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Exercise 5.4. Carl (C) and Diana (D) are two university students that have found that the department library is unoccupied overnight. It is a really good place to study and has a very fast Internet connection. So, they go there every night, but they do not coordinate or plan any action together. Upon their arrival every night, they independently decide whether: (S) study or (M) watch some movies on their laptop. If they both study, they both get utility 10. The individual benefit from watching a movie is instead 15 for C and 18 for D. However, if they both choose M, their individual benefit is halved (since they have half the connection speed). Also, trying studying while somebody else is playing a movie breaks the concentration, so $u_C(S, M) = u_D(M, S) = 0$ (C is written as the first player). Call $\mathbb{G}$ this game, and consider it in a repeated version $\mathbb{G}(T)$, where $\mathbb{G}$ is played every night for T nights. Individual payoffs are cumulated with discount factor δ. Finally, consider an *extended* game where a punishment strategy P is also available to both players. When either player chooses P, payoffs are -10 for *both* players (that would correspond, e.g., to do something really stupid in the library and get the library permanently closed). Call this game $\mathbb{G}'$. Note: despite P being weakly dominated, (P,P) is an NE for $\mathbb{G}'$.

$(M, M) \to 7.5, 9$

1. Find the Nash equilibria of $\mathbb{G}(3)$, for $\delta = 1$.

Normal form of the stage game:

		D	
		S	M
C	S	10, 10	0, 18
	M	15, 0	7.5, 9

Only 1 NE: Both players choose (M, M) at each round

2. What values of δ allow for sustaining a Nash equilibrium of $\mathbb{G}(\infty)$ via a "Grim Trigger" strategy where each player ends up in always choosing S?

GrT: Play S while the outcome is (S, S)
Otherwise play M

Coop:

$$u_C = u_D = \underbrace{10 + 10\delta + 10\delta^2}_{10(1+\delta+\delta^2+\cdots)} + \cdots = \frac{10}{1-\delta}$$

DEV:

$$u_C' = \underbrace{15 + 7.5\delta}_{15+7.5\delta(1+\delta+\cdots)} + 7.5\delta^2 + \cdots = 15 + \frac{7.5\delta}{1-\delta}$$

deviation NE

$$u_D' = 18 + 9\delta + 9\delta^2 + \cdots = 18 + \frac{9\delta}{1-\delta}$$

deviation NE

$$u_C \geq u_C' \quad , \quad u_D \geq u_D'$$

$$\frac{10}{1-\delta} \geq 18 + \frac{9\delta}{1-\delta}$$

$$10 \geq 18 - 18\delta + 9\delta$$

$$9\delta \geq 8 \rightarrow \boxed{\delta \geq \frac{8}{9}}$$

3. If you see an SPE of $\mathbb{G}'(2)$ where players may play S, state at which round do they play it, and what value of δ do you need to obtain it.

Normal form of G':

D

C

	S	M	P
S	10, 10	0, 18	-10, -10
M	15, 0	7.5, 9	-10, -10
P	-10, -10	-10, -10	-10, -10

For both: Play S at stage 1

If (S,S) is outcome of stage 1, play M; othw play P

Coop Dev

$10 + 9\delta \geq 18 - 10\delta$

Diana: $19\delta \geq 8 \rightarrow \boxed{\delta_D \geq \frac{8}{19}} \rightarrow \frac{8}{40} = 0.4$

Carl: $10 + 7.5\delta \geq 15 - 10\delta$

$17.5\delta \geq 5$

$\boxed{\delta_C \geq \frac{5}{17.5} = \frac{10}{35}} \leftarrow \frac{10}{50} = 0.25$

$\boxed{\delta_{min} = \frac{8}{19}}$