Fluid Mechanics Project

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Fluid Mechanics MECH 322

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Project Overview

The Waslala School initiative is committed to establishing a dependable and sustainable water supply system that meets the daily requirements of students, staff, and faculty. Our primary objective, as members of the design team, lies in crafting an efficient hydraulic system that ensures a steady water flow and sustains optimal pressure across the school premises. The utilization of schedule 40 PVC piping not only facilitates cost-effective and straightforward repairs but also safeguards the continuous functionality of the water supply system. Consistency in water supply is crucial for fundamental needs such as restroom facilities and drinking fountains within the school. Simultaneously, more critical aspects come into play, including access to medical attention and the ability to practice proper hand hygiene. Water for Waslala (WfW) is a non-profit that provides funding and technical expertise to create sustainable potable water projects [1]. Being in a remote region provides limited access to large industrial water systems requiring residents to outsource to rivers, wells and more. A dependable water source in Waslala contributes significantly to relieving students and staff of stress and challenges associated with daily access to water, fostering an improved quality of life for them within the school.

Design

The necessity for a pump arises when the input flow is not capable of reaching the desired output flow after losses, which creates an inequality between both sides of the Bernoulli equation where the initial side, on the left, is less than the final side, on the right. Disregarding unknown head losses, the transition from Pipe 0 to Pipe 1 applied to Bernoulli's equation is seen in Equation 1.

$$\frac{P_0}{\gamma} + \frac{V_0^2}{2g} + Z_0 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1$$

Equation 1: Bernoulli Inequality.

Using the given pressure, gamma, and flow rate, the Bernoulli equation for Pipe 0 yields 1.8502 meters which is below the height of all three pipe locations in the school. As a result, the water supply is inadequate, and pumps are required.

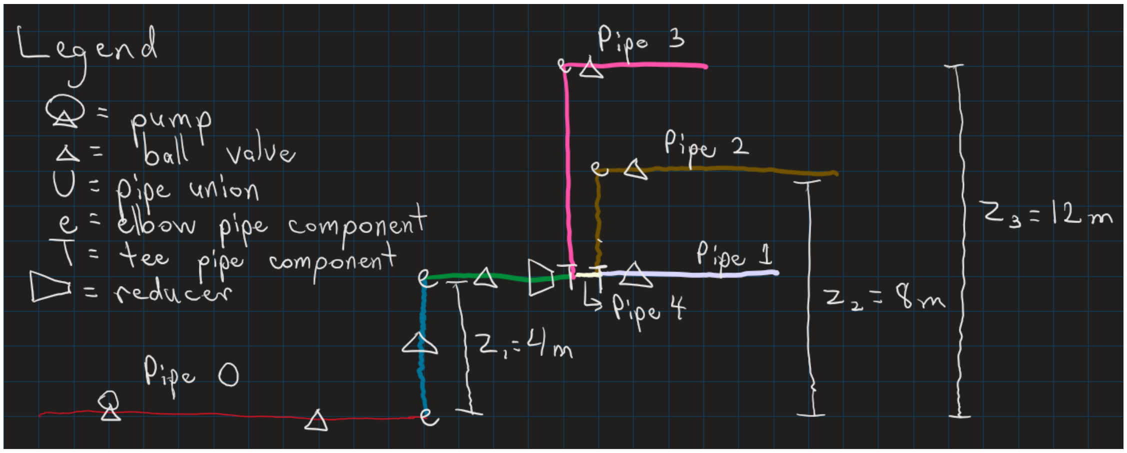

Figure 1: Pipe Schematic.

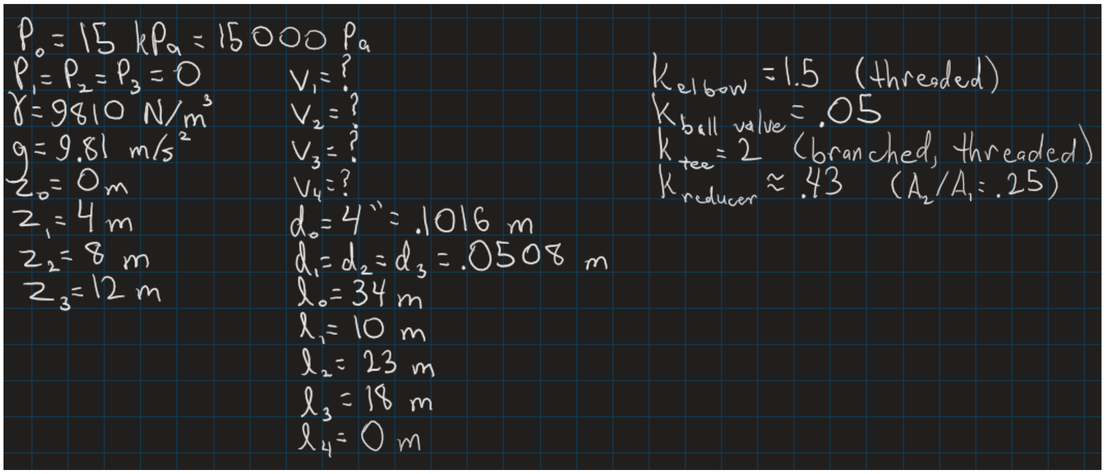

Figure 2: Variables & Unknowns.

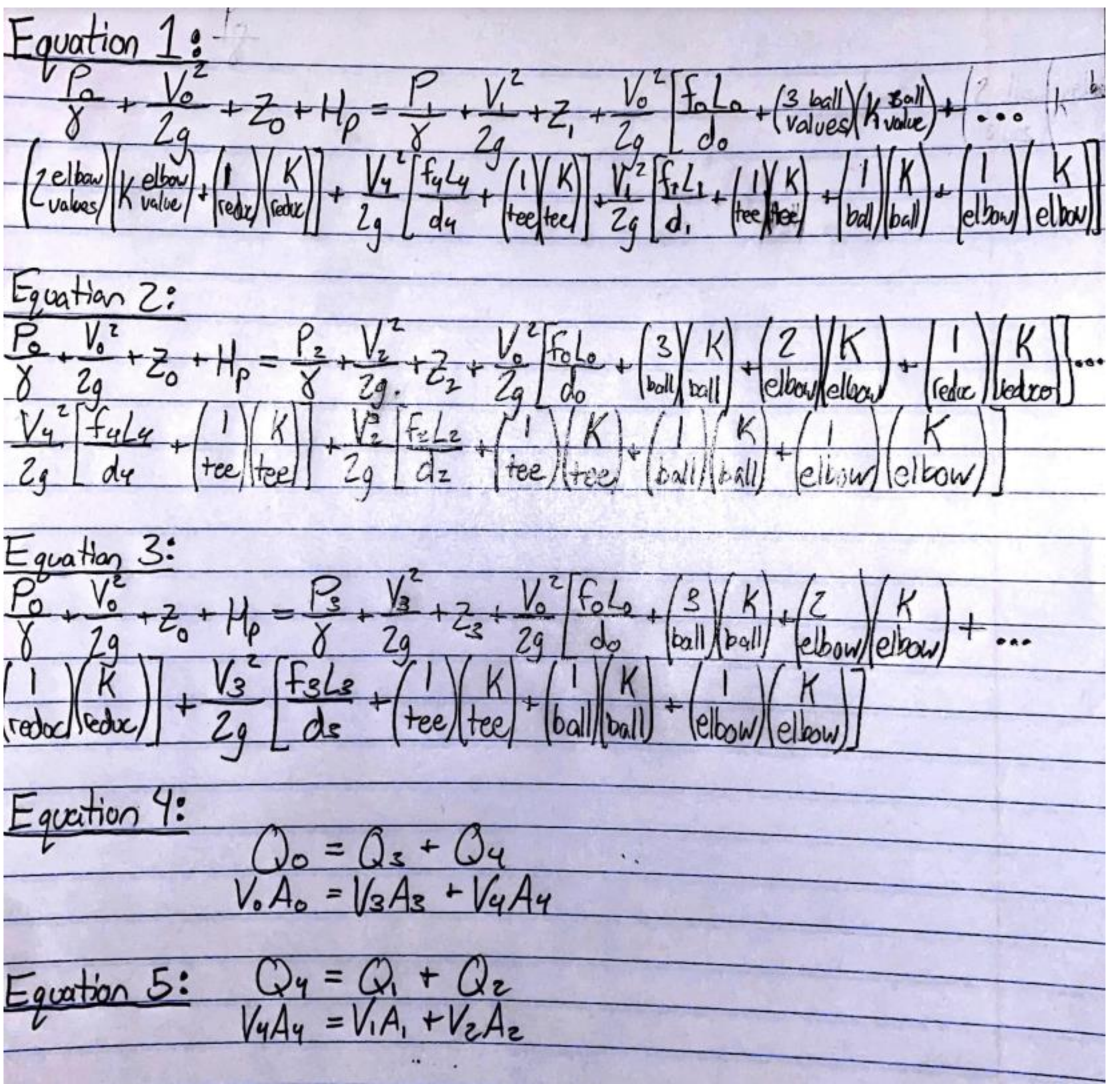

Figure 3: Equations.

The pump is positioned at the beginning of Pipe 0, contributing to the already flowing water in the pipe. Every piping section, except for Pipe 4, has ball valves to control the flow of water for maintenance, as well as an elbow pipe component at every corner. Pipe 4 is a small section of pipe but crucial to the functionality of the system, having two tee pipe components on each end, connecting Pipes 1, 2, 3 to Pipe 0. The K factor, or loss coefficient, used for a tee component is 2.0, which is an over-estimation as this value is the highest for a tee component. The length of Pipe 4 is negligible because the two tee components lie directly next to each other. A reducer is placed just before Pipe 4 to account for the change in pipe diameter between Pipe 0 and the rest of the piping. The velocity used to calculate the loss from the reducer is the input velocity, which is V_0. The K factor for the reducer was found using Figure 4. The material chosen for the design is 40 Gauge PVC. The determination of all friction factors will involve the application of the Colebrook Formula and calculation of Reynold's Number. MATLAB will be employed for the computation of the five variables V_1, V_2, V_3, V_4, and H_p, where H_p is the pump head in meters.

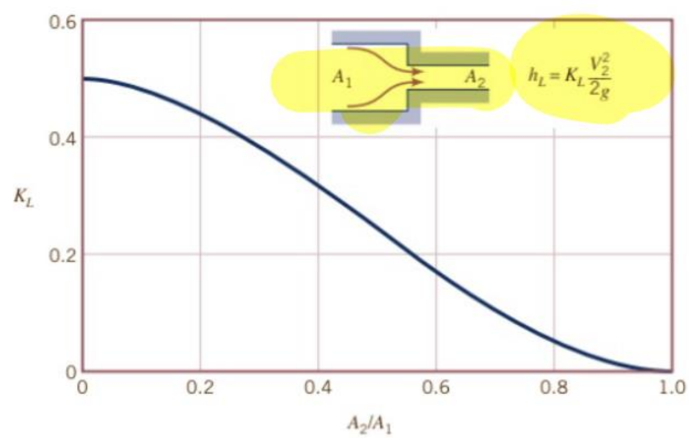

Figure 4: Loss coefficient for a sudden contraction [2].

<u>Equation 1</u> is in terms of V_0, V_1, V_4, and H_p. Number of friction factors f_0, f_1, and f_4 are to be calculated from Pipe 0 to Pipe 1.

Equation 2 is in terms of V_0, V_2, V_4, and H_p. Number of friction factors f_0, f_2, and f_4 are to be calculated from Pipe 0 to Pipe 2.

Equation 3 is in terms of V_0, V_3, and H_p. Number of friction factors f_0 and f_3 are to be calculated from Pipe 0 to Pipe 3.

Equation 4 is in terms of V_0, V_3, and V_4. Since volumetric flow rate of Pipe 3 and Pipe 4 are equal to the flow rate of the inlet Pipe 0.

Equation 5 is in terms of V_4, V_1, and V_2. Since volumetric flow rate of Pipe 1 and Pipe 2 are equal to the flow rate of the inlet Pipe 4.

The code will iterate through a sequence of calculations to solve for the five equations involving the five unknowns. The Reynolds number, $Re = \frac{pVD}{\mu}$, will undergo iterations with various velocities. The Colebrook Equation, $\frac{1}{\sqrt{f}} = -2Log_{10}\left[\left(\frac{\frac{\varepsilon}{D}}{3.7}\right) + \left(\frac{2.51}{Re\sqrt{f}}\right)\right]$, will incorporate these Reynolds numbers to compute friction factors. Notably, given the use of smooth PVC piping, the epsilon (ε) factor in the Colebrook equation is set to 1.5 e -7 meters, which is negligible because it is much less than the diameter of the piping, which ranges from 2 inches to 4 inches. The full code is in the appendix, and the following is the pseudocode:

```
Initialize all variables
Calculate initial Reynolds values using the initial velocity for every velocity value
If Reynolds Number > 2100, then the flow is turbulent:
        Calculate initial f values using the equation, -2*log10((epsilon / (diameter * 3.7)) +
        (2.51 / (Reynolds Number * sqrt(f)))) = 1/sqrt(f)
Else if Reynolds Number < 2100, then the flow is laminar:
        Calculate initial f values using the equation, f = 64 / Re
Solve the system of equations for initial values of velocity 1, velocity 2, velocity 3, velocity
4, and the pump head
```

While the change in velocity 1, velocity 2, velocity 3, velocity 4, or the pump head is greater than 5%:

Calculate Reynolds values using the velocities for each velocity value

If Reynolds Number > 2100, then the flow is turbulent:

Calculate f values using the equation, -2*log10((epsilon / (diameter * 3.7)) + (2.51 / (Reynolds Number * sqrt(f)))) = 1/sqrt(f)

Else if Reynolds Number < 2100, then the flow is laminar:

Calculate f values using the equation, f = 64 / Re

Solve the system of equations for values of velocity 1, velocity 2, velocity 3, velocity 4, and the pump head

Return the velocity 1, velocity 2, velocity 3, velocity 4, and the pump head

Results

The provided code in the Appendix yields initial values V_1 = 4.8812, V_2 = 2.2615, V_3 = 2.8977, V_4 = 7.1427, Ph= 3.4609 [meters]. The MATLAB program is designed to execute the specified calculations, involving Reynolds Number, new friction values, and new velocity values, for a complete iteration. The code will iteratively generate additional values, until all values have a change of less than 5% of the previous value.

With all these velocities we can calculate the volumetric flow rate, Q = V*A. The initial flow rate was calculated through other means, Q_0 = 0.02035 [m^3/sec]. The following are the calculated solutions for the rest of the flow rates: Q_1 = 0.009892, Q_2 = 0.004584, Q_3 = 0.005873, Q_4 = 0.01448 [m^3/sec]. Pipe sections 0, 1, 2, and 3 all intersect, and intuitively, the flow entering the intersections would split to pipes 1, 2, and 3, therefore, $Q_0 = Q_1 + Q_2 + Q_3$. Similarly for the convergence of sections 1, 2, and 4, the flow enters from 4 going to Pipes 1 and 2, providing $Q_4 = Q_1 + Q_2$. Therefore, the calculated values reflect a logical understanding. These flow rates indicate a well-designed system, as there is no excessive flow delivered to each pipe. This implies that the minimum required amount of water reaches each floor.

As a result of the code Q_0 = 322.55 GPM and H_p = 11.355ft. The system requires a pump that draws low power and low total head, this is a result of the system being efficient and having smooth piping. Instead, we will select a pump with a higher head around 20 ft, which was taken from figure 4. However, it is evident that this pump consumes excessive power for our system, with a head increase of nearly double the original.

The chosen pump, with a size of 6 by 8 by 11 and an impeller diameter of 7.25 inches, exhibits an efficiency of 52%. Specifications also from the pump curve include that the horsepower is 3 feet per second, and the power is 1180 rotations per minute. As seen from the graph, the motor is not efficient for our system and further research could reveal a more efficient motor. As a result of the low efficiency, each floor is receiving a higher flow rate than necessary. To enhance both flow rate and the efficiency of the system, an effective strategy involves optimizing the diameters of the piping sections. In the future, by strategically adjusting specific diameters, we can increase the flow rate to certain locations without necessity for an overhaul of the design.

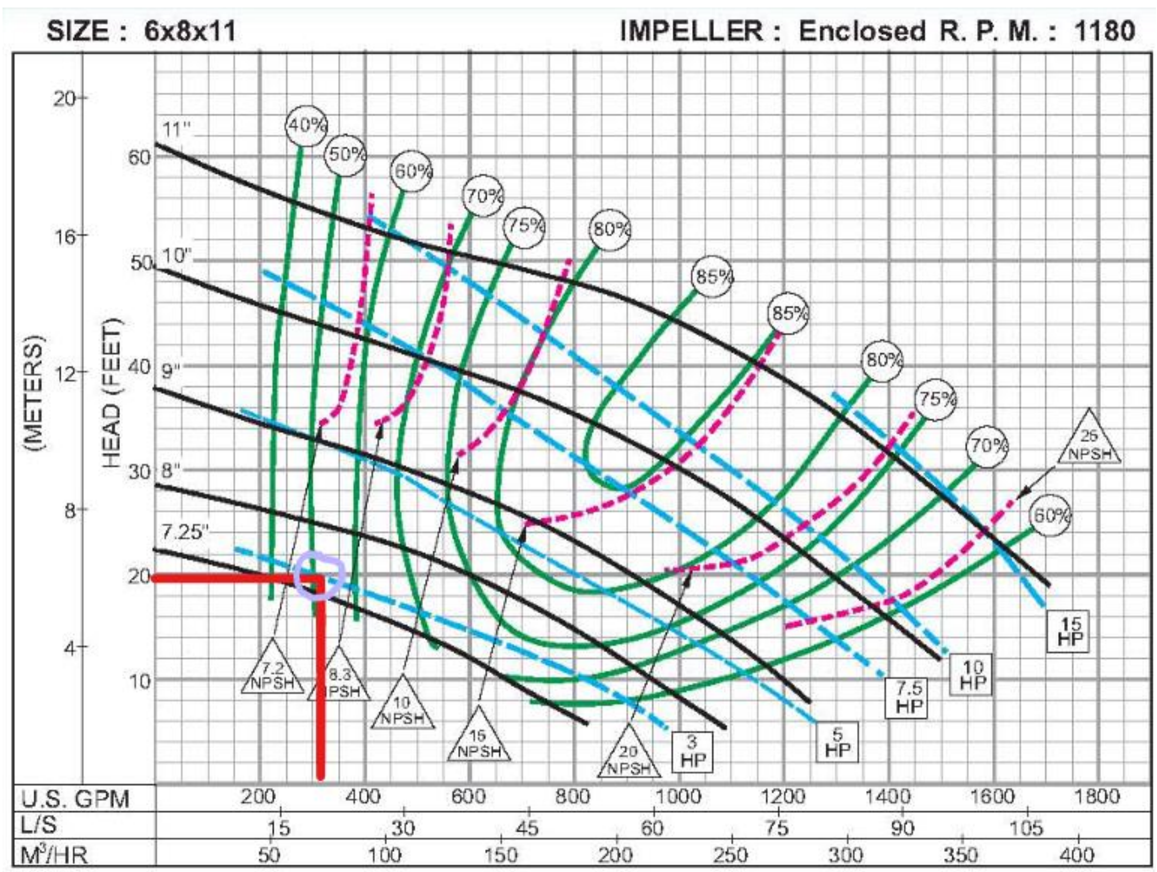

Figure 5: Pump Curve [3].

Conclusion

Opting for PVC as the selected material offers cost-effectiveness, but considering a thermoplastic pipe like HDPE proves to be a more sustainable choice, resistant to cracking over time. While the pump heads boast a notably high efficiency percentage, their operation demands a substantial amount of energy and financial investment. Introducing varied diameter sizes across different sections further enhances the overall system efficiency. Exploring alternative water sources has the potential to alleviate the total load placed on the natural source and enhance the sustainability of the design.

Appendix

```
% Mike Hennessy and Jack Michaelis
% Hydraulic System Design Project
clear
clc

% Knowns

% Fluid constants
g = 9.8; % m/s^2
density = 1000; % [kg/m^3]
gamma = 980; % rho * g
viscosity = .00084; % [Pa*s]
epsilon = .00000015; % [m]

% NPS converted to m
d0 = 4 * 0.0254;
d1 = 2 * 0.0254;
d2 = d1;
d3 = d1;
d4 = d1;

% Pipe Areas [m^2]
a0 = (pi*(d0/2)^2);
a1 = (pi*(d1/2)^2);
a2 = a1;
a3 = a1;
a4 = a1;

% Pipe Lengths [m]
l0 = 34;
l1 = 10;
l2 = 23;
l3 = 18;
l4 = 0;

% Elevations [m]
z0 = 0;
z1 = 4;
z2 = 8;
z3 = 12;

% Pressure Values [Pa]
p0 = 15000;
p1 = 0;
p2 = 0;
p3 = 0;

% K Values
k_elbow = 1.5; % threaded (overestimate)
k_ball_valve = .05;
k_tee = 2; % branched, threaded (overestimate)
k_reducer = .43; % where A_2 / A_1 = 0.25
```

```
% Quantities of Valves, Unions, Tees, and Reducer
ball_valves_0 = 3; % quantity of ball valves for pipe 0
ball_valves_1 = 1;
ball_valves_2 = ball_valves_1;
ball_valves_3 = ball_valves_1;
ball_valves_4 = 0;
elbow_valves_0 = 2;
elbow_valves_1 = 0;
elbow_valves_2 = 1;
elbow_valves_3 = elbow_valves_2;
elbow_valves_4 = 0;
tee_valves_0 = 0;
tee_valves_1 = 1;
tee_valves_2 = tee_valves_1;
tee_valves_3 = tee_valves_1;
tee_valves_4 = tee_valves_1;
reducer = 1;

% Flow Rate and Velocity Calculations
supply = .02 + ((3 + 4) / 2) * 10^(-4); % supplied flow rate with March and April
birthday months
v0 = supply / a0; % initial velocity

% Initial friction factor and Reynolds values
Re_0 = reynolds(density, v0, d0, viscosity);
f_0 = frictionFactor(Re_0, epsilon, d0);

% Unknowns
syms v1 v2 v3 v4 ph

% Matrix of all the diameters
d_all = [d0, d1, d2, d3, d4];

% Preallocating Reynolds Number and friction numbers
Re = [Re_0, Re_0, Re_0, Re_0, Re_0];
f = [f_0, f_0, f_0, f_0, f_0];

% Equation 1
Eq1 = p0/gamma + (v0^2)/(2*g) + z0 + ph == p1/gamma + (v1^2)/(2*g) + z1 +
((v0^2)/(2*g))*((f_0*l0)/d0 + ball_valves_0 * k_ball_valve + elbow_valves_0 * k_elbow
+ reducer * k_reducer) + ((v4^2)/(2*g))*((f(5)*l4)/d4 + tee_valves_4*k_tee) +
((v1^2)/(2*g))*((f(2)*l1)/d1 + tee_valves_1*k_tee + ball_valves_1*k_ball_valve);

% Equation 2
Eq2 = p0/gamma + (v0^2)/(2*g) + z0 + ph == p2/gamma + (v2^2)/(2*g) + z2 +
((v0^2)/(2*g))*((f_0*l0)/d0 + ball_valves_0 * k_ball_valve + elbow_valves_0 * k_elbow
+ reducer * k_reducer) + ((v4^2)/(2*g))*((f(5)*l4)/d4 + tee_valves_4*k_tee) +
((v2^2)/(2*g))*((f(3)*l2)/d2 + tee_valves_2*k_tee + ball_valves_2*k_ball_valve +
elbow_valves_2*k_elbow);

% Equation 3
Eq3 = p0/gamma + (v0^2)/(2*g) + z0 + ph == p3/gamma + (v3^2)/(2*g) + z3 +
((v0^2)/(2*g))*((f_0*l0)/d0 + ball_valves_0 * k_ball_valve + elbow_valves_0 * k_elbow
```

```
+ reducer * k_reducer) + ((v3^2)/(2*g))*((f(4)*l3)/d3 + tee_valves_3*k_tee +
ball_valves_3*k_ball_valve + elbow_valves_3*k_elbow);

% Equation 4
Eq4 = v0*a0 == v3*a3 + v4*a4;

% Equation 5
Eq5 = v4*a4 == v1*a1 + v2*a2;

% Solve initially for v1, v2, v3, v4, and ph
[v1, v2, v3, v4, ph] = solve([Eq1, Eq2, Eq3, Eq4, Eq5], [v1, v2, v3, v4, ph]);

response = filter(v1, v2, v3, v4, ph);

v1 = response(1);
v2 = response(2);
v3 = response(3);
v4 = response(4);
ph = response(5);

% Assign found variables
vguess = [v0, v1, v2, v3, v4];

% while loop variables
percent_change = 1;
old_values = zeros();
new_values = zeros();

% Loops through preselected iterations that runs the equations any number of times
while percent_change > .05
    % Store old values to calculate percent change at end of loop
    old_values = [v1, v2, v3, v4, ph];

    % Calculate Reynolds based on velocities
    for n = 2:5
        Re(n,:) = reynolds(density, vguess(:,n), d_all(:,n), viscosity);
        f(n, :) = frictionFactor(Re(n,:), epsilon, d_all(:,n));
    end

    syms v1 v2 v3 v4 ph

    % Equation 1
    Eq1 = p0/gamma + (v0^2)/(2*g) + z0 + ph == p1/gamma + (v1^2)/(2*g) + z1 +
((v0^2)/(2*g))*((f_0*l0)/d0 + ball_valves_0 * k_ball_valve + elbow_valves_0 * k_elbow
+ reducer * k_reducer) + ((v4^2)/(2*g))*((f(5)*l4)/d4 + tee_valves_4*k_tee) +
((v1^2)/(2*g))*((f(2)*l1)/d1 + tee_valves_1*k_tee + ball_valves_1*k_ball_valve);

    % Equation 2
    Eq2 = p0/gamma + (v0^2)/(2*g) + z0 + ph == p2/gamma + (v2^2)/(2*g) + z2 +
((v0^2)/(2*g))*((f_0*l0)/d0 + ball_valves_0 * k_ball_valve + elbow_valves_0 * k_elbow
+ reducer * k_reducer) + ((v4^2)/(2*g))*((f(5)*l4)/d4 + tee_valves_4*k_tee) +
((v2^2)/(2*g))*((f(3)*l2)/d2 + tee_valves_2*k_tee + ball_valves_2*k_ball_valve +
elbow_valves_2*k_elbow);

    % Equation 3
```

```
    Eq3 = p0/gamma + (v0^2)/(2*g) + z0 + ph == p3/gamma + (v3^2)/(2*g) + z3 +
((v0^2)/(2*g))*((f_0*l0)/d0 + ball_valves_0 * k_ball_valve + elbow_valves_0 * k_elbow
+ reducer * k_reducer) + ((v3^2)/(2*g))*((f(4)*l3)/d3 + tee_valves_3*k_tee +
ball_valves_3*k_ball_valve + elbow_valves_3*k_elbow);

    % Equation 4
    Eq4 = v0*a0 == v3*a3 + v4*a4;

    % Equation 5
    Eq5 = v4*a4 == v1*a1 + v2*a2;

    [v1, v2, v3, v4, ph] = solve([Eq1, Eq2, Eq3, Eq4, Eq5], [v1, v2, v3, v4, ph]);

    response = filter(v1, v2, v3, v4, ph);

    v1 = response(1);
    v2 = response(2);
    v3 = response(3);
    v4 = response(4);
    ph = response(5);

    % Reassigns vguess to contain new velocities
    vguess= [v0, v1, v2, v3, v4];
    new_values = [v1, v2, v3, v4, ph];

    percent_change = .05;
    for j = 1:5
        percent_change = max(percent_change, ((new_values(j) - old_values(j)) /
old_values(j)));
    end

end

disp(new_values)

function [Re] = reynolds(density, v, d, viscosity)
    Re = (density*v*d)/viscosity;
end

function [f] = frictionFactor(Re, epsilon, d)
    if(Re > 2100)
        syms f;
        f = vpa(solve(-2*log10((epsilon/(d*3.7)) + (2.51./(Re*sqrt(f)))) ==
1./sqrt(f), f));
    else
        f = 64 / Re;
    end
end

function [values] = filter(v1, v2, v3, v4, ph)
    for n = 1:4
        if(v1(n) >= 0 && v2(n) >= 0 && v3(n) >= 0 && v4(n) >= 0 && ph(n) >= 0)
            values = [v1(n), v2(n), v3(n), v4(n), ph(n)];
        end
    end
```

```
end
```

References

[1] *Dedicated to providing clean water to Waslala, Nicaragua*. Water for Wasala - Crosieres tout inclus. (n.d.). http://www.waterforwaslala.org/

[2] Ebrahim, M (2023). Lecture Notes.

[3] Understanding npsh & cavitation - pumps & systems. (n.d.). https://www.pumpsandsystems.com/sites/default/files/NPSH%20ebook-2018.pdf