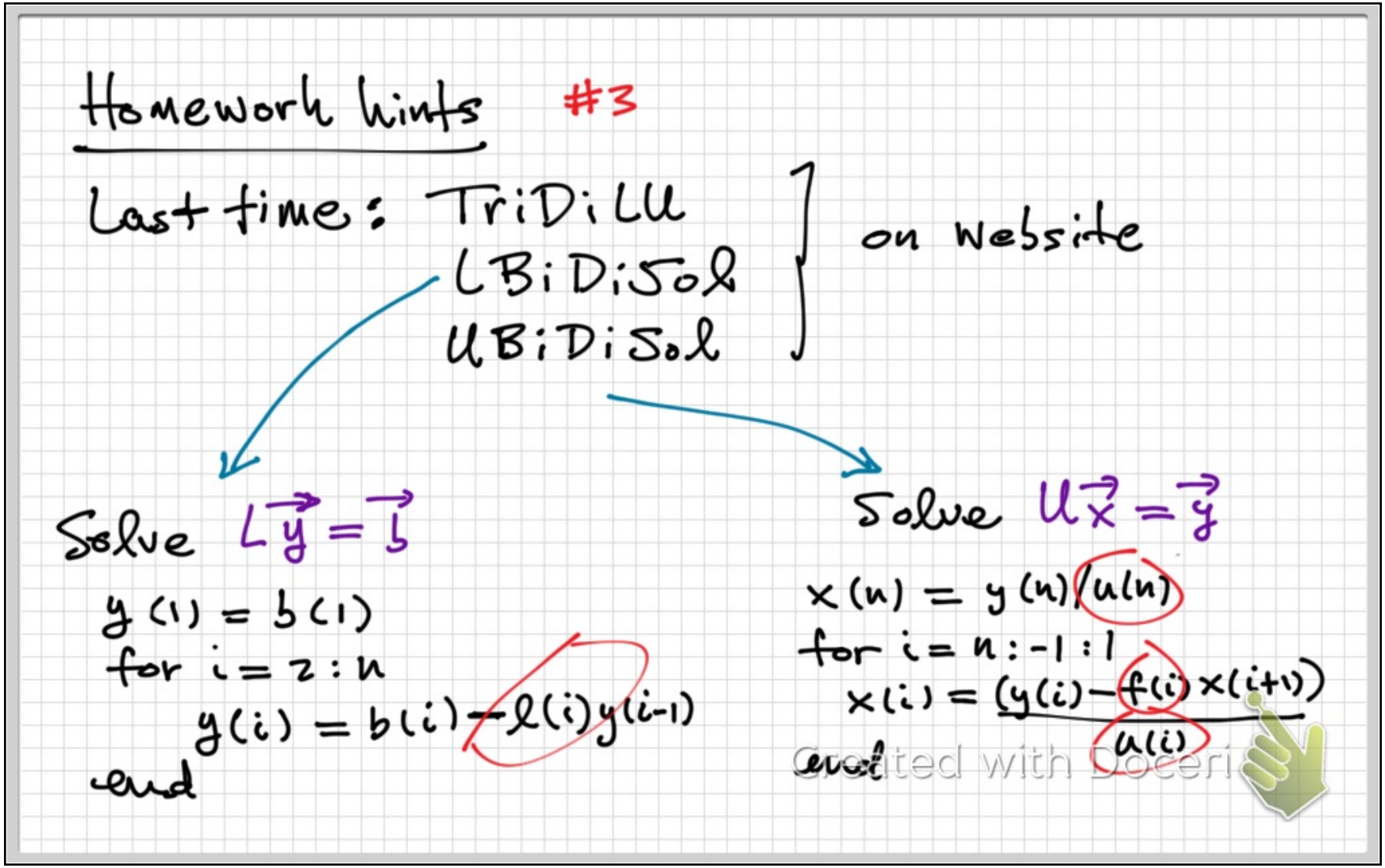
linalg 2 continuation

Note: refers to HW problem #3 in Set #6.
But J might change the # when J create the set


Homework hints #3
Last time: TriDiLU
LBiDiSol
UBiDiSol
on website
Solve $L\vec{y} = \vec{b}$
y(1) = b(1)
for i = 2 : n
y(i) = b(i) − ℓ(i)y(i−1)
end
Solve $U\vec{x} = \vec{y}$
x(n) = y(n)/u(n)
for i = n : −1 : 1
x(i) = (y(i) − f(i)x(i+1)) / u(i)
end
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Example from notes

$v''(x) = g(x)$ on $(a,b) = (0,\pi)$ subject to

$v(a) = \alpha$, $v(b) = \beta$

let $x_k = a + \frac{(k-1)}{(n-1)}(b-a)$ $k = 1, 2, \cdots, n$

↑ in notes

includes a, b

$x_1 = a$, $x_n = b$

Could instead take

$x_k = a + \frac{k}{n+1}(b-a)$ $k = 1, 2, \cdots, n$

now a, b not included

let $h = \frac{(b-a)}{n-1} = \Delta x$ and $v_h \simeq v(x_h)$

approximate $v''(x) = g(x)$ as

$$\frac{\frac{v_{i+1} - v_i}{h} - \frac{v_i - v_{i-1}}{h}}{h} = g_i$$

$$v_{i+1} - 2v_i + v_{i-1} = h^2 g_i$$

System (for $n=7$)

$$v_1 = \alpha$$

$$v_1 - 2v_2 + v_3 = h^2 g_2$$

$$v_2 - 2v_3 + v_4 = h^2 g_3$$

$$v_3 - 2v_4 + v_5 = h^2 g_4$$

$$v_4 - 2v_5 + v_6 = h^2 g_5$$

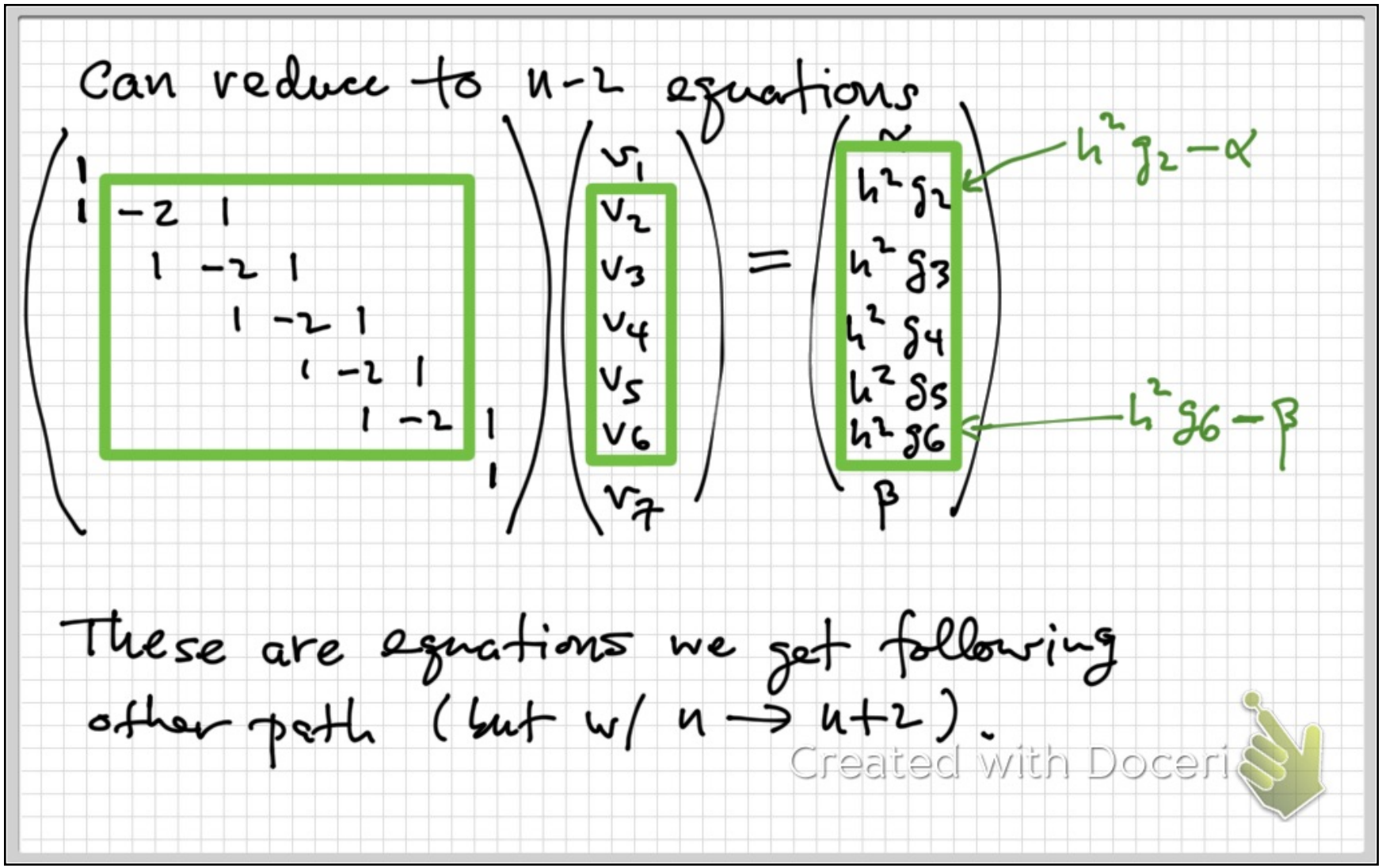
$$v_5 - 2v_6 + v_7 = h^2 g_6$$

$$v_7 = \beta$$

tridiagonal system

$$\begin{pmatrix} 1 & & & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & 1 & -2 & 1 & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & -2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{pmatrix} = \begin{pmatrix} \alpha \\ h^2 g_2 \\ h^2 g_3 \\ h^2 g_4 \\ h^2 g_5 \\ h^2 g_6 \\ \beta \end{pmatrix}$$

for $g(x) = -\sin x$, $(a,b) = (0,\pi)$ see notes.


Can reduce to n-2 equations
$\begin{pmatrix} 1 & & & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & 1 & -2 & 1 & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{pmatrix} = \begin{pmatrix} \tilde{} \\ h^2 g_2 \\ h^2 g_3 \\ h^2 g_4 \\ h^2 g_5 \\ h^2 g_6 \\ \beta \end{pmatrix}$
$h^2 g_2 - \alpha$
$h^2 g_6 - \beta$
These are equations we get following other path (but w/ $n \to n+2$).
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