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# Lecture 11
# Dynamic games

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- A **dynamic game** involves some players moving first, others moving later
- We still consider dynamic games of **complete information**: strategies and payoffs are common knowledge
- However, we make a further distinction:
    - **perfect information**: every player can make decision with full awareness
    - **imperfect information**: some decisions are "simultaneous" or Nature moves

# Battle of the Sexes (original)

Now it becomes a dynamic game of perfect information

- A and B agreed to go to either rock concert (R) or science night (S)
    - Let us denote B's options in lower case for clarity

|   |   | B r | s |
|---|---|-----|---|
| A | R | 2, 1 | 0, 0 |
|   | S | 0, 0 | 1, 2 |

- To frame this as a normal game, the two players must act unbeknownst of each other
- Not very realistic nowadays

# Battle of the Sexes (revisited)

- Let us add a more sensible time sequence
- Assume A decides (before B) which event to join, and texts B to let him know
    - Which event should she choose? R or S?
    - A knows (due to complete information) that whatever his/her choice, B's best response is to play along and choose the same thing
    - Since A prefers R over S, his/her best option is to choose R (no uncertainty on this outcome, we will see why)

- To unfold the time dimension, we may want more than just the bi-matrix and payoffs
- We need to link possible choices to the knowledge of previous events
    - e.g., we need to model the fact that B acts after receiving A's text (B knows that A is playing R)
- This can be expressed using the so-called **extensive form** of the game
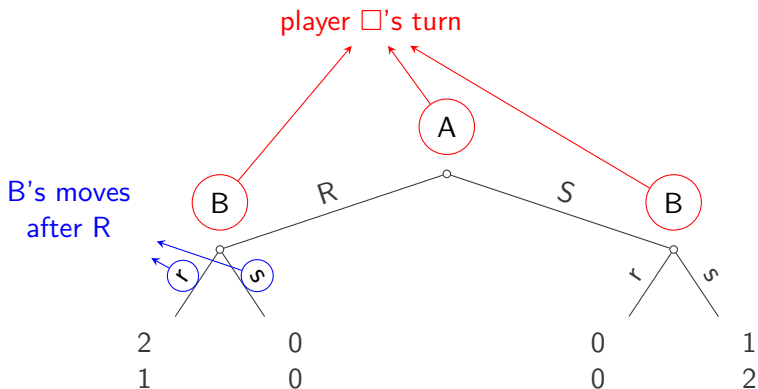    - Graphically, this can be represented as a decision tree

# Extensive form: ingredients

1. Set of players
2. Their utility functions
3. <u>Order of their move turns</u>
4. <u>Actions allowed to players when they can move</u>
5. <u>Information they have when they can move</u>
6. Probability of external events (lotteries)
7. All of this: common knowledge
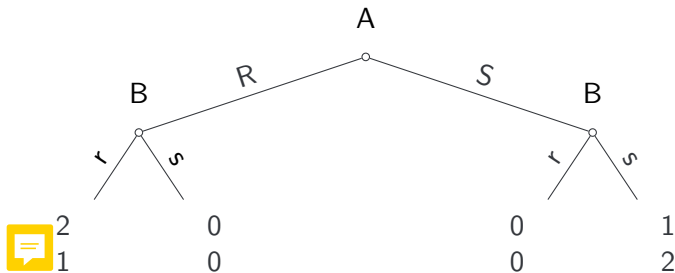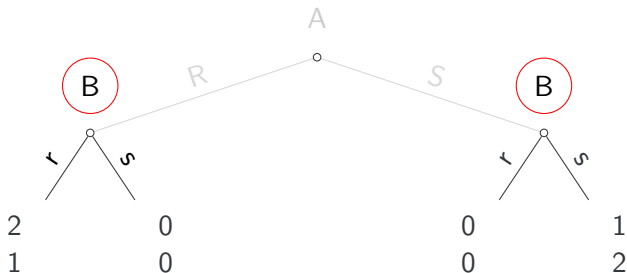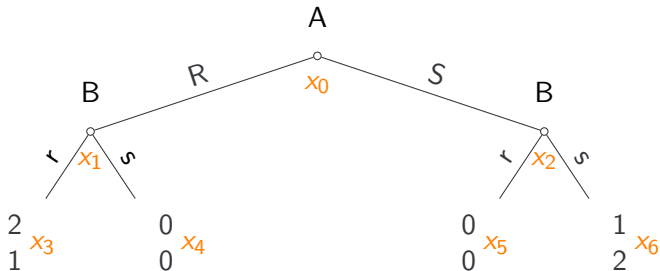
- From these two points on, B's options are the same, but the payoffs are different. Why?
- Because B knows A's choice
- Information is captured by different nodes

* priority on the left



- We can label the nodes of the tree starting from the root $x_0$ and proceeding until the leaves $x_3 \ldots, x_6$
- Here, in each node, players have different information

- Nodes go beyond denoting the game stage
- They also describe the **information set** $h_i$ available to player $i$ who is moving
- If the information set is a singleton $h_i = \{x_j\}$, then the node is fully aware of previous moves
- What if a player does not know?
    - In the original "battle of the sexes", B does not know A's choice
    - This means that B does not know if he/she is in $x_1$ or $x_2$
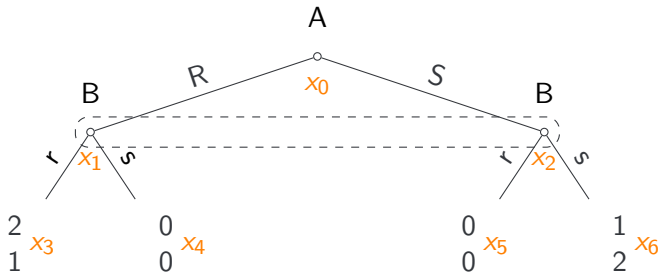
- We circle two nodes with a dashed line to show that a player is not able to distinguish between them (i.e., they belong to the same information set)
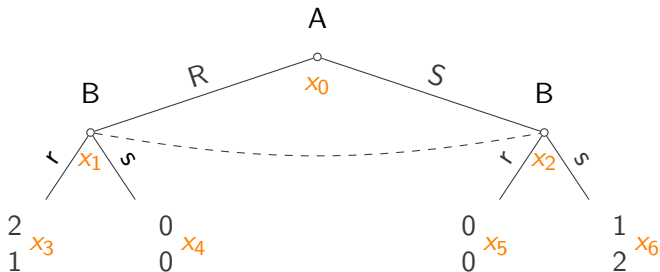
- Alternative notation: just use a dashed line connecting $x_1$ and $x_2$ to show that $h_B = \{x_1, x_2\}$

- In dynamic games with **perfect information**
  1. all information sets are singletons, and
  2. there is no choice of Nature
- Instead we have imperfect information in the following cases:
  - *endogenous uncertainty*: information sets contain multiple nodes (simultaneous moves)
  - *exogenous uncertainty*: there is a choice of Nature (lotteries)
- When we have imperfect information, players need to form beliefs

# Re-defining strategies

- In static games of complete information, we have that
    - pure strategy = action
    - mixed strategy = probability distribution of actions
- In dynamic games, we need to account for the **history of play** (through the information sets)
- A player's **pure strategy** specifies an action according to what happened in the game
- Think of it as an algorithm: you decide of a countermove for any possible case
    - e.g. "If A plays R I play r; if A plays S I play s"

- In the battle of Sexes, A is moving first and both players choose a move in the set of actions $A_1 = A_2 = \{R, S\}$
- B has 2 actions, but more strategies
- A **strategy** $s_B$ for B is a pair of elements of A

$$\underbrace{(a_R}_{\text{what to do if A plays R}} , \underbrace{a_S)}_{\text{what to do if A plays S}}$$

- $s_B = $(s,s) means "I go to S no matter what"
- $s_B = $(r,s) means "I do what A does"
- $s_B = $(s,r) means "I do the opposite of what A does"

- If A and B repeat the original (static) battle for two consecutive nights

- A **strategy** for both players is now a quintuple of moves

$$( \underbrace{a_1}_{\text{first move}} , \underbrace{a_{2Rr}}_{\text{answer to Rr}} , \underbrace{a_{2Rs}}_{\text{answer to Rs}} , \underbrace{a_{2Sr}}_{\text{answer to Sr}} , \underbrace{a_{2Ss}}_{\text{answer to Ss}} )$$

  - $s_i =$ (r,r,r,r,r) "Always go to R both nights"
  - $s_i =$ (r,s,r,r,r) "Go to R on the first night. If the outcome of the first night is Rr, go to S on the second night; otherwise, go to R again."

- In principle, we may describe an "algorithm" for all possible strategies
- Yet, even a simple game with two sequential moves and $|A_1| = |A_2| = 3$ has 27 possible joint (pure) strategies:
  - 3 strategies by player 1
  - 9 strategies by player 2
- Therefore, we will often rely on some implicit description, except for very simple cases

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Sorry, gotta bounce!
Send me questions via e-mail