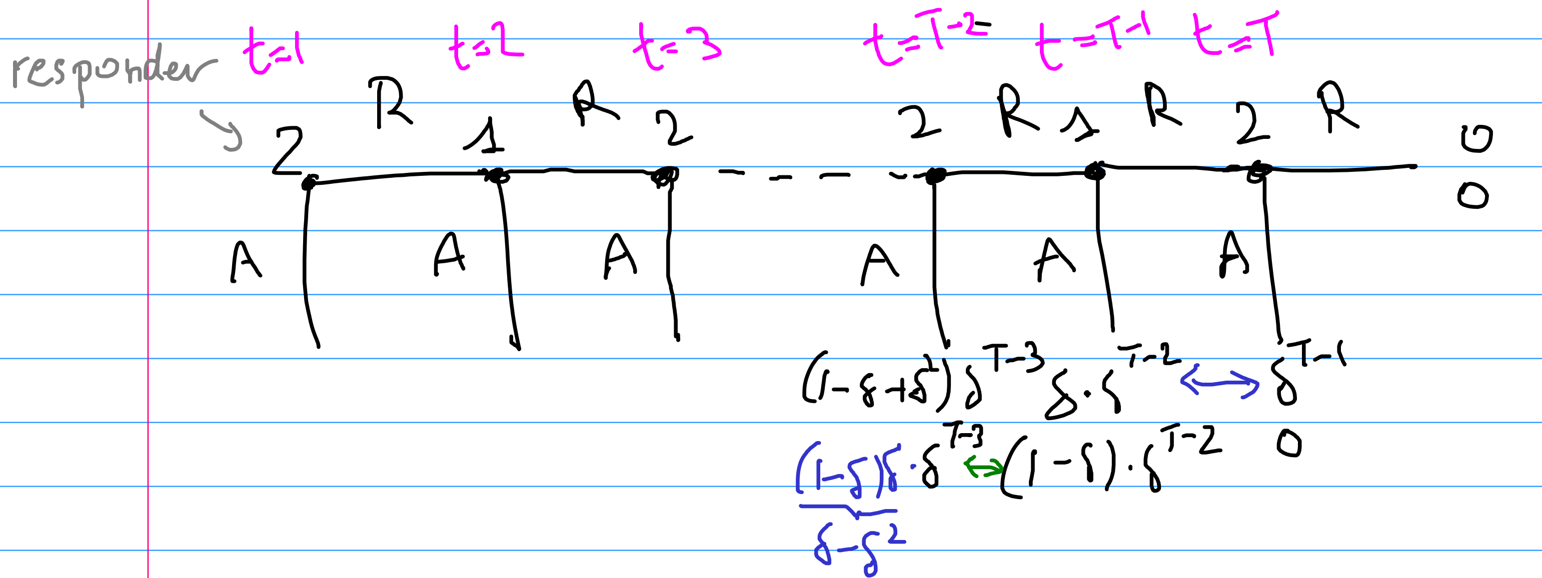
Lecture 17 (Simplified game)



If Player 1 is the last proposer

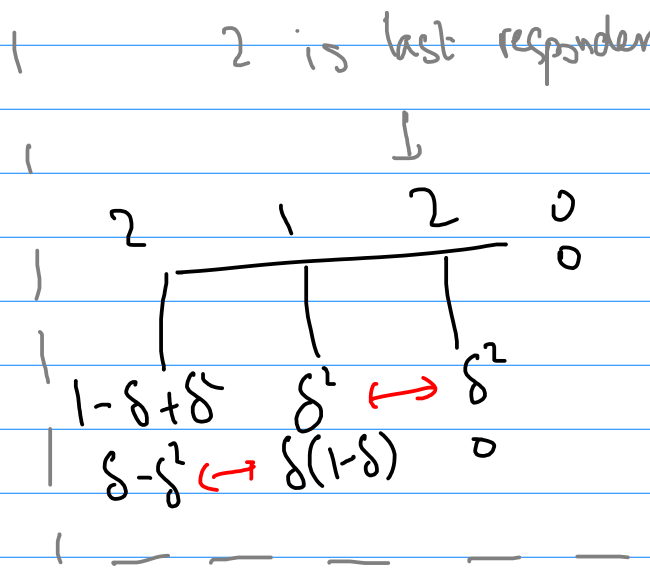
$T=1 \Rightarrow \underline{1} \quad 0$

$T=2 \Rightarrow \delta \quad \underline{1-\delta}$

$T=3 \Rightarrow \underline{1-\delta+\delta^2} \quad \delta-\delta^2$

$T=4 \Rightarrow \delta-\delta^2+\delta^3 \quad \underline{1-\delta+\delta^2-\delta^3}$

$\vdots$



First proposer gets $\underbrace{1}_{(-1)^0\delta^0}-\underbrace{\delta}_{(-1)^1\delta^1}+\delta^2-\delta^3+\cdots+(-1)^{T-1}\delta^{T-1}=x$

First responder get $\delta-\delta^2+\delta^3-\delta^4+\cdots+(-1)^T\delta^{T-1}=1-x$

$$\boxed{\sum_{t=0}^{\infty} \alpha^t = \frac{1}{1-\alpha} \quad |\alpha|<1}$$

First proposer

$$1-\delta+\delta^2-\delta^3+\cdots+(-1)^{T-1}\delta^{T-1} =$$

$$= (-\delta)^0+(-\delta)^1+(-\delta)^2+\cdots+(-\delta)^{T-1} =$$

$$= \sum_{t=1}^{T}(-\delta)^{t-1} = \sum_{t=0}^{T-1}(-\delta)^t = \sum_{t=0}^{\infty}(-\delta)^t - \sum_{t=T}^{\infty}(-\delta)^t =$$

$$= \frac{1}{1-(-\delta)} - \sum_{t=0}^{\infty}(-\delta)^{\overbrace{t+T}^{(-\delta)^t(-\delta)^T}} = \frac{1}{1+\delta} - (-\delta)^T\sum_{t=0}^{\infty}(-\delta)^t$$

$$= \frac{1}{1+\delta} - \frac{(-\delta)^T}{1+\delta}$$

If T is odd $\rightarrow$ First proposer gets $\dfrac{1+\delta^T}{1+\delta}$

First responder gets $\dfrac{\delta-\delta^T}{1+\delta}$