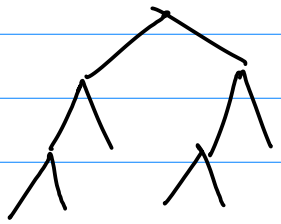
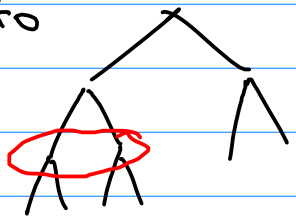
Nov 15, 2023

Recap

- Information sets → Perfect info

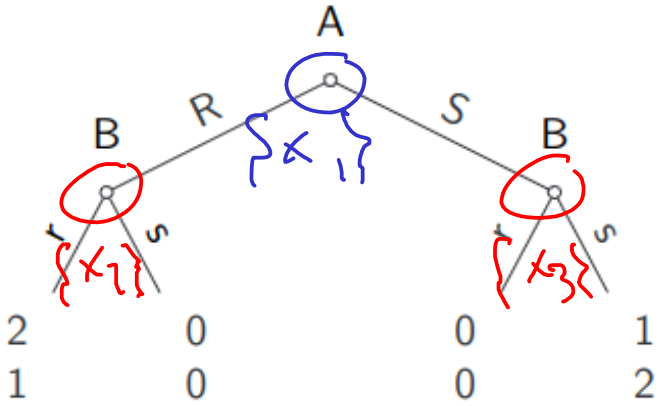


↘ Imperfect info



- Strategies: for each player

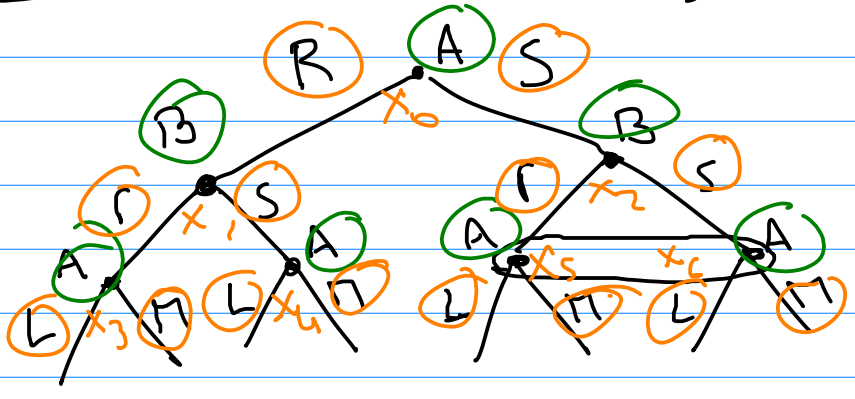
  1 action per info set



$S_A = a_1 \in \{R, S\}$

$S_B = a_2 a_3 \in \{rr, rs, sr, ss\}$

Q: What about this?

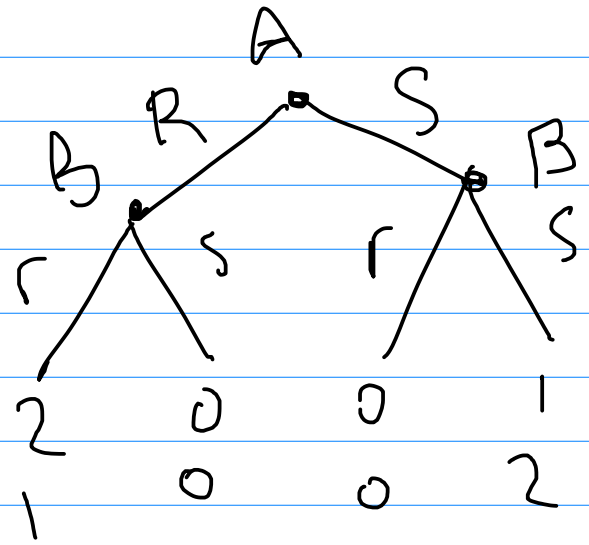


$S_A = (a_0, a_3, a_4, a_{56})$

$|S_A| = 16$

$\underbrace{m_0}_{2} \times \underbrace{m_3}_{2} \times \underbrace{m_4}_{2} \times \underbrace{m_{56}}_{2}$

Mixed strategies in dyn games



p_i : strategy → Prob[strategy]

e.g.

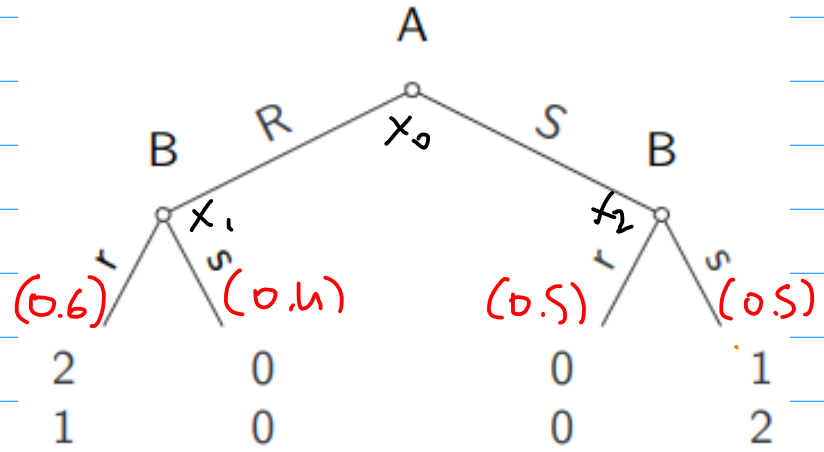
$p_1 = (0.3, 0.3, 0.2, 0.2)$

rr rs sr ss

Behavioral strategy σ_i (action | info set)

Example: $\sigma_B(r \mid \{x_1\}) = 0.6$ → Prob[B plays r | A played R]

$\sigma_B(r \mid \{x_2\}) = 0.5$



$P_B(rr) = 0.6 \times 0.5 = 0.3$

$P_B(rs) = 0.3$

$P_B(sr) = 0.4 \times 0.5 = 0.2$

$P_B(ss) = 0.2$

$$Prob[r \mid in\ x_1] = Prob[r \text{ in } x_1, r \text{ in } x_2] + Prob[r \text{ in } x_1, s \text{ in } x_2]$$

$$\sigma_B(r \mid \{x_1, x_2\}) = P_B(r) + P_B(s)$$

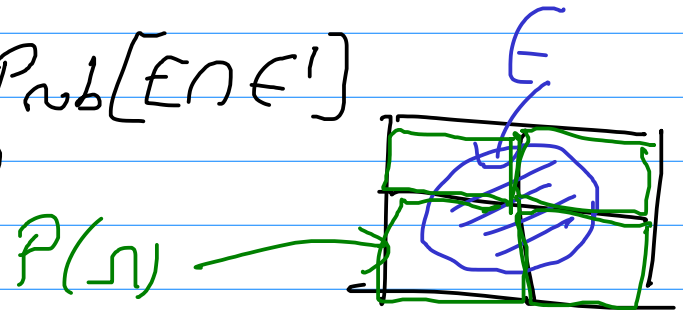
$$\sigma_B(r \mid \{x_1\}) = P_B(r) + P_B(s) = 0.5$$

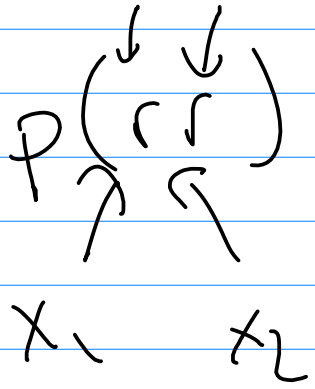
0.7 0.2

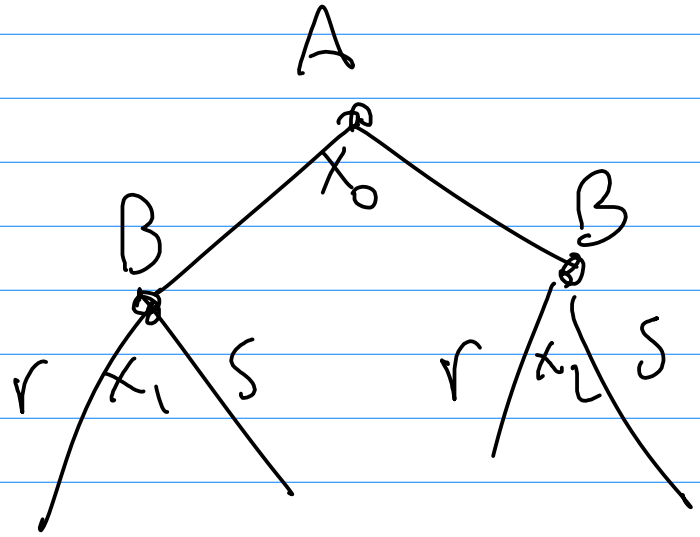
$$\sigma_B(s \mid \{x_1\}) = P_B(s) + P_B(s)$$

Law of total probability:

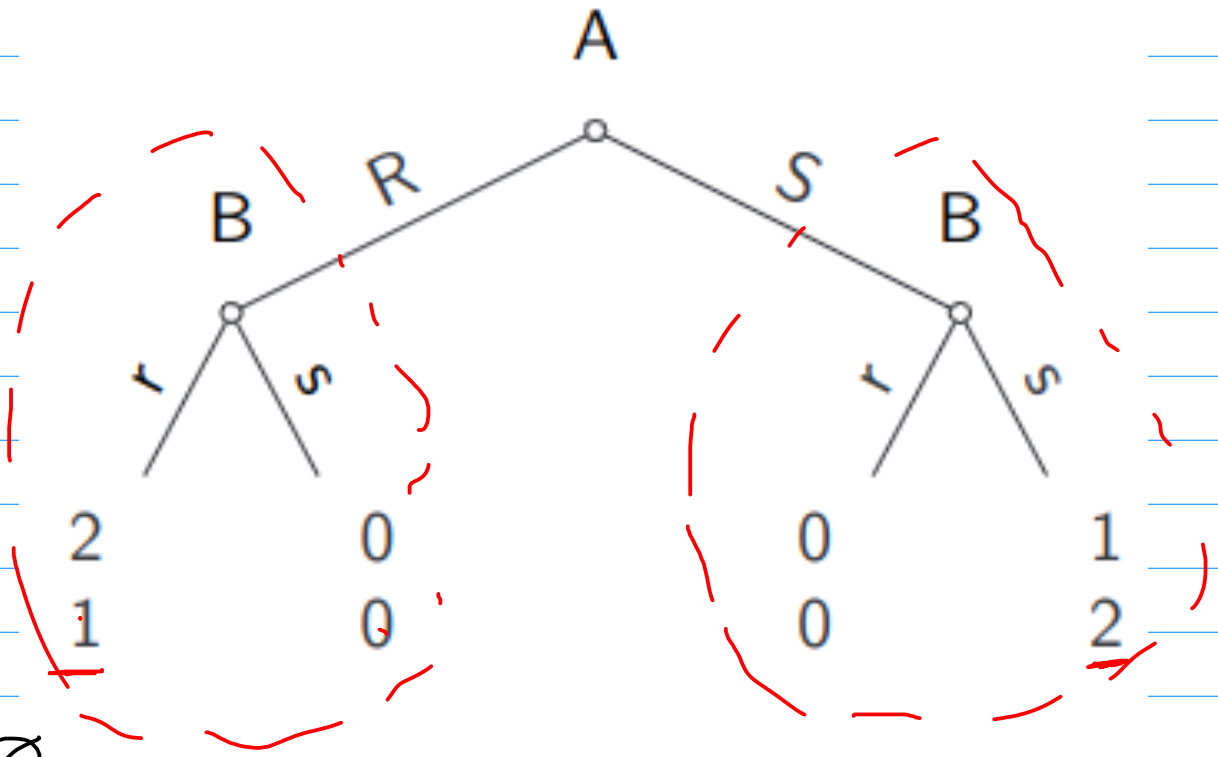
$$Prob[E] = \sum_{E' \in P(\Omega)} Prob[E \cap E']$$





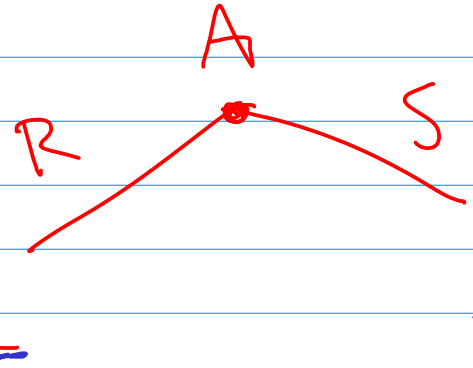


Backward induction



$S_B = rs$

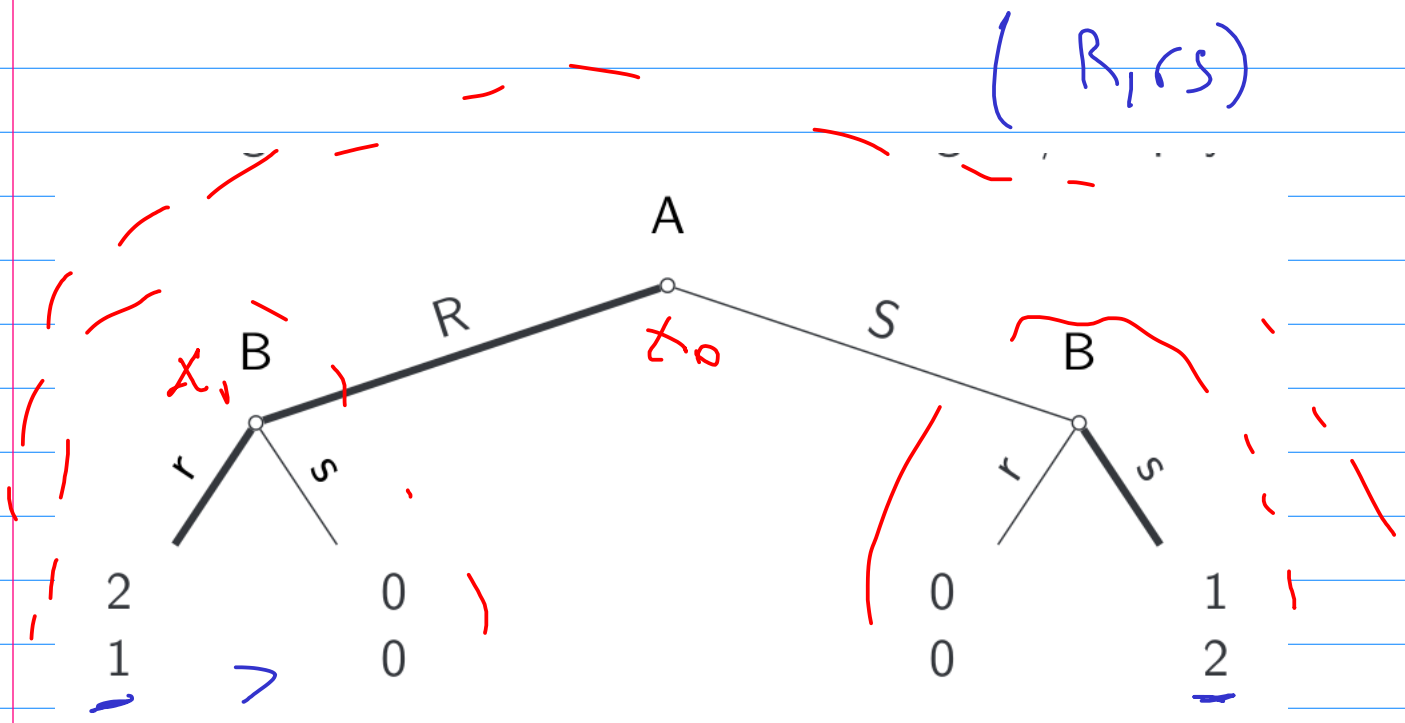
B to play -> Choose highest payoff for B



A to play -> Choose highest payoff for A

$S_A = R$

$(S_A, S_B) = (R, rs)$



To verify that a NE is subgame perfect using the definition, one needs to check that a NE is played in all subgames:
- overall game (x0 and descendants): (R, rs) is a NE
- x1: B maximizes his/her payoff by playing r -> NE
- x2: B maximizes his/her payoff by playing s -> NE