disjoint events + general addition rule

- disjoint events
- the general addition rule
- sample space
- probability distributions
- complementary events

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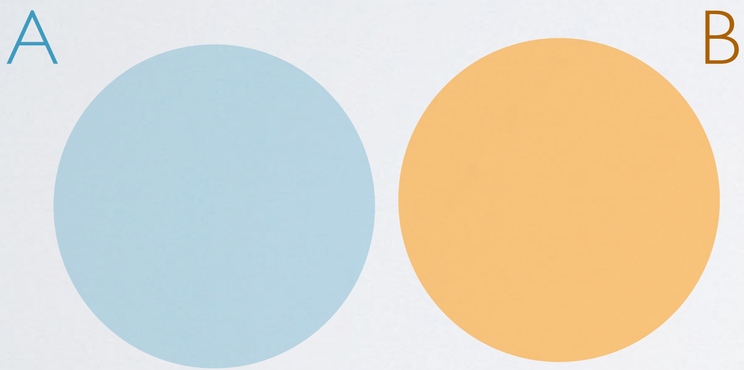
disjoint (mutually exclusive)

disjoint (mutually exclusive) events cannot happen at the same time.

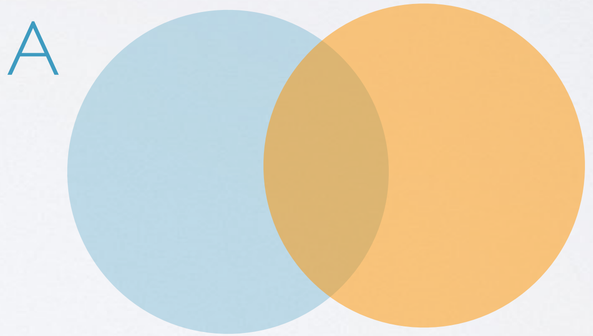
- the outcome of a single coin toss cannot be a head and a tail.
- a student can't both fail and pass a class.
- a single card drawn from a deck cannot be an ace and a queen.



$P(A \text{ and } B) = 0$

non-disjoint events can happen at the same time.

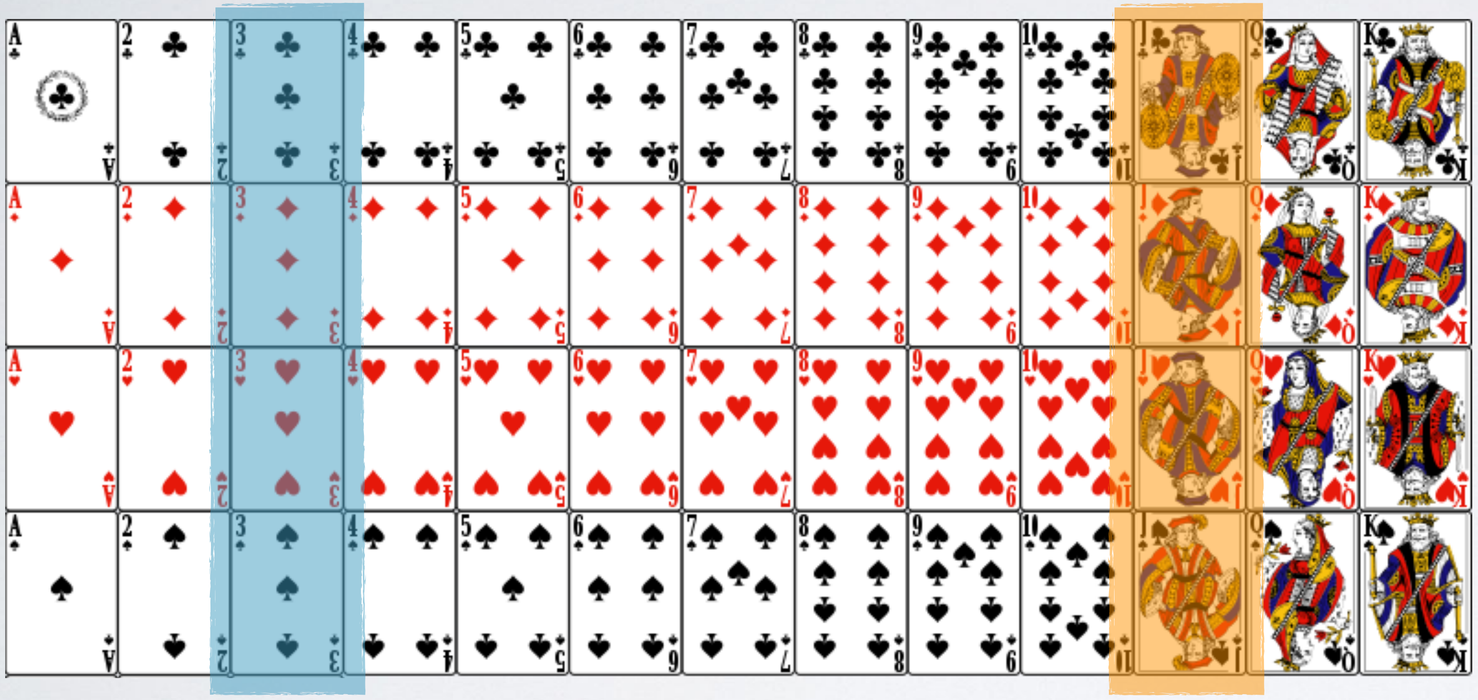
- a student can get an A in Stats and A in Econ in the same semester.



$P(A \text{ and } B) \neq 0$

union of disjoint events

What is the probability of drawing a Jack or a three from a well shuffled full deck of cards?



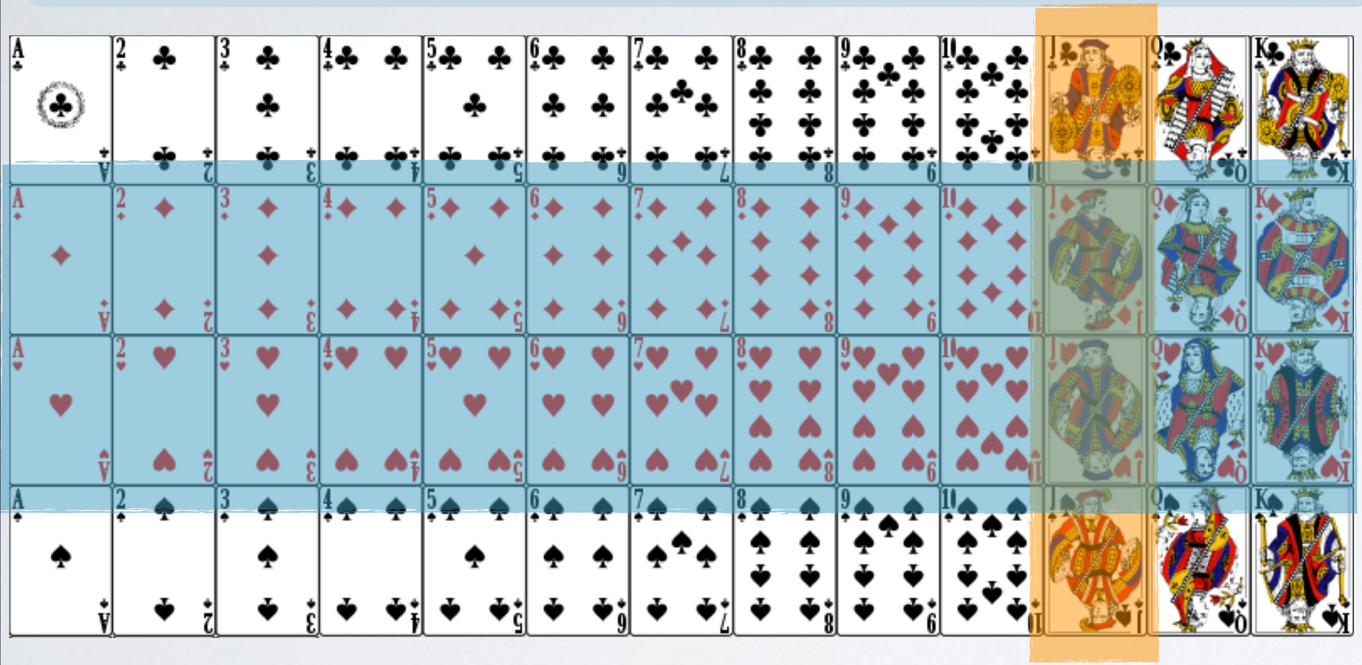
$$P(J \text{ or } 3)$$
$$= P(J) + P(3)$$
$$= (4/52) + (4/52)$$
$$\approx 0.154$$

For disjoint events A and B,
P(A or B) = P(A) + P(B)

Image source: http://svg-cards.sourceforge.net/

union of non-disjoint events

What is the probability of drawing a Jack or a red card from a well shuffled full deck of cards?



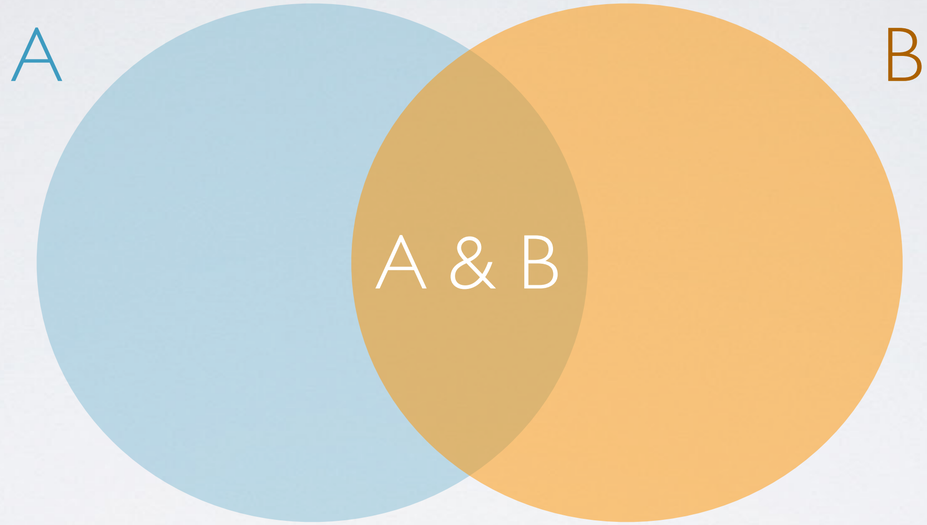
$$P(J \text{ or red})$$
$$= P(J) + P(\text{red}) - P(J \text{ and red})$$
$$= (4/52) + (26/52) - (2/52)$$
$$\approx 0.538$$

For non-disjoint events A and B,
P(A or B) = P(A) + P(B) - P(A and B)

Image source: http://svg-cards.sourceforge.net/

General addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Note: When A and B are disjoint, $P(A \text{ and } B) = 0$, so the formula simplifies to $P(A \text{ or } B) = P(A) + P(B)$.

sample space

a sample space is a collection of all possible outcomes of a trial.

A couple has two kids, what is the sample space for the sex of these kids? For simplicity assume that sex can only be male or female.

$$S = \{ MM, FF, FM, MF \}$$

probability distributions

a probability distribution lists all possible outcomes in the sample space, and the probabilities with which they occur.

one toss	head	tail
probability	0.5	0.5

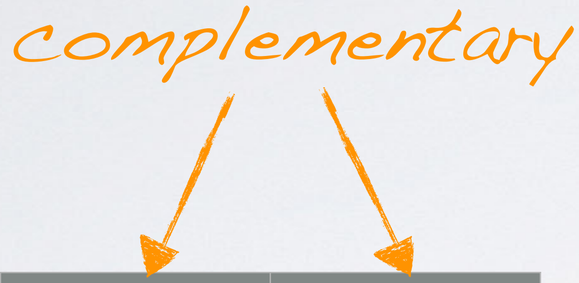
two tosses	head - head	tail - tail	head - tail	tail - head
probability	0.25	0.25	0.25	0.25

rules

1. the events listed must be disjoint
2. each probability must be between 0 and 1
3. the probabilities must total 1

complementary events

complementary events are two mutually exclusive events whose probabilities that add up to 1.



one toss	head	tail
probability	0.5	0.5

two tosses	head - head	tail - tail	head - tail	tail - head
probability	0.25	0.25	0.25	0.25

disjoint vs. complementary

Do the sum of probabilities of two disjoint outcomes always add up to 1?

Not necessarily, there may be more than 2 outcomes in the sample space.

Do the sum of probabilities of two complementary outcomes always add up to 1?

Yes, that's the definition of complementary.