Ex (Minimax) (of previous exam)

Exercise 2.2. Consider the following game in normal form. Players are denoted as 1 and 2 and their strategy sets are $\mathcal{S}_1 = \{A, B, C\}$ and $\mathcal{S}_2 = \{X, Y\}$. The payoff matrix is as follows:

		2	
		X	Y
1	A	9, -9	-5, 5
	B	-2, 2	7, -7
	C	8, -8	-1, 1

1. Describe what kind of game is that and name a specific solution concept you might use to find its Nash equilibria.

A: Zero-sum game → Can be solved using minimax / maximin

		2		
		X	Y	f_1
1	A	9, ~~-9~~	-5, ~~5~~	-5
	B	-2, ~~2~~	7, ~~-7~~	-2
	C	8, ~~-8~~	-1, ~~1~~	-1
		9	7	

$\text{maximin}_1 = -1$

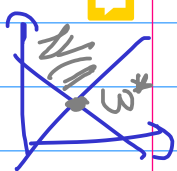
$\text{minimax}_1 = 7$

- zero-sum game
- $\text{minimax}_1 \neq \text{maximin}_1$ → No NE in pure strategies

2. What is the support within $\mathcal{S}_1$ of the mixed strategy played at the Nash equilibrium by 1?

OPTION 1

Minimax^{P}_{1}:



approach w^* from above

$$\min \; w$$

s.t. $9b_1 - 5b_2 \le w$ (A)

$-2b_1 + 7b_2 \le w$ (B)

$8b_1 - b_2 \le w$ (C)

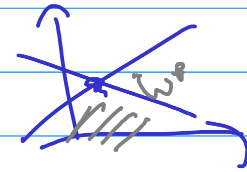
$b_1 + b_2 = 1, \; b_j \ge 0$

$\underbrace{\qquad\qquad\qquad}$

Lines → easy

OPTION 2

Maximin^{P}_{1}:



approach w^* from below

$$\max \; w$$

s.t. $9a_1 - 2a_2 + 8a_3 \ge w$ (X)

$-5a_1 + 7a_2 - a_3 \ge w$ (Y)

$a_1 + a_2 + a_3 = 1, \; a_k \ge 0$

$\underbrace{\qquad\qquad\qquad}$

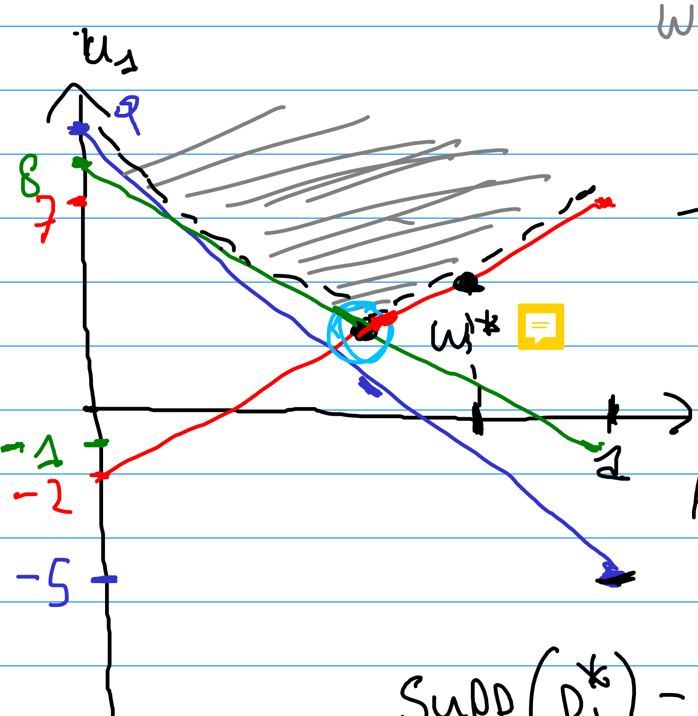
Planes → hard

$w \ge$ all lines

$b_1 = 1 - \beta$

$b_2 = \beta$

prob of 2 playing Y



$- - : F_1(\beta) = \max_{S_1} u_1(S_1, \beta)$

$\text{Supp}(p_1^*) = \{B, C\}$

3. Find the mixed strategies played by 1 and 2 at the Nash equilibrium.

- To find p_2^* : characterization theorem

$$u_1(B,\beta) = u_1(C,\beta) \qquad supp(p_1^*) = \{B,C\}$$

$$-2(1-\beta)+7\beta = 8(1-\beta)-\beta$$

$$-2+2\beta+7\beta = 8-8\beta-\beta$$

$$18\beta = 10$$

$$\rightarrow \boxed{\beta^* = \frac{5}{9}} \qquad \rightarrow p_2^* : \left(\frac{4}{9}, \frac{5}{9}\right)$$

- To find p_1^* : again, characterization theorem

Also, we know $p_1^*: (0, ?, ?)$

Let $a_2 = 1-\alpha$, $a_3 = \alpha$

$$-2(1-\alpha)+8\alpha = 7(1-\alpha)-\alpha$$

$$-2+2\alpha+8\alpha = 7-7\alpha-\alpha$$

$$18\alpha = 9$$

$$\boxed{\alpha^* = \frac{1}{2}} \qquad \rightarrow p_1^* : \left(0, \frac{1}{2}, \frac{1}{2}\right)$$

To verify if we have done all correctly we can check if the value of maximim and minimax are equal using their definition in mixed strategies:

$$\text{maximin}_1^{P.O}: -2\,\frac{1}{2} + 8\,\frac{1}{2} = -1 + 4 = 3$$

$$\text{minimax}_1^{P}: -2\,\frac{4}{9} + 7\,\frac{5}{9} = \frac{-8+35}{9} = \frac{27}{9} = 3$$