Fair Allocations

COMP 4418 – Assignment 3

Due 14 November 2024, 16:00
Total Marks: 100
Late Penalty: 10 marks per day

Worth: 15% of the course

Question 1 (20 marks) Consider a fair division instance $\langle N, M, v \rangle$ with n agents and m items. Prove or disprove the following:

1. **(5 marks)** Any Pareto Optimal allocation must also be Leximin Optimal.
2. **(5 marks)** Given any two allocations, one must pareto dominate the other.
3. **(5 marks)** For $n = 2$, any allocation that satisfies PROP is also EF.
4. **(5 marks)** Greedy round robin algorithm will return an EF1 allocation.

Question 2 (20 marks) Consider the following instance with $n = 4$ and $m = 8$.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
v_1	1	5	4	4	0	1	1	1
v_2	5	9	5	5	0	0	5	5
v_3	5	7	5	10	0	4	0	5
v_4	10	10	5	5	5	5	5	5

For this instance, consider running the standard round robin algorithm to find an EF1 allocation. We shall look at how different orderings over agents can lead to different allocations. For the given instance, identify:

1. **(10 marks)** The ordering over agents which leads to the following allocation: $A = (A_1, A_2, A_3, A_4)$, where $A_1 = \{g_1, g_5\}$, $A_2 = \{g_4, g_8\}$, $A_3 = \{g_3, g_7\}$ and $A_4 = \{g_2, g_6\}$.
2. **(5 marks)** An alternate EF1 allocation that can result from the same ordering which would Pareto dominate A.
3. **(5 marks)** An alternate ordering for the standard round robin algorithm that would result in the allocation identified in the previous part.

Question 3 (20 marks) Consider an indivisible item setting with $m > n$ where agents are indifferent between the items. That is, for any $i \in N$ and $g \neq g' \in M$, we have that $v_i(g) = v_i(g') > 0$. However, agent valuations are not (guaranteed to be) identical. That is, there may be $i \neq j$ and $g \in M$, s.t. $v_i(g) \neq v_j(g)$. For this setting:

1. **(5 marks)** Show that an MMS allocation always exists.
2. **(5 marks)** Show that an EF1 allocation will always be MMS.
3. **(10 marks)** Give examples of instances in this setting such that:
   a. Any allocation with maximum ESW is not MMS.
   b. There is at least one allocation with maximum USW which is $\frac{1}{2}-$MMS in this instance.

Question 4 (20 marks) Consider the random assignment problem with 3 agents with the following preferences over 3 items.

$$\succ_1: \quad g_1 \succ_1 g_2 \succ_1 g_3$$
$$\succ_2: \quad g_1 \succ_2 g_2 \succ_2 g_3$$
$$\succ_3: \quad g_2 \succ_3 g_1 \succ_3 g_3$$

Find the random assignment as a result of the following rules.

1. **(10 marks)** probabilistic serial (PS)
2. **(10 marks)** random serial dictator (RSD)

Question 5 (20 marks) Consider the following instance with $n = 4$ and $m = 8$.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
v_1	1	5	4	4	0	1	1	1
v_2	5	9	5	5	0	0	5	5
v_3	5	7	5	10	0	4	0	5
v_4	10	10	5	5	5	4	1	1

Consider the allocation A in which $A_1 = \{g_1, g_2\}$, $A_2 = \{g_3, g_4\}$, $A_3 = \{g_5, g_6\}$, and $A_4 = \{g_7, g_8\}$.

1. **(5 marks)** Prove or disprove that the allocation is envy-free.
2. **(5 marks)** Prove or disprove that the allocation is envy-freeable.
3. **(5 marks)** Compute the corresponding envy-graph with the amount of envy on the edge weights.
4. **(5 marks)** Find the subsidy needed to be given to each agent in order to make the allocation envy-free or show that no such subsidy exists.

SUBMISSION

- Submit your solution directly via Moodle in the assessment hub at the end of the Moodle page. Please make sure that your manuscript contains your name and zID.
- Your answers are to be submitted in a single PDF file. We will not accept any other file formats. Please make sure that your solutions are clearly readable.
- The deadline for this submission is 14th November 2024, 16:00.

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