PHY FUNDAMENTALS I

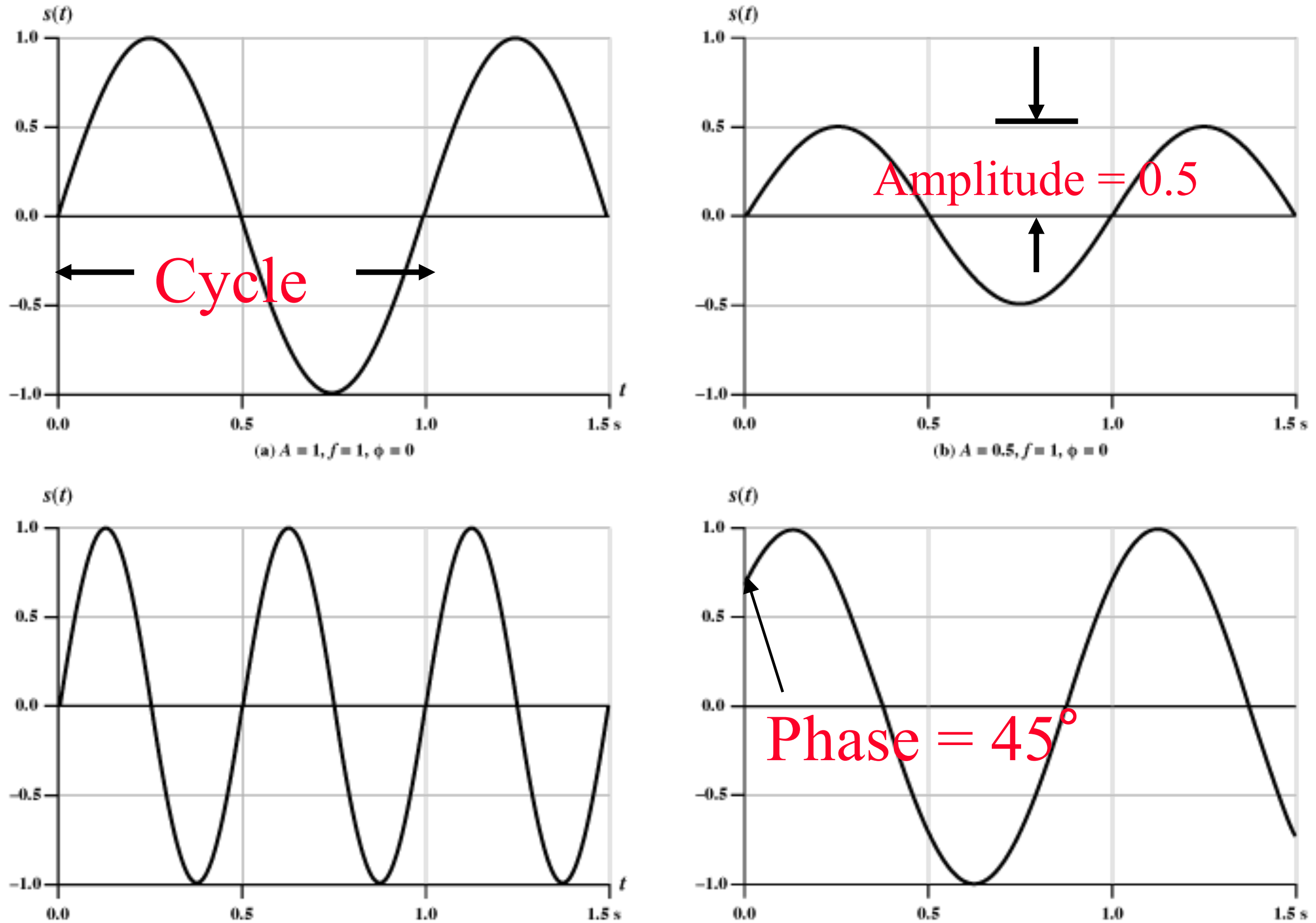
Coding and Modulation

Overview

1. Frequency, Wavelength, Amplitude, and Phase
2. Electromagnetic Spectrum
3. Time and Frequency Domains
4. Decibels
5. Coding and modulation
6. Channel Capacity (Nyquist’s and Shannon's Theorems)
7. Hamming Distance and Error Correction
8. Multiple Access Methods (TDMA, FDMA, CDMA)
9. Spread Spectrum (Frequency Hopping and Direct Sequence)
10. Doppler Shift, Doppler Spread, Coherence Time
11. Duplexing

Frequency, Period, and Phase

- A Sin(2pft + ϕ), A = Amplitude, f=Frequency, ϕ = Phase, Period T = 1/f, Frequency is measured in Cycles/sec or **Hertz**



(a) $A = 1, f = 1, \phi = 0$

(b) $A = 0.5, f = 1, \phi = 0$

(c) $A = 1, f = 2, \phi = 0$

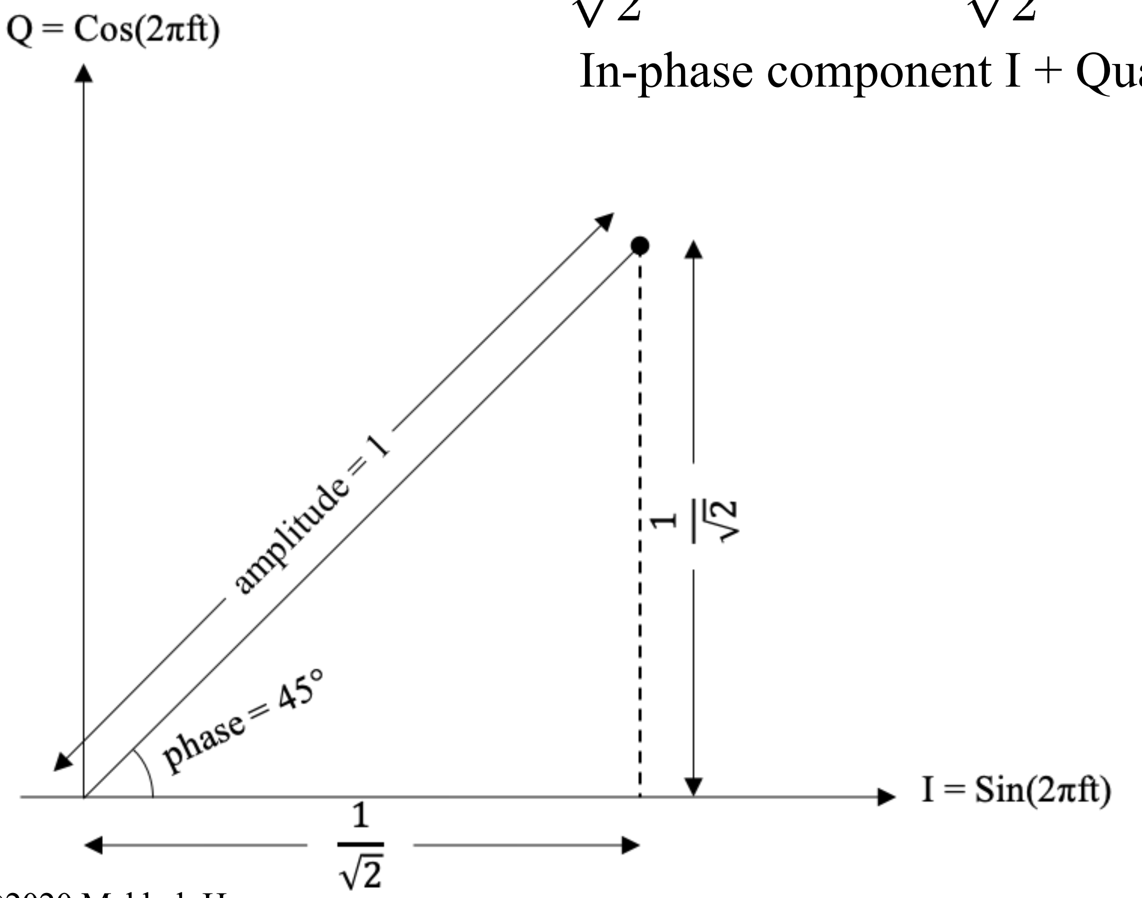
(d) $A = 1, f = 1, \phi = \pi/4$

Phase and Amplitude: 2D Representation

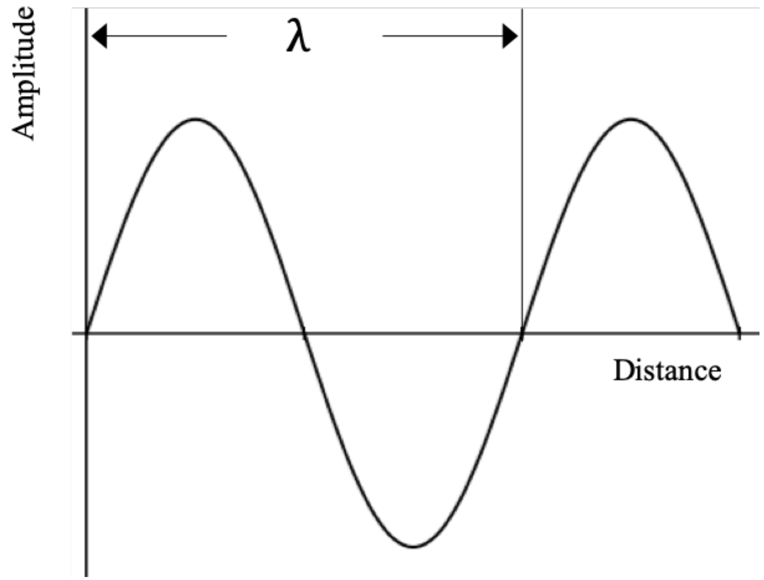
- Sine wave with a phase of 45°

$$\sin(2\pi ft + \frac{\pi}{4}) = \sin(2\pi ft)\cos(\frac{\pi}{4}) + \cos(2\pi ft)\sin(\frac{\pi}{4})$$
$$= \frac{1}{\sqrt{2}}\sin(2\pi ft) + \frac{1}{\sqrt{2}}\cos(2\pi ft)$$

In-phase component I + Quadrature component Q



Wavelength



- Distance occupied by one cycle
- Distance between two points of corresponding phase in two consecutive cycles
- Wavelength = λ
- Assuming signal velocity *v*
  - $\lambda = vT$
  - $\lambda f = v$
  - c = 3×10^8 m/s (speed of light in free space) = *300 m/μs*

Example: converting frequency to wavelength

- Frequency = 2.5 GHz

$$\begin{aligned}\text{Wavelength} = \lambda &= \frac{c}{f} \\ &= \frac{300\ \text{m}/\mu\text{s}}{2.5 \times 10^9} \\ &= 120 \times 10^{-3} = 120\ \text{mm} = 12\ \text{cm}\end{aligned}$$

Example: converting wavelength to frequency

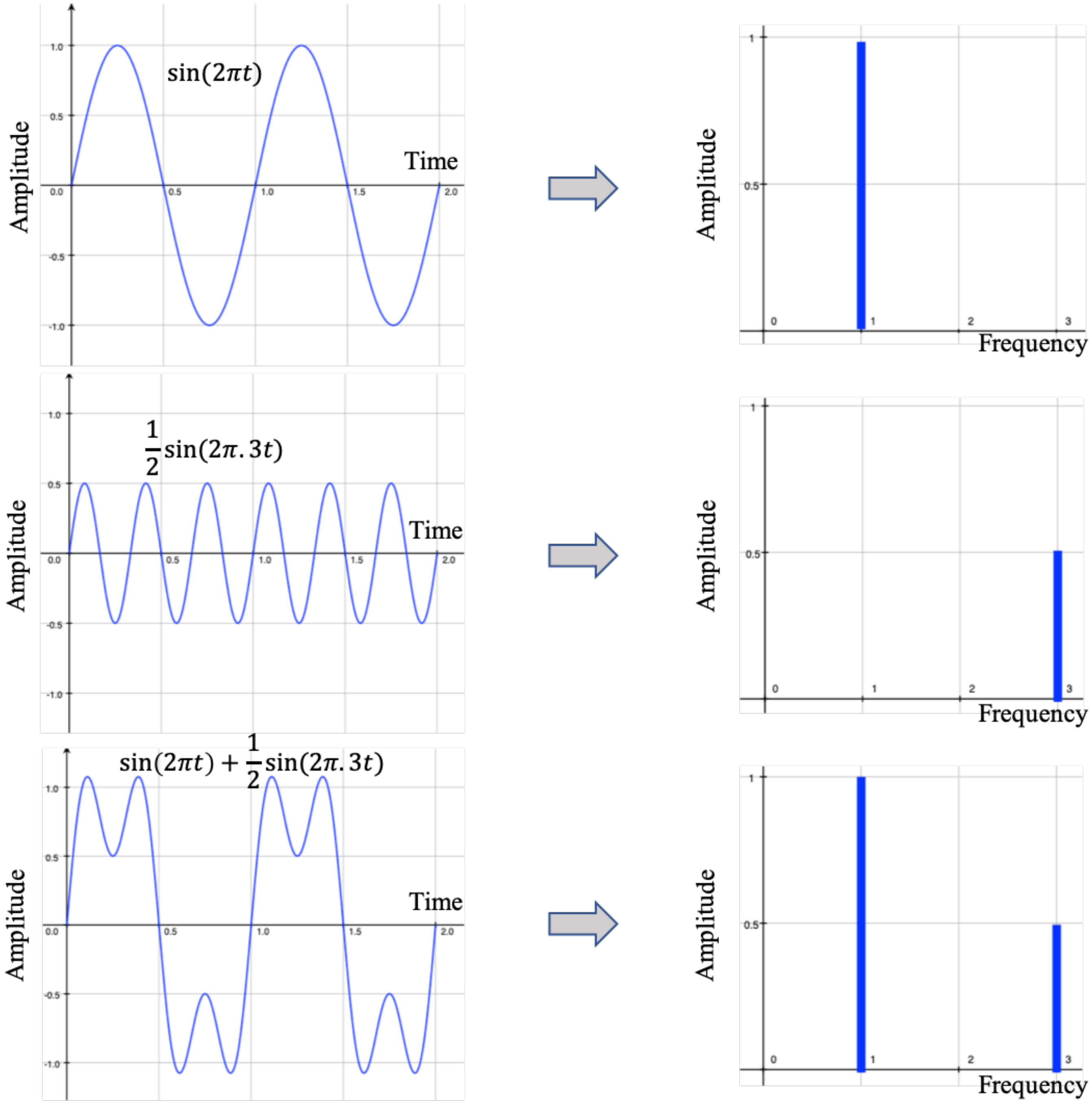
- Wavelength = λ = 5 mm

Frequency = f = c/λ
= $(3 \times 10^{8}$ m/s$) / (5 \times 10^{-3}$
m)
= $(300 \times 10^{9}) / 5 = 60$ GHz

Well-known frequencies & wavelengths

- The higher the frequency, the smaller the wavelength
- 900 MHz has a wavelength of 33.33 cm
- 2.4 GHz has a wavelength of 12.5 cm
- 60 Ghz has a wavelength of only 5 mm (this technology is called *millimeter wave or mmWave*)

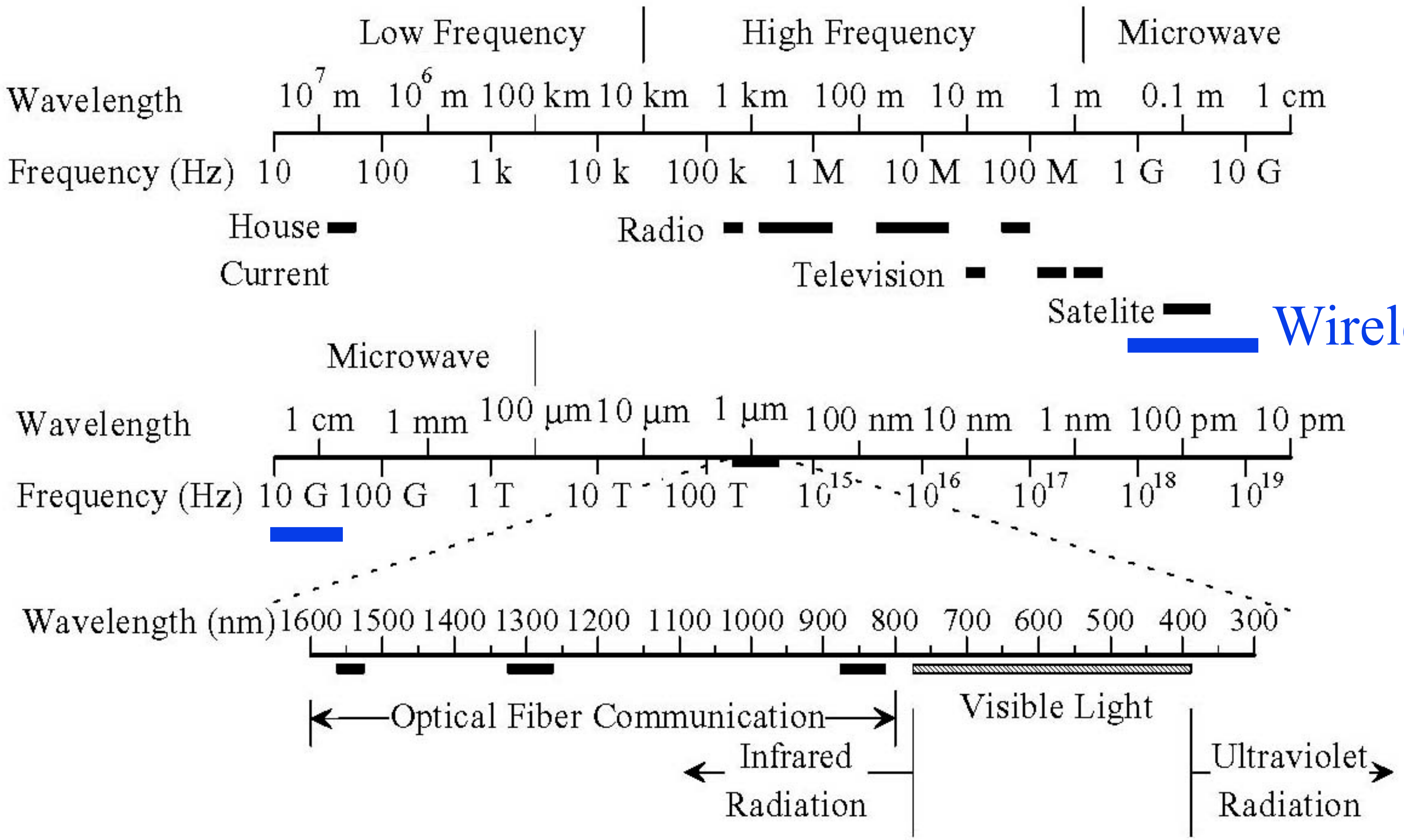
Time and Frequency Domains



Electromagnetic Spectrum

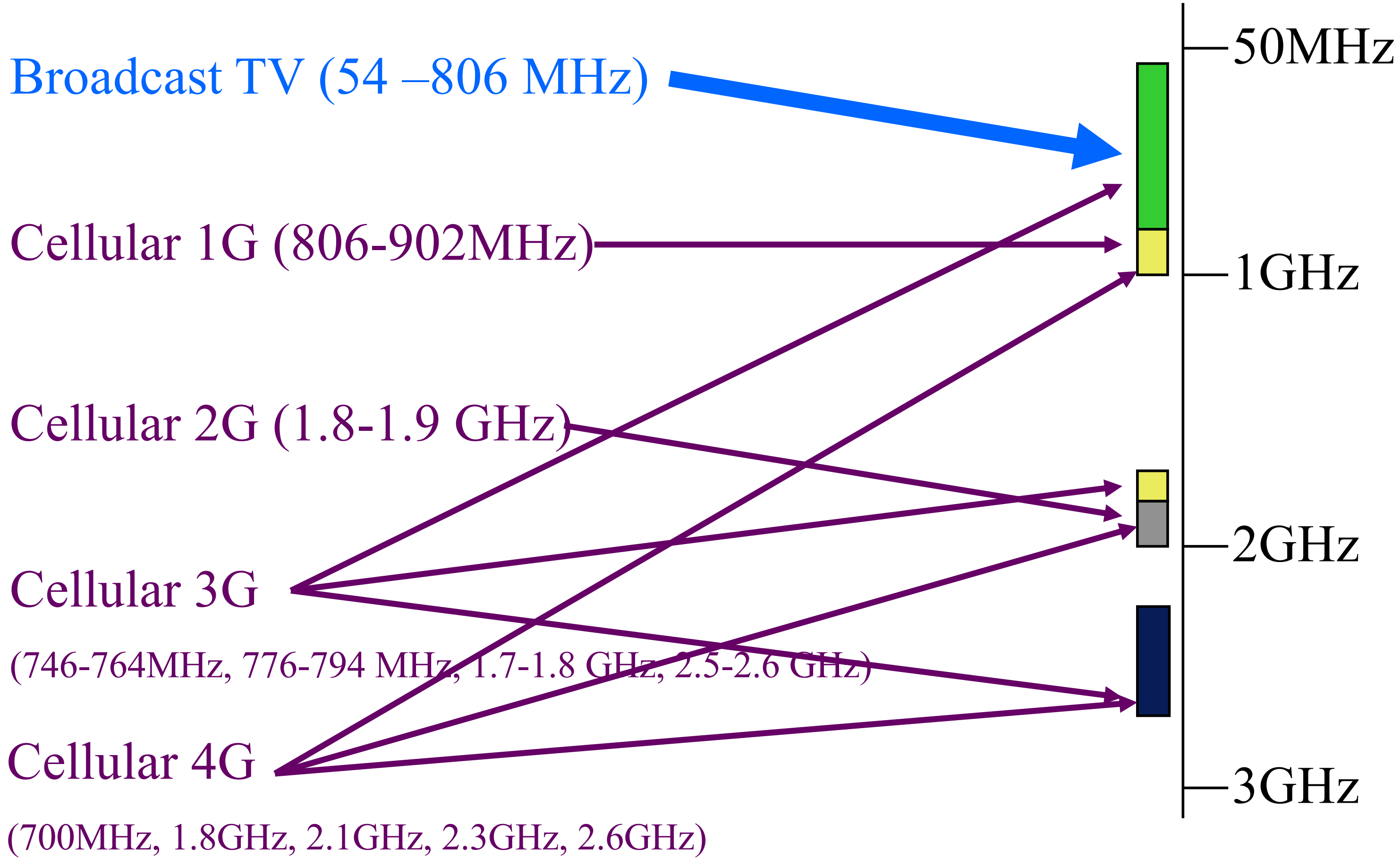
- Wireless transmissions use the airwaves
  - Airwaves are radio frequencies
- Useful frequencies constitute the Spectrum
- Spectrum is ‘virtual’
  - We cannot touch and feel
- A gift from nature (the force field)
  - Has been there ever since earth was created
- A (limited) natural resource

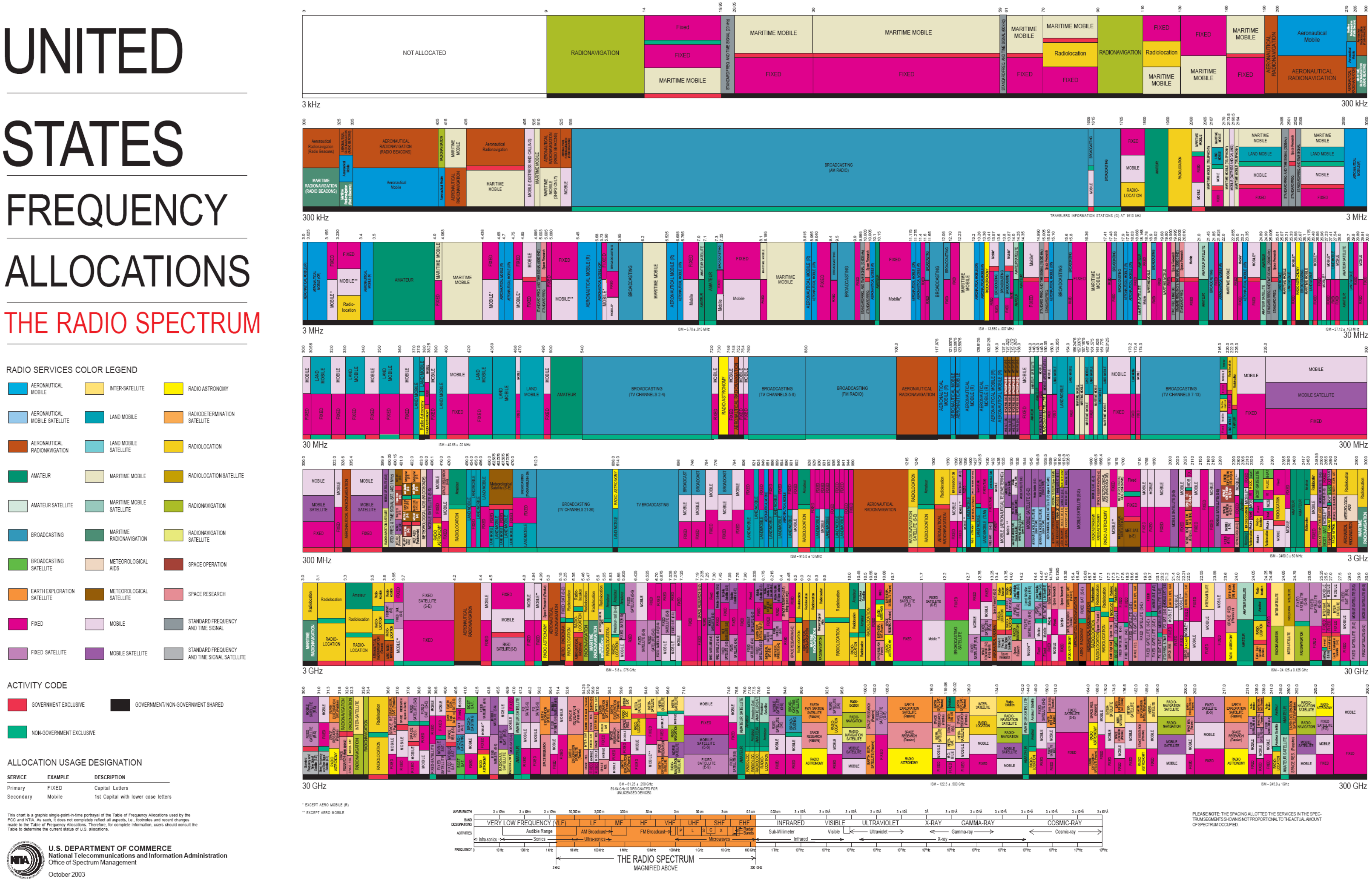
Electromagnetic Spectrum



- Wireless communication mostly uses 100 kHz to 6 GHz
- > 6 GHz are currently being explored and 60 GHz is used in some latest WiFi products

Spectrum use by TV and cellular services




UNITED STATES FREQUENCY ALLOCATIONS
THE RADIO SPECTRUM
RADIO SERVICES COLOR LEGEND
AERONAUTICAL MOBILE
INTER-SATELLITE
RADIO ASTRONOMY
AERONAUTICAL MOBILE SATELLITE
LAND MOBILE
RADIODETERMINATION SATELLITE
AERONAUTICAL RADIONAVIGATION
LAND MOBILE SATELLITE
RADIOLOCATION
AMATEUR
MARITIME MOBILE
RADIOLOCATION SATELLITE
AMATEUR SATELLITE
MARITIME MOBILE SATELLITE
RADIONAVIGATION
BROADCASTING
MARITIME RADIONAVIGATION
RADIONAVIGATION SATELLITE
BROADCASTING SATELLITE
METEOROLOGICAL AIDS
SPACE OPERATION
EARTH EXPLORATION SATELLITE
METEOROLOGICAL SATELLITE
SPACE RESEARCH
FIXED
MOBILE
STANDARD FREQUENCY AND TIME SIGNAL
FIXED SATELLITE
MOBILE SATELLITE
STANDARD FREQUENCY AND TIME SIGNAL SATELLITE
ACTIVITY CODE
GOVERNMENT EXCLUSIVE
GOVERNMENT/NON-GOVERNMENT SHARED
NON-GOVERNMENT EXCLUSIVE
ALLOCATION USAGE DESIGNATION
SERVICE EXAMPLE DESCRIPTION
Primary FIXED Capital Letters
Secondary Mobile 1st Capital with lower case letters
This chart is a graphic single-point-in-time portrayal of the Table of Frequency Allocations used by the FCC and NTIA. As such, it does not completely reflect all aspects, i.e., footnotes and recent changes made to the Table of Frequency Allocations. Therefore, for complete information, users should consult the Table to determine the current status of U.S. allocations.
U.S. DEPARTMENT OF COMMERCE
National Telecommunications and Information Administration
Office of Spectrum Management
October 2003
3 kHz
300 kHz
3 MHz
30 MHz
300 MHz
3 GHz
30 GHz
300 GHz
THE RADIO SPECTRUM
MAGNIFIED ABOVE

Spectrum regulation and licensing

- Many users use the same airspace
  - Recipe for collision
  - No one would get anything useful done
- Spectrum use is highly regulated
  - By govt. authorities (eg FCC in the USA)
- Spectrum is often licensed
  - By big companies, eg Telstra
  - Gives exclusive rights to certain freq. bands
  - Interference avoidance by regulation

Spectrum allocation

- Bulk of it reserved for government use
  - Scientific exploration
  - Public safety
  - Military
- Some for commercial services
  - TV broadcast
  - Mobile phone
- Some for free-to-use
  - High-speed wireless local area network (WiFi)
  - Cordless phone handsets at home
  - Can you name a few more?

Key principles of spectrum allocation

- Maximize spectrum utilization
- Spectrum made available to new technologies and services
  - Adapt to new market needs
- Fair licensing
- Promote competition
- Ensure spectrum availability for public safety, health, defense, scientific experiments…

Free/unregulated/license-exempt spectrum

- Not subject to license
- Has rules for products (eg *power limitation*)
- More frequencies are being released as license-exempt
- Some current license-exempt frequencies
  - 900 MHz
  - 2.4 GHz ISM band (WiFi, Microwave etc.)
  - 5.2/5.3/5.8 GHz (WiFi, Cordless phone etc.)
  - Can you identify more?

Decibel

- Tx_power for practical mobile systems vary by many orders of magnitude
  - 100kW or kilowatt (FM radio station)
  - 500mW or milliwatt (cellular phone tx power)
  - 2.5mW (Bluetooth with ~10m range)
  - 100pW or picowatt (typical WiFi rx threshold)
  - Femto watt? (nanosensor communication)
  - $1\ W = 10^3\ mW = 10^6\ \mu W = 10^9\ nW = 10^{12}\ pW$
- Decibel is a more convenient (logarithmic scale) unit to compare these powers, which are many orders of magnitude apart
- Also path loss (attenuation) can be many orders of magnitude
  - Path loss therefore is usually expressed in decibels

Decibel (dB) Formula

- In Honour of Graham Bell
- The number of decibels is ten times the logarithm to base 10 of the ratio of two power quantities ($dB = 10\log_{10}(P_1/P_2)$)
  - The quantity "Bel" would be $\log_{10}(P_1/P_2)$, but not used
- Decibel can be used for different purposes
  1. Path Loss: To express path loss or attenuation: [P_1 = transmit power; P_2 = receive power]
  2. SNR: To express signal (P_1) to noise (P_2) ratio at the receiver
  3. Signal Power: To express signal power (P_1), which can be either transmit or receive power, to a reference power (P_2)

Decibel Examples for Path Loss

- **Example 1**: P_t = 10 mW, P_r= 5 mW (power reduced by half)
  Attenuation (path loss) = 10 $\log_{10}$ (10/5) = 10 $\log_{10}$ 2 = 3 dB

- **Example 2**: P_t = 100 mW, P_r= 1 mW (power reduced by a factor of 100)
  Attenuation = 10 $\log_{10}$ (100/1) = 10 $\log_{10}$ 100 = 20 dB

Power Ratio	dB
10,000,000,000 (ten billion times)	100 (10 x 10)
1,000,000 (1 million times)	60 (10 x 6)
10 (ten times)	10 (10 x 1)
0.001 (10^{-3})	-30 (10 x -3)
0.0001 (10^{-4})	-40 (10 x -4)

Decibel Examples for Signal-to-Noise Ratio

- **Example 1**: P_{signal} = 1 mW (received signal strength), P_{noise}= 100 μW

  $SNR = 10 \log_{10} (1000/100) = 10 \log_{10} 10 = 10$ dB

- **Example 2**: Received signal strength is measured at 10 mW. What is the noise power if SNR = 10 dB?

  $SNR = 10 \text{ dB} = 10 \log_{10} (10\text{mW}/P_{noise})$

$P_{noise} = 1$ mW

Expressing Power in dBm

- dBm is in reference to 1 milliwatt
- First, express power in milliwatt
- Then apply the following formula to obtain dBm

$$\text{Power in dBm} = 10 \log (\text{power in milliwatt})$$

Conversion to dBW

- dBW is in reference to 1 watt
- First express power in watt
- Then apply the following formula to obtain dBW

$$\text{Power in dBW} = 10 \log (\text{power in watt})$$

Relationship between dBm & dBW

- Note that 1 W = 1000 mW
- This gives us following relationship
  - Note log(axb) = log(a)+log(b)

$$dBm = dBW + 30$$

If you've calculated a power in dBW, you can simply derive the equivalent dBm by adding 30 to dBW, and vice versa.

Examples for converting Watt to dBm/dBW

- Example 1: Express 1 mW power in units of dBm

$$10 \log (1) = 10 \text{x} 0 = 0 \text{ dBm}$$

So, ZERO dBm does not mean there is no power !

Example (2)

- Example 2: Express 50 W in
  - (a) dBW
  - (b) dBm

(a) $P(\text{dBW}) = 10 \log (50) = 17$ dBW

(b) $P(\text{dBm}) = 10 \log (50\text{x}1000)$ dBm

$= 10 \log (50) + 10 \log (1000)$ dBm

$= 17 + 30 = 47$ dBm

Coding Terminology

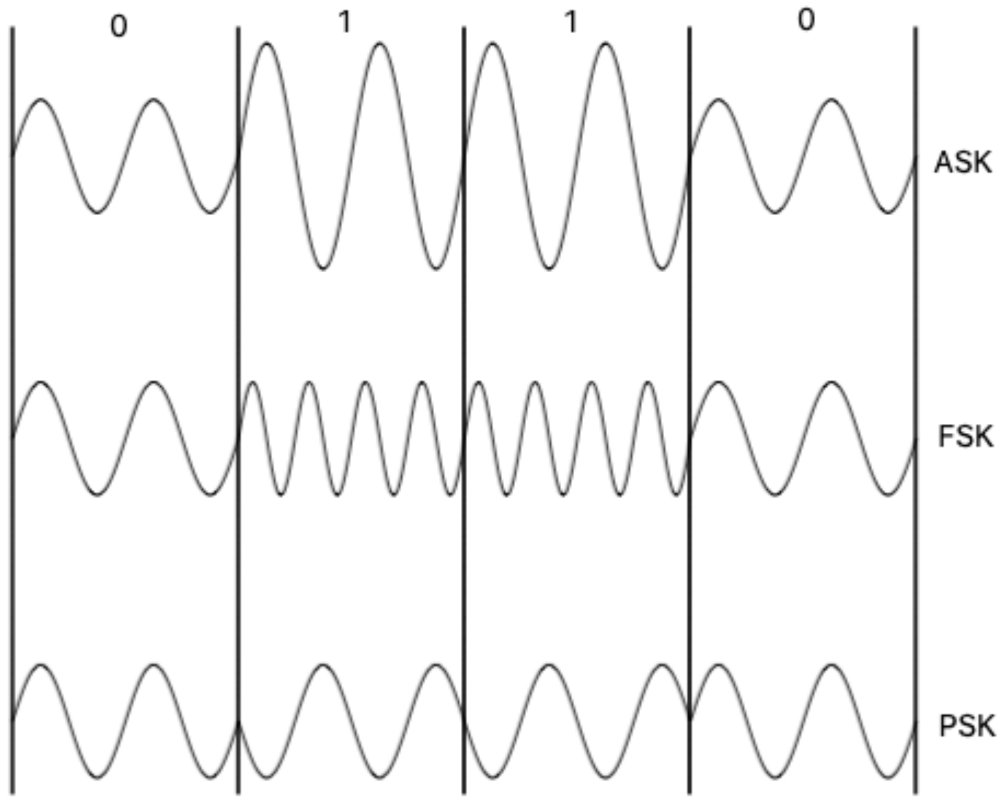
- **Symbol:** **the smallest element of a signal with a given amplitude, frequency, and phase that can be detected**

  1 0

- **Modulation Rate**: = 1/symbol_duration = Baud rate (or symbol rate)
- **Data Rate**: Bits per second (bps)
- A symbol may carry multiple bits
  - A binary signal with only two different symbols would carry 1 bit per symbol (baud rate = data rate)
  - For an M-ary signal, data rate = baud rate x $\log_2(M)$

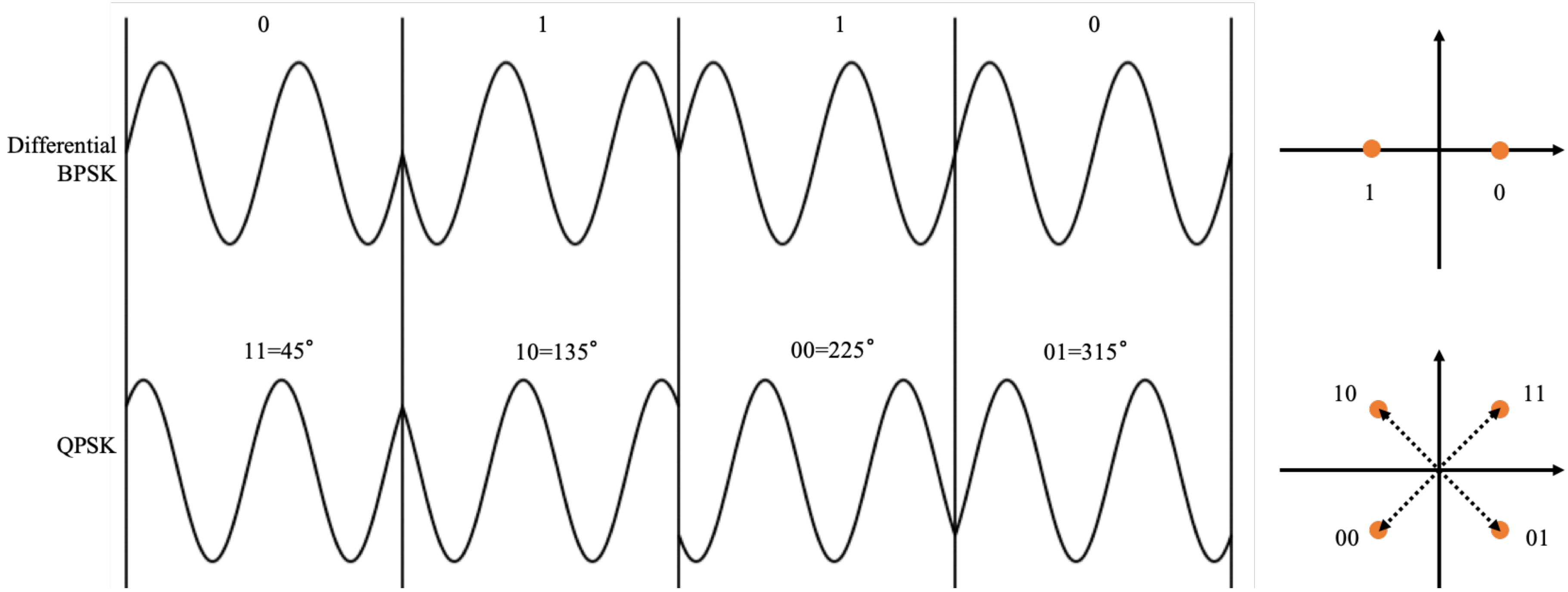
Modulation

- Digital version of modulation is called **keying**
  - **Amplitude Shift Keying (ASK)**
  - **Frequency Shift Keying (FSK)**
  - **Phase Shift Keying (PSK):** Binary PSK (BPSK)



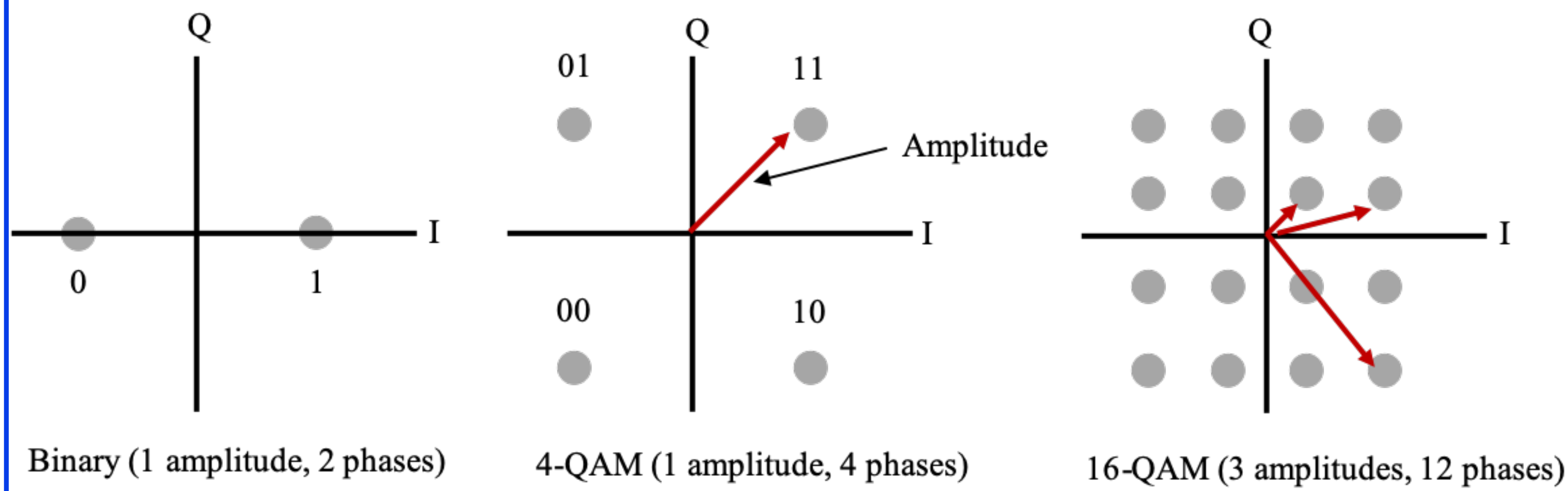
Modulation (Cont)

- **Differential BPSK:** Does not require reference signal
- **Quadrature Phase Shift Keying (QPSK)**



QAM

- Quadrature Amplitude and Phase Modulation
- 4-QAM, 16-QAM, 64-QAM, 256-QAM, …
- Used in DSL and wireless networks
- Constellation diagram (shows combinations of amplitudes and phases)



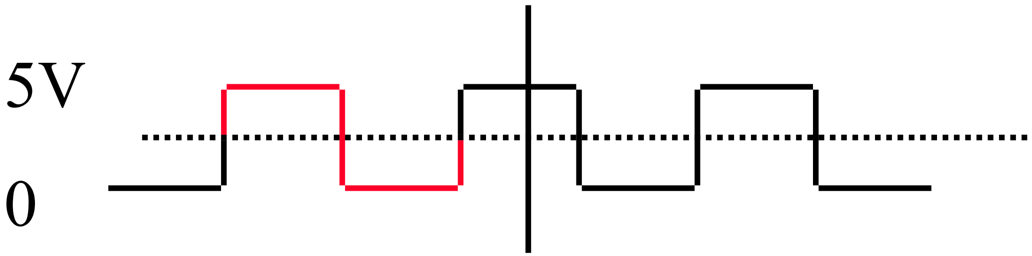
- 4-QAM ⇒ 2 bits/symbol, 16-QAM ⇒ 4 bits/symbol, …

QAM in Action

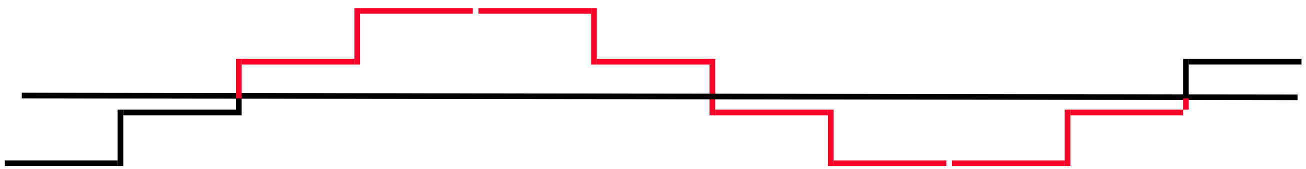
Wireless Technology	QAM Supported
4G	256 QAM
5G	1024 QAM
WiFi 802.11n	16 QAM, 64 QAM
WiFi5 802.11ac	256 QAM
WiFi6 802.11ax	1024 QAM

Channel Capacity

- Capacity = Maximum data rate (bps) for a channel
- **Nyquist Theorem (noiseless channel)**: Bandwidth = B Hz
  Baud rate ≤ 2 B
- Bi-level Encoding: Max. Data rate = $2 \times$ Bandwidth



- Multilevel: Capacity = $2 \times$ Bandwidth $\times \log_2 M$
  M = Number of levels



Example: M=4, Capacity = $4 \times$ Bandwidth

Example

Assume that you have discovered a novel material that has negligible electrical noise. What is the maximum data rate that this material could achieve over a phone wire having a bandwidth of 3100 Hz if data was encoded with 64-QAM?

Solution

We have B= 3100   M = 64

$$\text{Data rate} = 2 \times 3100 \times \log_2 64 = 37{,}200 \text{ bps}$$

Shannon's Theorem (noisy channel)

- Bandwidth = B Hz
  Signal-to-noise ratio = S/N
- Maximum number of bits/sec = B $\log_2$ (1+S/N) [error free communication]
- Example: Phone wire bandwidth = 3100 Hz

$$S/N = 30 \text{ dB}$$

$$10 \text{ Log}_{10} S/N = 30$$

$$\text{Log}_{10} S/N = 3$$

$$S/N = 10^3 = 1000$$

Capacity = 3100 $\log_2$ (1+1000) = 30,894 bps

Hamming Distance

- Hamming Distance between two sequences
  = Number of bits in which they disagree
- Example: 011011

  110001

  Difference 101010 $\Rightarrow$ Distance = 3

Error Correction Example

- 2-bit words transmitted as 5-bit/word

Data	Codeword
00	00000
01	00111
10	11001
11	11110

Received = 00100 ⇒ Not one of the code words ⇒ Error

Distance (00100,00000) = 1 Distance (00100,00111) = 2

Distance (00100,11001) = 4 Distance (00100,11110) = 3

⇒ Most likely 00000 was sent. Corrected data = 00

b. Received = 01010 Distance(…,00000) = 2 = Distance(…,11110)
Error detected but cannot be corrected

c. Three-bit errors will not be detected. Sent 00000, Received 00111.

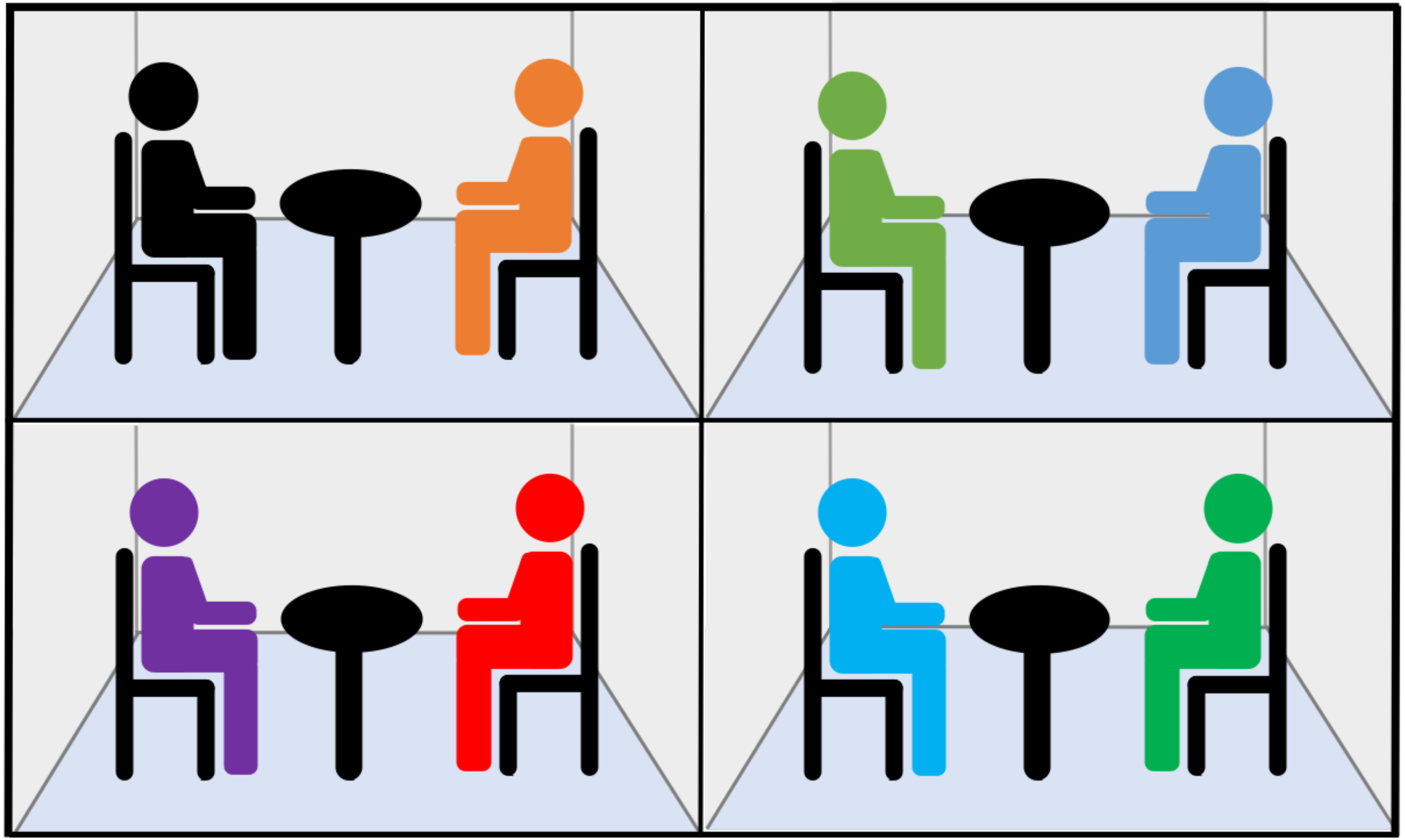
Multiple Access Methods

Time Division Multiple Access
(communicating groups are taking turns)

Code Division Multiple Access
(all communicating groups are talking at the same time)

Multiple Access

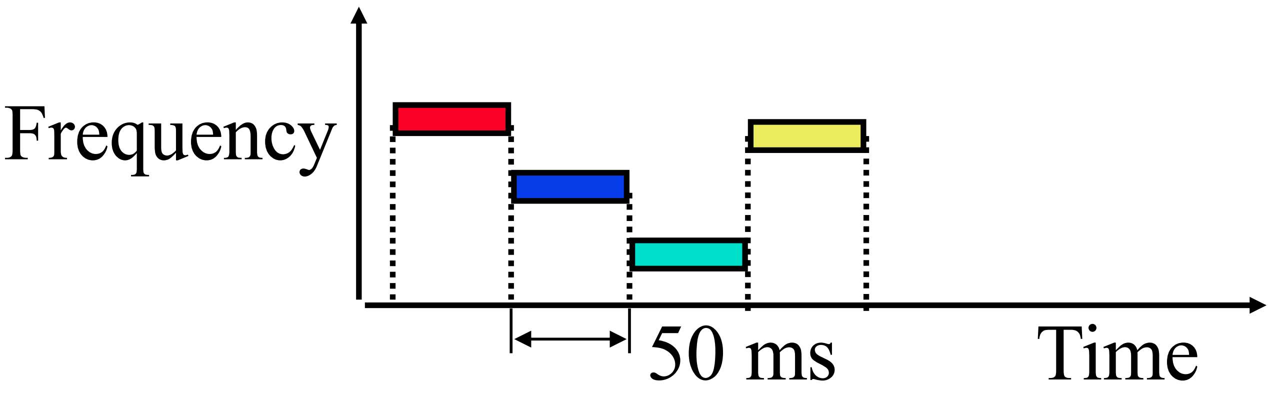
FDMA (frequency division multiple access)



CDMA

- Each communicating group is using a different "code"
- You can understand a conversation only if you know the code used in that conversation
- Much like a *multilingual party*, where people from different languages are all talking at the same time (code = language)
- Two popular coding methods for CDMA
  - Frequency hopping spread spectrum (FHSS)
  - Direct sequence spread spectrum (DSSS)

Frequency Hopping Spread Spectrum

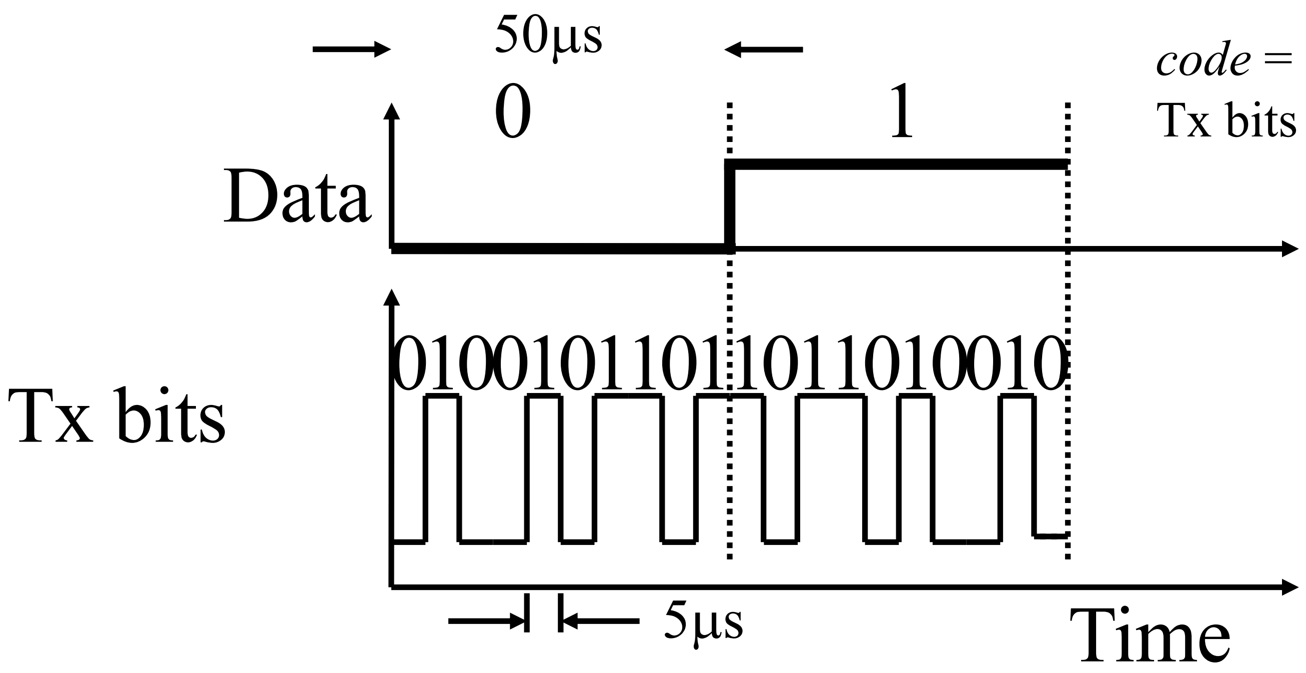


- Transmit over a narrowband, but continuously switch (hop) frequency over a wide spectrum
  - Spreads the transmission (power) over a wide spectrum
    ⇒ Spread Spectrum
- Pseudo-random frequency hopping (both transmitter and receiver use the same pseud-random number sequence = code)
  - Developed initially for military
  - Patented by actress Hedy Lamarr (idea came while playing a piano; tone changes continuously)

FHSS Advantages and Disadvantages

- Advantages
  - Difficult to intercept (appears as random 'blips')
  - Narrowband interference can't jam
- Disadvantages
  - Requires increased bandwidth (ability to randomly hop between 1000 frequencies → 1000 more bandwidth)
  - Both time and frequency synchronization

Direct-Sequence Spread Spectrum

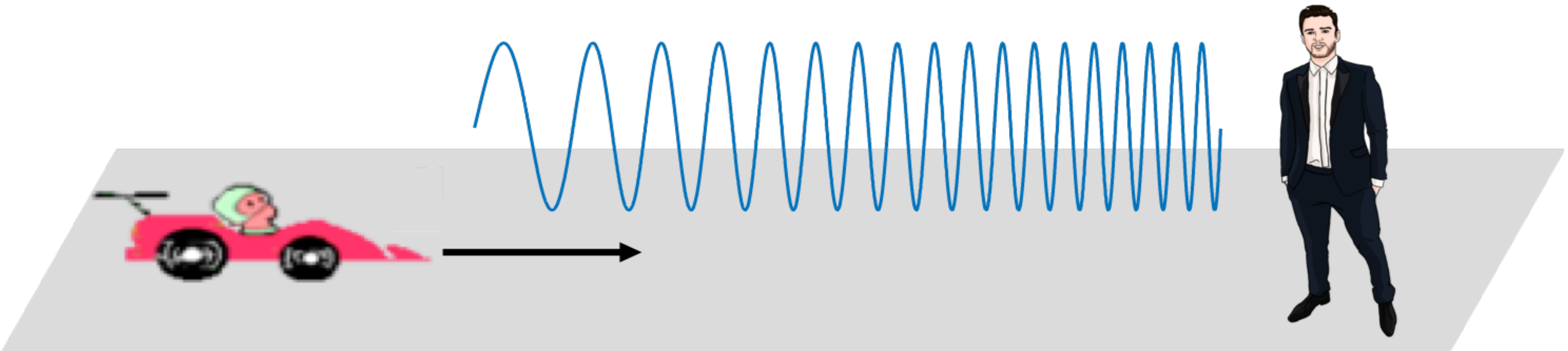


code = 0100101101

Tx bits = *data* **EXOR** *code*

- Many bits are transmitted for each data bit
- Spreading factor = Code bits/data bit, 10-100 commercial (Min 10 by FCC), 10,000 for military
- Signal bandwidth >10 × data bandwidth
- Code sequence synchronization
- Correlation between codes ⇒Interference (Orthogonal to avoid interference)

Doppler Shift



- If the transmitter or receiver or both are mobile the frequency of received signal changes
- Moving towards each other ⇒ Frequency increases
- Moving away from each other ⇒ Frequency decreases

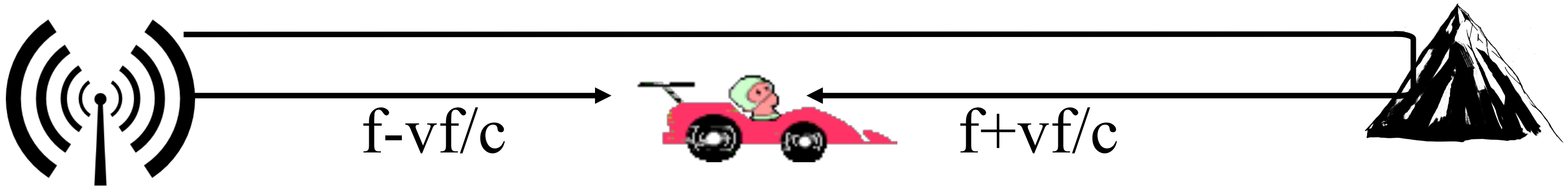
Frequency difference = velocity/Wavelength = v/λ = vf/c

Example: 2.4 GHz ⇒ $\lambda = 3\text{x}10^8/2.4\text{x}10^9 = 0.125$m

v = 120km/hr = 120x1000/3600 = 33.3 m/s

Freq diff (Doppler shift) = 33.3/0.125 = 267 Hz

Doppler Spread and Coherence Time



- Two rays will be received (*original+reflection*)
- **Doppler Spread** = 2vf/c = 2 × Doppler shift
- They will add or cancel-out each other as the receiver moves
- **Coherence time**: Time during which the channel response is constant = 1/Doppler spread = $c/2vf = \lambda/2v$

Example

- What is the *coherence time* for a 2.4 GHz wifi link connecting a car travelling at 72 km/hr?

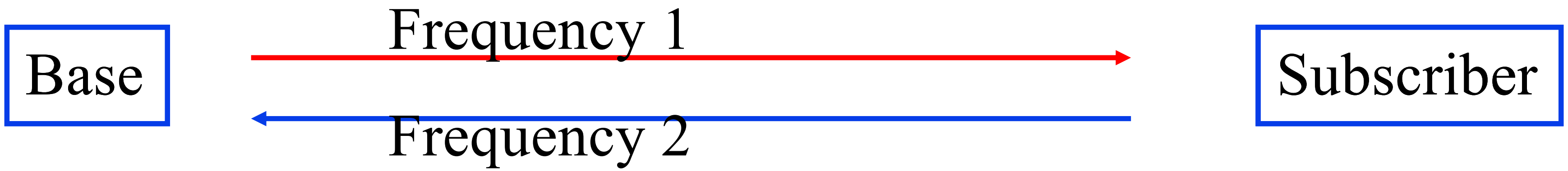
V=(72 x 1000)/3600 = 20 m/s

Doppler spread = $2vf/c$ = $(2\text{x}20\text{x}2.4\text{x}10^9)/(3\text{x}10^8)$ = 320 Hz

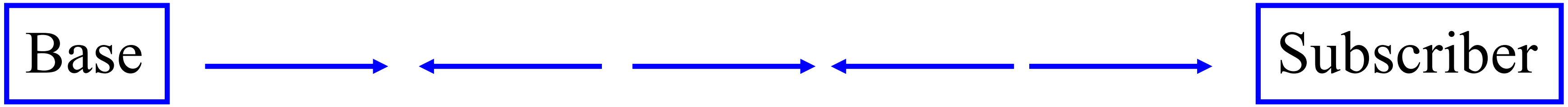
Coherence time = 1/320 = 0.003125 s = 3.125 ms

Duplexing

- Duplex = Bi-Directional Communication
- Frequency division duplexing (FDD) (Full-Duplex)

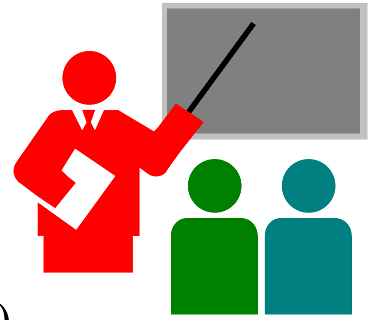


- Time division duplex (TDD): Half-duplex



- Many LTE deployments will use TDD.
  - Allows more flexible sharing of DL/UL data rate
  - Does not require paired spectrum
  - Easy channel estimation ⇒ Simpler transceiver design
  - Con: All neighboring BS should time synchronize

Summary



1. Electric, Radio, Light, X-Rays, are all electromagnetic waves
2. Wavelength and frequency are inversely proportional (wavelength = c/f)
3. Historically, wireless communications mostly used frequencies below 6 GHz, but beyond 6 GHz is actively explored in modern wireless networks.
4. Hertz and bit rate are related by Nyquist and Shannon's Theorems
5. Nyquist's theorem explains capacity for noiseless channels
6. Shannon's capacity takes SNR into consideration
7. By spreading the original signal bandwidth over a much wider band, spread spectrum can provide better immunity against interference and jamming as well allowing multiple parties to communicate over the same frequency at the same time.
8. FHSS and DSSS are two fundamental methods of realizing spread spectrum
9. Doppler effect explains the shift in frequency experienced by mobile objects
10. Doppler spread is twice the Doppler shift
11. Channel coherence time is inversely proportional to doppler spread
12. FDD and TDD are two fundamental methods of resource allocation between the transmitter and the receiver so they both can exchange information with each other