

$$\begin{aligned} a) \quad & 20'000x_1 + 30'000x_2 + 10'000x_3 = 5'200'000 \\ & 10'000x_1 + 17'000x_2 + 6'000x_3 = 3'000'000 \\ & 2'000x_1 + 3'000x_2 + 2'000x_3 = 760'000 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 20 & 30 & 10 & 5200 \\ 10 & 17 & 6 & 3000 \\ 2 & 3 & 2 & 760 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 2 & 3 & 2 & 760 \\ 10 & 17 & 6 & 3000 \\ 20 & 30 & 10 & 5200 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 2 & 3 & 2 & 5200 \\ 0 & 2 & -4 & -800 \\ 0 & 0 & -10 & -2400 \end{array} \right) \begin{array}{l} :2 \\ :10 \end{array}$$

$$\left(\begin{array}{ccc|c} 2 & 3 & 0 & 4720 \\ 0 & 1 & 0 & 80 \\ 0 & 0 & 1 & 240 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 2 & 3 & 2 & 5200 \\ 0 & 1 & -2 & -400 \\ 0 & 0 & 1 & 240 \end{array} \right) \begin{array}{l} :2 \\ :2 \end{array}$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 4000 \\ 0 & 1 & 0 & 80 \\ 0 & 0 & 1 & 240 \end{array} \right) :2 \quad \begin{array}{l} x_1 = 2000 \\ x_2 = 80 \\ x_3 = 240 \end{array}$$

b) LR-Zerlegung \rightarrow Gauss ohne Pivotierung

$$\left(\begin{array}{ccc} 20 & 30 & 10 \\ 10 & 17 & 6 \\ 2 & 3 & 2 \end{array} \right)$$

$$\begin{array}{l} z_1 \\ z_1 \\ z_3 \end{array} \quad \begin{array}{l} z_1 \\ z_2 - 0.5z_1 \\ z_3 \end{array} \quad \left(\begin{array}{ccc} 20 & 30 & 10 \\ 0 & 2 & 1 \\ 2 & 3 & 2 \end{array} \right) \begin{array}{l} \\ A_1 \\ \end{array}$$

$$\left(\begin{array}{ccc} 20 & 30 & 10 \\ 0 & 2 & 1 \\ 2 & 3 & 2 \end{array} \right)$$

$$\begin{array}{l} z_1 \\ z_2 \\ z_3 - 0.1z_1 \end{array} \quad \left(\begin{array}{ccc} 20 & 30 & 10 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{array} \right) \begin{array}{l} \\ A_2 \\ A_2 \end{array}$$

$$\cdot \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) = \left(\begin{array}{ccc} 20 & 30 & 10 \\ 10 & 17 & 6 \\ 2 & 3 & 2 \end{array} \right) \begin{array}{l} \\ L_1 \\ \end{array}$$

$$\cdot \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.1 & 0 & 1 \end{array} \right) = \left(\begin{array}{ccc} 20 & 30 & 10 \\ 0 & 2 & 1 \\ 2 & 3 & 2 \end{array} \right) \begin{array}{l} A \\ \\ A_1 \end{array}$$

$$\left(\begin{array}{ccc} 20 & 30 & 10 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{array} \right) \begin{array}{l} \\ \\ R \end{array}$$

$$\cdot \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.1 & 0 & 1 \end{array} \right) = \left(\begin{array}{ccc} 20 & 30 & 10 \\ 10 & 17 & 6 \\ 2 & 3 & 2 \end{array} \right) \begin{array}{l} L \\ \\ A \end{array}$$

$$c) \quad \left(\begin{array}{ccc|c} 20 & 30 & 10 & 5720 \\ 10 & 17 & 6 & 3300 \\ 2 & 3 & 2 & 836 \end{array} \right)$$

$$\begin{array}{l} z_1 \\ z_2 - 0.5z_1 \\ z_3 - 0.1z_1 \end{array} \quad \left(\begin{array}{ccc|c} 20 & 30 & 10 & 5720 \\ 0 & 2 & 1 & 940 \\ 0 & 0 & 1 & 764 \end{array} \right) \begin{array}{l} \\ R \\ L \end{array}$$

$$L \cdot y = b \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5720 \\ 0.5 & 1 & 0 & 3300 \\ 0.1 & 0 & 1 & 836 \end{array} \right) \begin{array}{l} \\ \\ b \end{array}$$

$$\begin{aligned} y_1 &= 5720 \\ y_2 &= 3300 - 0.5 \cdot 5720 = 940 \\ y_3 &= 836 - 0.1 \cdot 5720 = 264 \end{aligned}$$

$$y = \begin{pmatrix} 5720 \\ 940 \\ 264 \end{pmatrix}$$

$$Rx = y$$

$$\left(\begin{array}{ccc|c} 20 & 30 & 10 & 5720 \\ 0 & 2 & 1 & 940 \\ 0 & 0 & 1 & 264 \end{array} \right) \begin{array}{l} \\ \\ \end{array} \begin{array}{l} \\ \\ -10 \end{array}$$

$$\left(\begin{array}{ccc|c} 20 & 30 & 0 & 3080 \\ 0 & 2 & 0 & 176 \end{array} \right) \begin{array}{l} \\ \\ \end{array} \begin{array}{l} \\ \\ -20 \end{array}$$

$$\begin{pmatrix} 20 & 20 & 0 & | & 3080 \\ 0 & 2 & 0 & | & 176 \\ 0 & 0 & 1 & | & 264 \end{pmatrix} \xrightarrow{2} \begin{pmatrix} 20 & 20 & 0 & | & 3080 \\ 0 & 1 & 0 & | & 88 \\ 0 & 0 & 1 & | & 264 \end{pmatrix} \xrightarrow{-20}$$

$$\downarrow$$

$$\begin{pmatrix} 20 & 0 & 0 & | & 940 \\ 0 & 1 & 0 & | & 88 \\ 0 & 0 & 1 & | & 264 \end{pmatrix}$$

$$\begin{aligned} x_1 &= 22 \\ x_2 &= 88 \\ x_3 &= 264 \end{aligned}$$