

Nov 29, 2023

Exercise 5.4. Carl (C) and Diana (D) are two university students that have found that the department library is unoccupied overnight. It is a really good place to study and has a very fast Internet connection. So, they go there every night, but they do not coordinate or plan any action together. Upon their arrival every night, they independently decide whether: (S) study or (M) watch some movies on their laptop. If they both study, they both get utility 10. The individual benefit from watching a movie is instead 15 for C and 18 for D. However, if they both choose M, their individual benefit is halved (since they have half the connection speed). Also, trying studying while somebody else is playing a movie breaks the concentration, so $u_C(S, M) = u_D(M, S) = 0$ (C is written as the first player). Call \mathbb{G} this game, and consider it in a repeated version $\mathbb{G}(T)$, where \mathbb{G} is played every night for T nights. Individual payoffs are cumulated with discount factor δ . Finally, consider an *extended* game where a punishment strategy P is also available to both players. When either player chooses P , payoffs are -10 for *both* players (that would correspond, e.g., to do something really stupid in the library and get the library permanently closed). Call this game \mathbb{G}' . Note: despite P being weakly dominated, (P, P) is an NE for \mathbb{G}' .

$$(M, M) \rightarrow 7.5, 9$$

1. Find the Nash equilibria of $\mathbb{G}(3)$, for $\delta = 1$.

Normal form of the stage game:

		D	
		S	M
C	S	10, 10	0, <u>18</u>
	M	<u>15</u> , 0	<u>7.5</u> , <u>9</u>

Only 1 NE: Both players choose (M, M)
at each round

2. What values of δ allow for sustaining a Nash equilibrium of $G(\infty)$ via a "Grim Trigger" strategy where each player ends up in always choosing S?

GrT: Play S while the outcome is (S, S)
Otherwise play N

Coop:

$$u_c = u_D = 10 + 10\delta + 10\delta^2 + \dots = \frac{10}{1-\delta}$$

DEV:

$$u_c' = 15 + 7.5\delta + 7.5\delta^2 + \dots = 15 + \frac{7.5\delta}{1-\delta}$$

$$u_D' = 18 + 9\delta + 9\delta^2 + \dots = 18 + \frac{9\delta}{1-\delta}$$

$$u_c \geq u_c', \quad u_D \geq u_D'$$

$$\frac{10}{1-\delta} \geq 18 + \frac{9\delta}{1-\delta}$$

$$10 \geq 18 - 18\delta + 9\delta$$

$$9\delta \geq 8 \rightarrow \boxed{\delta \geq \frac{8}{9}}$$

3. If you see an SPE of $G'(2)$ where players may play S, state at which round do they play it, and what value of δ do you need to obtain it.

Normal form of G' :

		D		
		S	M	P
C	S	10, 10	0, <u>18</u>	<u>-10</u> , -10
	M	<u>15</u> , 0	<u>7.5</u> , <u>2</u>	<u>-10</u> , -10
	P	-10, <u>-10</u>	-10, <u>-10</u>	<u>-10</u> , <u>-10</u>

For both: Play S at stage 1

If (S,S) is outcome of stage 2,
play M; otherwise play P

Coop

Dev

$$10 + 9\delta \geq 18 - 10\delta$$

Diana: $19\delta \geq 8 \rightarrow \boxed{\delta \geq \frac{8}{19}} \rightarrow \frac{8}{19} = 0.4$

Carl: $10 + 7.5\delta \geq 15 - 10\delta$

$$17.5\delta \geq 5$$

$$\boxed{\delta \geq \frac{5}{17.5} = \frac{10}{35}}$$

$$\frac{10}{35} = 0.25$$

$$\boxed{\delta_{\min} = \frac{8}{19}}$$