Homework 2

ME 760

Due on *1:30 pm, October 6*

1. For the array

$$\mathbf{C} = \begin{pmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{pmatrix}$$

   calculate (a) $\mathbf{C}^2$, (b) $\mathbf{C}^T\mathbf{C}$, and (c) $\mathbf{C}\mathbf{C}^T$.

2. Solve $\mathbf{Ax} = \mathbf{b}$ for the following set of linear equations

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 8 & 6 \\ -2 & 4 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ 40 \end{pmatrix}$$

   (a) by Gauss elimination
   (b) by using Cramer's rule
   (c) by finding the inverse $\mathbf{x} = \mathbf{A^{-1}b}$

3. For each of the sets below determine if the set constitutes a vector space or not. Give your reason for your decision. If you conclude the set is a vector space, determine its dimension and provide a basis.

   (a) all vectors in R^3 satisfying $v_1 - 3v_2 + 2v_3 = 0$ where v_i are the components of a vector
   (b) all functions $y(x) = a\cos x + b\sin x$ with arbitrary real constants a and b
   (c) all skew-symmetric 2×2 matrices
   (d) all 2×2 matrices with $a_{11} + a_{22} = 0$
   (e) all $m \times m$ matrices with positive elements

4. Find the spectra and eigenvectors for the two matrices below. Show your work.

$$\mathbf{A} = \begin{pmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{pmatrix}$$

5. Find the matrix $\mathbf{A}$ for each of the indicated linear transformation $\mathbf{y} = \mathbf{Ax}$.
   Find its eigenvalues and eigenvectors.

   (a) Reflection about the x-axis in R^2. Here $\mathbf{x} = [x\ y]$.
   (b) Orthogonal projection of R^3 onto the plane $x = y$. Here $\mathbf{x} = [x\ y\ z]$.

6. Prove that trace of a square real or complex matrix $\mathbf{A}$ equals the sum of its eigenvalues. This fact is often a useful check on the accuracy of eigenvalue calculations. Demonstrate with an example of your choosing.

7. Prove that the eigenvectors of a real symmetric matrix corresponding to different eigenvalues are orthogonal.

8. Do there exist real symmetric 3×3 matrices that are orthogonal (except for the unit matrix $\mathbf{I}$)?

9. Prove that Hermitian, skew-Hermitian and unitary matrices are all normal matrices.

10. Find the similarity transformation that diagonalizes the following matrix. Show details of your work.

$$\mathbf{A} = \begin{pmatrix} 16 & 0 & 0 \\ 48 & -8 & 0 \\ 84 & -24 & 4 \end{pmatrix}$$