Università degli Studi di Padova

Dipartimento di Ingegneria dell'Informazione

Lecture 11
Dynamic games

Thomas Marchioro

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Dynamic games

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- A **dynamic game** involves some players moving first, others moving later
- We still consider dynamic games of **complete information**: strategies and payoffs are common knowledge
- However, we make a further distinction:
  - **perfect information**: every player can make decision with full awareness
  - **imperfect information**: some decisions are “simultaneous” or Nature moves

Battle of the Sexes (original)

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Now it becomes a dynamic game of perfect information

- A and B agreed to go to either rock concert (R) or science night (S)
  - Let us denote B's options in lower case for clarity

		B: r	B: s
A	R	2, 1	0, 0
	S	0, 0	1, 2

- To frame this as a normal game, the two players must act unbeknownst of each other
- Not very realistic nowadays

Battle of the Sexes (revisited)

- Let us add a more sensible time sequence
- Assume A decides (before B) which event to join, and texts B to let him know
  - Which event should she choose? R or S?
  - A knows (due to complete information) that whatever his/her choice, B's best response is to play along and choose the same thing
  - Since A prefers R over S, his/her best option is to choose R (no uncertainty on this outcome, we will see why)

Extensive form

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- To unfold the time dimension, we may want more than just the bi-matrix and payoffs
- We need to link possible choices to the knowledge of previous events
  - e.g., we need to model the fact that B acts after receiving A's text (B knows that A is playing R)
- This can be expressed using the so-called **extensive form** of the game
  - Graphically, this can be represented as a decision tree

Extensive form: ingredients

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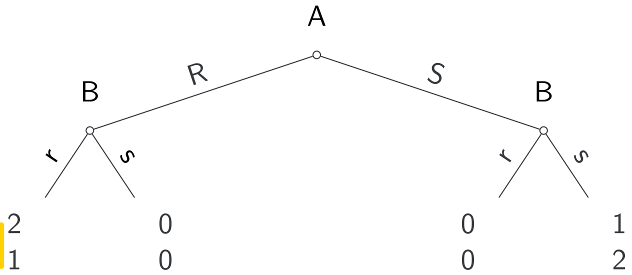
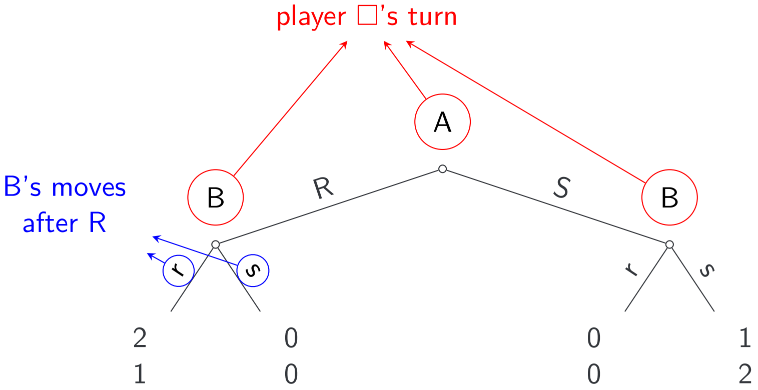
1. Set of players
2. Their utility functions
3. Order of their move turns
4. Actions allowed to players when they can move
5. Information they have when they can move
6. Probability of external events (lotteries)
7. All of this: common knowledge

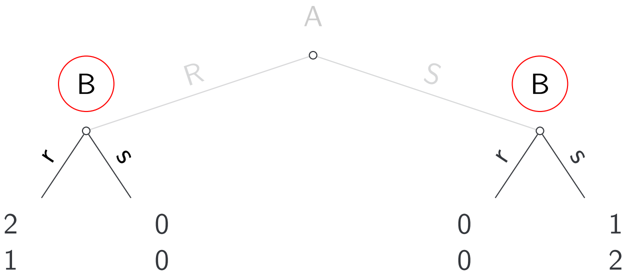
Extensive form representation





player □'s turn
A
B
B
R
S
B's moves
after R
r
s
r
s
2
1
0
0
0
0
1
2

Information sets



- From these two points on, B's options are the same, but the payoffs are different. Why?
- Because B knows A's choice
- Information is captured by different nodes

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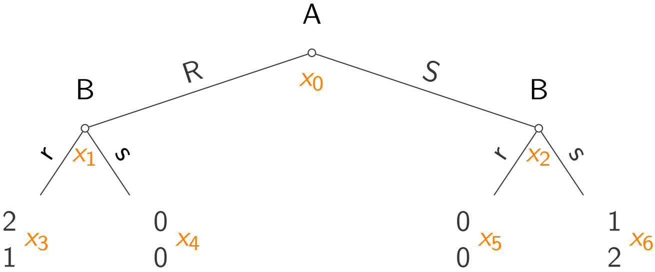
* priority on the left



- We can label the nodes of the tree starting from the root x_0 and proceeding until the leaves $x_3 \ldots, x_6$
- Here, in each node, players have different information

Information sets

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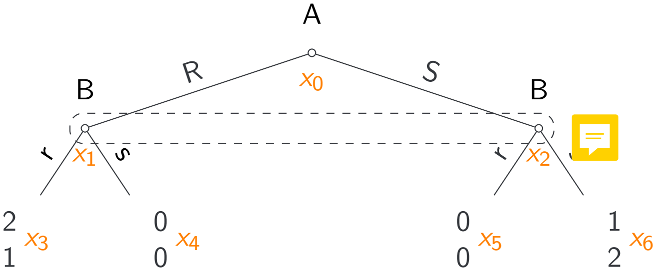
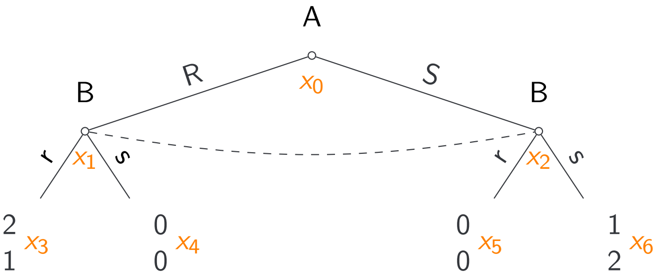
- Nodes go beyond denoting the game stage
- They also describe the **information set** h_i available to player i who is moving
- If the information set is a singleton $h_i = \{x_j\}$, then the node is fully aware of previous moves
- What if a player does not know?
  - In the original "battle of the sexes", B does not know A's choice
  - This means that B does not know if he/she is in x_1 or x_2

Information sets



- We circle two nodes with a dashed line to show that a player is not able to distinguish between them (i.e., they belong to the same information set)

Information sets



- Alternative notation: just use a dashed line connecting x_1 and x_2 to show that $h_\text{B} = \{x_1, x_2\}$

Perfect/imperfect information

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- In dynamic games with **perfect information**
  1. all information sets are singletons, and
  2. there is no choice of Nature
- Instead we have imperfect information in the following cases:
  - *endogenous uncertainty*: information sets contain multiple nodes (simultaneous moves)
  - *exogenous uncertainty*: there is a choice of Nature (lotteries)
- When we have imperfect information, players need to form beliefs

Re-defining strategies

- In static games of complete information, we have that
  - pure strategy = action
  - mixed strategy = probability distribution of actions
- In dynamic games, we need to account for the **history of play** (through the information sets)
- A player's **pure strategy** specifies an action according to what happened in the game
- Think of it as an algorithm: you decide of a countermove for any possible case
  - e.g. "If A plays R I play r; if A plays S I play s"

Re-defining strategies

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- In the battle of Sexes, A is moving first and both players choose a move in the set of actions $A_1 = A_2 = \{\mathrm{R}, \mathrm{S}\}$
- B has 2 actions, but more strategies
- A **strategy** s_B for B is a pair of elements of A

$$(\underbrace{a_R}_{\text{what to do if A plays R}} \quad , \quad \underbrace{a_S}_{\text{what to do if A plays S}})$$

  - s_B =(s,s) means "I go to S no matter what"
  - s_B =(r,s) means "I do what A does"
  - s_B =(s,r) means "I do the opposite of what A does"

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- If A and B repeat the original (static) battle for two consecutive nights
- A **strategy** for both players is now a quintuple of moves

$$\Big(\underbrace{a_1}_{\text{first move}},\ \underbrace{a_{2Rr}}_{\text{answer to Rr}},\ \underbrace{a_{2Rs}}_{\text{answer to Rs}},\ \underbrace{a_{2Sr}}_{\text{answer to Sr}},\ \underbrace{a_{2Ss}}_{\text{answer to Ss}}\Big)$$

  - s_i =(r,r,r,r,r) "Always go to R both nights"
  - s_i =(r,s,r,r,r) "Go to R on the first night. If the outcome of the first night is Rr, go to S on the second night; otherwise, go to R again."

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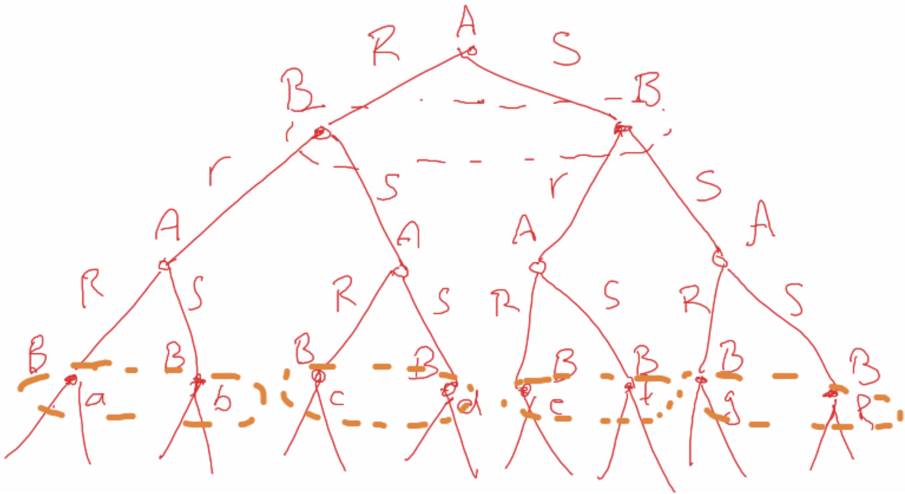
Prof example:



Re-defining strategies

- In principle, we may describe an "algorithm" for all possible strategies
- Yet, even a simple game with two sequential moves and $|A_1| = |A_2| = 3$ has 27 possible joint (pure) strategies:
  - 3 strategies by player 1
  - 9 strategies by player 2
- Therefore, we will often rely on some implicit description, except for very simple cases

Sorry, gotta bounce!
Send me questions via e-mail