Homework 4

ME 760

Due on *1:30 pm, November 10*

1. Verify that the contour integral $\int_C [2xy^2\,dx + 2x^2y\,dy + dz]$ is independent of the path. Evaluate this integral between the points $(0,0,0)$ and (a,b,c).

2. Given the parametric form of a cone $\mathbf{r}(u,v) = [u\cos v, u\sin v, cu]$
(a) find an explicit representation of the form $z = f(x,y)$,
(b) find and identify the *parameter curves* defined as $u = const$ and $v = const$, and
(c) find a normal vector $\mathbf{N}$ to the conical surface.

3. In class we discussed surface integrals without regard to orientation. By reparameterizing the surface integral could be written as

$$I = \iint_S G(\mathbf{r})dS = \iint_R G(r(u,v))|\mathbf{N}(u,v)|du\,dv$$

(a) Consider the case with $G = z$ and the surface S is the hemisphere $x^2+y^2+z^2 = 9$ with $z \geq 0$. Use polar coordinates and evaluate the right-hand side of the above result.
(b) The surface S is also given explicitly by $z = f(x,y) = \sqrt{9-x^2-y^2}$. For such cases the surface integral can be rewritten as

$$\iint_S G(\mathbf{r})dA = \iint_{R*} G(x,y,f(x,y))\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\,dx\,dy.$$

Evaluate the right-hand side of this result.

4. Evaluate $\iint_S \mathbf{F}\bullet\hat{\mathbf{n}}\,dA$ using the divergence (Gauss') theorem when (a) $\mathbf{F} = [x^3, y^3, z^3]$ and the surface S is the sphere $x^2+y^2+z^2 = 9$, and
(b) when $\mathbf{F} = [9x, y\cosh^2 x, -z\sinh^2 x]$ and S is the ellipsoid $4x^2+y^2+9z^2 = 36$.

5. Consider the vector function $\mathbf{F} = [e^z, e^z\sin y, e^z\cos y]$ and the surface $S: z = y^2,\ 0 \leq x \leq 4,\ 0 \leq y \leq 2$. Stokes's theorem states that

$$\iint_S (\nabla\times\mathbf{F})\bullet\hat{\mathbf{n}}\,dS = \oint_C \mathbf{F}\bullet d\mathbf{r}.$$

(a) Evaluate the left-hand side of this result, and (b) evaluate the right-hand side.