MATH 245 Project

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1 Determining the Spring Constant

$$mass = .9 \;\; grams = .009 \;\; kilograms$$

$$L = -20.8 \;\; inches = -.528 \;\; meters$$

$$F_{spring} = -k * L$$

$$\Sigma Force = F_{spring} - F_{gravity}$$

$$\Sigma Force = 0$$

$$0 = F_{spring} - F_{gravity}$$

$$F_{spring} = F_{gravity}$$

$$-k * L = m * g$$

$$-k * -.528 \;\; meters = .009 \;\; kilograms * 9.81 \;\; meters/second^2$$

$$k = .167 \;\; kilograms/second^2$$

2 Solving the IVP

Initial Conditions: $u(0) = -17.8$ inches $u'(0) = 0$ inches/second

$$m * u''(t) + \gamma * u'(t) + k * u(t) = 0$$

$$m * r^2 + \gamma * r + k = 0$$

$$r = \frac{-\gamma \pm \sqrt{\gamma^2 - 4 * m * k}}{2 * m}$$

$$r = \frac{-\gamma}{2 * m} \pm \frac{\sqrt{\gamma^2 - 4 * m * k}}{2 * m}$$

$$r_1 = \frac{-\gamma}{2*m} + \frac{\sqrt{\gamma^2 - 4*m*k}}{2*m} \qquad r_2 = \frac{-\gamma}{2*m} - \frac{\sqrt{\gamma^2 - 4*m*k}}{2*m}$$

$$u(t) = e^{\frac{-\gamma}{2*m}*t} * (c_1 * \cos(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t) + c_2 * \sin(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t))$$

$$u'(t) = e^{\frac{-\gamma}{2*m}*t} * (\frac{-\gamma}{2*m}) * (c_1 * \cos(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t) + c_2 * \sin(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t))$$
$$+ \mathrm{e}^{\frac{-\gamma}{2*m}*t} * (c_1 * -\sin(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t) * \frac{\sqrt{4*m*k-\gamma^2}}{2*m} + c_2 * \cos(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t) * \frac{\sqrt{4*m*k-\gamma^2}}{2*m})$$

$$u(0) = e^0 * (c_1 * \cos 0 + c_2 * \sin 0)$$
$$-17.8 = 1 * (c_1 * 1 + c_2 + 0)$$
$$c_1 = -17.8$$

$$\mathrm{u'(0)} = \mathrm{e}^0 * \frac{-\gamma}{2*m} * (c_1 * \cos 0 + c_2 * \sin 0) + e^0 * (c_1 * -\sin(0) * \frac{\sqrt{4*m*k-\gamma^2}}{2*m} + c_2 * \cos(0) * \frac{\sqrt{4*m*k-\gamma^2}}{2*m})$$
$$0 = \frac{-\gamma}{2*m} * -17.8 + c_2 * \frac{\sqrt{4*m*k-\gamma^2}}{2*m}$$
$$c_2 = -\frac{8.9*\gamma}{m} * \frac{2*m}{\sqrt{4*m*k-\gamma^2}}$$
$$c_2 = -\frac{17.8*\gamma}{\sqrt{4*m*k-\gamma^2}}$$

$$\boxed{\mathrm{u(t)} = \mathrm{e}^{\frac{-\gamma}{2*m}*t} * (-17.8 * \cos(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t) - \frac{17.8*\gamma}{\sqrt{4*m*k-\gamma^2}} * \sin(\frac{\sqrt{4*m*k-\gamma^2}}{2*m} * t))}$$

For $m = .009$ kg and $k = .167$ kg per second^2

$$\boxed{\mathrm{u(t)} = \mathrm{e}^{\frac{-\gamma}{.018}*t} * (-17.8 * \cos(\frac{\sqrt{.006-\gamma^2}}{.018} * t) - \frac{17.8*\gamma}{\sqrt{.006-\gamma^2}} * \sin(\frac{\sqrt{.006-\gamma^2}}{.018} * t))}$$

3 Data

Time (seconds)	Y (inches)
0	-17.832
1.000	-0.428
2.002	13.225
3.002	9.449
4.002	-6.089
5.004	-11.690
6.004	-2.320
7.005	8.709
8.005	6.555
9.007	-3.891
10.007	-8.978
11.007	-2.649
12.009	5.210
13.009	5.083
14.010	-2.066
15.010	-5.649
15.679	4.411
16.679	2.635
17.679	-2.485
18.680	-3.567
19.347	2.945
20.349	0.628
21.349	-1.870
22.349	-1.362
23.350	0.637

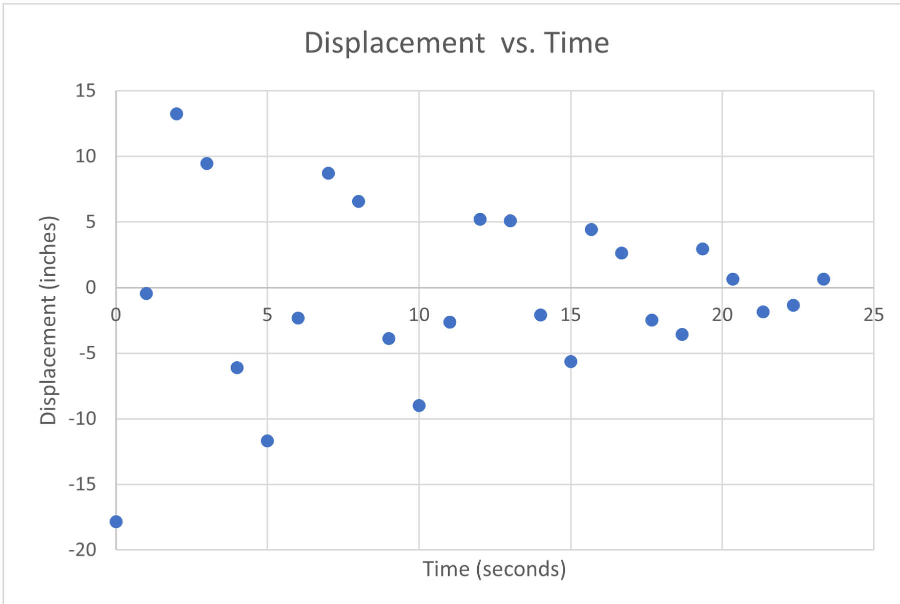

Figure 1: Excel graph

4 Determining γ

The first step of determining γ was to determine a range of values that it could possibly be. The range was found to be $0 - 0.077$. This range was determined because γ must be greater than zero and less than $\sqrt{4 * m * k} = 0.077$. This is because the value of $4 * m * k - \gamma^2$ must be a positive number, as it sits under a square root in the equation for $u(t)$ and the equation would not be reasonable if it contained complex values. The next step was to plug in the 25 values of time we used in our data to the determined equation $u(t)$. We then took those values and subtracted the subsequent actual displacement at each time. We square each difference, and then sum them all together. We did this for about 50 different values of γ between the determined range of $0 - 0.077$. We then chose the γ that had the minimum sum, because that proved the data and the graph of $u(t)$ to be the best fit.

5 Plot of Data and $u(t)$

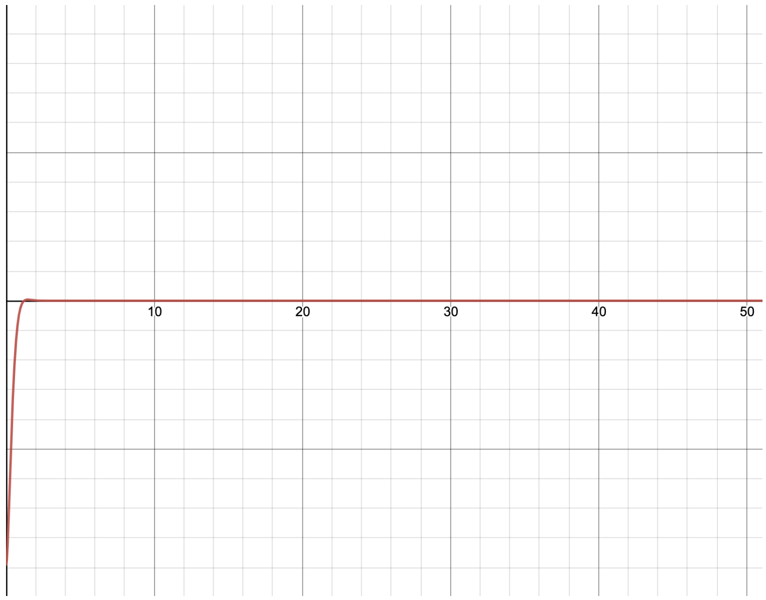

Figure 2: Graph of u(t) with the determined gamma

6 Accuracy of Estimation of γ

Our value of gamma is not an accurate estimation. Comparing the plot of data points and the graph of $u(t)$, they do not look similar. The initial value and first oscillation, for about one second, are very similar. After one second and for the rest of the time, the graphs are very different. Specifically, the graph of $u(t)$ dampens significantly faster than the plot of data. In fact, it dampens almost entirely after the first oscillation. On the other hand, as seen in the video, the spring we used oscillated heavily for the entire 25 seconds. The reasoning for the inaccuracy is because of the way we solved for gamma. While solving for gamma, we found the difference of the predicted displacement with $u(t)$ and the actual displacement in our data, then squared that difference. Since we squared the difference, the larger differences were inflated. We tested out all γ within a predetermined range, and looked for the smallest sum of these differences. The smallest sum of these differences will be more impacted by the first few data points that had larger displacements. Thus, the graph fit better when the first few data points were more similar. This caused the plot of data points and graph of $u(t)$ to then be extremely different in the following data points.