

Homework 1

ME 760

Due on 1:30 pm, September 18

1. For $\mathbf{a} \neq \mathbf{0}$, what does $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ imply about the vectors \mathbf{b} and \mathbf{c} ? Give a geometric interpretation.
2. In practical work the following formula is often quite useful.

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{(\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})^2}$$

Give a proof.

3. Determine the angles in the triangle formed by the three vertices $P_1 = (2, 2, 2)$, $P_2 = (3, 1, 1)$, and $P_3 = (3, 3, 3)$.
4. What is the equation of the plane containing the triangle of the previous problem?
5. Show that the three vectors $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ are coplanar.
6. For general vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} , show by expanding in component form

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$$

From this result deduce that the vector product is non-associative

7. Line L passes through the two points $P_1 = (-2, -2, -2)$ and $P_2 = (-1, -1, -1)$.
 - (a) What is the equation of this line?
 - (b) What are the coordinates of the two points of intersection of this line with a sphere of radius 2 centered at $(2, 2, 2)$?
8. Two fixed points A and B have position vectors \mathbf{a} and \mathbf{b} .
 - (a) Identify the plane P given by $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{r} = (a^2 - b^2)/2$ where a and b are the magnitudes of \mathbf{a} and \mathbf{b} , i.e. where is it and how is it oriented?
 - (b) Show that the equation

$$(\mathbf{a} - \mathbf{r}) \cdot (\mathbf{b} - \mathbf{r}) = 0$$

describes a sphere S of radius $R = |a - b|/2$. Hint: Cast this equation into the standard form $(\mathbf{r} - \mathbf{c})(\mathbf{r} - \mathbf{c}) = R^2$ where you should find $\mathbf{c} = (\mathbf{a} + \mathbf{b})/2$

- (c) Show that the intersection of P and S is also the intersection of two spheres centered on A and B each of radius $|\mathbf{a} - \mathbf{b}|/\sqrt{2}$. Hint: Add and subtract the equations for P and S and cast the results into the standard form $|\mathbf{r} - \mathbf{d}|^2 = R^2$.