Deep Learning

Optimizers

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Last session

- GD - SGD.
- MLP.
- Backprop.
- Multiple outputs.
- Activation functions.

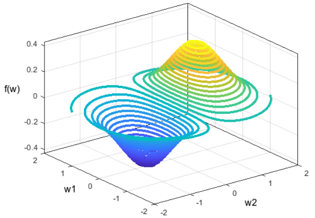
Today's outline

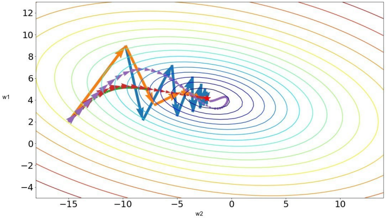
- Code revision.
- Paper 1 discussion.
- Paper 2 discussion.
- Optimization functions.
- New code.

Outline

Parameter space

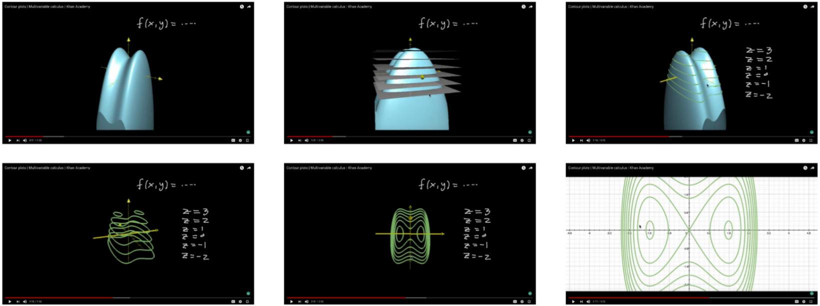
Example of different loss functions with 2 parameters.
Seen in 3D, and from above.





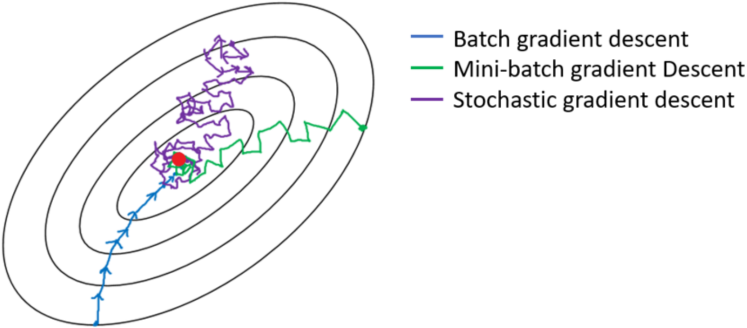
Colored arrows represent different possible paths to reach the local minimum, as followed by different optimization strategies.

Several local minima



Images from Video: Contour plots - Multivariable calculus - Khan Academy
https://www.youtube.com/watch?v=WsZj5Rb6do8

Example for SGD



SGD Limitation

$$\omega_i = \omega_i - \nabla_{\omega_i}\mathcal{L}(y, \hat{y}).$$

- Slows down around ravines.
- Oscillates across the slopes of the ravine.
- Limited progress towards the local minimum.
- Might never escape from saddle points.

Qian, 1999. "On the momentum term in gradient descent learning algorithms".

Outline

Notation

In this section, we will use subscripts to index time, e.g., ω_t refers to the value of parameter ω at time t.

Also, we omit the position index (i-th element) commonly seen as subscript, i.e, ω_i.

Momentum

$$\omega_t = \omega_{t-1} - v_t,$$

$$v_t = \gamma v_{t-1} + \eta \nabla_\omega \mathcal{L}(y, \hat{y}), \qquad v_0 = 0, \quad \gamma = 0.9, \eta \approx 0.001.$$

- Accelerates SGD in the relevant direction.
- Dampens oscillations.
- Includes a fraction of the historic direction.
- Momentum accelerates for gradients pointing in the same direction, and reduces for those in changing direction.

Sutskever et al., 2013. "On the importance of initialization and momentum in deep learning".

Nesterov Accelerated Gradient (NAG)

$$\omega_t = \omega_{t-1} - v_t,$$

$$v_t = \gamma v_{t-1} + \eta \nabla_{(\omega - \gamma v_{t-1})} \mathcal{L}(y, \hat{y}),$$

- $\nabla_{(\omega - \gamma v_{t-1})}$ approximates the next position of ω.
- Looks ahead by calculating the gradient w.r.t. future positions.
- Anticipates changes in the direction of the gradient.

Nesterov, 1983. "A method for unconstrained convex minimization problemwith the rate of convergence o(1/k2)".

Adaptive Gradient (AdaGrad)

$$\omega_t = \omega_{t-1} - \eta \frac{1}{\sqrt{G_t + \epsilon}} g_t, \qquad g_t = \nabla_\omega \mathcal{L}(y, \hat{y}),$$

$$G_t = \sum_{k=0}^{t} g_t^2, \qquad \epsilon \approx 1e^{-8}.$$

- G_t: sum of gradients^2 up to time t (heavy memory loads).
- Adapts η at each time step (always decreasing).
- Works well on sparse data and large models.

Duchi et al., 2011. “Adaptive Subgradient Methods for Online Learning and Stochastic Optimization”.

Adadelta

$$\omega_t = \omega_{t-1} - \eta \frac{1}{\sqrt{\mathbb{E}[g^2]_t}} g_t, \qquad g_t = \nabla_\omega \mathcal{L}(y, \hat{y}),$$

$$\mathbb{E}[g^2]_t = \gamma \mathbb{E}[g^2]_{t-1} + (1-\gamma) g_t^2, \qquad \gamma = 0.9.$$

- Addresses the issue of monotonically decreasing η.
- Restricts the past to a moving window.
- Recursively computes the sum of past gradients using exponential smoothing.

Zeiler, 2012. "ADADELTA: An Adaptive Learning Rate Method".
RMSprop: a variant by Hinton (unpublished).

Adaptive Momentum (Adam)

Adadelta + Momentum.

$$m_t = \beta_1 m_{t-1} + (1-\beta_1) g_t, \quad \text{first moment estimate (mean)},$$
$$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2, \quad \text{second moment estimate (stddv)}.$$

Correcting for bias towards zero:

$$\hat{m}_t = \frac{m_t}{1-\beta_1^t}; \qquad \hat{v}_t = \frac{v_t}{1-\beta_2^t}.$$

$$\omega_t = \omega_{t-1} - \eta \frac{1}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t,$$

Kingma & Ba, 2015. “Adam: a Method for Stochastic Optimization”.

Other variants: AdaMax, Nadam, AMSGrad.

To know more

Ruder, 2016. "An overview of gradient descent optimization algorithms". https://arxiv.org/abs/1609.04747

Q&A

Thank you!

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