$\boxed{\text{interp 3}}$ Lagrange approach

$$p(x) = \sum_{k=1}^{N} c_k \phi_k(x)$$

Monomial $\phi_k(x) = x^{k-1}$

Newton $\phi_k(x) = (x-x_1)(x-x_2)\cdots(x-x_{k-1})$

Lagrange $\phi_k(x) = \ell_k(x)$

$$p(x) = \sum_{k=1}^{N} c_k \ell_k(x)$$

Kronecker symbol

* $\ell_k(x)$ polynomials

* $\ell_k(x_j) = \delta_{kj} = \begin{cases} 1 & \text{if } k=j \\ 0 & \text{if } k \neq j \end{cases}$

Lagrange basis for 3-points x_1, x_2, x_3

$\ell_1(x_j) = \delta_{1j}$

$$\ell_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}$$

$\ell_2(x_j) = \delta_{2j}$

$$\ell_2(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}$$

$\ell_3(x_j) = \delta_{3j}$

$$\ell_3(x) = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$\boxed{\ell_k(x_j) = \delta_{kj}}$$

each $\ell_k(x)$ is a degree 2 polynomial

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$\ell_k(x_j) = \delta_{kj}$

$\phi_k(x) = \ell_k(x)$

Vandermonde system for Lagrange

$$\begin{pmatrix} \ell_1(x_1) & \ell_2(x_1) & \ell_3(x_1) \\ \ell_1(x_2) & \ell_2(x_2) & \ell_3(x_2) \\ \ell_1(x_3) & \ell_2(x_3) & \ell_3(x_3) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Vandermonde matrix = identity

$$V\vec{c} = \vec{y} \implies \vec{c} = \vec{y}$$

EX $D_4 = \{(-1,2),\ (0,1),\ (1,3),\ (2,2)\}$

$$\ell_1(x) = \frac{x(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} = -\frac{1}{6}x(x-1)(x-2)$$

$$\ell_2(x) = \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)} = \frac{1}{2}(x+1)(x-1)(x-2)$$

$$\ell_3(x) = \frac{(x+1)x(x-2)}{(1+1)(1-0)(1-2)} = -\frac{1}{2}x(x+1)(x-2)$$

$$\ell_4(x) = \frac{(x+1)x(x-1)}{(2+1)(2-0)(2-1)} = \frac{1}{6}x(x+1)(x-1)$$

$$P(x) = 2\ell_1(x) + \ell_2(x) + 3\ell_3(x) + 2\ell_4(x)$$

$$= \text{work it out.}$$

$$\ell_1(x) = -\frac{1}{6}x(x-1)(x-2)$$

$$\ell_2(x) = \frac{1}{2}(x+1)(x-1)(x-2)$$

$$\ell_3(x) = -\frac{1}{2}x(x+1)(x-2)$$

$$\ell_4(x) = \frac{1}{6}x(x+1)(x-1)$$

$$\Phi_N = \{(x_k, y_k)\}_{k=1}^{N}$$

In general

$$\ell_k(x) = \frac{\prod_{\substack{i=1 \\ i \neq k}}^{N} (x - x_i)}{\prod_{\substack{i=1 \\ i \neq k}}^{N} (x_k - x_i)}$$

c_k

$$P(x) = \sum_{k=1}^{N} y_k \ell_k(x)$$

$$P(x_j) = \sum_{k=1}^{N} y_k \delta_{kj} = y_j$$