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SLIP leg and ankle trajectory optimization problem statement

$$\begin{aligned} \underset{X}{\text{Minimize}} \quad & W_{actuators}(X) \\ \text{subject to} \quad & g_i(X) = 0,\ i = 1, \ldots, m. \\ & f_i(X) \leq 0,\ i = 1, \ldots, n. \\ & X_{low} \leq X \leq X_{high} \end{aligned}$$

where

$W_{actuators}(X) = W_{leg,m}(X) + W_{leg,e}(X) + W_{ankle,m}(X) + W_{leg,e}(X)$

where

$$W_{leg,m}(X) = \int_0^{t_F} \max(0,\ \tau_{leg}(X)\ n\ \dot{r}_0(X)\ dt$$

$$W_{leg,e}(X) = \int_0^{t_F} R_{leg}\ \tau_{leg}(X)^2\ dt$$

$$W_{ankle,m}(X) = \int_0^{t_F} \max(0,\ \frac{\tau_{ankle}(X)}{r(X)}\ v(X))\ dt$$

$$W_{ankle,e}(X) = \int_0^{t_F} R_{ankle}\ \tau_{ankle}(X)^2\ dt$$

Equality constraints
Initial conditions

$$g_1(X) = r(0) - r_{0,initial} \tag{1}$$
$$g_2(X) = r_0(0) - r_{0,initial} \tag{2}$$
$$g_3(X) = \dot{r}_0(0) \tag{3}$$
$$g_4(X) = \dot{x}(0) - \dot{x}_{initial\ apex} \tag{4}$$

Equilibrium gait conditions

$$g_5(X) = g \cdot (y_{initial\ apex} - y(0)) - \frac{1}{2} \cdot \dot{y}(0)^2 \tag{5}$$
$$g_6(X) = \dot{x}(t_F) - \dot{x}_{final\ apex} \tag{6}$$
$$g_7(X) = g \cdot (y_{finalapex} - y(t_F)) - \frac{1}{2}\dot{y}_{LO}^2 \tag{7}$$

End of stance condition

$$g_8(X) = r_0(t_F) - r(t_F) \tag{8}$$

Additional condition for analysis

$$g_9(X) = atan2(y(0),\ x(0)) - \theta_{TD} \tag{9}$$

Inequality constraints
Maximum and minimum ankle torque conditions

$$f_1(X) = T_{ankle}(t_k) - T_{max,\ ankle}(t_k) \tag{10}$$
$$f_2(X) = -(T_{ankle}(t_k) + T_{max,\ ankle}(t_k)) \tag{11}$$

where

$$T_{max,\ ankle}(t_k) = \frac{l_f\ k\ y(t_k)(r_0(t_k) - r(t_k)}{2\ r(t_k)} \tag{12}$$
$$r(X) = \sqrt{x(t_k)^2 + y(t_k)^2} \tag{13}$$
$$X = \begin{bmatrix} x_0 & \dots & x_{t_k} & \dots & x_{t_F} \\ y_0 & \dots & y_{t_k} & \dots & y_{t_F} \\ r_{0,0} & \dots & r_{0,t_k} & \dots & r_{0,t_F} \\ \dot{x}_0 & \dots & \dot{x}_{t_k} & \dots & \dot{x}_{t_F} \\ \dot{y}_0 & \dots & \dot{y}_{t_k} & \dots & \dot{y}_{t_F} \\ \dot{r}_{0,0} & \dots & \dot{r}_{0,t_k} & \dots & \dot{r}_{0,t_F} \\ T_{leg,0} & \dots & T_{leg,t_k} & \dots & T_{leg,t_F} \\ T_{ankle,0} & \dots & T_{ankle,t_k} & \dots & T_{ankle,t_F} \\ T_{stance} & 0 & \dots & \dots & 0 \end{bmatrix} \tag{14}$$

(1) Leg length starts at resting spring length
(2) Spring starts at resting spring length
(3) Spring starts with no velocity
(4) Horizontal velocity starts with initial apex velocity
(5) Position of COM at touchdown is along ballistic path from apex state
(6) Horizontal velocity ends with final apex velocity
(7) Position of COM at lift off is along ballistic path to final apex
(8) Stance ends when spring is undeflected
(9) In some scenarios this constraint is added to lock the touchdown angle
(10) Ankle torque must be smaller than max
(11) Ankle torque must greater than min
(12) Max ankle torque is a function of the applied spring force
(13) Short for full length of the spring
(14) Decision variables for collocation