Project 3: Solar Water Boiler

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Thermal Fluid Design

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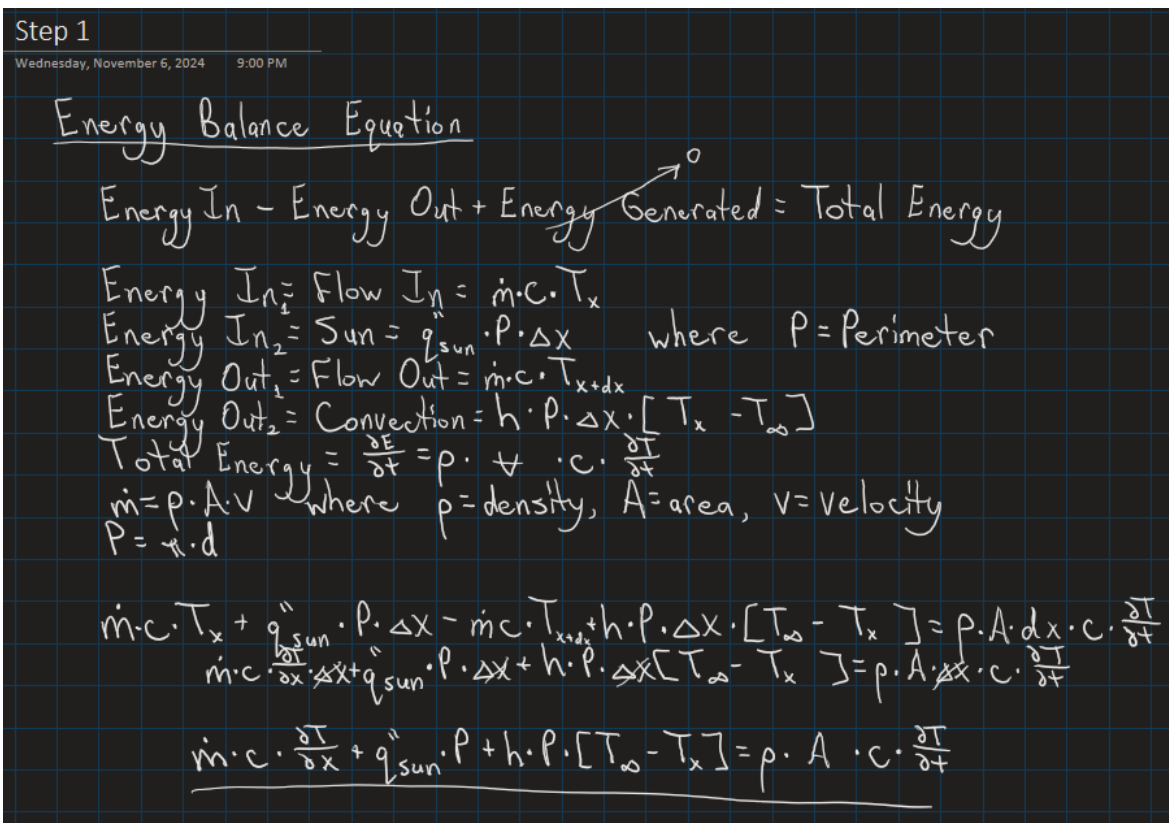
Design Specifications

- Boiler Volume = 40 gallons = 151.4 liters = .1514 m^3
- Desired Boiler T_B = 40 ^oC
- Piping Material: Copper
- Diameter = ¾ inch = 19.05 mm = .01905 m
- Wall Thickness = .065 inch = 1.65 mm = .00165 m
- Surface Emissivity = .03 (polished)

Assumptions

- Total length of HX Region: 20 meters
- No heat loss from the heat exchanger
- The temperature of water in the boiler is uniform
- The heat transfer coefficient is constant, calculated with an average temperature of 30 degrees Celsius
- Boiler height is 1 m

Part 1: Heat Exchanger Equation



Step 1

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Energy Balance Equation

Energy In − Energy Out + $\cancelto{0}{\text{Energy Generated}}$ = Total Energy

Energy In_1 = Flow In = $\dot{m}\cdot c\cdot T_x$

Energy In_2 = Sun = $q''_{sun}\cdot P\cdot\Delta x$ where P = Perimeter

Energy Out_1 = Flow Out = $\dot{m}\cdot c\cdot T_{x+dx}$

Energy Out_2 = Convection = $h\cdot P\cdot\Delta x\cdot[T_x - T_\infty]$

Total Energy = $\frac{\partial E}{\partial t} = \rho\cdot V\cdot c\cdot\frac{\partial T}{\partial t}$

$\dot{m} = \rho\cdot A\cdot v$ where ρ = density, A = area, v = velocity

$P = \pi\cdot d$

$$\dot{m}\cdot c\cdot T_x + q''_{sun}\cdot P\cdot\Delta x - \dot{m}c\cdot T_{x+dx} + h\cdot P\cdot\Delta x\cdot[T_\infty - T_x] = \rho\cdot A\cdot dx\cdot c\cdot\frac{\partial T}{\partial t}$$

$$\dot{m}\cdot c\cdot\frac{\partial T}{\partial x}\cdot\cancel{\Delta x} + q''_{sun}\cdot P\cdot\cancel{\Delta x} + h\cdot P\cdot\cancel{\Delta x}[T_\infty - T_x] = \rho\cdot A\cdot\cancel{dx}\cdot c\cdot\frac{\partial T}{\partial t}$$

$$\dot{m}\cdot c\cdot\frac{\partial T}{\partial x} + q''_{sun}\cdot P + h\cdot P\cdot[T_\infty - T_x] = \rho\cdot A\cdot c\cdot\frac{\partial T}{\partial t}$$

Figure 1: Steps to solve the heat exchanger equation.

Part 2: Heat Exchanger Equation Discretized

Step 2

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$$\dot{m} \cdot c \cdot \frac{\partial T}{\partial x} + \dot{q}''_{sun} \cdot P + h \cdot P \cdot [T_\infty - T_x] = \rho \cdot A \cdot c \cdot \frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t} = \frac{T_i^{n+1} - T_i^n}{\Delta t} \qquad \frac{\partial T}{\partial x} = \frac{T_i - T_{i-1}}{\Delta x}$$

$$\dot{m} \cdot c \cdot \left[\frac{T_i^n - T_{i-1}^n}{\Delta x}\right] + \dot{q}''_{sun} \cdot P + U \cdot P \cdot [T_\infty - T_i^n] = \rho \cdot A \cdot c \cdot \left[\frac{T_i^{n+1} - T_i^n}{\Delta t}\right]$$

$$\left[\dot{m} \cdot c \cdot \left[\frac{T_i^n - T_{i-1}^n}{\Delta x}\right] + \dot{q}''_{sun} \cdot P + U \cdot P \cdot [T_\infty - T_i^n]\right] \cdot \frac{\Delta t}{\rho \cdot A \cdot c} + T_i^n = T_i^{n+1}$$

Figure 2: Discretization of the heat exchanger equation.

Part 3: Boiler Equation

Step 3

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$$\dot{E}_{in} - \dot{E}_{out} = \frac{\partial E}{\partial t}$$

$$\dot{m} \cdot h_{in} - \dot{m} \cdot h_{out} = M_B \cdot c \cdot \frac{\partial T_B}{\partial t}$$

In = in the boiler

Out = out of the boiler

$$\dot{m} \cdot \cancel{c} \cdot T_{in} - \dot{m} \cdot \cancel{c} \cdot T_{out} = M_B \cdot \cancel{c} \cdot \frac{\partial T_B}{\partial t}$$

$$\dot{m} \cdot (T_{in} - T_{out}) = M_B \cdot \frac{\partial T_B}{\partial t}$$

Figure 3: Steps to solve the boiler equation.

Part 4: Boiler Equation Discretized

Step 4

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$$\dot{m} \cdot (T_{in} - T_{out}) = M_B \cdot \frac{\partial T_B}{\partial t}$$

Solve for T_B^{n+1}

$$\dot{m}\left(T^{n}_{B,in} - T^{n+1}_{B}\right) = M_B \cdot \frac{T_B^{n+1} - T_B^{n}}{\Delta t}$$

$$\dot{m} \cdot \Delta t \cdot T_{B,in} - \dot{m} \cdot \Delta t \cdot T_B^{n+1} = M_B \cdot T_B^{n+1} - T_B^{n} \cdot M_B$$

$$\dot{m} \cdot \Delta t \cdot T_{B,in} + T_B^{n} \cdot M_B = M_B \cdot T_B^{n+1} + \dot{m} \cdot \Delta t \cdot T_B^{n+1}$$

$$\dot{m} \cdot \Delta t \cdot T_{B,in} + T_B^{n} \cdot M_B = T_B^{n+1}\left(M_B + \dot{m} \cdot \Delta t\right)$$

$$T_B^{n+1} = \frac{\dot{m} \cdot \Delta t \cdot T_{B,in} + T_B^{n} \cdot M_B}{M_B + \dot{m} \cdot \Delta t}$$

Figure 4: Discretization of the boiler equation.

Part 5: Solving

See appendix for MATLAB code.

Step 5

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Solve for velocity:

Using Bernoulli's

$$v = \sqrt{2gh}$$

height of boiler: 1 m (vertical cylinder boiler)

$h = .5$ m

$$v = \sqrt{2gh} = \sqrt{2 \cdot 9.81\ m/s^2 \cdot .5} = 3.13\ m/s$$

Figure 5: Solving for velocity using Bernoulli's equation.

Solve for heat transfer coefficient:

$Re = \rho \cdot v \cdot d / \mu$

$\rho = 1000\ kg/m^3$

$d = .01905\ m$

$\mu = .001\ Pa \cdot s$

$Re = 1000 \cdot 3.13 \cdot .01905 / .001 = 59627$

Turbulent Flow

$Pr = \mu \cdot c_p / k$

$c_p = 4184\ J/kg°C$

$k = .6\ W/mK$ @ 30°C

$Pr = .001 \cdot 4184 / .6 = 6.97$

$Nu = .023 \cdot Re^{.8} \cdot Pr^{.4}$

$Nu = .023 \cdot 59627^{.8} \cdot 6.97^{.4} = 331$

$$h = \frac{Nu \cdot k}{d} = \frac{331 \cdot .6}{.01905} = 10414\ W/m^2K$$

Figure 6: Solving for the heat transfer coefficient, including Re, Pr, and Nu.

Solve for U:

$$\frac{1}{U \cdot P \cdot \Delta x} = \frac{1}{h \cdot P \cdot \Delta x} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi K_p \cdot \Delta x} + \frac{1}{h_o \cdot P}$$

$h_o = 5$

$D_o = .01905 + .00165 \cdot 2 = .0224\ m$

$D_i = .01905\ m$

$h = 10414\ W/m^2 K$

$A = \pi \cdot \left(\frac{D_i}{2}\right)^2 = \pi \cdot (.01905/2)^2 = 2.8502\ e{-4}$

$K = 400\ W/m \cdot K$

$\Delta x = .1\ m$

$P = \pi \cdot d_i = .0598\ m$

$$\frac{1}{UP} = \frac{1}{10414 \cdot .0598} + \frac{\ln\left(\frac{.0224}{.01905}\right)}{2 \cdot \pi \cdot 400} + \frac{1}{5 \cdot .0598}$$

$$\frac{1}{UP} = 3.346 \rightarrow UP = .2989$$

$U = 4.9975\ W/m^2 K$

Figure 7: Solving for U using the heat transfer coefficient.

Part 6: MATLAB Plots for $T_B(t)$ and $T(x)$

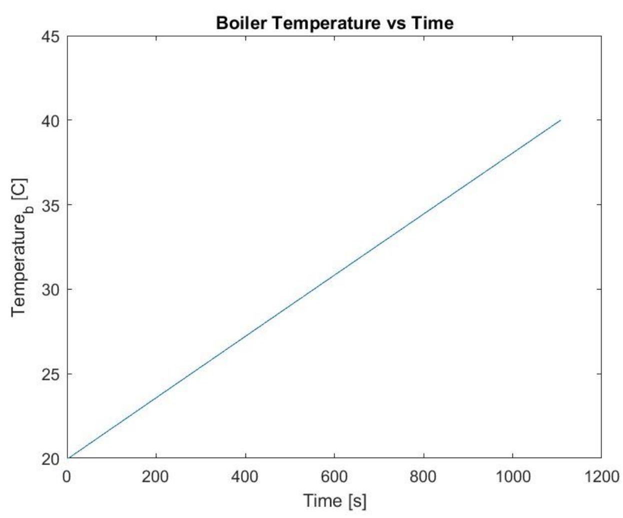

Figure 8: Plot of boiler temperature over time.

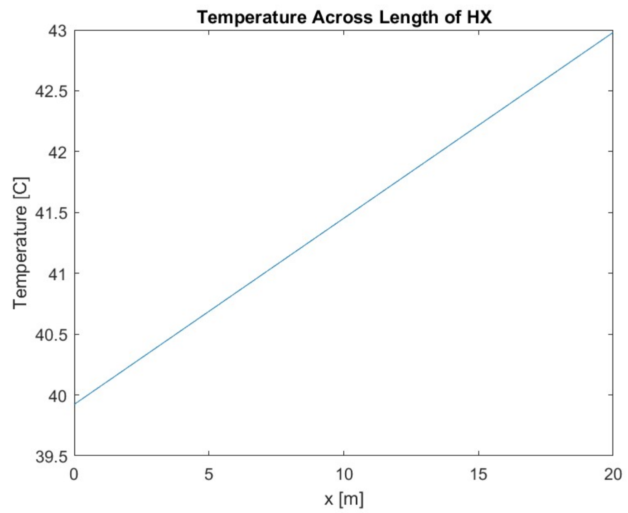

Figure 9: Plot of temperature of the water in the heat exchanger over the length of the heat exchanger.

Part 7: Discussion and Recommendation

This design was frustrating because there are a lot of variables and moving parts. If one variable was declared improperly, it would break the entire solution. Therefore, going through each step and ensuring that it is working is the best way of going about the boiler design.

Regarding the actual design, while the current copper piping offers good thermal conductivity, exploring alternative materials, like flat-plate collectors or evacuated tube collectors, could increase the heat absorption from solar radiation. Additionally, adding a surface coating on the heat exchanger could improve the absorption while minimizing heat loss.

The system's efficiency could be improved by incorporating insulation around the boiler to reduce heat losses to the environment. This would be particularly beneficial in maintaining the desired water temperature for longer periods, especially during times of low solar irradiance. Another consideration is adding a thermal storage system, which could store excess heat during peak sunlight hours for use during periods of low sunlight.

Constantly mixing the boiler could also prove useful in keeping the boiler temperature constant throughout the entire volume. Also, increasingly the length of the heat exchanger would prove the boiler to heat faster because there is more area where the water is getting heated.

Appendix A

MATLAB Code

```
% Design solar water boiler
clc
clear all

% declare constants
v_b = .1514; % meters^3
T_b_desired = 40; % degrees celsius
T_amb = 20; % degrees celsius
d = .01905; % meters
thickness = .00165; % meters
emissivity = .03; % polished copper
c_p = 4184; % J/kgK
rho = 1000; % kg/m^3
q_rad = 10000; % W/m^2
velocity = 3.13; % m/s, calculated
U = 4.9975; % W/m^2K, calculated

% declare variables
A = pi * (d/2)^2; % meters^2
m_dot = rho * A * velocity; % kg/s
P = pi * d; % meters
mass_b = v_b * rho; % kg

% Discretization
N = 20; % # of nodes in x dir.
M = 10000; % # nodes in time
L = 20; % meters

t_end = 1109;
dx = L/(N-1); % cell length [m]
dt = t_end/(M-1); % s

% setting intitial conditions
T = zeros(M, N);
T(1,1:N) = T_amb;
T_b = zeros(1, M);
T_b(1) = T_amb;

a = (dt*m_dot)/(rho*A*dx);
b = (P*dt)/(rho*A*c_p);
c = (U*P*dt)/(rho*A*c_p);
```

```
for k = 2:M
    % Heat exchanger loop
    for i = 2:N
        T(k,i) = T(k-1,i) + a*(T(k-1,i-1)-T(k-1,i)) + b*q_rad + c*(T_amb-T(k-1,i));
    end

    % Update boiler temperature
    T_b(k) = (m_dot*dt*T(k,N)+mass_b*T_b(k-1))/(mass_b+m_dot*dt);

    % boundary condition
    T(k,1)=T_b(k);
end

t_finish = find(T_b >= T_b_desired, 1) * dt;

if isempty(t_finish)
    fprintf('Desired temperature not reached within simulation time.\n');
else
    fprintf('Time to reach desired temperature: %.2f seconds\n', t_finish);
end

%% plotting data

t = [0:dt:t_end]; % time vector
x = [0:dx:L]; % x vector

figure (1)
plot(t, T_b)
xlabel('t [s]'); ylabel('T_b [C]')
title('Boiler Temperature vs time')

for i = 1:50:M
    figure(2)
    plot(x,T(i,:))
    hold off
    xlabel('x [m]'); ylabel('T [C]')
    title('Temperature across length of HX')
end
```