COMP 4418 – Exercise Sheet: Matching I

Exercise I: Painful Matchings

Given a one-one matching instance $\langle S, C, \succ \rangle$, define the **pain** of a matching μ to be the sum of the ranks of the matched partners under μ. For example, in the following instance, the matching $\mu = \{(s_1, c_1), (s_2, c_2)\}$ has **pain**$(\mu) = 5$.

$$s_1 : c_1 \succ c_2 \qquad\qquad c_1 : s_1 \succ s_2$$
$$s_2 : c_1 \succ c_2 \qquad\qquad c_2 : s_2 \succ s_1$$

For an arbitrary n, construct a one-one instance with n students and n colleges such that it admits a stable matching with **pain** of $n(n+1)$.

Exercise II: Optimal and Pessimal Partners

For an arbitrary n construct a one-one matching instance where exactly $\lfloor \frac{n}{2} \rfloor$ students have a unique achievable college. For all other students, they are their optimal college's least preferred student.

Exercise III: Number of Proposals

Given a many-to-one matching instance $\langle S, C, b, \succ \rangle$, let S_c be the top b_c most preferred students of $c \in C$. If these sets are mutually disjoint, how many proposals will occur under the College Proposing Deferred Acceptance algorithm?

Exercise IV: Optimal Manipulated Partner

Construct a one-one matching instance where under the Student Proposing Deferred Acceptance (SPDA) the optimal manipulation of a college c would not match it to its optimal partner.

Exercise V: Manipulation in Many-to-One Matchings

Prove that in a many-to-one matching instance, under the CPDA there is no beneficial manipulation for a college c s.t. $b_c = 1$.