02417: Time Series Analysis
Rune Grønborg Junker
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DTU Compute
Technical University of Denmark

Assignment 2: ARMA Processes and Seasonal Processes

NOTE that the model parametrizations in the assignment and in your favorite software package may not be identical. You will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions.

Question 2.1: Stability Consider a process X_t defined by the equation:

$$X_t - 0.8X_{t-1} = \epsilon_t + 0.8\epsilon_{t-1} - 0.5\epsilon_{t-2}$$

where ϵ_t is a white noise process with $\sigma = 0.4$.

1. Determine whether the process is stationary and invertible.
2. Analyze the second order moment representation of the process.
3. Simulate 10 realizations of the process with 200 observations each, and plot them on the same graph.
4. Estimate the autocorrelation function (ACF) for each realization, plot them on the same graph, and comment on the results.
5. Repeat the same steps for the partial autocorrelation function (PACF).
6. Calculate the variance of each of the realizations.
7. Compare and discuss the analytical and numerical results.

Question 2.2: Predicting the the number of sales of apartments The quarterly number of sales of apartments in the capital region of Denmark has been modelled.

Based on historical data the following model has been identified:

$$(1 - 1.04B + 0.2B^2)(1 - 0.86B^4)(Y_t - \mu) = (1 - 0.42B^4)\epsilon_t$$

where ϵ_t is a white-noise process with variance $\sigma_\epsilon^2 = 36963$ and μ was estimated to be 2070 based on historical data. The file `A2_sales.txt` contains five years' worth of data for Y_t.

1. Predict the values of Y_t for t = 2019Q1 and 2019Q2, along with 95% prediction intervals.
2. Plot the actual and predicted values of Y_t and comment on the results.

Question 2.3: Consider an ARMA(2,0) process given by the equation:

$\phi(B)X_t = X_t - 1.5X_{t-1} + \phi_2 X_{t-2} = \epsilon_t$

In this question, you will be examining four variations of this process, with $\phi_2 \in 0.52, 0.98$ and $\sigma^2 \in 0.1^2, 5^2$. You will simulate 300 observations of each of the four processes 100 times and estimate the model parameters based on the simulated sequences.

1. Calculate the roots of $\phi(z^{-1}) = 0$ for both values of ϕ_2.
2. For each process, make a histogram plot of the estimates of parameter ϕ_2 and indicate the 95
3. Analyze the effect of different values of ϕ_2 on the variance/distribution of the estimated ϕ_2.
4. Analyze the effect of different values of σ on the variance/distribution of the estimated ϕ_2.
5. Plot all the estimated pairs of parameters (ϕ_1, ϕ_2) for the four variations.
6. Consider the stability of the process and explain how this affects the distribution of the fitted estimated values.