Lecture 01
Introduction

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Game Theory 2023/24

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Are you in the right class?

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- Computer Engineering
- Data Science
- Cybersecurity
- …

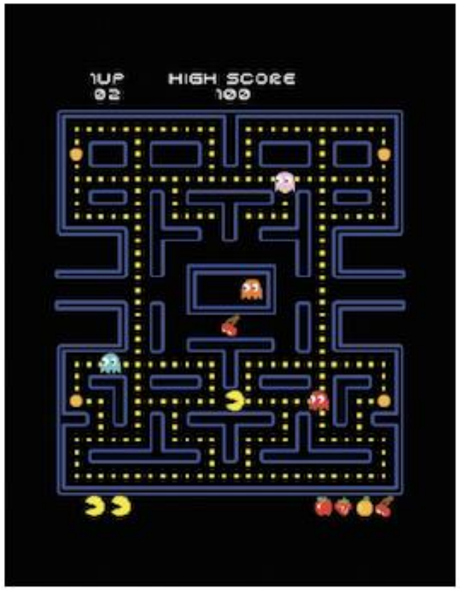
✓

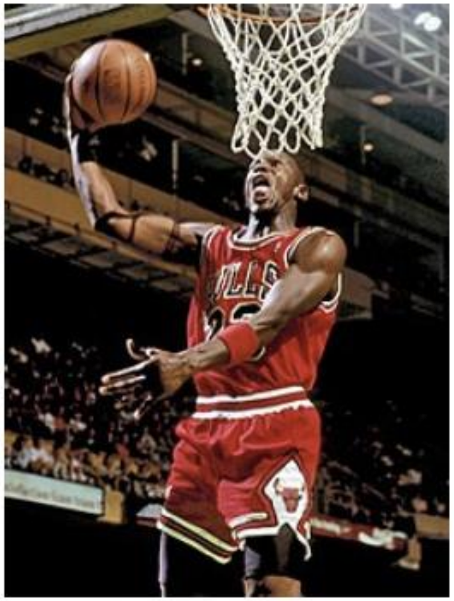
- ICT for Internet and Multimedia

✗

What game theory is NOT

- When you say “games” in everyday language, you typically mean





- Game theory is not about these ☹

What is game theory?

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- Game theory is a **mathematical framework** that allows to model specific type of problems
- The problems studied by game theory are called **games**

What is a game?

- **Definition:** A game is a multi-person multi-objective problem
  - Multi-person* = There are multiple agents (players) involved in a game
  - Multi-objective = Players have, in general, different goals
- The outcome of a game depends on the choices made by all players
- The purpose of game theory is to find the "best choice" for each of them according to their objectives

* Actually, players are not necessarily people

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COMPARISON WITH OPTIMIZATION

Optimization

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- In optimization, you need to find the best solution s^* in a set of possible choices S
- The best solution maximizes a certain utility function $u(s)$

- Here we have:
  - One player: the engineer who chooses the solution
  - One objective: the utility function

Optimization: example

- Find the cylinder with maximum volume having total surface 1 m^2
- **Player:** P = student
- **Set of possible choices:** S = all cylinders with total surface 1 m^2
- **Utility function:** $u(s)$ = volume of cylinder s

Distributed optimization

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- In distributed optimization, we have N players making independent decisions:
  - Player 1 chooses the best $s_1^* \in S_1$
  - Player 2 chooses the best $s_2^* \in S_2$
  - …
  - Player N chooses the best $s_N^* \in S_N$
- They all maximize the same function $u(s_1, \dots, s_N)$
- Here we have:
  - Multiple players
  - Single objective
  - Examples?

Game theory

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- In game theory the N players make independent decisions
  - Player 1 chooses the best $s_1^* \in S_1$
  - Player 2 chooses the best $s_2^* \in S_2$
  - …
- And maximize different utility functions
  - Player 1 wants to maximize $u_1(s_1, \ldots, s_N)$
  - Player 2 wants to maximize $u_2(s_1, \ldots, s_N)$
  - …

Characteristics of a game

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- Multiple players
- Multiple objectives
- Typical assumptions:
  - The rules of the game are known in advance to all the players
  - Players are *rational* (i.e., they always choose the best action/strategy to maximize their utility function)
  - The outcome of the game depends on the joint decision of all players

$$u_i\underbrace{(s_1, \ldots, s_N)}$$

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APPLICATIONS

Applications of game theory

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- Game theory originally starts as a branch of economics:
  - Micro level: trading, auctions, bargaining
  - Intermediate level: market, firms
  - Macro level: countries, monetary authorities
- Classic economic application:
  - Players = enterprises
  - Game = market

Applications of game theory

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- Biology (evolutionary game theory):
  - Players = animal species
  - Game = interaction between species
- Computer science:
  - Players = devices in a network
  - Game = resource allocation
- Engineering:
  - Players = autonomous vehicles
  - Game = road traffic

Predictions

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- Uncertainty in game theory: players know their own moves but can only make conjectures about other players' move
- Still, some outcomes can be predicted
  - Easier for machines and algorithms (rationality, clearly-defined utility function)

Application examples

- Resource/channel access
  - Users want to be served regardless of congestion
- Internet of Things
  - Devices with different capabilities and objectives
- Information security
  - Best attack strategy and best defense strategy
- Massive online platforms
  - Electronic voting, online auctions, social networks

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BACKGROUND CONCEPTS

Monopolies and oligopolies

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- Monopoly = single entity controlling the entire market for a product or good
- Oligopoly = few entities sharing the market control for a product or good
- Examples:
  - Internet Service Providers (ISPs)
  - Software: operating systems
  - Online services: search engines, maps, social networks
- Anti-trust punishes monopolists. Why?

Market

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- Adam Smith theory: a free market leads to the best social outcome
- Demand/offer laws, "invisible hand"
  - Competition leads to lower market price
- A <u>rational</u> monopolist pursues profit, not social welfare
- State monopoly admitted for public services and special goods (tobacco, alcohol, state lottery)

Substitute goods

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- The demand for a good generally decreases with its price. Notable exceptions:
  - Giffen goods: necessity goods (e.g., bread)
  - Veblen goods: luxury goods (e.g., expensive watches)
- Two goods are substitute if one can replace the other
  - E.g., butter and margarine; coffee and tea
- Economic perspective: x and y are substitute goods if an increase in the price of x leads to an increase in the demand for y

Discount factor

- Goods do not have the same value today and tomorrow: today's value is <u>higher</u>
- Intuitions behind this: greed of buyers, uncertainty, deterioration, price inflation…
- The discount factor $\delta \in (0, 1)$ is used to account for this phenomenon when calculating utility
- A euro is valued 1 at time 0, δ at time 1, δ^2 at time 2, …, δ^t at time t
- This can be seen as decreasing the value by a factor of $1 - \delta$ each time

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DECISION PROBLEMS

Decision problem

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- A decision problem is characterized by three elements:
  - Actions
  - Outcomes
  - Preferences
- Action a is selected from a set of possible actions A
- Action a results in a certain outcome
  - For 1-player problems actions = outcomes
- Preferences describe the relationship between different outcomes (i.e., which one is preferred)

Preferences

- 1-player decision problem case (action = outcome)
- A **preference** is a binary relationship $\succcurlyeq$ between elements of A (set of possible actions)
- If $a, b \in A$, $a \succcurlyeq b$ means that a is preferred to b
- A preference is always: reflexive, anti-symmetric
- A preference can also be:
  - **complete,** if for all $a, b \in A$ either $a \succcurlyeq b$ or $b \succcurlyeq a$
  - **transitive,** if for all $a, b, c \in A$, $a \succcurlyeq b \wedge b \succcurlyeq c \Rightarrow a \succcurlyeq c$
- A complete+transitive preference is called **rational**

Utility functions

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- Utility functions (also called payoff functions) are an arbitrary quantification of an outcome's goodness
- Utility functions reflect preferences
  - $u(a) \geq u(b) \Leftrightarrow a \succcurlyeq b$
- The exact formulation of u does not matter, it just needs to reflect all preferences

Rationality

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- **Definition**: Saying that players are **rational** means that they always maximize their utility function, i.e., choose the action that leads to their preferred outcome
- In other words, rational player act for their own good
- Does this mean that players always act in a "selfish" manner?

Rationality

- One may argue that humans are far from being rational (they can be crazy, selfless, or make mistakes)
  - This does not apply to machines/algorithms
- For problems that involve humans, the rationality assumption can still hold by modeling craziness, generosity and illogical preferences in the utility function

Decision trees

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- We can represent decision problems using tree diagrams
- **Example:** A student (S) goes to the university cafeteria for lunch. The cafeteria offers beef (B) or fish (F) as the main dish and polenta (P) or vegetables (V) as the side dish. The student prefers fish over beef. Also, being born and raised in Veneto, she clearly prefers polenta over vegetables.

Decision trees

- In this problem, the player S needs to take two decisions
  - B or F?
  - P or V?
- Her preferences are F$\succcurlyeq$B and P$\succcurlyeq$V
- We can model these with a utility function
- The actual payoff values are not important for the time being, they just need to reflect preferences
  - u(B)=2, u(F)=5, u(P)=3, u(V)=1

Decision trees

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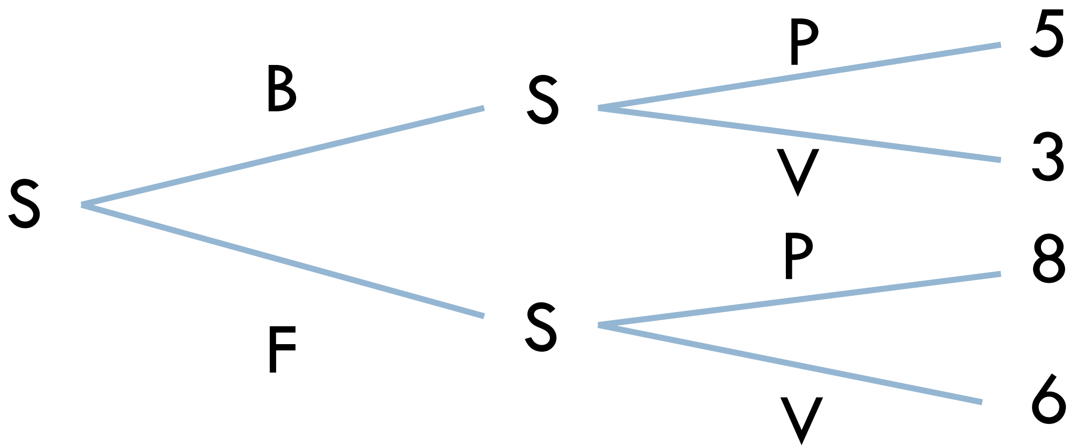
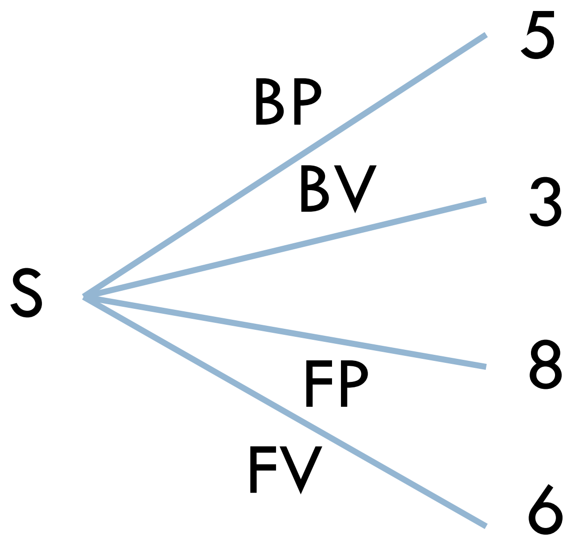
- A decision tree has the following structure:
  - On nodes, we write the player who needs to act
  - On branches, we write the actions
  - On leaves, we write payoffs
    - u(B)=2, u(F)=5, u(P)=3, u(V)=1



Decision trees

- We can always make a single-player decision tree collapse into a single layer



- This is a normal optimization problem
- Things get interesting when multiple players are involved

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SELF-ASSESSMENT

"Homework"

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- Consider a variation of the previous problem
- In this case, we account for the fact that the student has preferences for combinations of main and side dishes
- Suppose her preferences are:
  - Beef and polenta $\succcurlyeq$ fish and vegetables
  - Fish and vegetables $\succcurlyeq$ fish and polenta
  - Fish and polenta $\succcurlyeq$ beef and vegetables
- Assign payoffs according to the preferences and draw the decision tree

Self-assessment

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- What is a game in game theory?
- What does it mean for a preference to be rational?
- What does it mean for a player to be rational?
- What are the elements of a decision problem?

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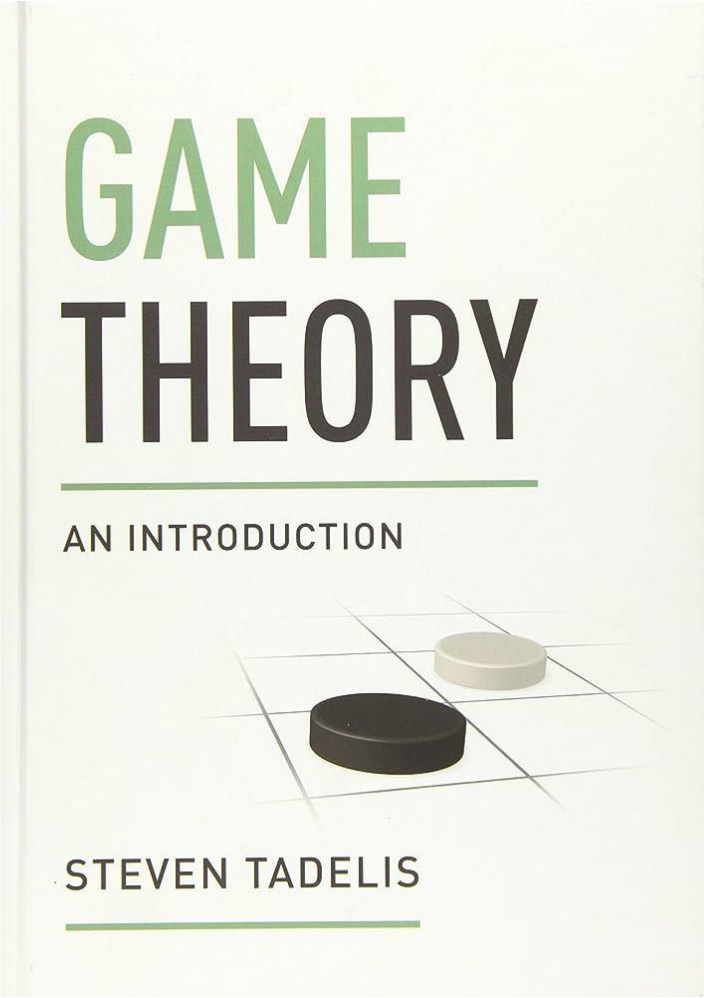
ABOUT THIS CLASS

Textbook

- S. Tadelis, Game Theory: an introduction. Princeton, 2013.



Handbook for exercises

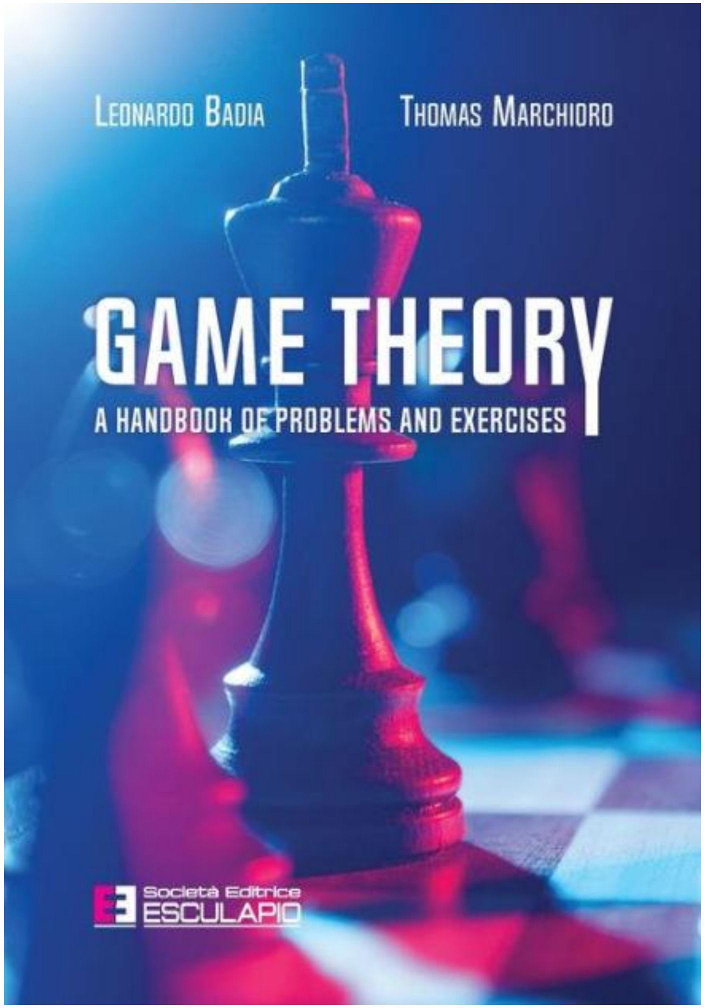
- L. Badia, T. Marchioro, Game theory - A handbook of problems and exercises, Esculapio, 2022.


★★★★★ Verified Purchase

5 stars!

I am the author and I think I did a pretty good job.

4 people found this helpful



Exam rules

- Written exam (0-27 points)
- Two options:
  - No project (3 points for free)
  - Project (0-7 points)
    - Groups of 1-3 people
- To get the "laude" (30L), you must score $\geq$31 total points