Game Theory

COMP 4418 – Assignment 1

Due 05/10/2024
Total Marks: 100
Late Penalty: 10 marks per day

Worth: 15% of the course

For all questions, proofs, possibly in the form of calculations, need to be given in order to obtain all points.

Question 1 (20 marks) Consider the following normal-form game, which is parameterized by a value $\alpha \in \mathbb{R}$.

	x	y
a	1, -1	-3, 3
b	-2, α	4, -4

a) For which value of α is the game zero-sum?

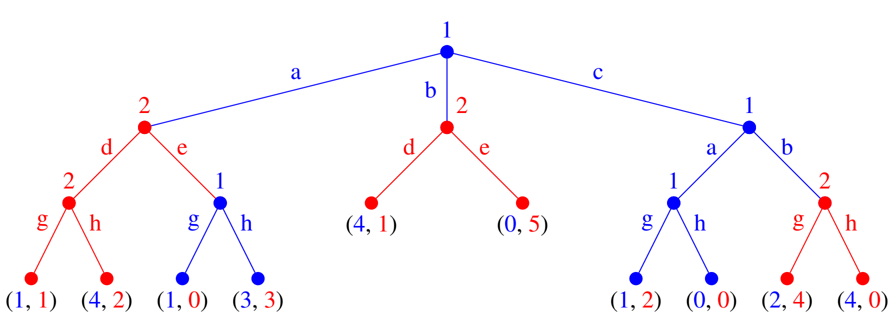
b) For which values of α is the outcome $(-2, \alpha)$ Pareto-optimal?

c) For which values of α can the game be solved by iterated strict dominance?

d) For which value of α is it the maximin strategy of the column player to play x with probability $\frac{1}{2}$?

e) For which value of α will the row player play a with probability $\frac{3}{4}$ in a Nash equilibrium?

Question 2 (20 marks) Consider the following extensive-form game.



a) Compute the subgame-perfect Nash equilibrium.

b) Is there a pure Nash equilibrium where player 1 has a utility of 4? Explain your answer!

c) What is the maximum utility that player 2 can obtain in a pure Nash equilibrium?

Question 3 (20 marks) Three pirates find a treasure of 90 gold coins. They decide on the following protocol to split the gold coins. First, all pirates $i \in \{1,2,3\}$ submit two numbers $t_i \in \{0,\dots,90\}$ and $k_i \in \{0,\dots,90\}$. Then, the pirates $1,2,3$ (in this order) take t_i coins from the treasure if there are sufficiently many gold coins left or all remaining coins otherwise. Next, pirate 2 checks the amount of coins pirate 1 took: if pirate 1 has more than k_2 gold coins, pirate 2 steals all coins of pirate 1; if pirate 1 has at most k_2 coins, he keeps his coins and nothing happens. Finally, pirate 3 checks how much coins pirate 2 has (including those he possibly stole from pirate 1): if this amount exceeds k_3, he steals all coins from pirate 2 and otherwise, pirate 2 keeps his coins.

Assume that the utility function of every pirate is equal to the number of gold coins he has in the end if he did not steal the gold coins of his predecessor and half of that otherwise.

a) Show that there is a pure Nash equilibrium where all pirates have 30 gold coins.

b) What is the maximal amount of gold coins that each pirate can obtain in a pure Nash equilibrium? Present for each pirate the corresponding strategy profile and reason why it is a pure Nash equilibrium.

Question 4 (10 marks) Let $A = \{a,b,c\}$ and $\succsim$ denote a rational preference relation over $\mathscr{L}(A)$ that is independent and satisfies that $[1:a] \succ [1:b]$ and $[\frac{1}{2}:a, \frac{1}{2}:c] \sim [1:b]$. Show that $[1:b] \succ [1:c]$.

Question 5 (30 marks) Prove the following statements.

a) Let $G_1 = (\{1,2\}, (A_i^1)_{i\in\{1,2\}}, (u_i^1)_{i\in\{1,2\}})$, $G_2 = (\{1,2\}, (A_i^2)_{i\in\{1,2\}}, (u_i^2)_{i\in\{1,2\}})$, and $G_3 = (\{1,2\}, (A_i^3)_{i\in\{1,2\}}, (u_i^3)_{i\in\{1,2\}})$ denote three two-player normal-form games such that $A_i^1 = A_i^2 = A_i^3$ for $i \in \{1,2\}$ and $u_i^3(a) = \frac{1}{2}(u_i^1(a) + u_i^2(a))$ for both players $i \in \{1,2\}$ and all action profiles $a \in A$. Show that, if a strategy profile s is a Nash equilibrium for G_1 and G_2, then it is a Nash equilibrium for G_3.

b) Let $G = (\{1,2\}, (A_i)_{i\in\{1,2\}}, (u_i)_{i\in\{1,2\}})$ denote a two-player normal-form game and let s^1 and s^2 denote two Nash equilibria for G such that $s_i^1(a_i) > 0$ if and only if $s_i^2(a_i) > 0$ for both players $i \in \{1,2\}$ and all actions $a_i \in A_i$. Show that the strategy profile s^3 given by $s_1^3(a_1) = \frac{1}{2}(s_1^1(a_1) + s_1^2(a_1))$ for all $a_1 \in A_1$ and $s_2^3(a_2) = s_2^1(a_2)$ for all $a_2 \in A_2$ is also a Nash equilibrium for G.

c) Let $G = (\{1,2\}, (A_i)_{i\in\{1,2\}}, (u_i)_{i\in\{1,2\}})$ denote a two-player zero-sum game such that $A_1 = A_2 = \{a_1,\dots,a_k\}$ for $k > 1$ and $u_1(a_i, a_j) = u_2(a_j, a_i)$ for all $a_i, a_j \in A$. Show that the value of G (i.e., the security level of player 1) is 0.
Hint: Show that for every strategy of player 2*, player* 1 *has a strategy that guarantees him an expected utility of at least* 0.

SUBMISSION

- Submit your solution directly via Moodle in the assessment hub at the end of the Moodle page. Please make sure that your manuscript contains your name and zID.
- Your answers are to be submitted in a single PDF file. We will not accept any other file formats. Please make sure that your solutions are clearly readable.
- The deadline for this submission is 5 October 2024, 23:55.

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