Optimality conditions: If I gave you a solution and claimed that this is the optimal solution, how could you check that?

Penalty Function

local min
strict local min
global min
strict global min

Hessian Matrix:

$$H(f) = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

example : $f(x,y) = e^{x/2} \cdot \sin(y)$

$$H_f = \begin{bmatrix} \frac{1}{4} \sin(y)\, e^{x/2} & \frac{\cos(y)}{2}\, e^{\frac{x}{2}} \\ \frac{\cos(y)}{2}\, e^{x/2} & -e^{x/2} \sin(y) \end{bmatrix}$$

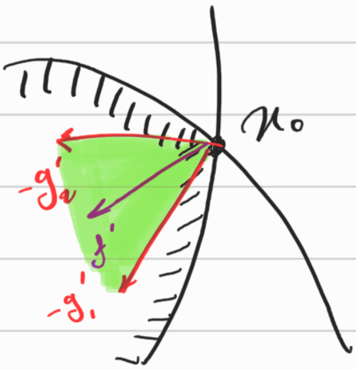
Hessian Matrix for n - variable functions :

$$H(f) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$$

for optimitization at x_0 :

① $f'(x_0) = 0$

② $H(x_0) > 0$



- $f' + u_1 g_1' + u_2 g_2' = 0$

  $\rightarrow f' = u_1(-g_1') + u_2(-g_2')$

- f' is in the conic hull of $[-g_1', -g_2']$

- every feasible move away from x_0 is increasing.

- g_1, g_2 are convex, and has strict interior