AM121/ES121: EO # 1

Mike Bao, Jonathan Huang, Luis Viceira

October 9, 2018

Task 1

1.1

Our goal is to find a balance between maximizing the radiation delivery to tumor areas and minimizing the delivery to critical areas. We address this goal in the constraints. We assume that an increase in beam intensity requires a cost c, which we normalize to 1. Furthermore, we assume that the patient, who pays for the treatment, is budget-constrained and would like to minimize the monetary costs of the treatment. As such, to represent this problem, we choose to minimize the sum of beam intensities used, which in turn minimizes the cost of the treatment.

We formulate our model to provide the required dosages to the tumorous cells and stay within the allowed dosages to the critical cells while minimizing the cost of the radiation treatment to both the patient and the oncologists.

Sets:	R	Rows of Cells
	C	Columns of Cells
	B	Beams
Parameters:	r	Required Dosage per Tumorous Cell
	a	Allowed Dosage per Critical Cell
	$X_{b,i,j} \quad b \in B, i \in R, j \in C$	Delivered Dosage per cell by Beam b
	$t_{ij} \quad i \in R, j \in C$	Binary parameters for tumor cells
		$t_{ij} = 1$ if cell (i, j) is tumorous
	$c_{ij} \quad i \in R, j \in C$	Binary parameters for critical cells
		$c_{ij} = 1$ if cell (i, j) is critical
Variables:	$I_b \quad b \in B$	Beam Intensities

$$\text{Minimize: } \sum_{b \in B} I_b$$

Subject to:

$$\begin{aligned}
r \cdot t_{ij} &\leq \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C && \text{minimum dose satisfied} \\
a &\geq c_{ij} \cdot \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C && \text{maximum dose allowed} \\
I_b &\geq 0 \quad \forall b \in B && \text{nonnegative beam intensities}
\end{aligned}$$

The first set of constraints represents the condition that all tumorous cells must receive the minimum dosage. If $t_{ij} = 0$, then cell (i, j) is not tumorous and so the condition is always met. The second set of constraints represents the condition that all critical cells must not receive more than the maximum dosage when $c_{ij} = 1$. If $c_{ij} = 0$, then the constraints become trivially true. The last constraint is a nonnegativity condition on the beam intensities.

We choose to minimize the total intensity of beams used not only because it allows the patient and clinic to minimize the cost of operating the radiation treatment but also because this makes the optimization problem bounded.

1.2

To allow for a slight variation on the limits to ensure feasibility, we introduce slack variables. It's unlikely that the oncologists can accurately predict limits that ensure feasibility, and so the slack variables allow us to violate the limits slightly to obtain a feasible solution. Our new LP aims to minimize the sum of the slack variables, as this represents an attempt to find a solution where the limits are only slightly violated. This will ensure that the solution we find is as close as possible to the best feasible solution.

Sets:		R	Rows of Cells
		C	Columns of Cells
		B	Beams
Parameters:		r	Required Dosage per Tumorous Cell
		a	Allowed Dosage per Critical Cell
	$X_{b,i,j}$	$b \in B, i \in R, j \in C$	Delivered Dosage per cell by Beam b
	t_{ij}	$i \in R, j \in C$	Binary parameters for tumor cells
			$t_{ij} = 1$ if cell (i, j) is tumorous
	c_{ij}	$i \in R, j \in C$	Binary parameters for critical cells
			$c_{ij} = 1$ if cell (i, j) is critical
Variables:	I_b	$b \in B$	Beam Intensities
	m_{ij}	$i \in R, j \in C$	Decrement in required dosage
	n_{ij}	$i \in R, j \in C$	Increment in allowed dosage

$$\text{Minimize: } \sum_{i \in R} \sum_{j \in C} (m_{ij} + n_{ij})$$

Subject to:

$$(r - m_{ij}) \cdot t_{ij} \leq \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{minimum dose satisfied}$$

$$(a + n_{ij}) \geq c_{ij} \cdot \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{maximum dose allowed}$$

$$I_b \geq 0 \quad \forall b \in B \quad \text{nonnegative beam intensities}$$

$$m_{ij} \geq 0, n_{ij} \geq 0 \quad \forall i \in R, \forall j \in C \quad \text{nonnegative slack variables}$$

The first constraint represents a slight variation on the minimum dose required for tumor cells - we can see that we are relaxing the constraint by subtracting m_{ij} from r. The second constraint relaxes the constraint for the maximum dose allowed for critical cells - we are relaxing the constraint by adding n_{ij} to a. Finally, we introduce nonnegativity constraints for m_{ij} and n_{ij}.

Once we solve this new LP and get the optimal values of m_{ij} and n_{ij}, we can translate these values back into the original LP to find a feasible solution. The values m_{ij} and n_{ij} switch from being variables to parameters as these values are now fixed. The only change we need to make to the model in subtask 1.1 is in the constraints of the model.

The new constraints need to reflect the relaxation of the radiation limits.

Subject to:

$$
\begin{aligned}
(r - m_{ij}) \cdot t_{ij} &\le \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C && \text{minimum dose satisfied} \\
(a + n_{ij}) &\ge c_{ij} \cdot \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C && \text{maximum dose allowed} \\
I_b &\ge 0 \quad \forall b \in B && \text{nonnegative beam intensities}
\end{aligned}
$$

1.3

Since imaging and radiation delivery is not perfect and there are often inaccuracies in the entire process, we need to penalize radiation delivery to parts of the non-critical area that border a critical area. In addition, because of the inaccuracies associated with delivery, radiation delivered to a specific critical cell can also affect neighboring critical cells. For example, the current radiation technology may not be able to concentrate all of the intensity on the intended cells; if a cell is designed to receive 5 units of intensity, the actual cell may only receive 4.5 units of intensity, and the remaining 0.5 units of intensity may be evenly spread over the neighboring cells.

Thus, we decide to penalize radiation delivery to all neighboring cells of a critical cell that receives radiation, regardless of if the neighboring cell is a critical cell or a non-critical cell. Based on the above logic and reasoning, we come up with the following formulation.

We use the indices i' and j' to iterate over the surrounding cells of the cell currently being targeted in the objective function. The objective function penalizes beams that deliver radiation close to areas of critical cells.

Sets:	R		Rows of Cells
	C		Columns of Cells
	B		Beams
Parameters:	r		Required Dosage per Tumorous Cell
	a		Allowed Dosage per Critical Cell
	$X_{b,i,j}$	$b \in B, i \in R, j \in C$	Delivered Dosage per cell by Beam b
	t_{ij}	$i \in R, j \in C$	Binary parameters for tumor cells $t_{ij} = 1$ if cell (i, j) is tumorous
	c_{ij}	$i \in R, j \in C$	Binary parameters for critical cells $c_{ij} = 1$ if cell (i, j) is critical
	m_{ij}	$i \in R, j \in C$	Decrement in required dosage
	n_{ij}	$i \in R, j \in C$	Increment in allowed dosage
Variables:	I_b	$b \in B$	Beam Intensities

$$\text{Minimize: } \sum_{b\in B}\sum_{i\in R}\sum_{j\in C}\sum_{i'=i-1}^{i+1}\sum_{j'=j-1}^{j+1} c_{i,j} I_b X_{b,i',j'}$$

Subject to:

$$(r - m_{ij}) \cdot t_{ij} \leq \sum_{b\in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{minimum dose satisfied}$$

$$(a + n_{ij}) \geq c_{ij} \cdot \sum_{b\in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{maximum dose allowed}$$

$$I_b \geq 0 \quad \forall b \in B \qquad \text{nonnegative beam intensities}$$

1.4

We consider the following three modifications and enhancements to our model.

Minimize the maximum intensity of any beam used: We believe this modification is an important consideration since different clinics might have machines and technology of different power (i.e. machines likely have an upper bound on the intensity level that any one beam can have). Some clinics may have tiers of pricing for the intensities of the beams used. For example, any beam with intensity between 0-10 may be charged a certain price, while any beam that uses an intensity above 10 may be charged a much steeper price. Thus if there are two optimal solutions, it may be better to pick the solution that uses beam intensities that are relatively close to each other in magnitude, rather than the one that uses an extremely high beam intensity and very low intensities for the other beams.

In addition, a high intensity beam may have unexpected consequences on the patients' health. It is likely that the high intensity of the beam may have larger-than-expected consequences on neighboring cells of its target area.

This formulation is able to be implemented without additional data. However, if we wanted to incorporate this model into real-life situations, we would need data regarding the technological capabilities of clinics in the local area of the patient so that we can assign patients to clinics that can best help them recover. This would make the process of receiving radiation more efficient for both patients and make the process of treating patients more efficient for clinics.

Our sets, parameters, and constraints would be the same as in subtask 1.3. We would add Z as a variable, and change our objective function to be:

$$\text{Minimize: } Z$$

with the additional constraints that

$$I_b \leq Z \quad \forall b \in B$$

Minimize the total radiation delivered to the critical areas: Our ultimate goal is to cure patients who have tumors by aiming radiation at tumorous cells, but we also have to consider potential negative effects of too much radiation on critical areas. Thus, dropping the budget constraint that we introduced in previous subtasks, we change our focus to only minimizing the total radiation delivered to critical areas. Since different patients have varying responses to radiation, we want to provide the necessary treatment without severely impacting the health of other parts of the patients' bodies.

Although no additional data is needed for the current formulation, if we wanted to provide individualized treatments, we could request data regarding the degree of harm that certain levels of radiation would inflict on a patient, given the patient's medical history and demographics. We would also want information on previous radiation history. For example if a patient just had a MRI, we would want to try to avoid delivering radiation to those areas as there may be some lingering effects.

The implementation of this modification would be fairly straightforward. We would again solve for m_{ij} and n_{ij} as done in subtask 1.2 and insert these values as parameters into our model. Our constraints would be the same as those in subtask 1.3. We could add in an extra parameter α_{ij} representing the degree of severity of each critical cell. In other words, if α_{ij} is relatively high (representing a cell that should be prioritized in not receiving radiation), then the objective function will heavily penalize any radiation that is delivered to cell (i,j). This could be more realistic in the sense that some critical cells are more "critical" than others and we want to incorporate this distinction into our model.

The objective function would be

$$\text{Minimize:} \quad \sum_{i\in R}\sum_{j\in C}\sum_{b\in B}\alpha_{ij}c_{ij}I_b X_{b,i,j}$$

Time Series Growth of Tumor Cells: We could introduce an element of time into our model, as we can reasonably assume that individual tumor cells behave differently over time. For example, some tumors may worsen over time if they haven't received radiation in the past 30 days, and different tumor cells may have different rates of deterioration. Thus, certain tumorous cells may require multiple radiation treatments over a time period. In addition, the effects of radiation treatment on tumorous cells are not immediate. Tumor cells likely take time to recover after being subject to radiation, so this would be another factor of complexity that this model would need to take into account. Both the time-dependence of recovery and deterioration would mean our model needs to be optimized over certain periods of time. If we also consider recovery and deterioration of critical cells, this introduction of time would complicate our model significantly.

We would need data and statistics from the clinics that provided information on how tumorous cells and critical cells behave over time. The data would tell us the rate of recovery of tumor cells as a function of the number of days that have passed since the last treatment it received. It would also tell us how tumor cells would deteriorate as a function of time. This would help us determine how to schedule the different radiation treatments over a period of time in addition to the specifics of each individual radiation treatment (which can be modeled by formulations given earlier).

In terms of implementation, we can model both the recovery and deterioration of cells with one set of parameters. For example, the parameters could be negative if the cells are recovering and positive if the cells are deteriorating. Our objective function could then be minimizing the amount of radiation that hits the critical areas over a period of time. We could introduce a new set of parameters that determine the influence or effectiveness of each treatment on the overall state of the patient's cells. Our constraints would then change to be a summation over a period of time.

We note that although this model introduces realistic concerns associated with radiation treatment, the added modifications seriously complicate the model.

1.5

We choose the first two modifications from subtask 1.4 to formulate below.
Model for minimizing the maximum beam intensity used

Sets:	R	Rows of Cells
	C	Columns of Cells
	B	Beams
Parameters:	r	Required Dosage per Tumorous Cell
	a	Allowed Dosage per Critical Cell
	$X_{b,i,j} \quad b \in B, i \in R, j \in C$	Delivered Dosage per cell by Beam b
	$t_{ij} \quad i \in R, j \in C$	Binary parameters for tumor cells
		$t_{ij} = 1$ if cell (i, j) is tumorous
	$c_{ij} \quad i \in R, j \in C$	Binary parameters for critical cells
		$c_{ij} = 1$ if cell (i, j) is critical
	$m_{ij} \quad i \in R, j \in C$	Decrement in required dosage
	$n_{ij} \quad i \in R, j \in C$	Increment in allowed dosage
Variables:	$I_b \quad b \in B$	Beam Intensities
	M	Maximum Beam Intensity

Minimize: M

Subject to:

$$(r - m_{ij}) \cdot t_{ij} \leq \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{minimum dose satisfied}$$

$$(a + n_{ij}) \geq c_{ij} \cdot \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{maximum dose allowed}$$

$$I_b \geq 0 \quad \forall b \in B \qquad \text{nonnegative beam intensities}$$

$$M \geq I_b \quad \forall b \in B \qquad \text{M is maximum intensity}$$

Model for minimizing the total radiation delivered to critical areas:
For this model, we decide to set $\alpha_{ij} = 1$ for all cells. In the case that the oncologists do not have significant knowledge of which cells are more "critical" than others, this simplification of setting $\alpha_{ij} = 1$ could be used.

Sets:	R	Rows of Cells
	C	Columns of Cells
	B	Beams
Parameters:	r	Required Dosage per Tumorous Cell
	a	Allowed Dosage per Critical Cell
	$X_{b,i,j} \quad b \in B, i \in R, j \in C$	Delivered Dosage per cell by Beam b
	$t_{ij} \quad i \in R, j \in C$	Binary parameters for tumor cells $t_{ij} = 1$ if cell (i, j) is tumorous
	$c_{ij} \quad i \in R, j \in C$	Binary parameters for critical cells $c_{ij} = 1$ if cell (i, j) is critical
	$m_{ij} \quad i \in R, j \in C$	Decrement in required dosage
	$n_{ij} \quad i \in R, j \in C$	Increment in allowed dosage
Variables:	$I_b \quad b \in B$	Beam Intensities

$$\text{Minimize: } \sum_{b \in B} \sum_{i \in R} \sum_{j \in C} c_{ij} X_{bij} I_b$$

Subject to:

$$(r - m_{ij}) \cdot t_{ij} \leq \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{minimum dose satisfied}$$

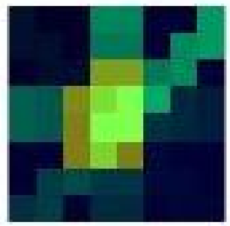
$$(a + n_{ij}) \geq c_{ij} \cdot \sum_{b \in B} I_b X_{b,i,j} \quad \forall i \in R, \forall j \in C \quad \text{maximum dose allowed}$$

$$I_b \geq 0 \quad \forall b \in B \quad \text{nonnegative beam intensities}$$

Task 2

2.1

After solving the given optimization problem for the optimal beam intensities given the beams we are allowed to use, we obtain the following visualization of the beam intensities and tumor cells[1].



As we can see, the beams that pass through only the tumorous region are prioritized with higher intensities than any beams that pass through the critical region (upper left corner) and any beams that only pass through non-critical cells. In this case, we were luckily able to avoid touching any of the critical cells while still being able to reach the threshold amount of 10 doses for each of the tumorous cells. The center of the tumor cell block is most radiated by the treatment. However, this was not intentional but rather because most of the beams we used passed through that central location.

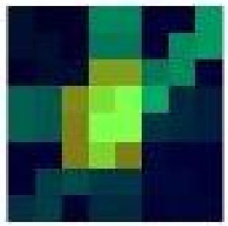
We chose our objective function to minimize the total intensity of beams used – in part to save cost to the patient for the treatment as well to minimize the amount of radiation that the patient is to be subjected to. This can be seen in the visualization as well. The cells at the edge of the tumor cell block are just radiated enough to reach the threshold with no excess radiation being applied to the cells. The optimal model achieves this solution by varying the intensities heavily to achieve a configuration in which the beam intensities used are minimized while all the cells are still

---

[1]See Appendix I for code

2.2

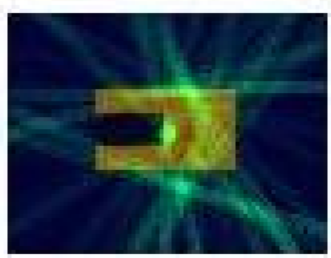
First, we applied the LP we formulated in order to find more feasible constraints on the smaller model. However, since the smaller model was already feasible, the feasibility LP we formulated did not transform the constraints whatsoever. Plugging in the results of the feasibility LP back into the objective function of minimizing total dosage given, we obtained the following visualization of the beam intensities and tumor cells in the smaller model[2]:



Note that these are essentially the exact same beams that we saw from the previous subtask. This is because it was unnecessary to change the constraints whatsoever in order to make the problem feasible. Thus, our solution has not changed by introducing our new feasibility LP. However, our feasibility LP will make a significant impact on the feasibility of the larger example.

Applying the same LP from the previous subtask to the larger actual cell model, we ran into a feasibility issue. The constraints on the minimum amount of radiation each cancer cell needed to receive and the maximum amount of radiation a critical cell could receive were too tight. Thus, we first used another LP to find how much we needed to loosen the constraints on each of the tumor and critical cells.

Plugging these new constraints into our revised LP, we were able to satisfy our new constraints while still minimizing the amount of total radiation intensity used on the patient. This yielded the following visualization for the beam intensities and tumor cells:



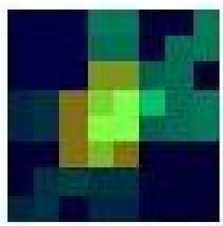
As shown in the figure, the tumorous area is generally very irradiated. However, there are sections of the tumorous region that cannot be irradiated fully without compromising the safety of the critical region. Thus, this therapy does not necessarily fulfill the original constraints given by the oncologists but does the best possible job of killing cancer cells given the available beams.

[2]See Appendix I for code

Also note that while the critical region is relatively void of radiation beams (area between the sideways "U"), there are areas of the critical region that are slightly irradiated. Again, this is because we had to loosen the constraints on that were given by the oncologist in order to make the LP feasible. Thus, we must make very slight sacrifices to the critical area in order to maximize the amount of cancer cells that we can kill using the beams available to the oncologist.

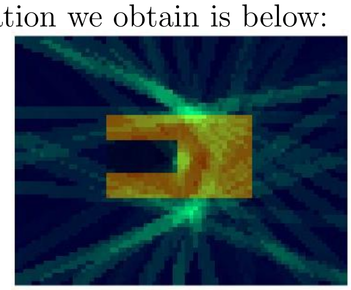
2.3

Using our results of the previous problem's loosen radiation constraints and combining that with a penalization for the amount of radiation given to neighboring cells of critical cells, we obtain a new LP formulation to simultaneously satisfy the renewed constraints that we found in subtask 2 while also minimizing the amount of radiation around the critical cells. Visualizing the outputted beam intensities and tumor cells from this new LP, we obtain the following image[3]:



As you can see, this image differs from the image that we visualized in subtasks 1 and 2. This is because we want to avoid beams that are too close to the boundary of the critical region for fear that inaccuracies in our beam technology will accidentally irradiate the critical cells. However, we are still able to irradiate all of the tumor cells to the necessary threshold by using greater intensities.

We obtain a similar result if we apply our new objective function and LP on the larger actual dataset. The visualization we obtain is below:



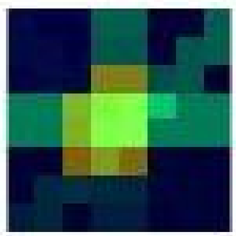
As we can see, in general, we obtain a much lower average intensity in the tumorous region than in the previous case. This makes sense because we are trying to take into account overflow and inaccuracy from each of the beams. Thus, we use fewer beams than we need since we do not know how the inaccuracies will affect the critical cells

[3]See Appendix I for code

and tumor cells. Additionally, note that there the radiation close to the critical cell region is significantly reduced. Because we want to penalize the amount of radiation to these regions, we want to hit just the bare minimum required to kill the tumor cells in these areas. Thus, we must use an array of different beams and likely higher intensities on some of the less “efficient” beams that only graze the tumorous region. However, in the end, our result is a beam intensity model that minimizes the risk to the critical region due to inaccuracies in beams.

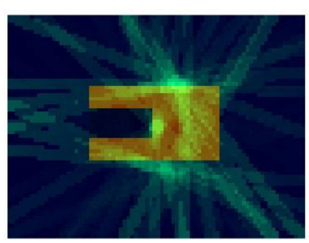
2.4

Our first modification is minimizing the maximum intensity of any beam. This is to make sure that even clinics with limited radiation power on their radiation equipment are able to provide the patient with optimal therapy. Using the LP we get from this modification, we are able to output the following visualization of the beam intensities and tumor cells for the smaller model[4].



This is a very different solution than the ones we have seen before in the previous subtasks. However, given the modifications to the LP that we made, it makes sense that we see this difference. In the past, we used one very strong vertical beam to kill basically all of the cancer cells, cleaning up with some lower intensity diagonal and horizontal beams. However, in this modification, we are able to use much more balanced intensities in the vertical, horizontal, and diagonal beams. We still arrive at a very optimal solution since we completely irradiate the tumorous region while completely avoiding the critical region in the upper left.

Our results are similar if we attempt to use our modification on the actual dataset. Setting up the new LP and solving, we obtain the following visualization of our solution:

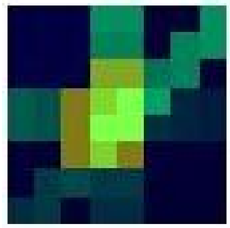


Again, note that the intensities of the beams in this case are much more evenly

[4]See Appendix I for code

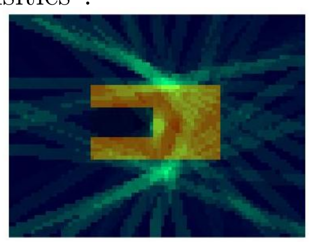
distributed across individual beams than in the previous cases. Thus, the highest intensity used can be minimized by making all the beams we use of similar intensity. We are not able to irradiate the tumor region as well as we were in subtask 2. However, this is expected as we are submitting our problem to additional constraints and penalties.

Our second enhancement to our radiation model is changing to objective function to minimize the radiation that the critical region receives, given that the constraints are satisfied. The solution to the small example when visualized is displayed below:



This looks very similar to the solutions we got in previous subtasks. However, this can be expected since in all subtasks, we are able to completely avoid irradiating the critical regions. Thus, our goal of minimizing the amount of radiation received by the critical cells is always fulfilled since no radiation is received.

However, we are able to see some changes if we apply this enhancement to our actual dataset. Applying our new enhanced LP to the actual dataset, we obtain the following visualization of the beam intensities[5]:



As we can see, the optimization in this case makes a conscious effort to avoid critical region at all costs. Although we inevitably receive some radiation at the very right edge of the critical region, we are able to avoid irradiating the critical region for the most part. Again, we are unable to irradiate the region as much as in subtask 2, which is expected. However, we are still able to hit our required thresholds while avoiding the critical region for the most part.

[5] See Appendix I for code

Appendix I

Subtask 2.1 - mod file

```
param num_matrices >= 1, integer; # Number of matrices in
   the data file to be read
param num_rows >= 1, integer; # Number of rows
param num_cols >= 1, integer; # Number of columns

set MATS := 1 .. num_matrices; # set of matrices
set ROWS := 1 .. num_rows; # set of rows
set COLUMNS := 1 .. num_cols; # set of columns

param req_dose >= 0; # required dosage for tumor cells
param all_dose >= 0; # allowed dosage for critical cells
param beam_dose {MATS, R, C} >= 0, default 0; # beam base
   dosage
param tumor {ROWS, COLUMNS} >= 0; # tumor matrix
param crit {ROWS, COLUMNS} >= 0; # critical cell matrix

var I {MATS} >= 0; # intensity of beams

minimize Dosage: sum{k in MATS} I[k];

subject to Satisfied {i in ROWS, j in COLUMNS}: req_dose *
   tumor[i,j] <= sum {k in MATS} I[k]*beam_dose[k,i,j];
subject to Allowed {i in ROWS, j in COLUMNS}: all_dose >=
   crit[i,j] * sum {k in MATS} I[k]*beam_dose[k,i,j];
```

Subtask 2.1 - dat file

```
param num_matrices := 5; # Number of matrices in the data
   file to be read
param num_rows := 8; # Number of rows
param num_cols := 8; # Number of columns
param req_dose := 10;
param all_dose := 2;

# Read from mat_raw.txt to fill matrix_value. Note that the
   order
# of the index variables matter; here one matrix is filled
   at a time,
# where for each row, the corresponding column values are
   assigned.
read {m in MATS, i in ROWS, j in COLUMNS} beam_dose[m,i,j]
   < beam_raw.txt;
read {i in ROWS, j in COLUMNS} tumor[i,j] < tumor_raw.txt;
read {i in ROWS, j in COLUMNS} crit[i,j] < critical_raw.txt;
```

Subtask 2.2 Feasibility - mod file

```
param num_matrices >= 1, integer; # Number of matrices in
   the data file to be read
param num_rows >= 1, integer; # Number of rows
param num_cols >= 1, integer; # Number of columns

set MATS := 1 .. num_matrices; # set of matrices
set ROWS := 1 .. num_rows; # set of rows
set COLUMNS := 1 .. num_cols; # set of columns

param req_dose >= 0; # required dosage for tumor cells
param all_dose >= 0; # allowed dosage for critical cells
param beam_dose {MATS, R, C} >= 0, default 0; # beam base
   dosage
param tumor {ROWS, COLUMNS} >= 0; # tumor matrix
param crit {ROWS, COLUMNS} >= 0; # critical cell matrix

var I {MATS} >= 0; # intensity of beams
var max_buf {ROWS, COLUMNS} >=0; # feasibility buffer
var min_buf {ROWS, COLUMNS} >= 0; # feasibility buffer

minimize Dosage: sum {i in ROWS, j in COLUMNS}
   (max_buf[i,j] + min_buf[i,j]);

subject to Satisfied {i in ROWS, j in COLUMNS}: (req_dose -
   max_buf[i,j]) * tumor[i,j] <= sum {k in MATS}
   I[k]*beam_dose[k,i,j];
subject to Allowed {i in ROWS, j in COLUMNS}: (all_dose +
   min_buf[i,j]) >= crit[i,j] * sum {k in MATS}
   I[k]*beam_dose[k,i,j];
```

Subtask 2.2 Feasibility - dat file

```
param num_matrices := 126; # Number of matrices in the data
   file to be read
param num_rows := 60; # Number of rows
param num_cols := 80; # Number of columns
param req_dose := 10;
param all_dose := 2;

# Read from mat_raw.txt to fill matrix_value. Note that the
   order
# of the index variables matter; here one matrix is filled
   at a time,
# where for each row, the corresponding column values are
   assigned.
read {m in MATS, i in ROWS, j in COLUMNS} beam_dose[m,i,j]
   < beam_raw.txt;
read {i in ROWS, j in COLUMNS} tumor[i,j] < tumor_raw.txt;
read {i in ROWS, j in COLUMNS} crit[i,j] < critical_raw.txt;
```

Subtask 2.2 Model - mod file

```
param num_matrices >= 1, integer; # Number of matrices in
   the data file to be read
param num_rows >= 1, integer; # Number of rows
param num_cols >= 1, integer; # Number of columns

set MATS := 1 .. num_matrices; # set of matrices
set ROWS := 1 .. num_rows; # set of rows
set COLUMNS := 1 .. num_cols; # set of columns

param req_dose >= 0; # required dosage for tumor cells
param all_dose >= 0; # allowed dosage for critical cells
param beam_dose {MATS, R, C} >= 0, default 0; # beam base
   dosage
param tumor {ROWS, COLUMNS} >= 0; # tumor matrix
param crit {ROWS, COLUMNS} >= 0; # critical cell matrix
param max_buf {ROWS, COLUMNS} >= 0; # feasibility buffer
param min_buf {ROWS, COLUMNS} >= 0; # feasibility buffer

var I {MATS} >= 0; # intensity of beams

minimize Dosage: sum{k in MATS} I[k];

subject to Satisfied {i in ROWS, j in COLUMNS}: (req_dose -
   max_buf[i,j]) * tumor[i,j] <= sum {k in MATS}
   I[k]*beam_dose[k,i,j];
subject to Allowed {i in ROWS, j in COLUMNS}: (all_dose +
   min_buf[i,j]) >= crit[i,j] * sum {k in MATS}
   I[k]*beam_dose[k,i,j];
```

Subtask 2.2 Model - dat file

```
param num_matrices := 126; # Number of matrices in the data
   file to be read
param num_rows := 60; # Number of rows
param num_cols := 80; # Number of columns
param req_dose := 10;
param all_dose := 2;

# Read from mat_raw.txt to fill matrix_value. Note that the
   order
# of the index variables matter; here one matrix is filled
   at a time,
# where for each row, the corresponding column values are
   assigned.
read {m in MATS, i in ROWS, j in COLUMNS} beam_dose[m,i,j]
   < beam_raw.txt;
read {i in ROWS, j in COLUMNS} tumor[i,j] < tumor_raw.txt;
read {i in ROWS, j in COLUMNS} crit[i,j] < critical_raw.txt;
read {i in ROWS, j in COLUMNS} max_buf[i,j] < max_buf.out;
read {i in ROWS, j in COLUMNS} min_buf[i,j] < min_buf.out;
```

Subtask 2.3 Model - mod file

```
param num_matrices >= 1, integer; # Number of matrices in
   the data file to be read
param num_rows >= 1, integer; # Number of rows
param num_cols >= 1, integer; # Number of columns

set MATS := 1 .. num_matrices; # set of matrices
set R := 0 .. (num_rows+1); # set of rows with padding
set C := 0 .. (num_cols+1); # set of columns with padding
set ROWS := 1 .. num_rows; # set of rows
set COLUMNS := 1 .. num_cols; # set of columns

param req_dose >= 0; # required dosage for tumor cells
param all_dose >= 0; # allowed dosage for critical cells
param beam_dose {MATS, R, C} >= 0, default 0; # beam base
   dosage
param tumor {ROWS, COLUMNS} >= 0; # tumor matrix
param crit {ROWS, COLUMNS} >= 0; # critical cell matrix
param max_buf {ROWS, COLUMNS} >= 0; # feasibility buffer
param min_buf {ROWS, COLUMNS} >= 0; # feasibility buffer

var I {MATS} >= 0; # intensity of beams

minimize Dosage: (sum {i in ROWS, j in COLUMNS, k in MATS,
   i2 in (i-1)..(i+1), j2 in (j-1)..(j+1)}
   crit[i,j]*I[k]*beam_dose[k,i2,j2]);

subject to Satisfied {i in ROWS, j in COLUMNS}: (req_dose -
   max_buf[i,j]) * tumor[i,j] <= sum {k in MATS}
   I[k]*beam_dose[k,i,j];
subject to Allowed {i in ROWS, j in COLUMNS}: (all_dose +
   min_buf[i,j]) >= crit[i,j] * (sum {k in MATS}
   I[k]*beam_dose[k,i,j]);
```

Subtask 2.3 Model - dat file

```
param num_matrices := 126; # Number of matrices in the data
   file to be read
param num_rows := 60; # Number of rows
param num_cols := 80; # Number of columns
param req_dose := 10;
param all_dose := 2;

# Read from mat_raw.txt to fill matrix_value. Note that the
   order
# of the index variables matter; here one matrix is filled
   at a time,
# where for each row, the corresponding column values are
   assigned.
read {m in MATS, i in ROWS, j in COLUMNS} beam_dose[m,i,j]
   < beam_raw.txt;
read {i in ROWS, j in COLUMNS} tumor[i,j] < tumor_raw.txt;
read {i in ROWS, j in COLUMNS} crit[i,j] < critical_raw.txt;
read {i in ROWS, j in COLUMNS} max_buf[i,j] < max_buf.out;
read {i in ROWS, j in COLUMNS} min_buf[i,j] < min_buf.out;
```

Subtask 2.4.1 Model - mod file

```
param num_matrices >= 1, integer; # Number of matrices in
   the data file to be read
param num_rows >= 1, integer; # Number of rows
param num_cols >= 1, integer; # Number of columns

set MATS := 1 .. num_matrices; # set of matrices
set R := 0 .. (num_rows+1); # set of rows with padding
set C := 0 .. (num_cols+1); # set of columns with padding
set ROWS := 1 .. num_rows; # set of rows
set COLUMNS := 1 .. num_cols; # set of columns

param req_dose >= 0; # required dosage for tumor cells
param all_dose >= 0; # allowed dosage for critical cells
param beam_dose {MATS, R, C} >= 0, default 0; # beam base
   dosage
param tumor {ROWS, COLUMNS} >= 0; # tumor matrix
param crit {ROWS, COLUMNS} >= 0; # critical cell matrix
param max_buf {ROWS, COLUMNS} >= 0; # feasibility buffer
param min_buf {ROWS, COLUMNS} >= 0; # feasibility buffer

var I {MATS} >= 0; # intensity of beams
var maxim >= 0; # maximum of intensities

minimize Dosage: maxim; # Minimax of dosage

subject to Satisfied {i in ROWS, j in COLUMNS}: (req_dose -
   max_buf[i,j]) * tumor[i,j] <= sum {k in MATS}
   I[k]*beam_dose[k,i,j];
subject to Allowed {i in ROWS, j in COLUMNS}: (all_dose +
   min_buf[i,j]) >= crit[i,j] * (sum {k in MATS}
   I[k]*beam_dose[k,i,j]);
subject to Maxi {k in MATS}: maxim >= I[k];
```

Subtask 2.4.1 Model - dat file

```
param num_matrices := 126; # Number of matrices in the data
   file to be read
param num_rows := 60; # Number of rows
param num_cols := 80; # Number of columns
param req_dose := 10;
param all_dose := 2;

# Read from mat_raw.txt to fill matrix_value. Note that the
   order
# of the index variables matter; here one matrix is filled
   at a time,
# where for each row, the corresponding column values are
   assigned.
read {m in MATS, i in ROWS, j in COLUMNS} beam_dose[m,i,j]
   < beam_raw.txt;
read {i in ROWS, j in COLUMNS} tumor[i,j] < tumor_raw.txt;
read {i in ROWS, j in COLUMNS} crit[i,j] < critical_raw.txt;
read {i in ROWS, j in COLUMNS} max_buf[i,j] < max_buf.out;
read {i in ROWS, j in COLUMNS} min_buf[i,j] < min_buf.out;
```

Subtask 2.4.2 Model - mod file

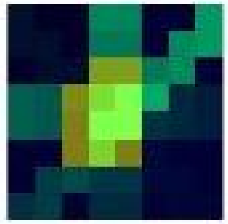
```
param num_matrices >= 1, integer; # Number of matrices in
   the data file to be read
param num_rows >= 1, integer; # Number of rows
param num_cols >= 1, integer; # Number of columns

set MATS := 1 .. num_matrices; # set of matrices
set R := 0 .. (num_rows+1); # set of rows with padding
set C := 0 .. (num_cols+1); # set of columns with padding
set ROWS := 1 .. num_rows; # set of rows
set COLUMNS := 1 .. num_cols; # set of columns

param req_dose >= 0; # required dosage for tumor cells
param all_dose >= 0; # allowed dosage for critical cells
param beam_dose {MATS, R, C} >= 0, default 0; # beam base
   dosage
param tumor {ROWS, COLUMNS} >= 0; # tumor matrix
param crit {ROWS, COLUMNS} >= 0; # critical cell matrix
param max_buf {ROWS, COLUMNS} >= 0; # feasibility buffer
param min_buf {ROWS, COLUMNS} >= 0; # feasibility buffer

var I {MATS} >= 0; # intensity of beams

minimize Dosage: sum {k in MATS, i in ROWS, j in COLUMNS}
   crit[i,j]*I[k]*beam_dose[k,i,j]; # Minimax of dosage

subject to Satisfied {i in ROWS, j in COLUMNS}: (req_dose -
   max_buf[i,j]) * tumor[i,j] <= sum {k in MATS}
   I[k]*beam_dose[k,i,j];
subject to Allowed {i in ROWS, j in COLUMNS}: (all_dose +
   min_buf[i,j]) >= crit[i,j] * (sum {k in MATS}
   I[k]*beam_dose[k,i,j]);
```

Subtask 2.4.2 Model - dat file

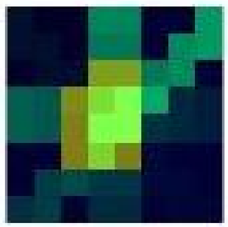
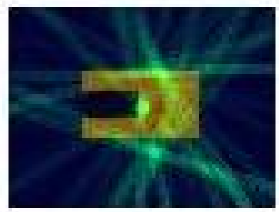
```
param num_matrices := 126; # Number of matrices in the data
   file to be read
param num_rows := 60; # Number of rows
param num_cols := 80; # Number of columns
param req_dose := 10;
param all_dose := 2;

# Read from mat_raw.txt to fill matrix_value. Note that the
   order
# of the index variables matter; here one matrix is filled
   at a time,
# where for each row, the corresponding column values are
   assigned.
read {m in MATS, i in ROWS, j in COLUMNS} beam_dose[m,i,j]
   < beam_raw.txt;
read {i in ROWS, j in COLUMNS} tumor[i,j] < tumor_raw.txt;
read {i in ROWS, j in COLUMNS} crit[i,j] < critical_raw.txt;
read {i in ROWS, j in COLUMNS} max_buf[i,j] < max_buf.out;
read {i in ROWS, j in COLUMNS} min_buf[i,j] < min_buf.out;
```

Appendix II

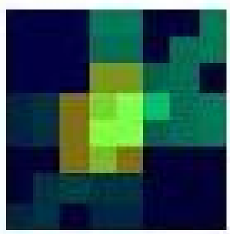
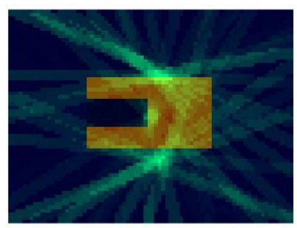
Subtask 1.1



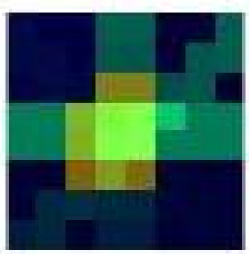
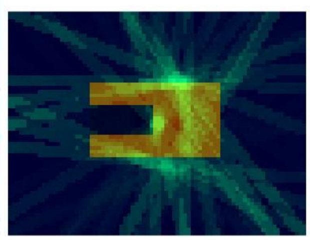
Subtask 1.2

Subtask 1.3

Subtask 1.4.1

Subtask 1.4.2