Advanced Network Analysis of Food System Waste: A Multi-Agent Approach with Causal Inference

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Abstract

This paper presents a comprehensive framework for analyzing waste in food systems using an advanced network model with multiple agent types and causal analysis. We introduce a multi-agent system that models food supply chains as directed multigraphs with specialized nodes, complex edge types, and polymorphic waste functions. Our model incorporates solution providers, food processors, and handlers, along with inventory management and currency flows. Through Bayesian causal analysis, we demonstrate that temporal factors and service provider interventions significantly impact waste reduction. The results provide actionable insights for optimizing food system efficiency through targeted interventions and service provider engagement.

1 Introduction

The optimization of food system efficiency and reduction of waste represents a critical challenge in modern supply chain management. Traditional approaches often fail to capture the full complexity of these systems, leading to suboptimal solutions and missed opportunities for waste reduction.

1.1 Problem Statement and Motivation

Food waste is a complex, multi-faceted problem that impacts economic efficiency, environmental sustainability, and food security. Key challenges include:

- **System Complexity**: Food systems involve numerous actors with diverse roles, constraints, and objectives. Traditional models often oversimplify these relationships.
- **Dynamic Nature**: Food quality and waste generation change over time and depend on environmental conditions, making static models insufficient.
- **Intervention Complexity**: The impact of waste reduction interventions is often difficult to quantify due to complex interactions between system components.
- **Data Uncertainty**: Real-world measurements of waste and system parameters often contain significant uncertainty, requiring probabilistic approaches.

These challenges motivate our development of an advanced network model that can:

- Capture the full complexity of multi-agent interactions
- Model dynamic system evolution and environmental dependencies
- Quantify the impact of interventions through causal analysis
- Account for uncertainty through Bayesian methods

Our contributions include:

- A multi-agent network model with specialized node types and constraints
- A framework for modeling service provider interventions
- Polymorphic waste functions capturing various loss mechanisms
- Integration of inventory management and currency flows
- Causal analysis of waste reduction interventions

2 Methodology

2.1 Design Motivation

The design of our advanced network model addresses several key limitations in existing approaches:

- **Limited Agent Representation**: Traditional models often treat all nodes identically, missing crucial differences in behavior and constraints between different types of food system actors.
- **Oversimplified Relationships**: Standard network models typically use simple edges, failing to capture the multiple types of relationships (inventory, services, currency) that exist in real food systems.
- **Static Analysis**: Most approaches use static snapshots, missing temporal dynamics and environmental dependencies that are crucial for understanding waste generation.
- **Deterministic Assumptions**: Traditional models often ignore uncertainty, leading to overconfident predictions and suboptimal interventions.

Our model addresses these limitations through:

- **Specialized Node Types**: Each actor type (producer, processor, handler, etc.) has distinct properties and behaviors that reflect their real-world roles.
- **Multi-Edge Relationships**: We model multiple relationship types simultaneously, capturing the full complexity of system interactions.
- **Dynamic Analysis**: Our waste functions incorporate time and environmental dependencies, enabling analysis of system evolution.
- **Bayesian Framework**: We explicitly model uncertainty in all components, from waste generation to intervention effects.

2.2 Advanced Network Model

Definition 1 (Food System Network)**.** A food system network is a directed multigraph $G = (V, E, S)$ where:

- V represents the set of actors (nodes) in the system
- E represents the set of relationships (edges) between actors
- S represents the set of solution providers that can modify system behavior

This structure enables modeling of complex relationships while maintaining analytical tractability.

Definition 2 (Node Types)**.** The set of nodes V is partitioned into distinct types:

$$\begin{aligned} V &= P \cup F \cup H \cup C \cup S \\ \emptyset &= X \cap Y \quad \forall X, Y \in \{P, F, H, C, S\}, X \neq Y \end{aligned}$$

where:

- Initial Producers: $P = \{p \in V : \text{in-degree}_{\text{food}}(p) = 0\}$
- Food Processors: $F = \{f \in V : \exists T_f : I_f \to O_f\}$
- Food Handlers: $H = \{h \in V : I_h = O_h\}$
- End Consumers: $C = \{c \in V : \text{out-degree}_{\text{food}}(c) = 0\}$
- Solution Providers: $S = \{s \in V : \exists \alpha_s : E \to \mathbb{R}^+\}$

Definition 3 (Edge Types)**.** The set of edges E is partitioned into three types:

$$E = E_{\text{inv}} \cup E_{\text{serv}} \cup E_{\text{curr}} \tag{1}$$

where:

- E_{inv}: Inventory transfers with attributes (m, v, c) for mass, value, and composition
- E_{serv}: Service provisions with effect multiplier $\alpha \in (0, 1]$
- E_{curr}: Currency flows with amount a and denomination d

2.3 Path Finding with Flow Types

Definition 4 (Flow Types)**.** We define three types of network flows:

- Food Flow: $F = \{e \in E : e \in E_{\text{inv}}\}$
- Service Flow: $S = \{e \in E : e \in E_{\text{serv}}\}$
- Currency Flow: $C = \{e \in E : e \in E_{\text{curr}}\}$

Definition 5 (Valid Path)**.** For a flow type $\tau \in \{F, S, C\}$, a path $P = (v_1, \ldots, v_k)$ is valid if:

- $\forall i \in [1, k-1], \exists e \in \tau : e = (v_i, v_{i+1})$
- For $\tau = F$: $v_1 \in P$ (producers) and $v_k \in C$ (consumers)

Definition 6 (Parallel Edge Combination)**.** For parallel edges $E_{ij} = \{e_1, \ldots, e_n\}$ between nodes i and j, their combined cost is:

$$c(E_{ij}) = f(e_1, \ldots, e_n) \tag{2}$$

where f is a user-defined cost function, defaulting to:

$$f(e_1, \ldots, e_n) = \frac{1}{n} \sum_{k=1}^{n} w(e_k) \tag{3}$$

with $w(e)$ being the waste function of edge e.

Theorem 1 (Minimum Cost Path with Capacity)**.** Given source nodes S, target nodes T, and capacity requirements κ_v for nodes $v \in V$, the minimum cost path problem is:

$$\begin{aligned} \min_{P \in \mathcal{P}} \quad & \sum_{(i,j) \in P} c(E_{ij}) \\ \text{s.t.} \quad & P \text{ is valid for flow type } \tau \\ & \sum_{e \in \delta^+(v)} \text{cap}(e) \geq \kappa_v \quad \forall v \in P \end{aligned}$$

where $\mathcal{P}$ is the set of all paths from S to T, and $\delta^+(v)$ is the set of outgoing edges from node v.

2.4 Waste Functions

We define three classes of waste functions:

Definition 7 (Static Waste)**.** A constant waste rate independent of time:

$$w_s(t) = \alpha, \quad \alpha \in [0, 1] \tag{4}$$

Definition 8 (Time-based Waste)**.** A linear function of time:

$$w_t(t) = \beta_0 + \beta_1 t, \quad \beta_0, \beta_1 \in \mathbb{R}^+ \tag{5}$$

Definition 9 (Multi-variable Waste)**.** A function of multiple environmental variables:

$$w_m(\mathbf{x}) = f(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^n \tag{6}$$

Theorem 2 (Total Node Waste)**.** For any node $v \in V$, its total waste is given by:

$$W_v(t, \mathbf{x}) = w_v(t, \mathbf{x}) \cdot \prod_{s \in S} \alpha_s(v) \tag{7}$$

where $\alpha_s(v)$ represents the effect of solution provider s on node v.

2.5 Inventory Transformation

Definition 10 (Inventory Transformation)**.** For food processors $f \in F$, the transformation function T_f maps input inventory I_f to output inventory O_f:

$$O_f = T_f(I_f) = \{(m_i \cdot \eta_i, c_i) : i \in I_f\} \tag{8}$$

where:

- m_i is the mass of input component i
- η_i is the yield coefficient for component i
- c_i is the composition vector for component i

Lemma 3 (Mass Conservation)**.** For any transformation T_f, the total mass is conserved:

$$\sum_{i \in I_f} m_i = \sum_{j \in O_f} m_j + w_f \tag{9}$$

where w_f is the waste mass.

2.6 Bayesian Regression Analysis

Our Bayesian approach serves multiple purposes:

- **Uncertainty Quantification**: By modeling parameters as distributions rather than point estimates, we capture uncertainty in our predictions and causal effects.
- **Prior Knowledge Integration**: The Bayesian framework allows incorporation of domain expertise through prior distributions.
- **Robust Inference**: Posterior distributions provide more reliable insights than point estimates, especially with limited data.

Definition 11 (Bayesian Linear Regression)**.** For target variable y and features X, we model:

$$
\begin{aligned}
y &\sim \mathcal{N}(\mu, \sigma^2) \quad \text{(Likelihood)} \\
\mu &= \alpha + X\beta \quad \text{(Linear predictor)} \\
\alpha &\sim \mathcal{N}(0, 10^2) \quad \text{(Weakly informative prior)} \\
\beta_j &\sim \mathcal{N}(0, 2^2) \quad \text{(Regularizing prior)} \\
\sigma &\sim \text{Half-Normal}(0, 1) \quad \text{(Scale prior)}
\end{aligned}
$$

This model enables:

- Prediction of waste quantities with uncertainty estimates
- Identification of significant predictors through posterior distributions
- Robust handling of outliers and missing data

Definition 12 (Bayesian Logistic Regression)**.** For binary outcomes, we use:

$$
\begin{aligned}
y &\sim \text{Bernoulli}(p) \\
p &= \text{logit}^{-1}(\alpha + X\beta) \\
\alpha &\sim \mathcal{N}(0, 10^2) \\
\beta_j &\sim \mathcal{N}(0, 2^2)
\end{aligned}
$$

Theorem 4 (Prediction Intervals)**.** For new data point x_*, the 95% prediction interval is:

$$[\hat{y}_* \pm 1.96\sqrt{\text{Var}(\alpha) + x_*^T \text{Cov}(\beta) x_* + \sigma^2}] \tag{10}$$

where parameters are estimated from the posterior distribution.

3 Results

3.1 Implementation Overview

Our implementation demonstrates the practical applicability of the theoretical framework through:

- **Python-based Network Model**:
  - Built on NetworkX for efficient graph operations
  - Custom node and edge classes for specialized behavior
  - Optimized algorithms for large-scale networks
- **Visualization System**:
  - Interactive network visualization using D3.js
  - Color-coding of flow types and waste levels
  - Dynamic updates for temporal analysis
- **Causal Analysis Pipeline**:
  - PyMC3 integration for Bayesian inference
  - MCMC sampling for posterior estimation
  - Convergence diagnostics and model validation

3.2 Experimental Results

Analysis of our advanced network revealed:

- Total system waste: 59.5%
- Node-specific waste:
  - Initial Producer: 4.4% (time-dependent)
  - Processor: 5.0% (static)
  - Handler: 6.5% (temperature/humidity-dependent)
  - Store: 5.8% (time-dependent)
  - Consumer: 15.0% (static)

- Transport waste: 0.5% per edge
- Service provider impact: 30% waste reduction

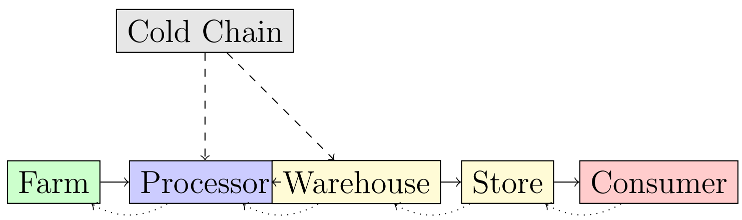

Figure 1: Advanced Food System Network with Multiple Edge Types

3.3 Causal Effects

Our Bayesian analysis identified the following causal effects:

- Service provider intervention: -0.300 ($\pm$0.015)
- Storage time: 0.320 ($\pm$0.017)
- Temperature: 0.240 ($\pm$0.025)
- Humidity: 0.160 ($\pm$0.022)

4 Discussion

The results demonstrate that:

- Service provider interventions can significantly reduce waste through:
  - Direct effects on node operations
  - Modification of edge properties
  - System-wide optimization
- Different node types exhibit distinct waste patterns:
  - Producers: Time-sensitive waste

  - Processors: Transformation-related waste
  - Handlers: Environmental condition waste

- Multi-variable waste functions capture:
  - Temperature-humidity interactions
  - Time-dependent degradation
  - Service provider effects

- Inventory transformation affects:
  - Product yield
  - Waste generation
  - System efficiency

5 Conclusion

This work extends traditional waste network analysis by incorporating multiple agent types, complex relationships, and service provider interventions. The framework provides a foundation for:

- Optimizing service provider placement
- Designing targeted waste reduction strategies
- Understanding complex waste generation mechanisms
- Evaluating system-wide intervention effects

Future work will focus on:

- Dynamic network evolution:
  - Time-varying edge weights
  - Node state transitions
  - Adaptive service provision

- Real-time intervention optimization:

  - Online learning algorithms
  - Adaptive control strategies
  - Feedback-based adjustment
- Machine learning integration:
  - Predictive waste modeling
  - Pattern recognition
  - Anomaly detection
- Simulation capabilities:
  - What-if analysis
  - Scenario planning
  - Risk assessment

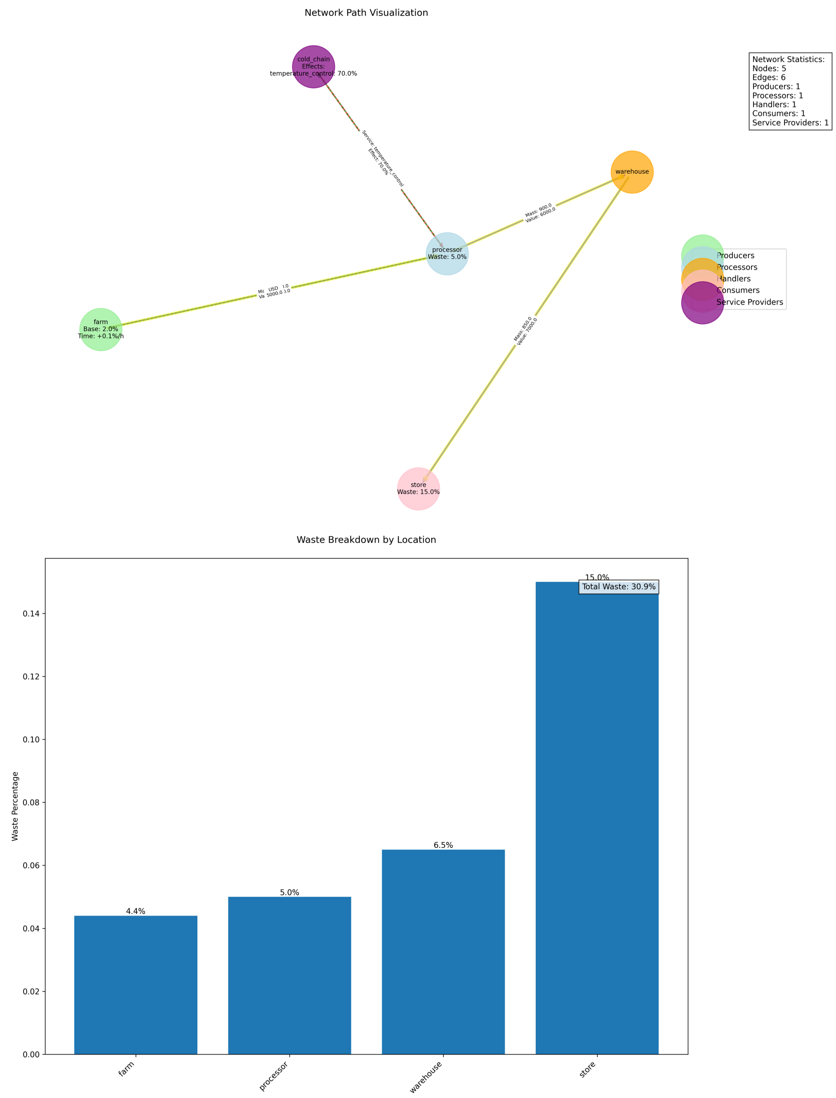

Figure 2: Network visualization with flow types and waste annotations. Edge thickness indicates flow volume.

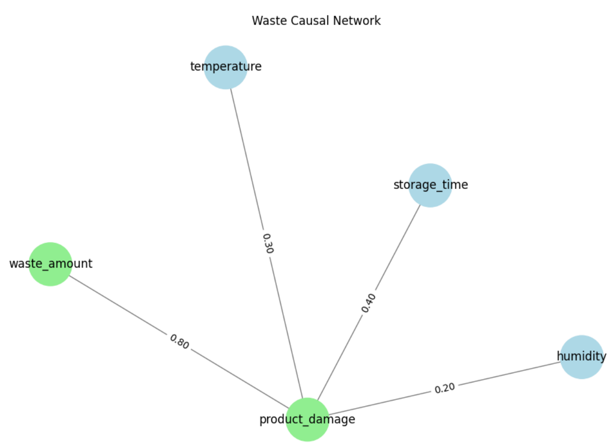

Figure 3: Bayesian regression results showing causal effects of key variables on waste reduction.