A Minimalist Set of Axiomatic and Supporting Expressions to Comprehensively Build and Discuss Economic Constructs in a Mathematically Robust and Rigorous Manner

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Abstract

LIST OF GOALS
Goal: Formalize a minimal subset of the possible expressions
Goal: Formalize language of discussing groups, societies, communities, and such
Goal: Formalize an immutable way of dealing with individuals

Introduction

All models are wrong; some are useful.

Definition Categories

Throughout this paper there will be three major categories of definitions; Universal, Structural, and Experienced.

Universal

Universal expressions are the axiomatic and supporting principles employed to form a mathematical foundation necessary to develop well-founded economic abstractions. They are fundamental concepts which have been developed over centuries of work by mathematicians, and thus are not deeply investigated within this body of work. Rather they are accepted as rigorous assumptions which have been throughly tested within other fields. Utilization of these commonly accepted definitions acts to scope conversations about expressions to those directly related to economic abstractions, thus allowing for deeper refinement of the concepts we strive to solidify within this work.

Structural

Structural expressions are the basic structures . These definitions are foundational to the Experienced definitions and are where most of the assumptions and by definition developments of the economics construct will be built. More often than not the structural definitions will not be employed in real world utility measurements, but rather they will be used to compare the experienced outcomes compared to what the actual experience could have felt like under perfect information. Their practical importance will become very obvious within the structural definition section of the paper as well as when discussed in the experienced definition section.

Experienced

Experienced expressions are those functions which describe what a person feels in the moment and are the most measurable and useful for determining utility measurements. As we will discuss later in this paper, corpus of information are in actuality an immutable set of facts, however how we recall them is a mutable set of facts with associated interpretations within the moment of recall meaning we can re-categorize events at a later time from bad events to good events, or refine our viewpoint on the issue itself. Experienced definitions deals with the imperfections of imperfect information and gives us a way to reason about the struggles that this can present .

Universal Expressions

Universal definitions often rely on other axiomatic constructs from various fields of mathematics including set theory, graph theory, number theory, algebra, and calculus. Universal definitions are the supporting structures upon which this math is built, often the minutia that goes into maintaining these mathematical structures is best suited for higher level mathematics, and in the scope of economic discussion can quickly become ineffectual to quibble about.

The sets, T and I, both contain every whole number starting from zero and counting up to and including the maximal element which exists for its respective domain. The maximal element is either less than or equal to the maximal element of the natural numbers, $\mathbb{N}^0$. Both I and T are subsets which may be equal to $\mathbb{N}^0$, but no claim about either set being a proper subsets will be made at the universal level. The ambiguity of the maximal element is intentional as the existence of elements in a finite or infinite manner is indeterminable. (**??**)

T represents the set of time starting at an initial reference point zero and progressing forward indeterminable. A specific time t exists whenever a property of the world changes. This is a reflective property which is used to identify a change in time in the real world which we are modeling, or in an imagined environment where we wish to specify changes. It is key to understand that time is universal in the sense that any time change for any individual element of the world results in a time change for every element of that world. A person could therefore have several time changes recorded without anything personally changing for them. This will become important in discussions of time-lag especially in comparative analysis of experienced situations. (**??**).

I represents the set of all possible indexes also starting at 0 and can continue indefinitely as well. The symbol i is used to represent an element from the total set of indexes, I (**??**).

$$T \overset{def}{=} \{t \in A \subseteq \mathbb{N}^0 \mid \forall t \geq 1 \exists a \in A.t - a = 1\} \tag{1}$$

$$I \overset{def}{=} \{i \in A \subseteq \mathbb{N}^0 \mid \forall i \geq 1 \exists a \in A.i - a = 1\} \tag{2}$$

$$T_o \overset{def}{=} \{t \in T \mid t_o\} \tag{3}$$

$$I_o \overset{def}{=} \{i \in I \mid i_o\} \tag{4}$$

$$T_o \subseteq T \tag{5}$$

$$I_o \subset I \tag{6}$$

$$\mathcal{T}_o \overset{def}{=} \{\mathfrak{t}_o \in T \mid \exists \mathfrak{t}_o = |T_o| \wedge \forall \mathfrak{t}_o.1 \leq \mathfrak{t}_o \leq |T_o| \exists \mathfrak{a}_o \in T.\mathfrak{t}_o - \mathfrak{a}_o = 1\} \tag{7}$$

$$\mathcal{I}_o \overset{def}{=} \{\mathfrak{i}_o \in I \mid \exists \mathfrak{i}_o = |I_o| \wedge \forall \mathfrak{i}_o.1 \leq \mathfrak{i}_o \leq |I_o| \exists \mathfrak{a}_o \in I.\mathfrak{i}_o - \mathfrak{a}_o = 1\} \tag{8}$$

$$O \overset{def}{=} \{o\} \tag{9}$$

Ranges, The use of *, Locations and other refinement/scoping variables, the use of A and a and $\mathfrak{a}$, the convention used for the majority of the paper and exceptions to these rules. How reflexive properties apply to things like indexes and time indexes. The world variable.

The choice to start at 0 for these sets was a somewhat arbitrary choice representing the author's preference to not discuss negative time, nor indexes. This decision was not made without some consideration. Any discussion of negative time, negatively indexed items, or the absence of an agreed upon initial reference point seems to only muddle productive conversation within the scope of economic thought. Still it is possible to use the constructs later discussed by utilizing any set that progress indefinitely in at least one direction, and may stretch indefinitely in both directions if so desired.

Structural Definitions

Person

A person, P_i, is the set of all elements $p_{i,t}$ that exist for the specified player, i. Each element represent a snapshots of that individual at any given time t. (10)

A person at time, t, denoted by $P_{i,t}$, is the set of all elements $p_{i,t}$ that exist for the specified player, i, where each snapshot time is less than or equal to the given time, t. (11)

A person between two times, t_0 and t_1, is denoted by P_i

$$P_i \stackrel{def}{=} \{p_{i,t}\} \tag{10}$$

$$P_{i,t} \stackrel{def}{=} \{p \in P_i \mid p_{i,t^* \leq t}\} \tag{11}$$

$$P_{i,r_{t_0,t_1}} \stackrel{def}{=} \{p \in P_i \mid p_{i,t_0 \leq t^* \leq t_1}\} \tag{12}$$

Populations

The population, P, is the set of all people, and a person, p, is any individual element of the population. The population is an immutable construct which contains individual elements that do not change over time. Rather each element stores a unique identification index, i, for each physical person, and several time indexes, t, allowing access to the unique states, functions, and values of the individual at any given time t (**??**).

A population at any selected time t, denoted by P_t, contains all the elements from the population, P, where for any person, i, they have at least one time index, t^*, which is less than or equal to the selected time, t (**??**).

A population from any selected time t_0 to any other selected time t_1 is the symmetric difference of the populations P_{t_0} and P_{t_1}. The symmetric difference is simply an XOR function between the two populations.The resulting population includes all people starting at t_0 and ending at t_1.

$$\mathbb{P} \stackrel{def}{=} \{P_i\} \tag{13}$$

$$\mathbb{P}_t \stackrel{def}{=} \{P \mid P_{i,t^*=t}\} \tag{14}$$

$$\mathbb{P}_{r_{t_0,t_1}} \stackrel{def}{=} \{P \mid P_{i,r^*_{t0,t1}=r_{t0,t1}}\} \tag{15}$$

Groups

$$\mathbb{G} \stackrel{def}{=} \{a \in A \in \mathbb{P} \mid \forall t \exists a_t |A| \geq 2\} \tag{16}$$

Societies

A Society, $\mathbb{S}$, is the power set of the total population, P (17).

$$\mathbb{S} \stackrel{def}{=} \wp(P) \tag{17}$$

A community, C, is one of the possible subset of the society, S (**??**), where g is the power set of g^* (**??**), and where g^* is a set selected from all possible subsets of the population P with cardinality greater than one, but less than the cardinality of the total population (16).

Experienced Definitions

Alphabet

a: Arbitrary element
i: Relative index element
p: Person element
t: universal time element
t: Relative time element
A: Arbitrary Set

Conventions

Capital letter sets will typically contain elements that are all the same type. The element