a) $A = \begin{pmatrix} 1 & -2 & 3 \\ -5 & 4 & 1 \\ 2 & -1 & 3 \end{pmatrix}$

$v_1 = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} + \operatorname{sgn}(1)\sqrt{1^2+5^2+2^2}\,\sqrt{30}\,\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+\sqrt{30} \\ -5 \\ 2 \end{pmatrix}$

$\frac{2}{(1+\sqrt{30})^2+5^2+2^2}\begin{pmatrix} 1+\sqrt{30} \\ -5 \\ 2 \end{pmatrix}\begin{pmatrix} 1+\sqrt{30} & -5 & 2 \end{pmatrix}\begin{pmatrix} 1+\sqrt{30} \\ -5 \\ 2 \end{pmatrix} = \frac{2}{60+2\sqrt{30}}$

$= \frac{2}{60+2\sqrt{30}}\begin{pmatrix} 31+2\sqrt{30} & -5-5\sqrt{30} & 2+2\sqrt{30} \\ -5-5\sqrt{30} & 25 & -10 \\ 2+2\sqrt{30} & -10 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1.182574186 & -0.912870232 & 0.365148372 \\ -0.912870232 & 0.704677426 & 0.281870304 \\ 0.365148372 & -0.281870304 & 0.112748352 \end{pmatrix}$

$Q_1 = I - V = \begin{pmatrix} -0.182574186 & 0.912870232 & -0.365148372 \\ 0.912870232 & 0.295322574 & 0.281870304 \\ 0.365148372 & 0.281870304 & 0.887251648 \end{pmatrix}$

$Q_1 A = \begin{pmatrix} -5.477 & 4.382 & -0.730 \\ 0 & -0.526 & 3.880 \\ 0 & 0.3705 & 1.848 \end{pmatrix}$

$a_2 = \operatorname{sgn}(-5)\cdot\sqrt{0.526^2+0.3705^2} = -1.341$

$v_2 = \begin{pmatrix} -0.526 \\ 0.3705 \end{pmatrix} - 1.341\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2.267 \\ 0.3705 \end{pmatrix}$

$\frac{2vv^t}{v^tv} = \frac{2}{(-2.267)^2+0.3705^2}\begin{pmatrix} -2.267 \\ 0.3705 \end{pmatrix}\begin{pmatrix} -2.267 & 0.3705 \end{pmatrix} = \frac{2}{6.081}\begin{pmatrix} 5.1392 & -2.2001 \\ -2.2001 & 0.5973 \end{pmatrix}$

$= \begin{pmatrix} 1.6902 & -0.7236 \\ -0.7236 & 0.3098 \end{pmatrix}$

$\tilde{Q}_2 = I - \frac{2vv^t}{v^tv} = \begin{pmatrix} -0.6902 & 0.7236 \\ 0.7236 & 0.6902 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.6902 & 0.7236 \\ 0 & 0.7236 & 0.6902 \end{pmatrix} = Q_2$

$Q_2 \cdot Q_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.6902 & 0.7236 \\ 0 & 0.7236 & 0.6902 \end{pmatrix}\cdot\begin{pmatrix} -5.477 & 4.382 & -0.730 \\ 0 & -0.526 & 3.880 \\ 0 & 0.3705 & 1.848 \end{pmatrix}$

$= \begin{pmatrix} -5.477 & 4.382 & -0.730 \\ 0 & 1.341 & -1.341 \\ 0 & 0 & 4.084 \end{pmatrix} = R$

$Q_2 Q_1 A = R \iff Q_2^T Q_2 Q_1 A = Q_2^T R \iff A = \underbrace{Q_1^T \cdot Q_2^T}_{Q} \cdot R$

$Q = Q_1^T \cdot Q_2^T = \begin{pmatrix} -0.1826 & 0.9129 & -0.3651 \\ 0.9129 & 0.2953 & 0.2819 \\ -0.3651 & 0.2819 & 0.8873 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.6902 & 0.7236 \\ 0 & 0.7236 & 0.6902 \end{pmatrix}$

$= \begin{pmatrix} -0.1826 & -0.8943 & 0.4082 \\ 0.9129 & 0 & 0.4082 \\ -0.3651 & 0.4472 & 0.8170 \end{pmatrix}$

b) $R\cdot x = Q^T b$

$Q^T = \left(\begin{array}{ccc|c} -0.1826 & 0.9129 & -0.3651 & -0.1826+8.2161-1.8255 = 6.208 \\ -0.8943 & 0 & 0.4472 & -0.8943+2.2375 = 1.3432 \\ 0.4082 & 0.4082 & 0.8170 & 0.4082+3.6756+4.085 = 8.1688 \end{array}\right)$ (b = (1, 5, 5))

$R\cdot x = b$

$\left(\begin{array}{ccc|c} -5.477 & 4.382 & -0.730 & 6.208 \\ 0 & 1.341 & -1.341 & 1.3432 \\ 0 & 0 & 4.084 & 8.1688 \end{array}\right)$ (8.1688 ↔ b)

$x_3 = \frac{8.1688}{4.084} = 2$

$x_2 = \frac{1.3432+2(1.341)}{1.341} = 3.002$

$x_1 = \frac{6.208-3.002(4.382)+2(0.730)}{-5.477} = 1.002$

$x = \begin{Bmatrix} 1.002 \\ 3.002 \\ 2 \end{Bmatrix}$