

Exercise set #1

Game Theory 2023/24

Exercise 1 Consider the following static games of complete information played between A and B where the normal-form representation is given. For all of them, find the entire set of Nash equilibria.

| | | B | |
|---|---|------|------|
| | | M | N |
| A | F | 2, 4 | 0, 1 |
| | G | 1, 6 | 3, 5 |

| | | B | |
|---|---|------|------|
| | | M | N |
| A | F | 0, 4 | 3, 0 |
| | G | 6, 0 | 0, 5 |

| | | B | |
|---|---|------|------|
| | | M | N |
| A | F | 9, 3 | 2, 2 |
| | G | 0, 0 | 3, 9 |

| | | B | |
|---|---|------|------|
| | | M | N |
| A | F | 2, 2 | 0, 6 |
| | G | 6, 0 | 1, 1 |

Exercise 2 Consider the following static game of complete information played between A and B where the normal-form representation is given below.

| | | B | | | |
|---|---|------|------|------|------|
| | | J | K | L | M |
| A | X | 6, 7 | 5, 5 | 3, 8 | 8, 1 |
| | Y | 4, 9 | 9, 2 | 0, 4 | 2, 3 |
| | Z | 8, 4 | 2, 8 | 4, 2 | 3, 6 |

1. Show that there is no Nash equilibrium in pure strategies.
2. Prove that the joint mixed strategy $(\mathbf{p}_A, \mathbf{p}_B)$ is a mixed Nash equilibrium if $\mathbf{p}_A = (2/3, 0, 1/3)$ and $\mathbf{p}_B = (5/11, 4/11, 2/11, 0)$.
3. List all the joint pure strategies that are Pareto optimal ("Pareto optimal" is the same as "Pareto efficient").

Exercise 3 (*Exercise 1.7 in the handbook*) Two firms (F1 and F2) work on a joint project from the European Commission. They can allocate an integer number of employees on the project, from 0 to infinity. They decide independently and without consulting with each other. The outcome of the project is that, if the number of employees allocated by each firm is identical (even zero!), both firms get a funding of 290 k€ from the European Commission. If the two firms assign a different number of employees, the European Commission gives them different fundings: 700 k€ to the one with more employees, and 320 k€ to the one with fewer employees. However, assigning employees costs 200 k€ per employee. The *utility* of a firm is funding minus costs.

1. Show that no rational firm will allocate more than 2 employees on the project.
2. Draw the normal form of this game (considering then up to 2 employees) and find its Nash equilibria in pure strategies.
3. Find the additional Nash equilibria in mixed strategies.

Exercise 4 A strategic interaction takes place between the taxpayer T and the tax inspector I. T is supposed to pay a share S of his income so as to have a net income equal to R after paying taxes. However, T is considering two alternatives: hide part of his income (**H**) so as to pay $S - L$ instead of S (thus getting a net income of $R + L$), or pay all due taxes in full (**P**). I also has two options: check T for tax fraud (**C**) or not (**N**). Performing a check has a cost equal to E . If the inspector finds out that T has hidden part of the income, then the taxpayer will have to pay a fine of F , and an equal amount is collected by the inspector. The probability of being caught after a tax inspection is p . The goal of the taxpayer is to get the maximum possible amount of money. The objective of the inspector is to collect as much money as possible. Formalize this conflict in the form of a static game of complete information and find the Nash equilibria in pure strategies.