		J	K	L	M
A	X	6, 7	5, 5	3, 8	8, 1
	Y	4, 9	9, 2	0, 4	2, 3
	Z	8, 4	2, 8	4, 2	3, 6

(Columns: player B)

1. Show that there is no NE in pure strategies
2. Show that $(\mathbf{p}_A, \mathbf{p}_B)$ with $\mathbf{p}_A = (2/3, 0, 1/3)$ and $\mathbf{p}_B = (5/11, 4/11, 2/11, 0)$ is a mixed NE
3. List all joint pure strategies that are Pareto optimal (i.e., Pareto efficient)

1)

		B			
		J	K	L	M
A	X	6, 7	5, 5	3, 8	8, 1
	Y	4, 9	9, 2	0, 4	2, 3
	Z	8, 4	2, 8	4, 2	3, 6

2)

Use characterization theorem: for each player i

$$u_i(s_i, \mathbf{p}_{-i}) = u_i(\mathbf{p}_i, \mathbf{p}_{-i}) \text{ for each } s_i \in \text{supp}(\mathbf{p}_i)$$
$$u_i(s_i, \mathbf{p}_{-i}) \leq u_i(\mathbf{p}_i, \mathbf{p}_{-i}) \text{ for each } s_i \notin \text{supp}(\mathbf{p}_i)$$

- *first equation:* This part of the theorem ensures that each player's expected utility when playing their mixed strategy is equal to their utility when playing the mixed strategy prescribed by the equilibrium.
- *second equation:* This part of the theorem ensures that each player has no incentive to unilaterally deviate from their chosen mixed strategy. If a player were to play a pure strategy outside the support of their mixed strategy, their expected utility should be less than or equal to the utility they get from playing the equilibrium mixed strategy

So know we have to take only the utility values for pure strategies with prob >0 ⇒ this means that they are in support of pi.

For player A

$$u_A(X, \mathbf{p}_B) = 6 \cdot \frac{5}{11} + 5 \cdot \frac{4}{11} + 3 \cdot \frac{2}{11} = \frac{56}{11}$$
$$u_A(Y, \mathbf{p}_B) = 4 \cdot \frac{5}{11} + 9 \cdot \frac{4}{11} + 0 \cdot \frac{2}{11} = \frac{56}{11}$$
$$u_A(Z, \mathbf{p}_B) = 8 \cdot \frac{5}{11} + 2 \cdot \frac{4}{11} + 4 \cdot \frac{2}{11} = \frac{56}{11}$$

All pure strategies yield equal payoff, so $\mathbf{p}_B$ is a sustainable NE strategy

N.B:

- To satisfy the first part we've checked that the prob of pure strategies in support of the mixed strategies knowing that the other player is playing a mixed strategy is the same (*1st part of the theorem)*
- Note that here M is not considered since pb has prob=0, so it is not in support of the mixed strategy (*2nd part of the theorem)*

For player B

$$u_B(\mathbf{p}_A, J) = 7 \cdot \frac{2}{3} + 4 \cdot \frac{1}{3} = \frac{18}{3} = 6$$
$$u_B(\mathbf{p}_A, K) = 5 \cdot \frac{2}{3} + 8 \cdot \frac{1}{3} = \frac{18}{3} = 6$$
$$u_B(\mathbf{p}_A, L) = 8 \cdot \frac{2}{3} + 2 \cdot \frac{1}{3} = \frac{18}{3} = 6$$
$$u_B(\mathbf{p}_A, M) = 1 \cdot \frac{2}{3} + 6 \cdot \frac{1}{3} = \frac{8}{3} \leq 6$$

M does not belong to the support and yields lower payoff, so $\mathbf{p}_A$ is a sustainable NE strategy

Since both players satisfy the conditions (pA,pB) is a mixed NE.