \frac{3}{2} \quad y \le \frac{3}{2}x$$

$$y - \frac{3}{2}x \le 0$$

$$w = 2400x + 1100y \rightarrow 1100y = -2400x + w \rightarrow y = \frac{-24}{11}x + \frac{w}{1100}$$

(1) $x + y_1 \le 45 \rightarrow y_1 \le -x + 45$

(2) $x + y_2 \ge 30$

$y_2 \ge -x + 30$

(3) $y_3 \ge 10 \rightarrow$ h line

(4) $\cancel{y_3 \le \frac{3}{2}x}$

$y_4 \le \frac{3}{2}x$

Find Corner Points:

C_1: $\frac{3}{2}x = -x + 30 \rightarrow \frac{5}{2}x = 30 \rightarrow x = 12,\ y = 18$ (12, 18)

C_2: $\frac{3}{2}x = -x + 45 \rightarrow \frac{5}{2}x = 45 \rightarrow x = 18,\ y = 27$ (18, 27)

C_3: $-x + 45 = 10 \rightarrow -x = -35 \rightarrow x = 35,\ y = 10$ (35, 10)

C_4: $-x + 30 = 10 \rightarrow -x = -20 \rightarrow x = 20,\ y = 10$ (20, 10)

#7.)

- From Graph, can identify min corner point @ (12, 18)

12 teachers + 18 TA's

cost = \$2,400(12) + \$1,100(18) = \$48,600

- Should hire 12 teachers, 18 TA's for a minimum cost of \$48,600

8. To be at his best as an athlete, Roger needs at least 10 units of vitamin A, 12 units of vitamin B, and 20 units of vitamin C per day. Pill #1 contains 4 units of A and 3 of B. Pill #2 contains 1 unit of A, 2 of B, and 4 of C. Pill #3 contains 10 units of A, 1 of B, and 5 of C. Pill #1 costs 6 cents, pill #2 costs 8 cents, and pill #3 costs 1 cent. How many of each pill must Roger take to minimize his cost, and what is that cost?

   This is a standard minimization problem which can be solved via the simplex method as shown by hand.

   The problem can also be solved using the pulp module in Python:

`Problem 8`

To be at his best as an athlete, Roger needs at least 10 units of vitamin A, 12 units of vitamin B, and 20 units of vitamin C per day. Pill #1 contains 4 units of A and 3 of B. Pill #2 contains 1 unit of A, 2 of B, and 4 of C. Pill #3 contains 10 units of A, 1 of B, and 5 of C. Pill #1 costs 6 cents, pill #2 costs 8 cents, and pill #3 costs 1 cent. How many of each pill must Roger take to minimize his cost, and what is that cost?

```python
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize

# declare your variables
y1 = LpVariable("y1", 0, None) # y1>=0
y2 = LpVariable("y2", 0, None) # y2>=0
y3 = LpVariable("y3", 0, None) # y3>=0

# defines the problem
prob = LpProblem("problem", LpMinimize)

# defines the constraints
prob += 4*y1 + y2 + 10*y3 >= 10
prob += 3*y1 + 2*y2 + y3 >= 12
prob += 4*y2 + 5*y3 >= 20

# defines the objective function to optimize
prob += 6*y1 + 8*y2 + y3

# solve the problem
status = prob.solve()
LpStatus[status]

#Calculate value of objective function

OF = 6*value(y1) + 8*value(y2) + value(y3)

# print the results
print("Pulp solution for the quantity of each pill Roger must take is: ")
print('# of Pill #1: ', value(y1))
print('# of Pill #2: ', value(y2))
print('# of Pill #3: ', value(y3))
print('Minimized Cost: Cents',round(OF,2))
```

```
Pulp solution for the quantity of each pill Roger must take is:
# of Pill #1:  0.0
# of Pill #2:  0.0
# of Pill #3:  12.0
Minimized Cost: Cents 12.0
```

   Both methods provide the solution that Roger shouldn’t purchase and take either Pill # 1 or #2. Roger should only purchase and take Pill #3 to meet his nutritional demands at a minimized cost of 12 cents per day by taking 12 Pill #3's a day.

#8.)

Pill#1: V.A = 4, V.B = 3 ; 6¢

Roger: vitamin A: 10 units
vitamin B: 12 units
vitamin C: 20 units

Pill#2: V.A = 1, V.B = 2, V.C = 4, 8¢
Pill#3: V.A = 10, V.B = 1, V.C = 5, 1¢

y_1 = # Pill #1 (count) ; $y_1 \geq 0$
y_2 = # Pill #2 (count) ; $y_2 \geq 0$
y_3 = # Pill #3 (count) ; $y_3 \geq 0$

$w = 0.06y_1 + 0.08y_2 + 0.01y_3$ ※ standard minimization problem.

V.A: $4y_1 + y_2 + 10y_3 \geq 10$ ↳ ¢

V.B: $3y_1 + 2y_2 + y_3 \geq 12$ $w = 6y_1 + 8y_2 + 1y_3$

V.C: $0y_1 + 4y_2 + 5y_3 \geq 20$

Dual → matrix:

$$\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \left[\begin{array}{ccc|c} 4 & 1 & 10 & 10 \\ 3 & 2 & 1 & 12 \\ 0 & 4 & 5 & 20 \\ \hline 6 & 8 & 1 & 0 \end{array}\right] \end{array} \xrightarrow[\text{to Dual}]{\text{transpose}} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 4 & 3 & 0 & 6 \\ 1 & 2 & 4 & 8 \\ 10 & 1 & 5 & 1 \\ \hline 10 & 12 & 20 & 0 \end{array}\right] \end{array}$$

$x_1 \geq 0$
$x_2 \geq 0$
$x_3 \geq 0$

$z = 10x_1 + 12x_2 + 20x_3$

$4x_1 + 3x_2 + 0x_3 \leq 6$

$1x_1 + 2x_2 + 4x_3 \leq 8$

$10x_1 + 1x_2 + 5x_3 \leq 1$

#8 cont.)

(+) don't take (−) multiple of row being changed.

$z - 10x_1 - 12x_2 - 20x_3$

$4x_1 + 3x_2 + 0x_3 + s_1 = 6 \; ; \; s_1 \geq 0$

$x_1 + 2x_2 + 4x_3 + s_2 = 8 \; ; \; s_2 \geq 0$

$10x_1 + x_2 + 5x_3 + s_3 = 1 \; ; \; s_3 \geq 0$

Initial tableau

$$\begin{array}{l} \\ \\ 8/4 = 2 \\ 1/5 = 1/5 \\ \\ \end{array}
\begin{array}{c} \begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \end{array} \\
\left[\begin{array}{ccccccc|c} 4 & 3 & 0 & 1 & 0 & 0 & 0 & 6 \\ 1 & 2 & 4 & 0 & 1 & 0 & 0 & 8 \\ 10 & 1 & \boxed{5} & 0 & 0 & 1 & 0 & 1 \\ \hline -10 & -12 & -20^* & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}
\quad -4R_3 + 5R_2 \to R_2$$

$$\begin{array}{c} \begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \end{array} \\
\left[\begin{array}{ccccccc|c} 4 & 3 & 0 & 1 & 0 & 0 & 0 & 6 \\ -35 & 6 & 0 & 0 & 5 & -4 & 0 & 36 \\ 10 & 1 & 5 & 0 & 0 & 1 & 0 & 1 \\ \hline -10 & -12 & -20 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}
\quad 4R_3 + R_4 \to R_4$$

$$\begin{array}{l} \\ 6/3 = 2 \\ 36/6 = 6 \\ 1/1 = 1 \\ \\ \end{array}
\begin{array}{c} \begin{array}{ccccccc|c} x_1 & x_2 & x_3^B & s_1^B & s_2^B & s_3 & z^* & \end{array} \\
\left[\begin{array}{ccccccc|c} 4 & 3 & 0 & 1 & 0 & 0 & 0 & 6 \\ -35 & 6 & 0 & 0 & 5 & -4 & 0 & 36 \\ 10 & \boxed{1} & 5 & 0 & 0 & 1 & 0 & 1 \\ \hline 30 & -8^* & 0 & 0 & 0 & 4 & 1 & 4 \end{array}\right] \end{array}
\quad -3R_3 + R_1 \to R_1$$

#8 cont

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\ \hline -26 & 0 & -15 & 1 & 0 & -3 & 0 & 3 \\ -35 & 0 & 0 & 0 & 5 & -4 & 0 & 36 \\ 10 & 1 & 5 & 0 & 0 & 1 & 0 & 1 \\ \hline 30 & -8 & 0 & 0 & 0 & 4 & 1 & 4 \end{array} \quad -6R_3 + R_2 \rightarrow R_2$$

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\ \hline -26 & 0 & -15 & 1 & 0 & -3 & 0 & 3 \\ -95 & 0 & -30 & 0 & 5 & -10 & 0 & 30 \\ 10 & 1 & 5 & 0 & 0 & 1 & 0 & 1 \\ \hline 30 & -8 & 0 & 0 & 0 & 4 & 1 & 4 \end{array} \quad 8R_3 + R_4 \rightarrow R_4$$

$$\begin{array}{ccccccc|c} x_1 & x_2^* & x_3 & s_1^* & s_2^* & s_3 & z & \\ \hline -26 & 0 & -15 & 1 & 0 & -3 & 0 & 3 \\ -95 & 0 & -30 & 0 & 5 & -10 & 0 & 30 \\ 10 & 1 & 5 & 0 & 0 & 1 & 0 & 1 \\ \hline 110 & 0 & 40 & \boxed{0} & \boxed{0} & \boxed{12} & 1 & 12 \end{array}$$

$\boxed{y_1 = 0 \; ; \; y_2 = 0 \; ;}$ $\boxed{y_3 = 12 \; ; \; z = 12¢}$

- Roger should take 0 pill #1 + #2, take 12 pill #3 to meet his nutritional demands @ a minimized cost of 12¢ per day

9. Better Buy stocks Blu-ray DVD players, surround sound systems, and Smart TVs. They have limited storage space and can stock a maximum of 210 of these three machines. They know from past experience that they should stock twice as many DVD players as stereo systems and at least 30 TVs. If each DVD player sells for $450, each surround sound system sells for $2000, and each TV sells for $750, how many of each should be stocked and sold for maximum revenues? What is the maximum revenue?

   The simplex method was used by hand to solve this non-standard maximization problem as shown below. Also, the pulp Python module was solved with the code and output below as well.

`Problem 9`

Better Buy stocks Blu-ray DVD players, surround sound systems, and Smart TVs. They have limited storage space and can stock a maximum of 210 of these three machines. They know from past experience that they should stock twice as many DVD players as stereo systems and at least 30 TVs. If each DVD player sells for 450 dollars, each surround sound system sells for 2000 dollars, and each TV sells for 750 dollars, how many of each should be stocked and sold for maximum revenues? What is the maximum revenue?

```python
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize

# declare your variables
x1 = LpVariable("x1", 0, None) # x1>=0
x2 = LpVariable("x2", 0, None) # x2>=0
x3 = LpVariable("x3", 0, None) # x3>=0

# defines the problem
prob = LpProblem("problem", LpMaximize)

# defines the constraints
prob += x1 + x2 + x3 <= 210
prob += x1 - 2*x2 >= 0
prob += x3 >= 30

# defines the objective function to optimize
prob += 450*x1 + 2000*x2 + 750*x3

# solve the problem
status = prob.solve()
LpStatus[status]

#Calculate value of objective function

OF = 450*value(x1) + 2000*value(x2) + 750*value(x3)

# print the results
print("Pulp solution for stock quantities of DVD players, surround sound systems, and Smart TVs at Better Buy: ")
print('Blu-ray DVD players: ', value(x1))
print('Surround Sound Systems: ', value(x2))
print('Smart TVs: ', value(x3))
print('Maximum Revenue: $',round(OF,2))
```

```
Pulp solution for stock quantities of DVD players, surround sound systems, and Smart TVs at Better Buy:
Blu-ray DVD players:  120.0
Surround Sound Systems:  60.0
Smart TVs:  30.0
Maximum Revenue: $ 196500.0
```

Both methods yield the same solution that Better Buy should stock 120 Blu-ray DVD players, 60 surround sound systems, and 30 Smart TVs for a maximum revenue of $196,500.

#9.)

x_1 = Blu-ray → DVD ; $x_1 \geq 0$ → \$450

x_2 = Surround sound ; $x_2 \geq 0$ → \$2,000

x_3 = Smart TVs ; $x_3 \geq 0$ → \$750

$x_3 \geq 30$

Nonstandard Maximization

$\frac{x_1}{x_2} \geq 2 \rightarrow x_1 \geq 2x_2$

$x_1 - 2x_2 \geq 0$

$z = 450x_1 + 2000x_2 + 750x_3$

$x_1 \geq 2x_2$

$x_1 + x_2 + x_3 \leq 210$

$x_3 \geq 30$

$x_1 - 2x_2 + 0x_3 \geq 0$

Reformulate:

$z = 450x_1 + 2000x_2 + 750x_3 \rightarrow z - 450x_1 - 2000x_2 - 750x_3 = 0$

1 $x_1 + x_2 + x_3 \leq 210$

2 $x_1 - 2x_2 + 0x_3 \geq 0$

3 $x_3 \geq 30$

add slack + surplus variables

1 $x_1 + x_2 + x_3 + s_1 = 210$; $s_1 \geq 0$

2 $x_1 - 2x_2 - s_2 = 0$; $s_2 \geq 0$

3 $x_3 - s_3 = 30$; $s_3 \geq 0$

Initial tableau:

$$\begin{array}{c} \\ 210/1 = 210 \\ \\ 30/1 = 30 \\ \\ \end{array}
\left[\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1^* & s_2^* & s_3^* & z & \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 210 \\ 1 & -2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & \boxed{1}^{\text{Pivot}} & 0 & 0 & \boxed{-1} & 0 & 30 \\ \hline -450 & -2000 & -750 & 0 & 0 & 0 & 1 & 0 \end{array}\right]
\begin{array}{l} s_1 = 210 \\ s_2 = 0 \\ s_3 = -30 \\ \\ -R_3 + R_1 \rightarrow R_1 \end{array}$$

#9 cont.)

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \end{array}$$
$$\left[\begin{array}{ccccccc|c} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 180 \\ 1 & -2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\ \hline -450 & -2000 & -750 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$750R_1 + R_4 \rightarrow R_4$

check in spot

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3^* & s_1^* & s_2^* & s_3^* & z & \end{array}$$
$$\left[\begin{array}{ccccccc|c} 1 & \boxed{1} & 0 & 1 & 0 & 1 & 0 & 180 \\ 1 & -2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\ \hline -450 & -2000 & 0 & 0 & 0 & -750 & 1 & 22,500 \end{array}\right]$$

$x_3 = 30$

$s_1 = 180$

$s_2 = 0$

$2R_1 + R_2 \rightarrow R_2$

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \end{array}$$
$$\left[\begin{array}{ccccccc|c} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 180 \\ 3 & 0 & 0 & 2 & -1 & 2 & 0 & 360 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\ \hline -450 & -2000 & 0 & 0 & 0 & -750 & 1 & 22,500 \end{array}\right]$$

$2000R_1 + R_4 \rightarrow R_4$

$$\begin{array}{ccccccc|c} x_1 & x_2^* & x_3^* & s_1 & s_2^* & s_3 & z & \end{array}$$
$$\left[\begin{array}{ccccccc|c} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 180 \\ 3 & 0 & 0 & 2 & -1 & 2 & 0 & 360 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\ \hline 1550 & 0 & 0 & 2000 & 0 & 1250 & 1 & 382,500 \end{array}\right]$$

$x_2 = 180$

$x_3 = 30$

$-s_2 = 360$ $\quad s_2 = -360$

$s_2 < 0$ X

9. cont)

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\
\hline
\boxed{1} & 1 & 0 & 1 & 0 & 1 & 0 & 180 \\
1 & -2 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\
\hline
-450 & -2000 & 0 & 0 & 0 & -750 & 1 & 22{,}500
\end{array}$$

$-R_1 + R_2 \rightarrow R_2$

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\
\hline
\boxed{1} & 1 & 0 & 1 & 0 & 1 & 0 & 180 \\
0 & -3 & 0 & -1 & -1 & -1 & 0 & -180 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\
\hline
-450 & -2000 & 0 & 0 & 0 & -750 & 1 & 22{,}500
\end{array}$$

$450R_1 + R_4 \rightarrow R_4$

4

$180/1 = 180$

$-180/-3 = 60$

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\
\hline
1 & 1 & 0 & 1 & 0 & 1 & 0 & 180 \\
0 & \boxed{-3} & 0 & -1 & -1 & -1 & 0 & -180 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\
\hline
0 & -1550 & 0 & 450 & 0 & -300 & 1 & 103{,}500
\end{array}$$

$x_1 = 180$

$x_3 = 30$

$s_2 = 180$

$3R_1 + R_2 \rightarrow R_1$

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\
\hline
3 & 0 & 0 & 2 & -1 & 2 & 0 & 360 \\
0 & -3 & 0 & -1 & -1 & -1 & 0 & -180 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\
\hline
0 & -1550 & 0 & 450 & 0 & -300 & 1 & 103{,}500
\end{array}$$

$-1550R_2 + 3R_4 \rightarrow R_4$

#9 cont

$$\begin{array}{cccccccc} x_1^* & x_2^* & x_3^* & s_1 & s_2 & s_3 & z^* & \\ \end{array}$$

$$\left[\begin{array}{ccccccc|c} 3 & 0 & 0 & 2 & -1 & 2 & 0 & 360 \\ 0 & -3 & 0 & -1 & -1 & -1 & 0 & -180 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 30 \\ \hline 0 & 0 & 0 & 2700 & 1550 & 650 & 3 & 589{,}500 \end{array}\right]$$

$3x_1 = 360 \rightarrow x_1 = 120$

$-3x_2 = -180 \rightarrow x_2 = 60$

$x_3 = 30 \qquad x_3 = 30$

$3z = 589500 \quad z = \$196{,}500$

10. McDowell's is conducting a sweepstakes, and ships two boxes of game pieces to a particular franchise. Box A has 4% of its contents being winners, while 5% of the contents of box B are winners. Box A contains 27% of the total tickets. The contents of both boxes are mixed in a drawer and a ticket is chosen at random. Find the probability it came from box A if it is a winner.

This problem can be solved both using Bayes' Theorem and by checking with an arbitrary number of total tickets as was done.

A tree diagram was constructed in order to better visualize and utilize Bayes' Theorem. The work can be shown on the following page. The solution for $P(A|W) = 0.2283$. This tells us that there is a 22.83% chance that the ticket came from box A if it was a winner.

The same solution was found using an arbitrary total number of tickets at 10,000. Using the percentages given, this indicates that Box A has 2,700 tickets with 108 winning tickets. Box B therefore has 7,300 total tickets and 365 winning tickets. Then, using the concept of Bayes' Theorem, I was able to find $P(A|W)$ by dividing the number of winning tickets from box A by the total number of winning tickets. This method provided the same solution that there is a 22.83% chance that the ticket came from box A if it was a winner.

#10.)

2 boxes.

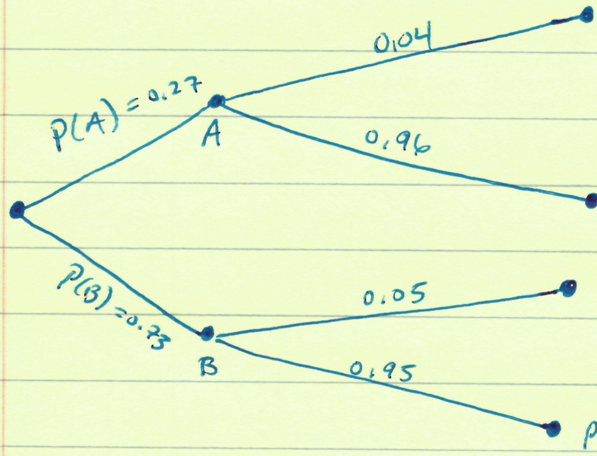
Box A: 4% winners , 27% total tickets

Box B: 5% winners ; 73% total tickets

$P(A|W)$ = Probability it came from box A given it is a winner.

LET'S first try a tree diagram

Box   Win / Lose   branch



$P(W|A) \rightarrow \underline{P(A) * P(W|A)} = P(W \cap A) = 0.0108$

$P(L|A) \rightarrow P(A) * P(L|A) = P(L \cap A) = 0.2592$

$P(W|B) \rightarrow \underline{P(B) * P(W|B)} = P(B \cap W) = 0.0365$

$P(L|B) \rightarrow P(B) * P(L|B) = P(B \cap L) = 0.6935$

$$P(A|W) = \frac{P(A \cap W)}{P(W)} = \frac{P(A \cap W)}{P(A) * P(W|A) + P(B) * P(W|B)}$$

$$P(A|W) = \frac{0.0108}{0.0108 + 0.0365} = 0.2283$$

$$\boxed{P(A|W) = 0.2283}$$

#10 cont.)

Total = ~~10000~~

Box A : 4% winners

~~270 tickets~~

~~108 winners~~

2,700 tickets

108 winners

Box B: 5% winners

730 tickets

7,300 tickets

365 winners.

} mix this up!

Box A

$$\frac{108}{108 + 365} = 0.2283 = \boxed{P(A|w)}$$