## Additional Notes for the AAMAS 2023 Paper "Separating and Collapsing Electoral Control Types" by Carleton, Chavrimootoo, Hemaspaandra, Narváez, Taliancich, and Welles

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Introduction to Additional Notes: All these notes are intended to be read within the context of the abovementioned paper. In particular, all references to definitions, theorems, and corollaries are to those in that paper (and use the same numbering scheme). And comments in these Additional Notes such as "as mentioned earlier" mean earlier in that paper than where the given Additional Note is called out from.

ADDITIONAL NOTE 1. A number of our election examples showing separations were found with the aid of computer searches. Other examples were human generated.

In addition to the verification code within the computer-search programs, we coded—with a different person doing the coding—stand-alone verification routines, as a double-check of the correctness of all separation examples used in this paper. Whether computer-generated or human-generated, every example was checked via those separate verification programs. Also, every computer-generated example was verified by a human.

To support reproducibility, and to aid researchers who might wish to carry our study to other cases, we have made the computer-search programs, and their inputs and outputs, available within the same repository that contains these Additional Notes.

Our search programs use randomization, and this repository captures and documents the randomization used in each run generating our examples. Thus skeptical researchers could, if they wanted, simulate our code, using the same randomization, to assure themselves that our codes indeed produced the examples we claim they did. This repository also includes some information about what systems the programs were run on and how long certain runs took. (In fact, each program was run once, and the run itself randomly tried many examples until a counterexample was found; even the longest-running program ran for at most 21 CPU seconds.)

This repository additionally includes the separate verification programs mentioned above.

ADDITIONAL NOTE 2. The treatment of V as a multiset—which we use to match [16]'s models—in effect limits the election systems the model covers to so-called (voter-)anonymous election systems. In fact, all general-case collapses of [16] and all new general-case containments of our paper also hold (in the natural nonanonymous model) for all nonanonymous election systems, as one can easily see from those results' proofs. Also, we mention that all three concrete election systems that we study in fact are anonymous.

ADDITIONAL NOTE 3. Regarding the partition-based control types, in Definition 1 and elsewhere in the paper we sometimes will speak of elections whose candidate set is C' but whose votes, due to candidate partitioning and/or first-round candidate eliminations, are over a set  $C \supseteq C'$ . As is standard in the literature, in such cases we always take this to mean that the votes are each masked down to just the candidates in C'.

ADDITIONAL NOTE 4. We note that for some cases we achieve strong incomparability in our general-case results, always via 13\*. In such cases, incomparability still holds, since strong incomparability is simply a specific subcase of incomparability.

ADDITIONAL NOTE 5. We mention in passing that, as mentioned earlier, a benefit of collapsing compatible types (especially when we do so in the general case) is that doing so automatically establishes that the control complexity of the types is the same, rather than having to separately determine the control complexity of each of the types. This can save work.

Regarding the "race" to find systems that are resistant to a large number of control types, one might ask whether it still is fair to count it as multiple strengths if the elements of collapsing pairs (or larger equivalence classes) are resistant (and so NP-hard), or to count it as multiple weaknesses if the elements of collapsing pairs (or larger equivalence classes) are vulnerable (and thus in P). In some sense, one could argue that this is unfair, as it is putting extra weight on the "same" type.

However, one could also argue that having a collapsing control type foursome count as four is fair and natural, since the actual mechanisms of the four are different, and so one either is fighting off attacks through NP-completeness, or is vulnerable to multiple attack lines.

In any case, if one focuses solely on number of resistances, one is implicitly saying that all control types are equally important, and we do not think that saying that is correct. Rather, what is most important is that the field knows, for the full palette of control types, which ones are resistant and which ones are vulnerable with respect to a given election system. That will allow the people choosing which election system to use in a given setting to choose an election system that is resistant to as many as possible of the attacks that they expect are the most likely within the setting. And for that, there is no doubt that avoiding duplicate work through exploiting collapses is helpful.

Finally, we mention that merging collapsed types on each system's "score card" would be problematic, since different elections have different collapses, and so different election systems' score cards would not even have the same number of entries.