Dyscalculia: A Behavioural Vision

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Abstract. *Learning Disabilities* (*LD*) constitute a diverse group of disorders in which children who generally possess at least average intelligence have problems processing information or generating output, i.e., *LD* may be interpreted as a neurologically-based processing problem. The causes and treatment of *LD*, namely reading disorders has been the subject of considerable thought and study. Being one among others, this is the reason why this work will focus on dyscalculia and in its different manifestations and how they may interfere with the children natural development. It will be assessed it in terms of a measurement of the child`s entropy, a thermodynamic quantity representing the unavailability of a child brain energy for conversion into mental work, and seen as the degree of disorder or randomness in the brain, i.e., lack of order or predictability; gradual decline into disorder; an arena where entropy reigns supreme. In one`s work it reigns in a specific interval, i.e., one may have two scenarios, namely the worst and the best one. The formal background will be grounded in the use of *Logic Programming* to set the architecture of a *Function Machine* to assess *LD* and built on base of a *Deep Learning* approach to *Knowledge Representation and Reasoning*.

Keywords: Learning Disabilities; Entropy; Logic Programming; Knowledge Representation and Reasoning; Deep Learning; Function Machine.

1 Introduction

Learning Disabilities (*LD*) are one of the main concerns when it comes to scholar ratings of success. Specific mathematical *LD* are, yet, not so deeply approached when there is an attempt to mitigate the learning ones affecting the rates. Even so, there is more research work on this issue that will make a sustainable difference to the quality and consistency with which safe and therapeutic services for people with *LD*, not only regarding the children evolution. Indeed, *developmental dyscalculia* is a specific mathematical *LD* that has been studied and it is understood as *a difficulty in leading with arithmetical issues*. There is not, yet, a standard screening test, but there are several tools to help assess the type of *LD* that affect children.

Following Kosc [1], one may be faced to six distinct types of dyscalculia that comprehend the *lexical one*, which concerns troubles reading and understanding mathematical symbols and numbers, as well as mathematical expressions or equations. The children who has *lexical dyscalculia* can understand spoken views, but have trouble in writing or understanding them, presenting difficulty in reading symbols, such as numerals, and cannot understand them when they occur in number sentences or equations; *verbal dyscalculia*, in which children have problems in naming and comprehending the mathematical concepts exposed verbally. The children are able to read or write a number, but cannot recognize them when they are revealed verbally – they present some strain when talking about mathematical concepts or relationships; *graphical dyscalculia*, manifested as not easy task when writing mathematical symbols. The children that have this type of dyscalculia are able to understand the mathematical concepts but do not have the ability to read, write, or use the mathematical symbols – a difficulty with writing such icons including but not limited to numbers; *operational dyscalculia*, which presents itself with a difficulty to complete arithmetical operations or mathematical computations, both written and verbal. Someone with *operational dyscalculia* will be able to understand the numbers and the relationships between them, but will have trouble manipulating numbers and mathematical symbols in computational process; *practognostic dyscalculia*, which denotes difficulty in the process of translating their abstract-mathematical concepts to real and ideal aspects of human life. These children are able to understand mathematical concepts but have trouble in listing, comparing or manipulating mathematical equations, demonstrating difficulty in translating their abstract mathematical knowledge into real-world actions or procedures; and *ideagnostic dyscalculia*, a snag when carrying out mental operations without using numbers to arrive at a solution or to understand concepts or ideas related to mathematics or arithmetic. These children have a challenging time in remembering mathematical concepts once having learned them and difficulties with tasks that require understanding of mathematical notions and relationships, such as identifying which sequence of numbers is larger or smaller.

As stated above, the distinction between each type of dyscalculia can be done through tasks directed towards a certain objective and its assessment. So, the board games, puzzles and other type of educative tools are the best instruments to use in order to evaluate the stage and type of *LD*.

Additionally, questionnaires filled by teachers, educators or relatives (preferentially someone that deals daily with children) are an asset to complete a primary diagnosis of the several types of dyscalculia.

On the other hand, *Artificial Intelligence* (*AI*), in touch with reality, is in the evolving and testing of theories and aspects of intelligent behaviour, including *Knowledge Representation and Reasoning*, *Learning*, *Decision-making*, *Communication*, *Coordination*, *Action*, *Interaction*, where *Machine Learning* (*ML*) is concerned with the scientific study, design, analysis, and applications of algorithms that learn concepts, predictive models and behaviours. Indeed, *AI* is transforming the world of different disciplines. *AI* can help teachers, doctors and other practitioners to make faster, more accurate diagnoses. It will be used here to improve children care, supporting a *Deep Learning* approach to *Knowledge Representation and Reasoning* (*KRR*).

Therefore, a brief description of an innovative *KRR* it is set in the next section, followed by the presentation of a case study focused on screening the types of dyscalculia in children. Finally, conclusions are gathered and directions for future work are outlined.

2 Knowledge Representation and Reasoning

On the one hand, many approaches to integrate *Deep Learning* with *Knowledge Representation and Reasoning* (*KRR*) are based on the fact that one must give up on having a fixed symbolic *structure*, i.e., it must be set a process of *relaxation* when going from symbolic to sub-symbolic, where the *KRR`s* process is induced by learning algorithms, with an outcome mostly opaque to the users. This view stands for the key distinction between such approaches where it is asserted that when it is used symbolic logic in vector spaces, the essential features of a universe of discourse remain discrete, and as a result nothing is gained. On the other hand, in one`s approach, although presenting a symbolic logic in vector spaces, the functions` elements or attributes go from discrete to continuous, allowing for *unknown*, *incomplete*, *forbidden* and even *self-contradictory information* or *knowledge*, with no opaqueness at all for the users. Thus, the universe of discourse in this work will be engendered according to predicate`s extensions of the type:

$$predicate_i - \bigcup_{1 \leq j \leq m} clause_j \left(\left([A_{x_1}, B_{x_1}] (QoI_{x_1}, DoC_{x_1}) \right), \cdots, \left([A_{x_n}, B_{x_n}] (QoI_{x_m}, DoC_{x_m}) \right) \right) :: QoI_j :: DoC_j$$

where $\left[A_{x_j}, B_{x_j}\right]$, QoI_{x_j} and DoC_{x_j} denote, respectively, the scope where the unknown *attribute*$_j$ for *predicate*$_i$ is expected to appear, the attribute`s *Quality-of-Information* and, finally, the *Degree-of-Confidence* that one may have on such a value. Therefore, we will look at approaches to *KRR* that have been proposed using the *Logic Programming* (*LP*) epitome, namely in the area of *Model Theory* [2], [3] and *Proof Theory* [4], [5]. In the present work, the *Proof Theoretical* approach in terms of an extension to the *LP* language is followed. An *Extended Logic Program* is, therefore, given by a finite set of clauses, in the form:

{

$\neg\, p \leftarrow not\ p, not\ exception_p$

$p \leftarrow p_1, \cdots, p_n, not\ q_1, \cdots, not\ q_m$

$?\,(p_1, \cdots, p_n, not\ q_1, \cdots, not\ q_m) \ \ (n, m \geq 0)$

$exception_{p_1}, \ \cdots \ , exception_{p_j} \ \ (0 \leq j \leq k), \qquad being\ k\ an\ integer\ number$

$\} :: scoring_{value}$

where the first clause stand for predicate’s closure, “,” denotes “*logical and*”, while “*?*” is a domain atom denoting falsity, the p_i, q_j, and p are classical ground literals, i.e., either positive atoms or atoms preceded by the classical negation sign $\neg$ [5]. Indeed, $\neg$ stands for a strong declaration that speaks for itself, and *not* denotes *negation-by-failure*, or in other words, a flop in proving a given statement, once it was not declared explicitly. Under this formalism, every program is associated with a set of *abducibles* [2], [3], given here in the form of exceptions to the extensions of the predicates that make the program, i.e., clauses of the form:

$exception_{p_1}, \ \cdots \ , exception_{p_j} \ (0 \leq j \leq k), \ \ being\ k\ an\ integer\ number$

that stand for data, information or knowledge that cannot be ruled out. On the other hand, clauses of the type:

$$?\,(p_1, \cdots, p_n, not\ q_1, \cdots, not\ q_m) \ \ (n, m \geq 0)$$

also named *invariants*, allows one to set the context under which the universe of discourse has to be understood. The term $scoring_{value}$ stands for the relative weight of the extension of a specific predicate with respect to the extensions of peers' ones that make the inclusive or global program.

In order to evaluate the data or knowledge's assets that may be associated with a logical program, an assessment of it is given in terms of the *Quality-of-Information* (*QoI*) and *Degree-of-Confidence* (*DoC*) metrics, that range between 0 and 1 and lead to predicates or logical functions in the form[6]–[8]:

$$predicate_{new} - \bigcup_{1 \leq j \leq m} new_j \left(\left([A_{x_1}, B_{x_1}] (QoI_{x_1}, DoC_{x_1}) \right), \cdots \right.$$

$$\left. \cdots, \left([A_{x_n}, B_{x_n}] (QoI_{x_m}, DoC_{x_m}) \right) \right) :: QoI_j :: DoC_j$$

which answer our question stated above, i.e., *AI* is no more riding a one-trick pony.

3 Case Study

3.1 Data Collection

On the one hand, one's approach to the problem referred to above will focus on the study of sensory problems in children diagnosed with specific *LD*, where cognition of visual stimuli is of the utmost importance, i.e., a special attention would be given not only to the sensing of the children physical characteristics but also to their sentiments and emotions or even our own soul. On the other hand, it is also imperative to focus on the social perspective and its assessment, which will be done in terms of a *Function Machine (*i.e., a writing board for presentations), set as a computational environment comprising children, teachers and other practitioners and technology, interacting and producing actions and information that would not be possible to extract without having all parties present. Therefore, regarding the factors that influence *LD*, focus should be given to national policies, such as economic strategies and the conditions in which children live and learn.

Regarding data collection, the technique used was observation. Teachers from *N* schools evaluated *N*N* students following a criteria list. The target group, the *N*N* students, is characterized by a set of children between 5 to 8 years-old, attending the first, second and third grades. These annotations focused not only on students that were already diagnosed with, or suspected of having dyscalculia but also on non-affected ones. The process consisted in fill a data registry regarding each student where the teachers and other practitioners assess the student`s difficulty in performing the tasks described in the criteria list, based on the previous knowledge and daily contact with the child. The criteria list contained mixed tasks concerning the six types of dyscalculia, delivered to the lecturers and other experts and built by the research team. Such records are stored in a *Case Base*, according to productions of the type:

$$Case\ Records = \{\, Raw_{data}, Normalized_{data}, Description_{data} \,\}$$

where the attributes Raw_{data} and $Normalized_{data}$ stand for themselves, and the last one, $Description_{data}$ denotes a set of procedures that may be used in order to set a diagnosis or even free text that explains the *Case Records* in more detail, namely presenting a diagnosis and validating it.

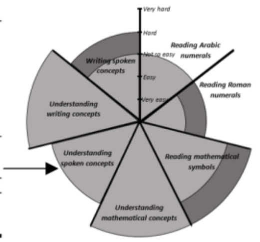
3.2 Feature Extraction

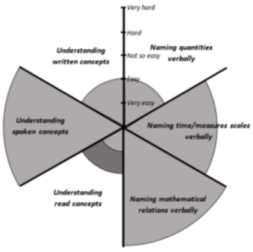
The feature extraction`s process focused on the more relevant tasks associated with each type of dyscalculia. Given the criteria list that guided the teachers, it were selected five to eight tasks related to a type of dyscalculia and stored in the respective table, as shown in
Fig. 1. The remaining data from the criteria list was discarded.

The data used in this study will be given in terms of the extensions of the relations/tables *Lexical Dyscalculia*, *Verbal Dyscalculia*, *Graphical Dyscalculia*, *Operational Dyscalculia*, *Practognostic Dyscalculia* and *Ideagnostic Dyscalculia* (
Fig. 1), where the attributes ‘values speak for themselves. For example, *Understanding Relationships Between Numbers* stands for an attribute that belongs to the *Operational Dyscalculia* table, a situation that occurs when the educator presents the numbers 5 and 10 to a child, and asks him/her about the relationship between them, expecting an answer that should sustain of the greatness of one be above the other, or even the use of term *double of*.

The qualitative values also used to classify the children difficulties are given in terms of the scale *very easy, easy, not so easy, hard* and *very hard*. These values are posteriorly converted in quantitative ones, according to the method described in the work of Ramalhosa *et al.* [9]. It is now possible to set the *degree of uncertainty in the entropy variation* associated to the diverse types of dyscalculia that may affect each child (
Fig. 1). It must also be denoted that the scenario that was workout assume that the distinct types of dyscalculia understudy affect every child.

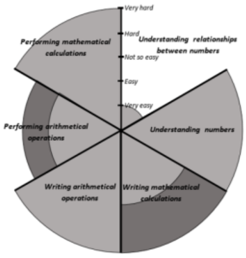
Lexical Dyscalculia										
Child Assessment	*Reading Arabic numerals*	*Reading Roman numerals*	*Reading mathematical symbols*	*Understanding mathematical expressions*	*Understanding spoken concepts*	*Understanding written concepts*	*Writing spoken concepts*	***Degree of uncertainty in the entropy variation***	***QoI***	***DoC***
1	not so easy	easy/not so easy	hard, hard/very hard, very hard	⊥	hard	⊥	not so easy/hard	**[0.17, 0.54]**	**0.670**	**0.673**
…	…	…	…	…	…	…	…	…	…	…
n	not so easy	hard	not so easy/hard	very hard	easy	hard/very hard	hard	**[0.23, 0.29]**	**1**	**0.976**



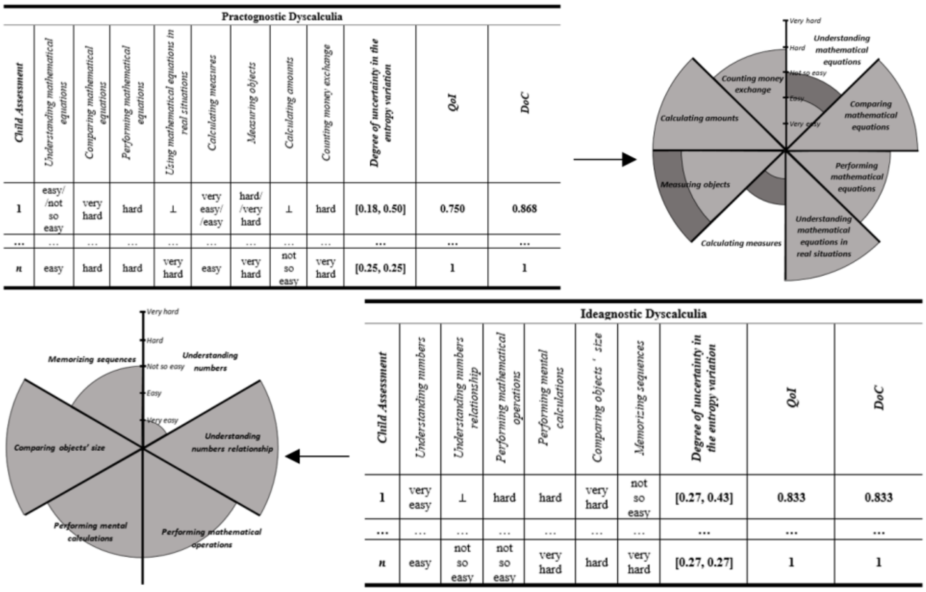


Verbal Dyscalculia									
Child Assessment	*Naming quantities verbally*	*Naming time/measures /scales verbally*	*Naming mathematical relations verbally*	*Understanding read concepts*	*Understanding spoken concepts*	*Understanding written concepts*	***Degree of uncertainty in the entropy variation***	***QoI***	***DoC***
1	easy	hard	⊥	very easy/easy	⊥	easy	**[0.33, 0.70]**	**0.667**	**0.429**
…	…	…	…	…	…	…	…	…	…
n	easy	not so easy	not so easy	easy	hard	very easy	**[0.37, 0.37]**	**1**	**1**

Graphical Dyscalculia								
Child Assessment	*Writing mathematical symbols*	*Understanding mathematical concepts*	*Reading mathematical symbols*	*Using mathematical symbols*	*Writing numbers*	***Degree of uncertainty in the entropy variation***	***QoI***	***DoC***
1	not so easy	easy	⊥, very easy	hard/very hard	not so easy/hard	**[0.24, 0.52]**	**0.967**	**0.792**
…	…	…	…	…	…	…	…	…
n	hard	easy	very hard	very hard	hard	**[0.20, 0.20]**	**1**	**1**



Operational Dyscalculia									
Child Assessment	*Understanding relationships between numbers*	*Understanding numbers*	*Writing mathematical calculations*	*Writing arithmetical operations*	*Performing mathematical calculations*	*Performing arithmetical operations*	***Degree of uncertainty in the entropy variation***	***QoI***	***DoC***
1	very easy	⊥	not so easy/very hard	⊥	not so easy/hard	very hard	**[0.17, 0.60]**	**0.667**	**0.650**
…	…	…	…	…	…	…	…	…	…
n	easy	very easy	hard	very hard	not so easy	hard	**[0.37, 0.37]**	**1**	**1**



Practognostic Dyscalculia											
Child Assessment	*Understanding mathematical equations*	*Comparing mathematical equations*	*Performing mathematical equations*	*Using mathematical equations in real situations*	*Calculating measures*	*Measuring objects*	*Calculating amounts*	*Counting money exchange*	***Degree of uncertainty in the entropy variation***	***QoI***	***DoC***
1	easy/ /not so easy	very hard	hard	⊥	very easy/ /easy	hard/ /very hard	⊥	hard	**[0.18, 0.50]**	**0.750**	**0.868**
…	…	…	…	…	…	…	…	…	**…**	**…**	**…**
n	easy	hard	hard	very hard	easy	very hard	not so easy	very hard	**[0.25, 0.25]**	**1**	**1**

Ideagnostic Dyscalculia									
Child Assessment	*Understanding numbers*	*Understanding numbers relationship*	*Performing mathematical operations*	*Performing mental calculations*	*Comparing objects ' size*	*Memorizing sequences*	***Degree of uncertainty in the entropy variation***	***QoI***	***DoC***
1	very easy	⊥	hard	hard	very hard	not so easy	**[0.27, 0.43]**	**0.833**	**0.833**
…	…	…	…	…	…	…	**…**	**…**	**…**
n	easy	not so easy	not so easy	very hard	hard	very hard	**[0.27, 0.27]**	**1**	**1**

Fig. 1. A knowledge based fragment of an extension of the relational database for the different types of dyscalculia's screening.

Table 1. Overall degree of uncertainty in the dyscalculia assessment.

Overall Dyscalculia Assessment								
Child Assessment	*Lexical Dyscalculia*	*Verbal Dyscalculia*	*Graphical Dyscalculia*	*Operational Dyscalculia*	*Practognostic Dyscalculia*	*Ideagnostic Dyscalculia*	*Degree of Uncertainty Worst Scenario*	*Degree of Uncertainty Best Scenario*
1	[0.17, 0.54]	[0.33, 0.70]	[0.24, 0.52]	[0.17, 0.60]	[0.18, 0.50]	[0.27, 0.43]	0.975	0.99989
…	…	...	…	…	…	…	…	…
n	[0.23, 0.29]	[0.37, 0.37]	[0.20, 0.20]	[0.37, 0.37]	[0.25, 0.25]	[0.27, 0.27]	0.99946	0.99957

Therefore, one may have:

$$\text{Entropy Worst Scenario} = 0.54 \times 0.70 \times 0.52 \times 0.60 \times 0.50 \times 0.43 = 0.025$$

$$Degree\ of\ Uncertainty\ Worst\ Scenario = 1 - 0.025 = 0.975$$

$$\text{Entropy Best Scenario} = 0.17 \times 0.33 \times 0.24 \times 0.17 \times 0.18 \times 0.27 = 0.00011$$

$$Degree\ of\ Uncertainty\ Best\ Scenario = 1 - 0.00011 = 0.99989$$

i.e., the ***degree of uncertainty*** with respect to the 1st child listed in Table 1 and in terms of the different types of dyscalculia that he/she may present is set in the interval $0.975 \ldots 0.99989$. Therefore, the degree of uncertainty for the children' set will be given in the form:

$$DegreeOfUncertaintyWorstScenario = \frac{\sum_{i=1}^{N} \left(1 - \prod_{j=1}^{K} Maximum_{area_j}\right)_i}{N}$$

$$DegreeOfUncertaintyBestScenario = \frac{\sum_{i=1}^{N} \left(1 - \prod_{j=1}^{K} Minimum_{area_j}\right)_i}{N}$$

where *K* and *N* stand, respectively, for the cardinality of the set of the different types of dyscalculia and the cardinality of the children`s set.

4 Conclusions

LD can affect neurocognitive processes and may manifest as an imperfect ability to listen, speak, read, spell, write, reason, concentrate, solve mathematical problems, or organize information. It may interfere with children reaching their full potential. In particular, the inability to read and comprehend is a major obstacle to learning that may have long-term educational, social, and economic implications. Teaching children with reading difficulties is a challenge for the student, parents, and educators.

Indeed, *LD* has become a major cause of concern. This in itself shows the increased attention paid to improving safety of the most vulnerable people on the society, our children. Thus, considering our previous studies on this subject [10], it becomes essential to accommodate the system with the ability to reason on data that may be unknown, incomplete or even self-contradictory. One`s approach not only proved successful in such a task, but also explain workings on qualitative data. Considering how social factors may put children at risk, we have gone further and yield result on data from a set of immaterial variables that glimpse social perception and how to expose children. Focusing on such attributes, which may be indicative of dissimilarities in the cognition arena, we were able to quantify the degree of disorder, i.e., its level of entropy. Undeniably, it was set the fundamentals of a *Function Machine* based on a *Deep Learning* approach to *Knowledge Representation and Reasoning*. This is extremely valuable as it strengths the systems' confidence, reliability, reduces unpredictability and ensures stability among all the actors. It is now possible to start employing different *AI* based techniques, such as *Artificial Neural Networks* or *Case-Based Reasoning* for the construction of predictive models to handle such situations. Such models will be essential to reveal possible dangerous situations and behaviours, allowing the enhancement of the *children`s natural development*.

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