COMP 4418 – Exercise Sheet: Social Choice Theory II

Exercise I: Compute RSCFs

Compute the lottery chosen by the uniform random dictatorship, the randomized Borda rule (which randomizes proportional to the Borda scores), and a maximal lottery for the two subsequent profiles.

a) R^1: 2: $b \succ c \succ d \succ a$
2: $a \succ b \succ c \succ d$
2: $c \succ d \succ a \succ b$
1: $a \succ d \succ c \succ b$

b) R^2: 2: $a \succ b \succ c \succ d$
2: $d \succ b \succ c \succ a$
1: $c \succ a \succ b \succ d$

Exercise II: Strategyproofness for RSCFs

a) Show that no maximal lottery rule is strategyproof.

b) Show that the randomized Borda rule is strategyproof.

c) Given a preference relation $\succ$ and an alternative x, let $U(\succ,x) = \{x|\} \cup \{y \in A\colon y \succ x\}$. Show that, for all preference relations $\succ$ and all lotteries $p,q \in \Delta(A)$, it holds that $\mathbb{E}[p(u)] \geq \mathbb{E}[q(u)]$ for all u that are consistent with $\succ$ if and only if $\sum_{y \in U(\succ,x)} p(y) \geq \sum_{y \in U(\succ,x)} q(y)$ for all $x \in A$.

Exercise III: Computing ABC voting rules

Compute *AV*, *PAV*, *CCAV*, *Phragmen*, and *MES* for the subsequent profile and the target committee size $k = 3$.

3: $\{a,b\}$ 3: $\{a,c\}$ 2 : $\{a,b,d\}$ 2: $\{e\}$ 1: $\{f\}$

Exercise IV: Extended Justified Representation

a) Show that *PAV* satisfies EJR.

b) Show that *MES* satisfies EJR.