Logarithm Notes

By Mike Rapp

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Log Arrangement

https://www.mathsisfun.com/algebra/logarithms.html
http://tutorial.math.lamar.edu/Classes/Alg/LogFunctions.aspx

This usually read as "log base *b* of *x*":

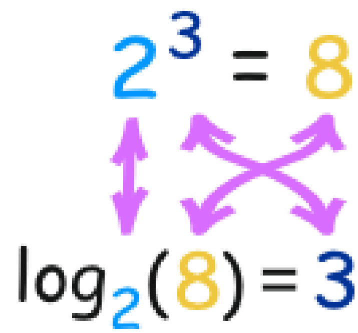
$$\log_b(x)$$

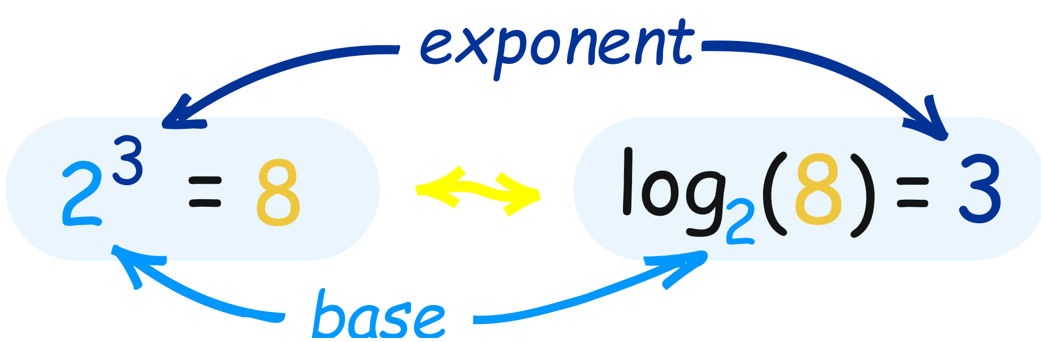
Provided that:

$$b > 0 \; and \; b \neq 1 \; and \; x > 0$$

$$\text{Log}_2(X) = 8 \;\; == \;\; 2^3 = X$$

$$X == 2^3 == 8$$





Logarithm Facts

https://www.vitutor.com/alg/log/antilogarithm.html

https://www.mathsisfun.com/algebra/logarithms.html

(1) A logarithm, positive or negative, corresponds to every positive number.

$$\log_b(x) = y <==> \ b^y = x$$

Provided that:

$$b > 0 \ and \ b \ \neq 1 \ and \ x > 0$$

(2) Cologarithm

The cologarithm of a number is the logarithm of its inverse, that is to say, the cologarithm of a number is the opposite of its logarithm.

$$colog\ N = \log \frac{1}{N} = -\log N$$

$$colog\ 200 = -\log 200 = -2.3010$$

(3) Negative Logarithms

A negative logarithm means how many times **to divide** by the number.

Example-1:

What is $\log_8(0.125)$ ... ?

Well, $1 \div 8 = 0.125$,

So $\log_8(0.125) = -1$

Example-2:

What is $\log_5(0.008)$ ... ?

$\mathbf{1 \div 5 \div 5 \div 5 = 5^{-3}}$,

So $\log_5(0.008) = -3$

(4) Logarithm - Origins

"Logarithm" is a word made up by Scottish mathematician John Napier (1550-1617), from the Greek word *logos* meaning "proportion, ratio or word" and *arithmos* meaning "number", ... which together makes "ratio-number"

Common Logarithm

https://www.mathsisfun.com/algebra/logarithms.html

Sometimes a logarithm is written **without** a base, like this:

$$\log(1000)$$

This ***usually*** means that the base is really ***10***.

Engineers love to use it.

On a calculator it is the "***log***" button.

It is how many times we need to use ***10*** in a multiplication, to get our desired number.

Example:

$$\log(1000) = \log_{10}(1000) = 3$$

Log To Base e

https://en.wikipedia.org/wiki/Natural_logarithm

https://www.mathsisfun.com/algebra/logarithms.html

$$\log_e = ?$$

is equivalent to:

$$\ln = ?$$

For Natural Logarithms *log_e* and *ln* expressions denote natural logarithms.

Natural Logarithms: Base "e"

Another base that is often used is e (Euler's Number) which is about 2.71828.

This is called a "natural logarithm". Mathematicians use this one a lot.

On a calculator it is the "ln" button.

Example:

$$\mathbf{\ln(7.389) = \log_e(7.389) \approx 2}$$

Because $\mathbf{2.71828^2 \approx 7.389}$

Logarithm Calculator

http://rapidtables.com/calc/math/Log_Calculator.htm

Calculate logarithm of a number to any base:

Description:

* Use e for scientific notation. E.g: 5e3, 4e-8, 1.45e12

When:

$$b^y = x$$

Then the base *b* logarithm of a number *x*:

$$\log_b(x) = y$$

Or

$$\log_b x$$

Antilogarithm

https://www.vitutor.com/alg/log/antilogarithm.html

https://www.rapidtables.com/math/algebra/Logarithm.html

The inverse logarithm (or anti logarithm) is calculated by raising the base ***b*** to the logarithm ***y***:

If:

$$Y = \log_b(X)$$

Then the antilog is:

$$X = \log^{-1}(y) == b^y$$

To find the antilog of a number in a given base, raise the base to the number result.

$$\log_2(8) = 3$$

$$Anti\ \log_2 3 = 2^3 = 8$$

Antilog = Inverse Log

Define an antilogarithm in terms of a logarithm. The antilogarithm is the inverse function of a logarithm, so $\log_b(x) = y$ means that $anti\log_b(y) = x$. You write this with exponential notation such that antilog $_b$(y) = x implies $b^y = x$.

Examine Antilog Notation

Examine a specific example of antilog notation. Because $\log_{10}(100) = 2$, $\text{antilog}_{10}(2) = 100$ or $10^2 = 100$.

Calculate an Antilog

Solve a specific antilog problem. Given log (2) 32 = 5, what is antilog (2) 5? $2^5 = 32$, so antilog (2) 5 = 32.

Anti-logarithm calculator

To calculate $\log^{-1}(y)$ on the calculator, enter the base ***b*** (10 is the default value, enter *e* for e constant), enter the logarithm value *y* and press the **=** or calculate button:

Result:

When

$$y = \log_b(x)$$

The antilogarithm (or inverse logarithm) is calculated by raising the base ***b*** to the logarithm ***y***:

$$x = \log_b^{-1}(y) = b^y$$

If the logarithm is known, a calculator can be used to find the antilog by pressing the ***10^x*** key or ***e^x***. This is usually the second function of the **log** key.

$$\log_{10}(x) = 2.4572$$

$$x = 10^{2.4572} = 286.55$$

Logarithm Rules

https://www.youtube.com/watch?v=AAW7WRFBKdw&t=6s

https://en.wikipedia.org/wiki/Logarithm

Logarithm Product Rule

$$\log_b(j * k) = \log_b(j) + \log_b(k)$$

Logarithm Quotient Rule

$$\log_b \frac{j}{k} = \log_b(j) - \log_b(k)$$

Logarithm Power Rule

$$\log_b(j^k) = k * \log_b(j)$$

Logarithm Base Switch Rule

$$\log_b(k) = \frac{1}{\log_k(b)}$$

Logarithm Base Change Rule

$$\log_b(x) = \frac{\log_j(x)}{\log_j(b)}$$

Logarithm Root Rule

$$\log_b \sqrt[p]{x} = \frac{\log_b X}{p}$$

Logarithm Equality Rule

$$\log_b(m) = \log_b(n) == (m == n)$$

Inverse Properties

http://www.mathwords.com/l/logarithm_rules.htm

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

Watch Out

http://www.mathwords.com/l/logarithm_rules.htm

$$\log_b(x + y) \neq \log_b(x) + \log_b(y)$$

$$\log_b(x - y) \neq \log_b(x) - \log_b(y)$$

Logarithm Rule Examples

Product

$$\log_b(x * y) = \log_b x + \log_b y$$

$$\log_3(243) = \log_3(9 * 27) = \log_3(9) + \log_3 27 = 2 + 3 = 5$$

Quotient

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_2 16 = \log_2 \frac{64}{4} = \log_2 64 - \log_2 4 = 6 - 2 = 4$$

Power

$$\log_b(x^p) = p\log_b x$$

$$\log_2(64) = \log_2(2^6) = 6\log_2 2 = 6$$

Root

$$\log_b \sqrt[p]{x} = \frac{\log_b x}{p}$$

$$\log_{10}\sqrt{1000} = \frac{\log_{10}(1000)}{2} = \frac{3}{2} = 1.5$$

Change of Base

The logarithm $\mathbf{log}_{\boldsymbol{b}}(\boldsymbol{x})$ can be computed from the logarithms of $\boldsymbol{x}$ and $\boldsymbol{b}$ with respect to an arbitrary base $\boldsymbol{k}$ using the following formula:

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$$

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https://en.wikipedia.org/wiki/Logarithm

Derivation of the conversion factor between logarithms of arbitrary base:

Typical scientific calculators calculate the logarithms to bases 10 and $\boldsymbol{e}$. Logarithms with respect to any base $\boldsymbol{b}$ can be determined using either of these two logarithms by the previous formula:

$$\log_b(x) = \frac{\log_{10}(x)}{\log_{10}(b)} = \frac{\log_e(x)}{\log_e(b)}$$

Given a number $\boldsymbol{x}$ and its logarithm $\boldsymbol{\log_b(x)}$ to an unknown base $\boldsymbol{b}$, the base is given by:

$$b = x^{\frac{1}{\log_b(x)}}$$

which can be seen from taking the defining equation

$$x = b^{\log_b(x)}$$

to the power of

$$\frac{1}{\log_b(x)}$$

Properties of Logarithms

https://www.youtube.com/watch?v=AAW7WRFBKdw&t=6s

$\log_b(X) = y$

$b^y = X$

$\log_{10}(1000) = y$

$10^y = 1000$

$\text{Log}_{10}(1000) = y = 3$

$10^3 = 1000$

$\log_b(X) = y$

$b = \sqrt[y]{X}$

$10 = \sqrt[3]{1000}$

$$b = x^{\frac{1}{\log_b(x)}}$$

$$10 = 1000^{\frac{1}{3}}$$

Examples

https://en.wikipedia.org/wiki/Logarithm#Power_series

https://www.codeproject.com/tips/311714/natural-logarithms-and-exponent

https://www.youtube.com/watch?v=AAW7WRFBKdw&t=6s

Example - 1

$$\log_5\left(\frac{1}{25}\right) = X = -2$$

$$5^X = \frac{1}{25} = \frac{1}{5^2} = 5^{-2}$$

Example - 2

$$log_{10}(\text{X}) = \text{Y}$$

$$log_{10}(X) \ = 3$$

X = 0.477121254719662

$10^{0.477121254719662}$ = 3

$$\sqrt[.4777121254719662]{3} \ \ = 10$$

$$3^{1/0.4777121254719662} = 10$$

$$\sqrt[x]{3} \ = 10$$

$$3^{\frac{1}{x}} \ = 10$$

Exponent Relationships

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$$

$$a^{-m} = \left(\frac{1}{a}\right)^m$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^m = a^n => m == n$$