

One-stage deviation principle

- Suppose we have a strategy that involves a sequence of moves "play a_1 at stage 1, a_2 at stage 2, ..., a_T at stage T " and we want to verify whether it is a NE or not

One-stage deviation principle: We only need to verify deviation at each stage individually

Example:

- Suppose we have a joint strategy that involves both players choosing actions a_1, a_2, carrot on three consecutive stages
- If sequence a_1', a_2', stick improves (by unilateral deviation) one player's payoff, then either a_1', a_2, stick improves the payoff or a_1, a_2', stick improves the payoff

Intuition: since stages are independent, either a_1' must improve the payoff of stage 1 or a_2' must improve the payoff of stage 2

$$\begin{aligned} \text{If } & u(a_1', a_1) + \delta u(a_2', a_2) + \delta^2 u(\text{carrot}) \\ & > u(a_1, a_1) + \delta u(a_2, a_2) + \delta u(\text{stick}) \end{aligned}$$

> ? > ? > (known)

We can't have both

$$\begin{aligned} & u(a_1', a_1) < u(a_1, a_1) \\ \text{and } & u(a_2', a_2) < u(a_2, a_2) \end{aligned} \quad \left. \vphantom{\begin{aligned} & u(a_1', a_1) < u(a_1, a_1) \\ & u(a_2', a_2) < u(a_2, a_2) \end{aligned}} \right\} \begin{array}{l} \text{otherwise} \\ u(a_1', a_1) + \delta u(a_2', a_2) \\ < u(a_1, a_1) + \delta u(a_2, a_2) \end{array}$$