ZJU

summer

camp

$\mathrm{Proj}(h) \circ r^H - \mathrm{Proj}(t) \circ r^T$

$t, h, r^T, r^H \in \mathbb{C}$ [证明仅考虑一维]

$\mathrm{Proj}(\cdot)$ 仅在传递性关系生效，在其它关系中不生效，退化为：

$$h \circ r^H - t \circ r^T$$

1. Symmetry / Anti-Symmetry

$(e_1, r, e_2) \quad (e_2, r, e_1)$

$e_1 \circ r^H = e_2 \circ r^T \quad e_2 \circ r^H = e_1 \circ r^T$

两式相乘，得：

$$(e_1 \circ r^H) \circ (e_2 \circ r^H) = (e_2 \circ r^T) \circ (e_1 \circ r^T)$$

$$\therefore r^H \circ r^H = r^T \circ r^T$$

反之 $r^H \circ r^H \neq r^T \circ r^T$

证毕

2. Inversion

$$(e_1, r_1, e_2) \quad (e_2, r_2, e_1)$$

$$e_1 \circ r_1^H = e_2 \circ r_1^T \quad e_2 \circ r_2^H = e_1 \circ r_2^T$$

两式相乘得：

$$r_1^H \circ r_2^H = r_1^T \circ r_2^T$$

3. Composition

$$(e_1, r_1, e_2) \ (e_2, r_2, e_3) \ (e_1, r_3, e_3)$$

$$e_1 \circ r_1^H = e_2 \circ r_1^T \quad e_2 \circ r_2^H = e_3 \circ r_2^T$$

$$e_1 \circ r_3^H = e_3 \circ r_3^H$$

三式相乘得：

$$r_1^T \circ r_2^T \circ r_3^H = r_1^H \circ r_2^H \circ r_3^T$$

4. Subrelation

$(h, r_1, t) \longrightarrow (h, r_2, t)$

example:

父亲$(h,t) \longrightarrow$ 家人(h,t)

目标: Score(家人) > Score(父亲)

证:

$$h \circ r_1^H = t \circ r_1^T \qquad h \circ r_2^H = t \circ r_2^T$$

$$\therefore \frac{h}{t} = \boxed{\frac{r_1^T}{r_1^H} = \frac{r_2^T}{r_2^H}} \qquad \frac{r_2^T}{r_1^T} = \frac{r_2^H}{r_1^H} = \alpha$$

$\alpha = x + yi$

$\sqrt{x^2+y^2} \leq 1$

$$f_{r_2}(h,t) - f_{r_1}(h,t)$$

$$= -\|h \circ r_2^H - t \circ r_2^T\| + \|h \circ r_1^H - h \circ r_1^T\|$$

$$= \|h \circ r_1^H - t \circ r_1^T\| - \|\underbrace{\alpha \circ (h \circ r_1^H - h \circ r_1^T)}_{\text{缩放} \leq 1 \text{即可}}\|$$

$$\geq 0$$

5. Multiple

$i \in [1, N] \quad (h, r_i, t)$

$$h \circ r_i^H = t \circ r_i^T$$

$$\frac{h}{t} = \frac{r_i^T}{r_i^H} = K$$

6. Transitivity

$$e_1 \xrightarrow{r} e_2 \xrightarrow{r} \cdots \xrightarrow{r} e_N$$

$$e_1 \circ r^H = e_2 \circ r^T = e_3 \circ r^T = \cdots = e_N \circ r^T$$

$$e_2 \circ r^H = e_3 \circ r^T = \cdots = e_N \circ r^T$$

$$\vdots$$

$$e_{N-1} \circ r^H = e_N \circ r^T$$

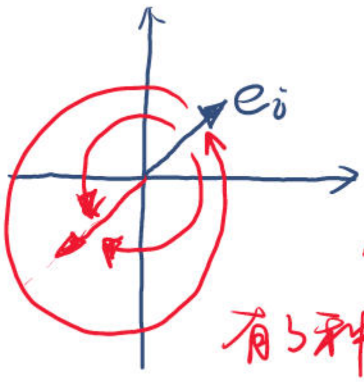
相等.

$$\therefore \underbrace{e_i \circ r^H = e_i \circ r^T}_{①} = e_j \circ r^T \quad i<j$$

(with ② denoting $e_i \circ r^T = e_j \circ r^T$)

由②得 $e_i = e_j$

由①得



$\therefore r^H = k_1\pi$

$r^T = k_2\pi$

有5种旋转

↓加入投影

即

$$Pro(h) \circ r^H - Pro(t) \circ r^T$$

$$= \underline{M_\theta \begin{bmatrix} x \\ y \end{bmatrix}} \circ r^H - \underline{M_\theta \begin{bmatrix} m \\ n \end{bmatrix}} \circ r^T$$

$M_\theta \begin{bmatrix} x \\ y \end{bmatrix} = M_\theta \begin{bmatrix} m \\ n \end{bmatrix}$ 有无穷多组解

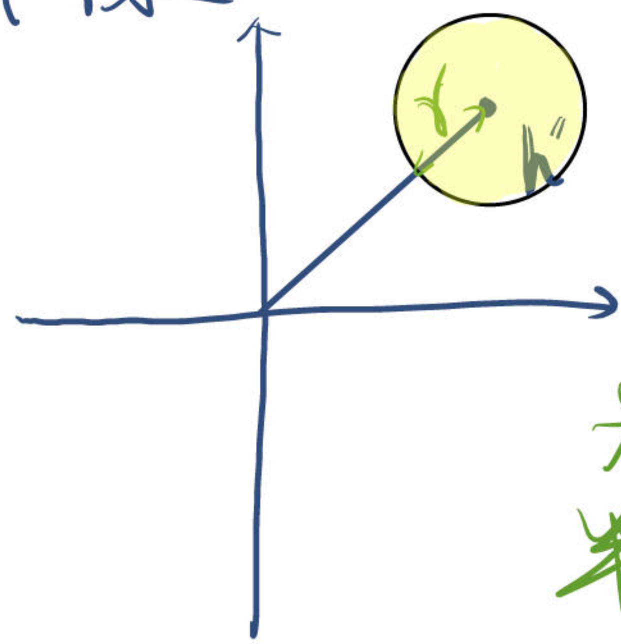
$\therefore$ 证毕

1、1-N

(h, r, S_t) $S_t = \{t_1, \dots, t_k\}$

$\|\underbrace{h \circ r_H} - t_i \circ r_T\| < \gamma$

h' 固定



h'为圆上的点，

则 $t_i \circ r_T$ 的解

为以 h'为圆心、γ为

半径内的黄色区域

的点。

证毕