MEMORANDUM

TO: Dr. Siniawski
FROM: Tristan Minter, Nicolai Meiswinkel, Michael Hennessy, Lorenzo Remy
DATE: November 10, 2021
SUBJECT: Cardboard Beam Design

Introduction:
The objective of this project was to construct a beam made entirely of cardboard with a maximum structural efficiency while maintaining outlined specifications. This objective had to be accomplished by carrying out the engineering process, relevant engineering analyses, and calculations discussed below to design, fabricate, test and evaluate the prototype.

Design Process:
The objective of the beam design challenge is to fabricate a beam with highest efficiency of maximum load to weight. The challenge included four design constraints, indicated in table 1: overall nominal length of 16.0 +/- 0.5 inches, beam width of 2.0 +/- 0.125 inches, and a maximum beam height of 4.0 inches. The design of the final prototype maximized all three constraints, indicated in figure 7: a length of 16.0 inches, height of 4.0 inches, and width of 2.0 inches. The final prototype measured to be 16.0 inches in length, 2.001 inches in width, and the height of the front side 4.025 inches, with the height of the back side 3.953 inches. The height constraint was not met, by approximately .025 inches on one side, while the other requirements were met.

Requirement	Nominal Value(in)	Measured Value(in)	Within Requirement?	Percent Error (%)
Nominal Length:	16.0 ∓ .5	16.0	Yes	0
Beam Width:	2.0 ∓ .0125	2.001	Yes	0.0005
Max. Beam Height:	4.0	Side 1: 4.0250 Side 2: 3.953	No	Side 1:0.00625 Side 2:-0.01175

Table 1: Validation of beam design requirements.

The chosen design, as seen in figure 3 and figure 7, was an I-beam with a web width of 0.84 inches and a flange width of 0.56 inches on the top and bottom. Initially, their were four design alternatives for the cross-section of the beam to choose between: an I-beam with a web width of 0.56 inches, an I-beam with a web width of 0.84 inches, a rectangular beam with an ‘X’ cross section through the center, and a square beam with an ‘X’ cross section through the center (figure 2). The square beam alternative was eliminated almost immediately, as the smaller height creates an efficiency significantly less than the rectangular beam. The ensuing calculations for the moment of inertia of each design are as follows: 7.80 inches^4 for the smaller I-beam, 8.35 inches^4

for the larger I-beam, and 8.17 $inches^4$ for the rectangular beam. The ensuing calculations for the surface area of the cross-sections of each design are as follows: 3.96 $inches^2$, 4.76 $inches^2$, and 6.78 $inches^2$. The efficiency for each beam design was then calculated by dividing the moment of inertia by the surface area: 1.97 for the smaller I-beam, 1.75 for the larger I-beam, and 1.21 for the rectangle. Each of the three final design alternatives maximized the full width, height, and length constraints. For extra analysis, the beams were drawn in AutoDesk Fusion 360 in order to perform a stress test. The results of the stress test are seen in figure two, and the material used in the stress test was plastic because cardboard was not an option. Based on the calculated efficiency and the stress test results, the smaller I-beam performed best. However, taking into account the sturdiness and strength of cardboard, which is significantly less than the strength of common beam materials like steel and the plastic from the stress test, the larger I-beam was chosen as the final design. One final choice came with the fabrication of the beam. The options were to glue together all pieces of the beam separately, or to bend parts of the beam at 90 degree angles, so that the beam's support comes from the strength of the cardboard rather than the strength of the glue. Concluding that cardboard is stronger than hot glue, the decision was made to bend the cardboard between the vertical support and the top and bottom supports in order to ensure maximum strength, as seen in figure three. Glue was used to put the top and bottom layer of cardboard on the bent pieces, and one layer of cardboard in between the bent pieces in the vertical support.

Prototype Fabrication:
After reviewing the design analysis matrix, the practicality of physical construction was taken into consideration. The model that utilized 3 pieces of 0.28 inch thick corrugated cardboard in the web of the I Beam was determined to be the most practical and potentially most structurally efficient. Considering the rigidity and malleability of the corrugated cardboard, and the bend from the web on the top and bottom toe of the I beam while maintaining the beam height and width restrictions, the 3 layer web model proved to be the most practical to construct. The necessary dimensional calculations to ensure all parameters were adhered to and a 2D engineering drawing was drafted, as seen in figure 7.

The beam consisted of 5 strips of cardboard, all 16 inches in length. 3 pieces were used in the web, with the centerpiece having a width of 3.44 inches, and the two other web pieces having a width of 4.6 inches to allow for bending of 0.58 inches on either end for reinforcement at the flange. The flange on either end was made up of 1 layer of 2 inch width strips. These pieces were all fused in congruence with figure 7 using hot glue.

Final Design:
A final orthographic drawing of our design can be found in figure 6. Some final design aspects we considered included the orientation of the currigation in the cardboard and the utilization of bending to further strengthen our flanges. We conclude that the corrugation is prone to bending

when orientated perpendicularly to the length of the beam. With this conclusion we decided to orientate the corrugation parallel to the length of the beam. This conclusion also led to our decision to utilize bending to strengthen the flanges of our beam. By cutting slits into the cardboard at the point of the bend we were able to cleanly bend the cardboard without deforming the cardboard forming the flanges. Though we did our best to stay true to the original design specifications, due to the innate inaccuracy of human fabrication we inherited some error. As seen in Table 1 the height of our beam was slightly distorted. This effect was the result of inaccurate bending which left room for deformation when glueing the top and bottom pieces to the beam.

Quality Control:
With the final design of the beam completed, the first component of testing involved conducting a quality control check. This check was meant to validate that the final beam design was within all of the design requirements for length, width, and height as well as determining the weight of the beam.

After analyzing the measurements in table 1 and comparing them to the design constraints, the length and width were almost exactly to specification and both met the requirements for the beam challenge. The height of the beam had more discrepancy in the measurement as the height measured on both sides was different and the first side was 0.025 inches larger than the 4-inch requirement. This likely happened because when cutting the cardboard, a slight angle of the ruler caused one end to exceed the desired height of 4 inches. In future fabrication, the ninety degree angles on the rectangular cardboard pieces will be measured with greater efficiency.

Load Capacity Testing:
We subjected our cardboard beam to a load capacity test using the Instron 4505. Testing parameters include: 10in support span and 0.5in/min extension rate. The test was set up by placing the beam in the instrument (figure 1). As the test proceeded the mounting supports began to move upwards at the selected extension rate. The force on the node and the length of the extension were recorded in 0.1 second intervals. Our load test was performed over a 413.96 second span. As seen in the load vs displacement graph (figure 8) our max load capacity was 180.41713lbs which we achieved at 160.7 seconds and an extension of 1.339in. Using the weight of our beam from the quality control process (166.7g) we can determine the structural efficiency of our beam. Since it was able to hold 180.41713lbs with a weight of 0.368lbs its structural efficiency is 491.

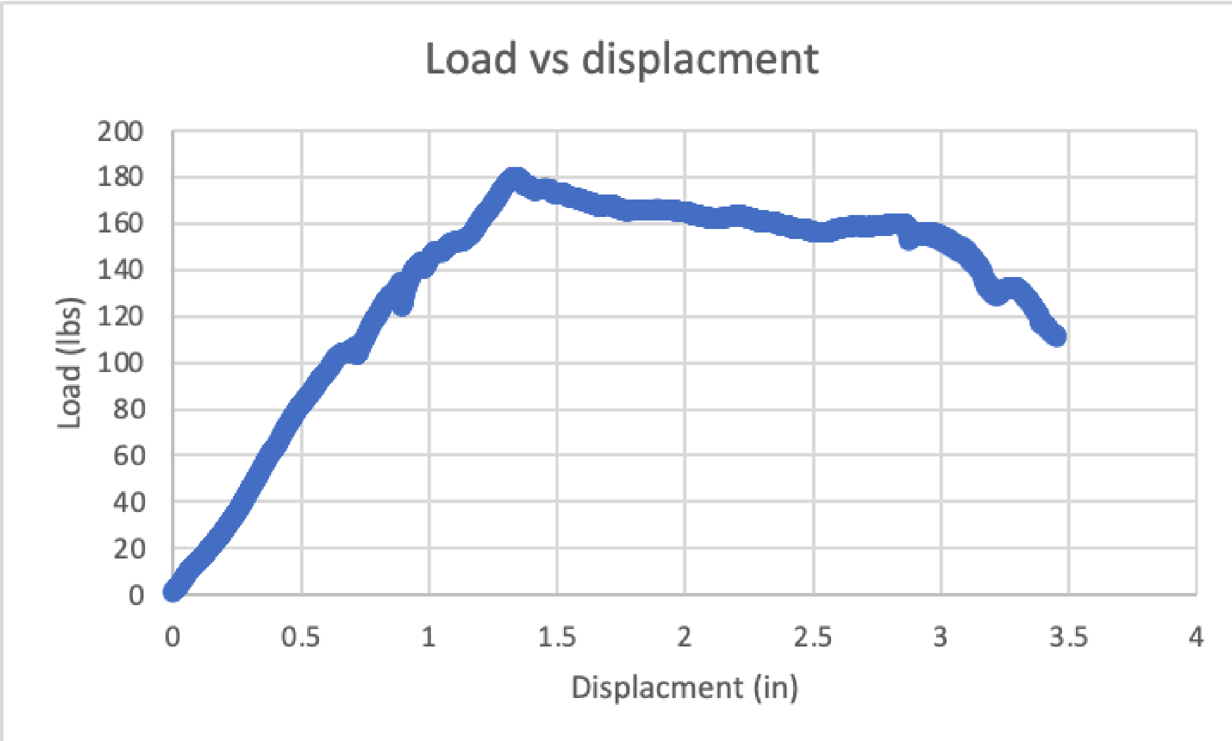

Figure 8: Load vs Displacement graph for load capacity test

Conclusions:

In this module, useful knowledge was gained on the basics of the design process, design evaluation, fabrication processes, and testing beams using a bend test. Throughout these steps, a new perspective was gained in terms of beams and how to work as a team in order to create a successful design. While the goal of 200 pounds using the I-beam design was not reached, and the beam height was .025 inches over the constraint, the beam held up against 180 pounds and stayed relatively intact. These design flaws also showed the importance of creating clear designs so the fabrication process is completed accurately and within the constraints. They also prove that designs should account for a small fabrication error when finalizing measurements, making sure to be extra careful in staying within constraints. After analyzing the effects of weight on the beam, several improvements were suggested. When conducting the test, some of the glue seemed to be lightly applied and poorly secured (refer to figure 4). This caused some of the cardboard pieces to tear apart, which likely contributed to an increased displacement. The top and bottom cardboard layers were also cut with the ridged inner layers facing sideways instead of facing the edges of the beam. This caused the horizontal top and bottom pieces to be bent more easily. This limited the effectiveness of the beam at resisting displacement. When constructing the beam this slight error was overlooked and would be changed if the beam was rebuilt.

Appendices:

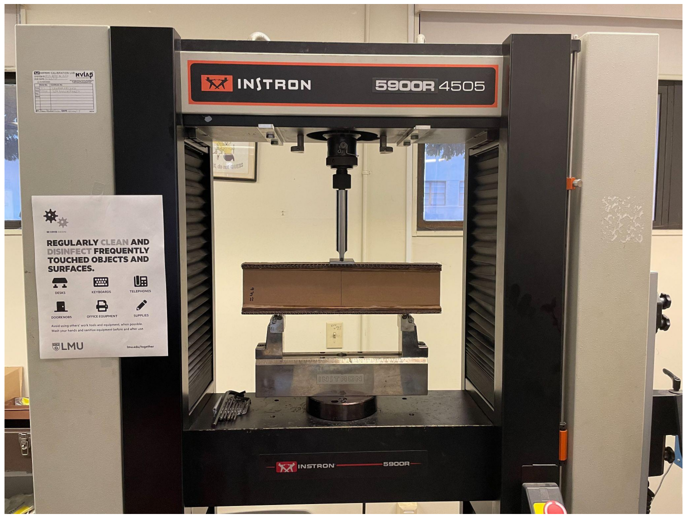

Figure 1: Setup of the beam on the Instron 4505 Test Frame prior to testing.

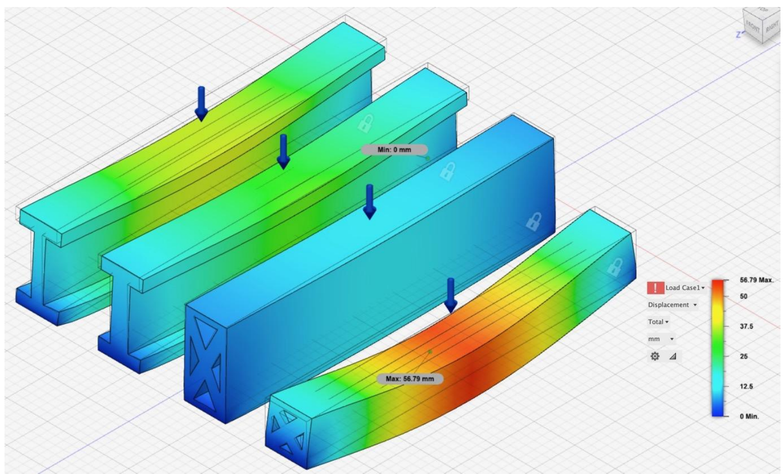

Figure 2: Stress test results from four design alternatives.

Figure 3: Side view of the final fabricated prototype.

Figure 4: Front view of the prototype following testing.

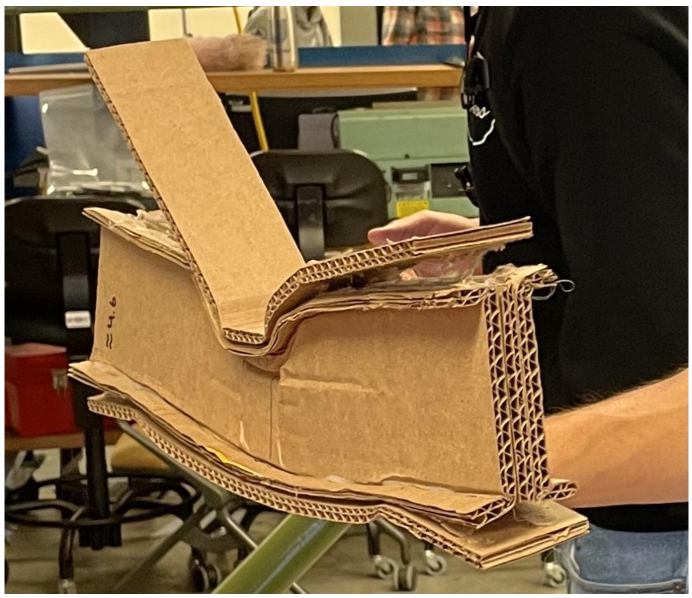

Figure 5: Front side view of the prototype following testing.

Figure 6: Back side view of the prototype following testing.

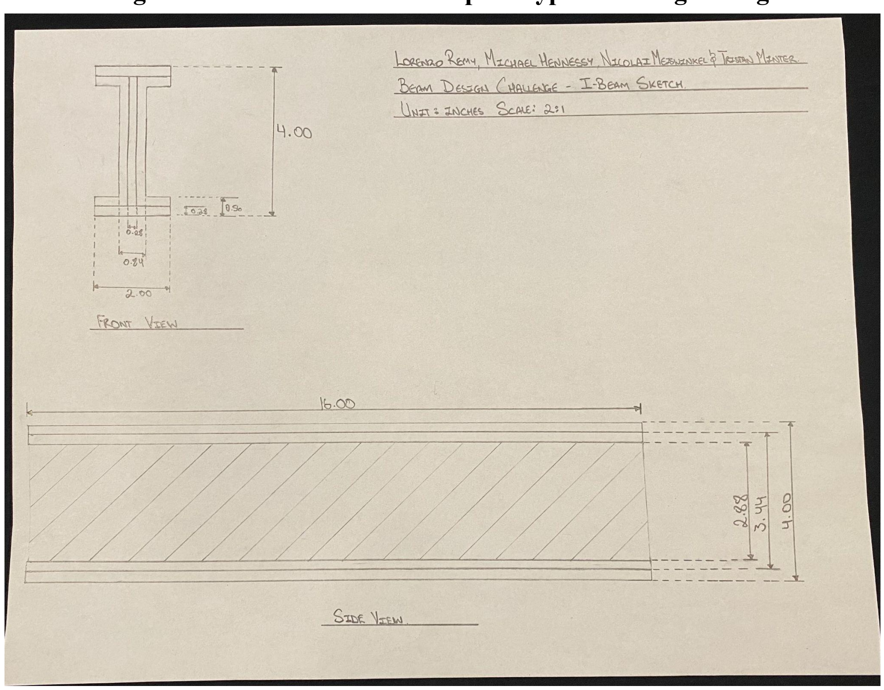

Figure 7: Two-dimensional orthographic drawing of the final design.