

comparisons
or $\Theta(n \log n)$

Euclid Alg

$$\text{gcd}(m, n) = \text{gcd}(n, m \bmod n) \quad \text{until } n=0$$

BST starts
with middle num

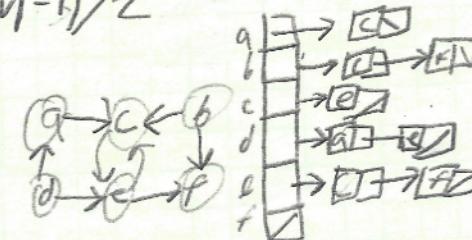
ISP

Find all combinations,
calculate each

Graph Def
A graph $G = \langle V, E \rangle$ = Finite, non-empty set V
Set E (sets cannot have edges)

$$\sum_i = \frac{n(n+1)}{2}, \quad 0 \leq |E| \leq |V|(|V|-1)/2$$

$$\begin{array}{c} a \rightarrow b \\ a \rightarrow c \\ a \rightarrow d \\ b \rightarrow c \\ b \rightarrow d \\ c \rightarrow d \\ d \rightarrow e \\ d \rightarrow f \\ e \rightarrow f \end{array}$$
$$a \left[\begin{array}{cccccc} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right] =$$



$$T > \log n > n > n \log n > n^2 > n^3 > 2^n > n!$$

Big Oh - Set of all functions with lower/same order of growth n ($n \in O(n^2)$)

Big Omega - Set of all func with higher/same

Big Theta - Set of all with same ($n^3 \in \Omega(n^2)$)

($n^3 \in \Theta(n^2)$)

Recursion:

$M(n)$: Number of multiplications executed for the input number n .

Algorithm F(n)
if $n=0$
return 1

$M(0) = 0$ (initial)
 $M(n) = M(n-1) + 1$ (recursion)

$$\begin{aligned} M(n) &= M(n-1) + 1 \\ &= [M(n-2) + 1] + 1 \\ &= M(n-2) + 2 \end{aligned}$$

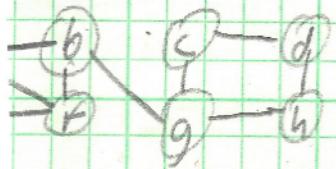
$M(n)$ is the amount of multiplications for each n .

if $n=1$
return 1 (2 mults)
else
return $M(n-1) + n \cdot n \cdot n$

$$\begin{aligned} M(n) &= M(n-3) + 3 \\ &= M(n-i) + i \quad i \in \mathbb{Z} \\ &= \dots \\ &= M(0) + n \\ &= 0 + n \\ &= n \in \Theta(n) \end{aligned}$$

$$\begin{aligned} M(n-1) + 2 \\ &= M(n-i) + 2i \quad (i=n-1) \\ &= M(i) + 2 \cdot (n-1) \\ &= 0 + 2(n-1) \\ &\in \Theta(n) \end{aligned}$$

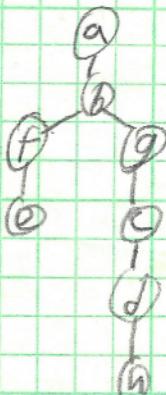
S Uses Stack
Mark array to track visited vertices



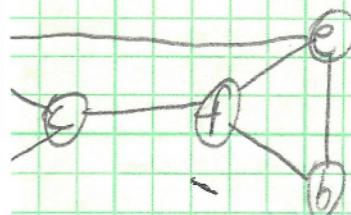
Mark Array

a	1
b	2
c	6
d	7
e	4
f	3
g	5
h	8

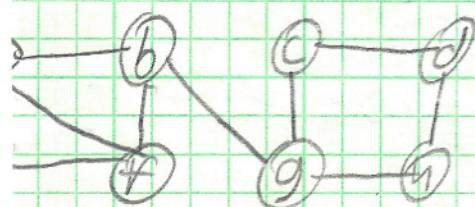
DFS Tree



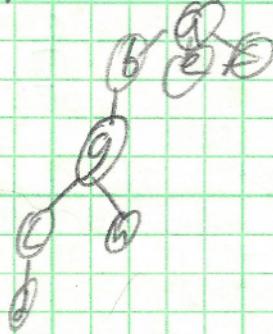
S Uses Queue $\Theta(|V|^2)$



$a \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow b$



$a \rightarrow b \rightarrow e \rightarrow f \rightarrow g \rightarrow c \rightarrow h \rightarrow d$



TC Force

```
function sort
    int i, j
    int a.length
    for (i = 0; i < a.length - 1; i++) {
        int jMin = i
        for (j = i + 1; j < a.length; j++) {
            if (a[i] > a[jMin]) {
                jMin = j
            }
        }
        if (jMin != i) {
            swap(a[i], a[jMin])
        }
    }
}
```

Bubble Sort +

func bubbleSort(A[])

n = length(A)

while (swapped == false)

for (i = 1; i < n - 1)

if A[i-1] > A[i]

swap(A[i-1], A[i])

swapped = true

n = n - 1

OK

for i = 0 to n - 2 do

for j = 0 to n - 2 do

i ≠ A[j+1] < A[j]

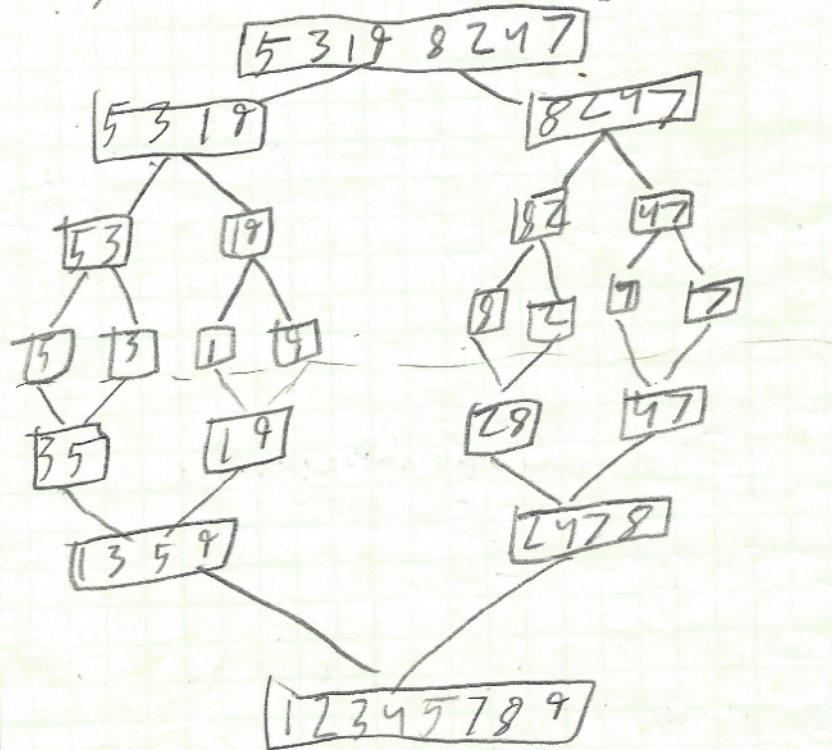
swap(A[j], A[j+1])

Text NOBODY

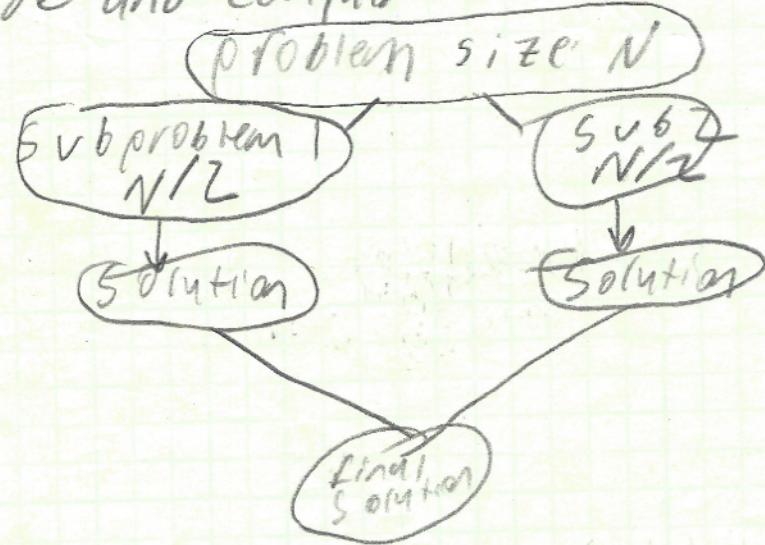
NOT
NOT
NOT
NOT

$B \leq T$
 $D = N$
 $B \geq N$

Merge Sort - $O(n \log(n))$



Divide and Conquer



start the array

$b = \text{number of subproblems}$

$g = \text{Number of subproblems to be solved}$

$d =$ is the number in $\Theta(n^d)$ in which $\Theta(n^d)$ is the time to divide & conquer the original problem of size n

```

int sum(A[], start, end)
    if (start == end)
        return A[start]
    else
        int sum1 = sum(A, start, (start+end)/2)
        int sum2 = sum(A, (start+end)/2, end)
        return (sum1 + sum2)

```

$b = 2$

$g = 2$

$d = 0$

Case 1 IF $a < b^d$ then $\Theta(n^d)$
 Case 2 IF $a = b^d$ then $\Theta(n^d \cdot \log n)$
 Case 3 IF $a > b^d$ then $\Theta(n^{\log_2 a})$

$$T(n) = 2T(n/2) + 5n^3 + 7n$$

$$\begin{aligned} a &= 2 \\ b &= 2 \\ d &= 3 \quad (5n^3) \end{aligned}$$

$$2 < 2^3, \text{ therefore } \Theta(n^3)$$