Signalling Game - How to draw the normal form matrix?

Asked 6 years, 8 months ago Modified 6 years, 8 months ago Viewed 2k times

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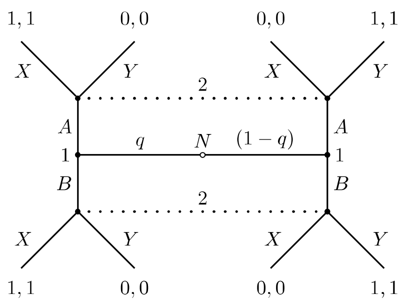
I am having trouble converting a signalling game with an extensive form representation to the normal form matrix.

E.g. this one from wikipedia:



I know each player has four strategies: P1: $\{AA, AB, BA, BB\}$ and P2: $\{XX, XY, YX, YY\}$ so we'll have a 4 by 4 table but then I am stuck.

For example, if player 1 plays AA, we are in the top part of the game, now if player 2 plays X we get 2 possible payoffs, how do I discern which payoff is related to strategy XX and which is XY ?

game-theory bayesian nash-equilibrium

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asked Apr 14, 2017 at 19:13

guy
367 5 22

3 Answers

Sorted by: Highest score (default)

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The *first* step in the game is the central node labeled "N". It encodes that Nature (or the referee, or Lady Luck) makes a random choice (with probabilities q and $1-q$) of whether we're in the left or right part of the game. Player 1 knows the outcome of this step, but player 2 doesn't (indicated by the dotted lines in the diagram).

If player 1's strategy is AA, then the second part of player 2's strategy (namely what P2 plays if P1 plays B) will never matter, and the outcome is the same in both cases.

However, since there is randomness involved, the payoff matrix should contain *expected* payoffs.

So in the case AA, XX (or AA, XY), the payoff is 1 to each player with probability q and 0 to each player with probability $1-q$. The *expected* payoff for these strategies is therefore q.

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answered Apr 14, 2017 at 19:32
hmakholm left over Monica
284k 24 425 685

I think I get it now, I will go off and run the calculations and come back with an answer so we can verify and close out this question. – guy Apr 14, 2017 at 19:52

Can you explain why AA, XX and AA, XY are both q? I got a payoff of q for both players for AA, XX but for AA, XY I got a payoff of 1 for both players. My reasoning being is P1 plays A if he's type 1 or type 2 then we are looking at the top half of scenarios. Then for P2 playing XY which means play X if P1 is type 1 and play Y if P1 is type 2, results in two payoff possibilities of (1,1) for both players i.e. the top left most payoff and top right most payoff, so the resulting expected payoffs is $q+1-q=1$ for each player. Where am I going wrong? – guy Apr 14, 2017 at 20:42

@tbone: No, because player 2 doesn't know which type P1 is. The XY strategy means the player 2 plays X in the top half of the game (the dotted line between those two nodes means that Y has to play identically in each of them) and plays Y in the bottom half of the game. – hmakholm left over Monica Apr 14, 2017 at 21:24

Okay so P1 plays according to his type given by Nature. And P2 plays according to P1's move where his strategy profiles are of the form (play this if P1 plays A, play this if P1 plays B), correct? – guy Apr 14, 2017 at 21:40

@tbone: Correct. – hmakholm left over Monica Apr 14, 2017 at 21:42

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If player 1 (the sender) plays AA then the payoffs from player 2 (the receiver) playing XX will be the same as playing XY.

In other words, if player 1 plays A whichever type they are, and player 2 plays X when player 1 plays A, then the world is unaffected by the hypothetical question of how player 2 might react to player 1 playing B

If player 1 has a strategy of AA and player 2 has a strategy of XX (or player 2 has a strategy of XY) then the payoff is $1, 1$ when player 1 is of the left type, and the payoff is $0, 0$ when player 1 is of the right type. That makes the expected payoff q, q.

By comparison, if player 1 has a strategy of AB (so top left and bottom right) and player 2 has a strategy of XX then then the payoff is $1, 1$ when player 1 is of the left type, and the payoff is $0, 0$ when player 1 is of the right type, so the expected payoff is still q, q.

Meanwhile, if player 1 has a strategy of AB (so top left and bottom right) and player 2 has a strategy of XY then then the payoff is $1, 1$ when player 1 is of the left type, and the payoff is $1, 1$ when player 1 is of the right type, so the expected payoff is now $1, 1$. This shows the benefit of signalling

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answered Apr 14, 2017 at 19:32

Henry
155k 9 124 252

Can you explain why AA,XXAA,XX and AA,XYAA,XY are both qq ? I got a payoff of qq for both players for AA,XXAA,XX but for AA,XYAA,XY I got a payoff of 11 for both players. My reasoning being is P1 plays AA if he's type 1 or type 2 then we are looking at the top half of scenarios. Then for P2 playing XYXY which means play XX if P1 is type 1 and play YY if P1 is type 2, results in two payoff possibilities of (1,1) for both players i.e. the top left most payoff and top right most payoff, so the resulting expected payoffs is q+1−q=1q+1−q=1 for each player. Where am I going wrong? – guy Apr 14, 2017 at 20:44

P2's choices is playing X or Y if P1 plays A, and playing X or Y when P1 plays B. P2 does not know P1's type. So "XY" would mean P2 plays X if P1 plays A, and P2 plays Y when P1 plays B. Since P1 is playing "AA", they always get the A&X outcomes, making the expectation $q(1,1) + (1-q)(0,0) = q, q$ – Henry Apr 14, 2017 at 21:09

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To summarize, it is important to remember P1 chooses his strategies based on his type and P2 chooses his strategies based on P1's choice.

So for example the strategy AB for P1 means, if I am type 1 (left side of the graph) choose A as an action, if I am type 2 (right side of the graph) choose B as an action. Remember, P1 observes his type based on Natures probabilities of being type 1 or type 2. For P2, the strategy XY would mean, if P1 plays A play X, if P1 plays B play Y. P2 does not know what P1's type is so his strategies must be based on P1's actions.

Since we don't know what side we are on because this always decided by Nature, there will always be two possible payoffs for any situation. Meaning, we need to calculate each player's **expected payoff** based on a situation.

Let's calculate a few to demonstrate this. If P1 plays AA, we are essentially in the top part of the graph as regardless of his type, he will play A. Now if P2 plays XX, he is basically saying, whatever action P1 takes, I'll play X, so this leaves only the X branches to consider. And since we are only in the top part of the graph, the branches to consider are the X branches yielding a payoff of (1,1) and (0,0). The expected playoff of P1 is $u_1 = 1 \times q + 0 \times (1-q) = q$ and for P2 it is $u_2 = 1 \times q + 0 \times (1-q) = q$.

Using the same logic we can repeat this for all 16 scenarios. Here are a few more examples:

- BA and YX results in $u_1 = u_2 = 1 \times q + 1 \times (1-q) = 1$

- AB and XY results in $u_1 = u + 2 = 1 \times q + 1 \times (1 - q) = 1$

Computing these for all the strategy profiles, we can create the following normal form representation:

	XX	XY	YX	YY
AA	q, q	q, q	$1 - q, 1 - q$	$1 - q, 1 - q$
AB	q, q	$1, 1$	$0, 0$	$1 - q, 1 - q$
BA	q, q	$0, 0$	$1, 1$	$1 - q, 1 - q$
BB	q, q	$1 - q, 1 - q$	q, q	$1 - q, 1 - q$

Big thanks to Henry and Henning for providing intuition for this understanding.

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edited Apr 15, 2017 at 14:58

answered Apr 14, 2017 at 22:36

guy
367 5 22

Your matrix seems to have swapped the outcomes for BA,XY and BA,YX. – hmakholm left over Monica Apr 15, 2017 at 10:29