語音處理 HW1 Hidden Markov Models

Homework 1

- A three-state Hidden Markov Model for the *Dow Jones Industrial average*

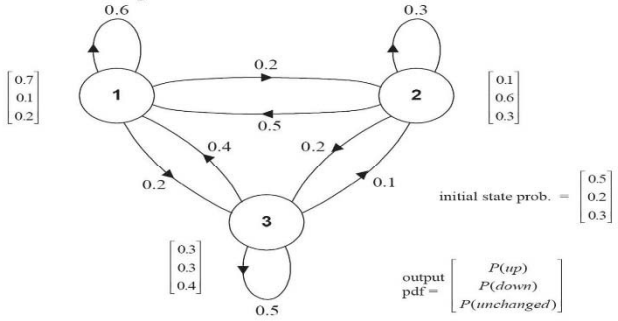

Figure 8.2 A hidden Markov model for the Dow Jones Industrial average. The three states no longer have deterministic meanings as the Markov chain illustrated in Figure 8.1.

- *Find the probability:*
  P(up, up, unchanged, down, unchanged, down, up|λ)
- *Fnd the optimal state sequence of the model which generates the observation sequence: (up, up, unchanged, down, unchanged, down, up)*

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-Find the probability P(up, up, unchanged, down, unchanged, down, up|λ)

Question1:

P(up, up, unchanged, down, unchanged, down, up|λ)

=∑P(S|λ)P(O|S, λ)

BruteForce:

= 0.00037439866445840796

Forward:

= 0.0004967268975999998

-Find the optimal state sequence of the model which generates the observation sequence: (up, up, unchanged, down, unchanged, down, up)

Question2:

Best case of observation sequence: (up, up, unchanged, down, unchanged, down, up)

= State sequence['s1', 's1', 's3', 's3', 's3', 's3', 's1']

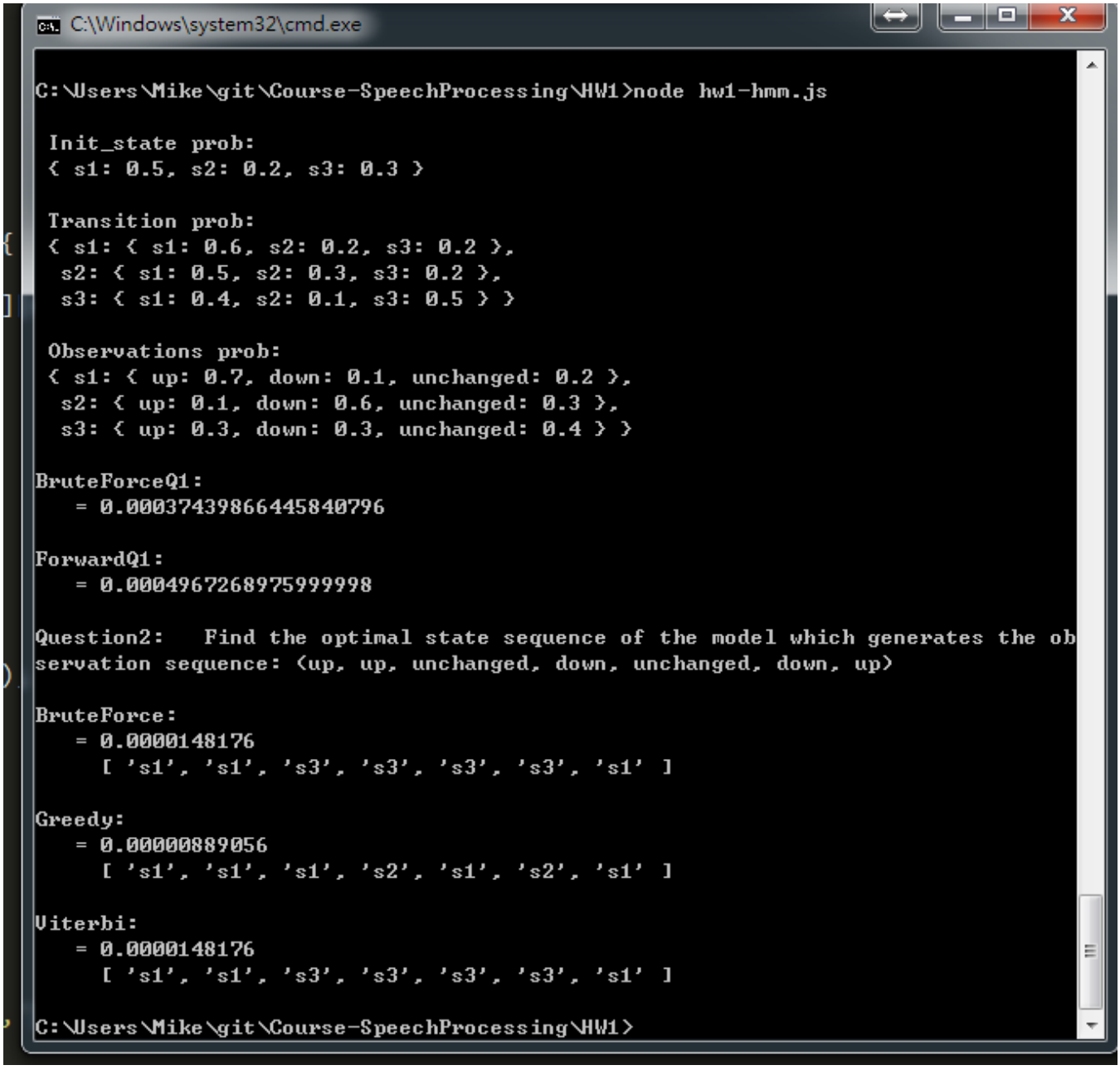
= P(S1,up)P(S1, up |S1)P(S3, unchanged |S1)P(S3, down |S3)P(S3, unchanged |S3)P(S3, down |S3)P(S1, up |S3)

= 0.7 0.5 0.6 0.7 0.2 0.4 0.5 0.3 0.5 0.4 0.5 0.3 0.4 0.7

= 0.0000148176

B: 程式運算結果

使用 javascript 進行撰寫，運行結果如下



C: 程式碼

細節步驟及詳細撰寫紀錄已放上 git

https://github.com/Mike-Zheng/Course-SpeechProcessing/tree/master/HW1

隱馬可夫數學模型:

```
//2016 NTU mike
//隱馬可夫數學模型
var HMM = {
    data: {
        //觀測符號物件
        //observations
        obs: [],
        //狀態物件
        //states
        states: [
            's1',
            's2',
            's3'
        ],
        //初始機率物件
        //init_state_prob
        init_prob: {
            's1': 0.5,
            's2': 0.2,
            's3': 0.3
        },
        //狀態轉移機率物件
        //transition_prob
        trans_prob: {
            's1': { 's1': 0.6, 's2': 0.2, 's3': 0.2 },
            's2': { 's1': 0.5, 's2': 0.3, 's3': 0.2 },
            's3': { 's1': 0.4, 's2': 0.1, 's3': 0.5 }
        },
        //觀測符號出現機率
        //observations_prob
        obs_prob: {
            's1': { 'up': 0.7, 'down': 0.1, 'unchanged': 0.2 },
            's2': { 'up': 0.1, 'down': 0.6, 'unchanged': 0.3 },
            's3': { 'up': 0.3, 'down': 0.3, 'unchanged': 0.4 }
        }
    },
    setObservations: function(arr) {
        //以物件導向方式設定 observations 值
        this.data.obs = arr;
    },
    printInit: function() {
        //將模型內數學機率印出
        console.log("\n Init_state prob: \n", this.data.init_prob);
        console.log("\n Transition prob: \n", this.data.trans_prob);
        console.log("\n Observations prob: \n", this.data.obs_prob);
    },
    //第一題
    question1: function(prob) {
        this.setObservations(prob);
        //暴力解
```

```
        BruteForceQ1(this.data);
        //Forward
        ForwardQ1(this.data);

    },
    //第二題
    question2: function(prob) {
        this.setObservations(prob);
        var q2 = "\nQuestion2:   Find the optimal state sequence of the model which generates the
observation sequence: (up, up, unchanged, down, unchanged, down, up)";
        console.log(q2);
        //暴力解
        BruteForce(this.data);
        //貪婪
        greedy(this.data);
        //Viterbi 演算法
        Viterbi(this.data);
    }
}
```

執行:

```
HMM.printInit();
HMM.question1(['up', 'up', 'unchanged', 'down', 'unchanged', 'down', 'up']);
HMM.question2(['up', 'up', 'unchanged', 'down', 'unchanged', 'down', 'up']);
```

Q1 Forward:

```
function ForwardQ1(data) {
    var path = [];
    var step = data.obs;

    //初始化路徑
    path[0] = {
        's1': accMul(data.obs_prob['s1'][step[0]], 0.5),
        's2': accMul(data.obs_prob['s2'][step[0]], 0.2),
        's3': accMul(data.obs_prob['s3'][step[0]], 0.3)
    };
    for (var i = 1; i < step.length; i++) {
        //addAll(i,'s1')
        //找出最大的路徑權重
        path[i] = {
            's1': accMul(data.obs_prob['s1'][step[i]], addAll(i, 's1')),
            's2': accMul(data.obs_prob['s2'][step[i]], addAll(i, 's2')),
            's3': accMul(data.obs_prob['s3'][step[i]], addAll(i, 's3'))
        };
    }

    // console.log(path);
    var ans = path[6]['s1'] + path[6]['s2'] + path[6]['s3'];

    console.log("\nForwardQ1: ");
    console.log("   =", ans);
```

```
    //symbol 轉 state
    function syToSt(inp) {
        var state;
        if (inp == 'up' || inp == 0) state = 's1';
        else if (inp == 'down' || inp == 1) state = 's2';
        else if (inp == 'unchanged' || inp == 2) state = 's3';
        return state;
    }

    function addAll(i, nowSt) {
        //用 forward 法 加入
        // var stateTemp;
        var sum = 0;

        for (var j = 0; j < 3; j++) {

            sum += accMul(path[i - 1][syToSt(j)], data.trans_prob[syToSt(j)][nowSt]);
        }
        return sum;
    }


}
```

Q1 BruteForce 演算法(暴力解):

```
function BruteForceQ1(data) {

    var results = getBruteforceList();
    // console.log(results.length);
    //初始化最大值暫存變數

    //初始化佇列
    var nowAns = 1;
    var sum = 0;

    //從 1~3^7 將所有路徑納入考慮
    for (var i = 0; i < 2187; i++) {

        var dig = results[i].split(',')
        nowAns = 1;
        for (var j = 0; j < 7; j++) {
            //將路徑上機率相乘
            nowAns = accMul(data.obs_prob[dig[j]][data.obs[j]], nowAns);
            //乘上一開始初始機率
            if (j == 0) nowAns = accMul(data.init_prob[dig[j]], nowAns);
            if (j < 6) nowAns = accMul(data.trans_prob[dig[j]][dig[j + 1]], nowAns);
        }
        // console.log(nowAns);
        sum += nowAns;

    }
    console.log("\nBruteForceQ1: ");
    console.log("   =", sum);

}
```

Q2 BruteForce 演算法(暴力解):

```
function BruteForce(data) {


    //排列組合用的維度 BEGIN
    //參考 http://blog.darkthread.net/post-2012-03-17-recursion-game.aspx
    var dimensions = [];
    for (var i = 0; i < 7; i++) {
        dimensions.push(["s1", "s2", "s3"]);
    }
    //用以存放結果的陣列
    var results = [];
    //使用遞迴方式排列出所有組合
    function explore(curDim, prefix) {
        //取出下一層維度
        var nextDim = dimensions.shift();
        for (var i = 0; i < curDim.length; i++) {
            if (nextDim)
            //若仍有下一層維度，繼續展開
                explore(nextDim, prefix + curDim[i] + ",");
            else
            //若已無下一層，則傳入字首加上目前維度選項成為結果
                results.push(prefix + curDim[i]);
        }
        //將下層維度存回，供上層維度其他選項使用
        if (nextDim) dimensions.push(nextDim);
    }
    //傳入第一層維度開始演算
    explore(dimensions.shift(), "");
    //排列組合用的維度 END

    //初始化最大值暫存變數
    var MAX = 0;
    //初始化佇列
    var sq;
    var nowAns = 1;

    //從 1~3^7 將所有路徑納入考慮
    for (var i = 0; i < 2187; i++) {
        var dig = results[i].split(',')
        nowAns = 1;
        for (var j = 0; j < 7; j++) {
            //將路徑上機率相乘
            nowAns = accMul(data.obs_prob[dig[j]][data.obs[j]], nowAns);
            //乘上一開始初始機率
            if (j == 0) nowAns = accMul(data.init_prob[dig[j]], nowAns);
            if (j < 6) nowAns = accMul(data.trans_prob[dig[j]][dig[j + 1]], nowAns);
        }
        if (MAX < nowAns) {
            //找出可能機率放入暫存變數
            MAX = nowAns;
            sq = results[i];
        }

    }
    console.log("\nBruteForce: ");
    console.log("   =", MAX);
```

```
    console.log("    ", sq.split(',').join().split(','));

    //檢查答案局部乘法用
    // console.log('Check: ');
    // var qdi = sq.split(',')
    // // console.log(data.obs_prob[qdi[0]]);
    // nowAns = 1;
    // for (var j = 0; j < 7; j++) {
    //     console.log(data.obs_prob[qdi[j]][data.obs[j]]);
    //     if (j == 0) console.log(data.init_prob[qdi[j]]);
    //     if (j < 6) console.log(data.trans_prob[qdi[j]][qdi[j + 1]]);
    // }

}
```

Q2 Greedy 演算法:

```
function greedy(data) {
    var sq = [];
    var nowAns = 1;
    var step = data.obs;

    //加入路徑
    sq.push(syToSt(step[0]));
    // 初始狀態出現機率
    nowAns = accMul(data.obs_prob[syToSt(step[0])][step[0]], data.init_prob[syToSt(step[0])]);

    for (var i = 1; i < step.length; i++) {
        //將 7 個節點抓該點最大的相乘
        nowAns = accMul(findMAX(data.trans_prob, sq[sq.length - 1], step[i]), nowAns);
    }

    //symbol 轉 state
    function syToSt(inp) {
        var state;
        if (inp == 'up' || inp == 0) state = 's1';
        else if (inp == 'down' || inp == 1) state = 's2';
        else if (inp == 'unchanged' || inp == 2) state = 's3';
        return state;
    }

    //找這個節點最大的
    function findMAX(arr, now, nxtObs) {
        var stateTemp;
        var max = 0;
        var fd;

        for (var i = 0; i < data.states.length; i++) { //state 有 3 種
            //考慮結點挑最大的
            fd = syToSt(i);
            if (accMul(data.trans_prob[now][fd], data.obs_prob[fd][nxtObs]) > max) {
                max = accMul(data.trans_prob[now][fd], data.obs_prob[fd][nxtObs]);
                stateTemp = fd;
            }

        }
```

```
        //若為最大加入路徑
        sq.push(stateTemp);
        return max;
    }

    console.log("\nGreedy: ");
    console.log("   =", nowAns);
    console.log("    ", sq);


}
```

Q2 Viterbi 演算法:

```
function Viterbi_a(data) {
    var path = [];
    var step = data.obs;
    var sq = []; //答案路徑
    var Ans; //最佳答案
    //[{s1:w,s2:w,s3:w},{}]
    //初始化路徑
    path[0] = {
        's1': accMul(data.obs_prob['s1'][step[0]], 0.5),
        's2': accMul(data.obs_prob['s2'][step[0]], 0.2),
        's3': accMul(data.obs_prob['s3'][step[0]], 0.3)
    };
    for (var i = 1; i < step.length; i++) {
        //findMAX(i,'s1')
        //找出最大的路徑權重
        path[i] = {
            's1': accMul(data.obs_prob['s1'][step[i]], findMAX(i, 's1')),
            's2': accMul(data.obs_prob['s2'][step[i]], findMAX(i, 's2')),
            's3': accMul(data.obs_prob['s3'][step[i]], findMAX(i, 's3'))
        };
    }

    //權重步驟
    // console.log(path);

    //最後算出來最大的權重就是答案
    Ans = max(path[step.length - 1]['s1'], path[step.length - 1]['s2'], path[step.length -
1]['s3'])[0];
    //向前找路徑
    //初始化最後路徑
    var tempAns = Ans;
    var tempSt = max(path[step.length - 1]['s1'], path[step.length - 1]['s2'], path[step.length -
1]['s3'])[1];
    sq.unshift(max(path[step.length - 1]['s1'], path[step.length - 1]['s2'], path[step.length -
1]['s3'])[1]);

    //往回找路
    for (var i = 5; i >= 0; i--) {
        // accMul(path[i ]['s1'], data.trans_prob['s1'][nowSt])
        for (var k = 0; k < 3; k++) {
            // console.log(accMul(path[i][syToSt(k)], data.trans_prob[syToSt(k)][tempSt]));
```

```
            if (accMul(data.obs_prob[tempSt][step[i + 1]], accMul(path[i][syToSt(k)],
data.trans_prob[syToSt(k)][tempSt])) == tempAns) {
                tempAns = path[i][syToSt(k)];
                tempSt = syToSt(k);
                sq.unshift(syToSt(k));
            }
        }


    }

    console.log("\nViterbi: ");
    console.log("   =", Ans);
    console.log("    ", sq);




    function max(a, b, c) {
        var M = Math.max(a, b, c);
        if (M == a) return [M, 's1'];
        else if (M == b) return [M, 's2'];
        else if (M == c) return [M, 's3'];
    }

    //symbol 轉 state
    function syToSt(inp) {
        var state;
        if (inp == 'up' || inp == 0) state = 's1';
        else if (inp == 'down' || inp == 1) state = 's2';
        else if (inp == 'unchanged' || inp == 2) state = 's3';
        return state;
    }

    //找這個節點最大的
    function findMAX(i, nowSt) {
        // var stateTemp;
        var max = 0;
        // 找出(前一節點)X(轉移機率)權重最大的
        for (var j = 0; j < 3; j++) {
            if (accMul(path[i - 1][syToSt(j)], data.trans_prob[syToSt(j)][nowSt]) > max)
                max = accMul(path[i - 1][syToSt(j)], data.trans_prob[syToSt(j)][nowSt]);
        }
        return max;
    }

}
```

D: Q2 時間複雜度計算

這次 Q2 部分演算法的撰寫，主要針對三種演算法進行攻略

演算法	攻略細節	預計時間複雜度
BruteForce 暴力解	暴力解，將 3^7 路徑都進行進算	O(n^m)
Greedy 貪婪演算法	透過找尋下一個最大的可能進行計算，算出來的結果是錯的，但卻很快可以找到一個可能的答案。	O(n^2)
Viterbi dynamic programming	Viterbi 透過路徑權重記錄進行運算，算是用空間取得時間的一種方式，但由於從中無法紀錄運算的步驟(太浪費空間)，所以最後還要從解果逆推回去計算進行的步驟。	O(2 n^2)

E: 參考

陳柏琳教授課程網站
http://berlin.csie.ntnu.edu.tw/Courses/Speech%20Processing/Speech%20Processing_Main_2016S.htm

生物信息学课件 Hidden Markov Model 隐马尔可夫模型
http://www.tudou.com/programs/view/cBZqDKojiw8/

HMM 学习笔记_1(从一个实例中学习 DTW 算法)
http://www.cnblogs.com/tornadomeet/archive/2012/03/23/2413363.html

演算法筆記- Hidden Markov Model
http://www.csie.ntnu.edu.tw/~u91029/HiddenMarkovModel.html

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F: 心得

這次撰寫過程著實花了一些時間，從理解 HMM 到攻略演算法，都花了不少時間推導。

主要撰寫的 pattern 是將各個機率做成一個數學模型，再透過不同的演算法 function，進行運算。

基於目前的實習工作及預計想走的方向，使用 javascript(nodejs)目前前後端都流行的語言進行撰寫開發，其中**浮點數相乘**遇到的問題解法，及**暴力解的狀態排列組合序列**(遞回部分)，有參考網路上的解法，剩下部分都是自行完成，程式碼的撰寫有上 git，如需運行需要安裝 nodeJs 做為直譯工具。

https://github.com/Mike-Zheng/Course-SpeechProcessing/tree/master/HW1

2016/3/24

因為答案有誤而更正，第一提用了兩種解法，暴力解和 forward，兩種算出來的答案差了 0.001，在次檢查後估計是記算小樹點遇到的問題，故此也列出兩種寫法。

力文