Game Theory exam –December 21, 2022

Set the value X as the last digit of your *student number.*

Exercise 1 Brother (B) and Sister (S) are both coming home for Christmas, and they also want to buy gifts to their parents. They both know that their parents like Chianti (C) wine and it is the most preferable choice that provides utility ($u_C = 70$) but, at the same time, is the most costly choice ($c_C = 25$). The second best choice is some local Christmas sweets such as Panettone (P) that takes a lot of space in the luggage and provides utility ($u_P = 50$), nevertheless, it's the less costly choice ($c_P = 15$). Alternatively, they can always bring a generic small souvenir (S) from their countries. It's a less joyful gift ($u_S = 15$) that can be delivered with the lowest cost ($c_S = 5$). The utility is defined by the level of satisfaction of their parents that is the sum of the joy/utilities provided by two gifts from (B) and (S) minus individual cost. If they buy the same gift, the utility will not be doubled, but equal to $(0.1X + 1) \cdot u$ minus individual cost. Brother and sister are going to buy gifts without consulting with each other, and have all of this information as common knowledge.

1. Write down the normal form of this game
2. Find all NEs in pure strategies.
3. Find all NEs in mixed strategies.

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Exercise 2 Investor (I) wants to finance a fast growing startup (S) hoping to get a percentage of the profit, but he is not sure if he should really do a big investment (B) or a small one (S). Startup really needs these money and has to prepare plan how to distribute the potential investments beforehand. They can plan either a higher split with an investor (B) or a low one (L). The best situation is when the share of a profit corresponds to the amount of received investments. If investments are big and the split is high as well, the payoff of S is $u_H = 6$, while if the split is low $u_L = 3$. If received investments are small, then paying a big percentage will be catastrophic for a startup and that would result in the utility of S $u_H = -1 - X$, but if the split of a profit is also low then $u_L = 1$. For investor, instead, getting the high split of a profit is the best scenario that provides a profit of 5 and 3 as a return from big and small investments, respectively.If the proposed spit is low and I made a big investment, it is a disappointment situation that provides utility $u_B = -1 - X$. If the split is low but also the investment is small as well then $u_S = 1$.

1. Write down the normal form of this game and find pure NE

2. If this game is played repeatedly, describe "carrot" and "stick" strategies of the game

3. If the game is played repeatedly over infinite time horizon and partial payoffs are discounted with discount factor δ, and Investor plays a Grim Trigger strategy. What would be the value of δ so that the startup prefers to deviate from cooperative strategy.

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Exercise 3 Home heating cost is rising, and any Household (H) prefers to save money on heating, i.e. to switch it off and use a Bonfire (B) which is illegal. Otherwise, H has to keep the heating on and keep Paying for it (P). The Government (G) can check (C) from time to time if the household is using a bonfire, or, instead, G can choose to trust (T) a Household and do not perform a check. With probability p the household is honest and prefers not to use the bonfire, and with probability $1-p$ the household is sneaky and prefers to save money on heating. In both cases, if the household pays for the heating, his payoff will be equal to 5, and the payoff of the government will be equal to 5 if it did not perform a check and equal to 3 otherwise, since the monitoring is costly. If the Household is honest but it uses the bonfire without being revealed, then H will get the same payoff equal to 5, but if the household is sneaky, then having the satisfaction that he tricked the government will increase the payoff on +1, and the utility of the government will be equal to -5. In case if the household is revealed (i.e. being checked by the government), it will achieve the payoff -5 if he is honest and -3 if he is sneaky, while the government will carry the cost of $-5 - X$. The probability of being honest is $p = 0.5$.

1. Represent the game in extensive form
2. Represent the game in normal form after stating H's and G's strategies.
3. Find all the equilibria and discuss them.