

Nov 23, 2023

ex Multistage games

Ashley and Brook live together. During the winter break they contemplate giving each other a nice gift (G) for Christmas or not (N). They know each other's preferences so they are able to buy a gift for 10 euros that is worth like 100 euros for the other. They make this decision independently and without telling each other. After Christmas, they also consider whether to celebrate New Year's eve downtown (D) or stay home (H).

For the New Year's eve celebration, they decide independently of each other in a coordination-game fashion. Staying home has utility of 0 for both. Going downtown has utility of 50. However, spending New Year's eve apart from each other has utility of -100 for both. The total payoff of the players is the sum of the partial payoffs in each stage with a discount factor of δ for the second stage.

1) Normal form of stage games

Stage 1

	G	N
G	90 90	-10 100
N	100 -10	0 0

prisoner's dilemma-like

Only NE: (N,N)

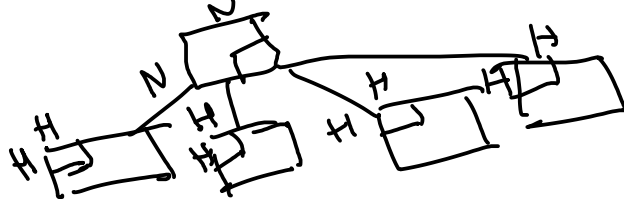
Stage 2

	H	D
H	0 0	-100 -100
D	-100 -100	50 50

coordination game

NE:

- (H, H) \rightarrow stick
- (D, D) \rightarrow carrot



2) Trivial SPE: playing stages independently

Stage 1: (N, N)

Stage 2: (H, H) regardless of outcome in stage 1

$(N H H H H, N H H H H)$ is SPE

Q: Other trivial SPE?

$(N D D D D, N D D D D)$

3) Cooperative SPE: "Play G at stage 1. If (G, G) at stage 1, play D; otherwise, play H"

Sustainable if unilateral deviation

$$90 + 50\delta \geq 100 + 0\delta$$

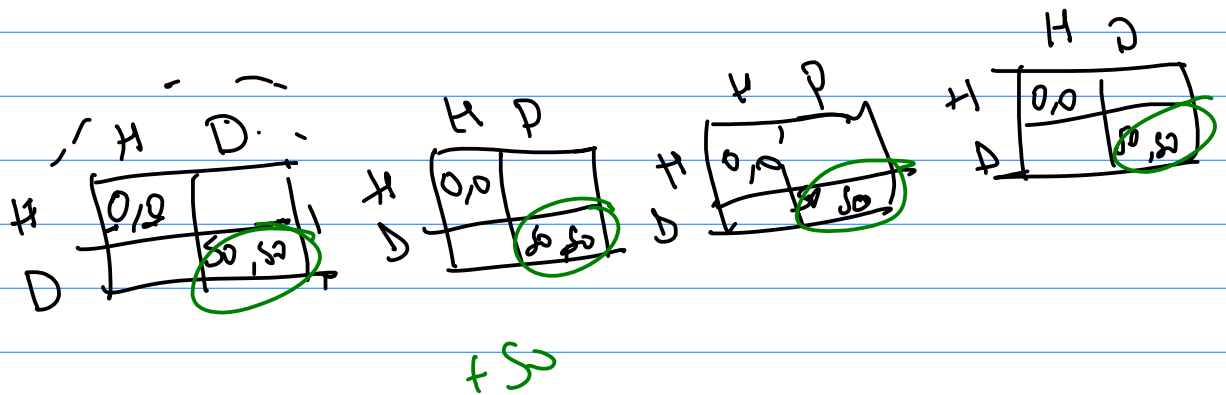
$$\delta \geq \frac{1}{5}$$

\Rightarrow

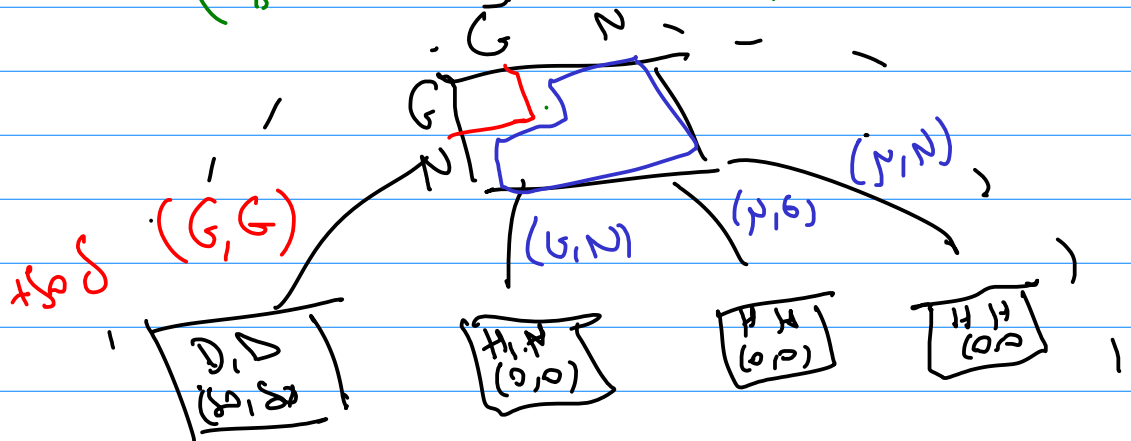
$$\boxed{\delta_{\min} = \frac{1}{5}}$$

$$u(\text{cooperation}) + \delta u(\text{carrot}) \geq u(\text{unilateral deviation}) + \delta u(\text{stick})$$

	G	N
G	10, 10	10, 0
N	0, 0	0, 0



(? DDDDD, ? DDDDD)



(? DHHH, ? DHHH)

This diagram is not an "officially accepted" representation, but should give you an intuition of how a cooperative strategy is defined in a 2-stage game:

- each outcome of the first stage leads to a different subgame in stage 2
- a stick-and-carrot cooperative strategy plays the "carrot" strategy (D) at stage 2 if players cooperated at stage 1, i.e., if the outcome was (G, G); otherwise, any different outcome in stage 1 leads to the "stick" strategy (H) played at stage 2