Least squares and QR factorization Linaly 5 A E TR MXn, m>n, full rank minimize || Ax-3|| over all Z∈ TR" x one solution: salve ATAX = ATB BAT NUMERICALLY IC (ATA) tends to be large * New approach OR factorization

Write A = QR where Q is an orthogonal matrix and R is upper Δ . Will discuss how to do this later. @ orthogonal means PT = PT THU assume Q is orthogonal. Then
for any V, we have ||QV|| = ||V||(norm preserving)

Proof: $||QV||^2 = (QV)^T (QV) = V = QVQV = V$

If
$$Q$$
 is orthogonal than so is QT .

 $\overrightarrow{r} = A\overrightarrow{x} - \overrightarrow{J} = QR\overrightarrow{x} - \overrightarrow{J}$

Because \overrightarrow{r} and $QT\overrightarrow{r} = R\overrightarrow{x} - QT\overrightarrow{J}$

have same longth, following problems are equivalent:

Minimize $||A\overrightarrow{x} - \overrightarrow{J}|| \iff \text{Minimize } ||R\overrightarrow{x} - QT\overrightarrow{J}||$

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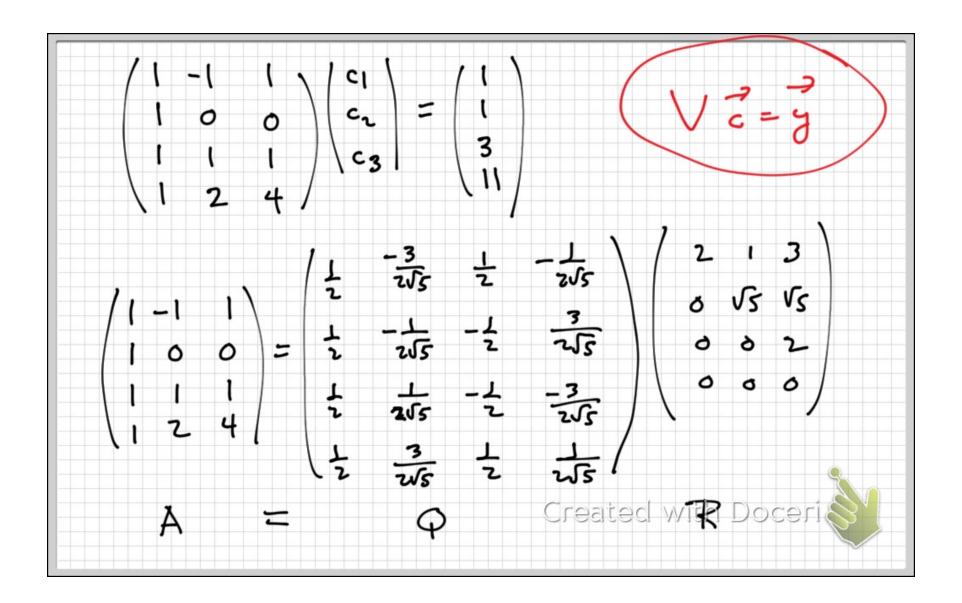
Beneric
$$4 \times 3$$
 example
$$\begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{21} & a_{23} \\
a_{31} & a_{32} & a_{33} \\
a_{41} & a_{41} & a_{43}
\end{pmatrix}
\begin{pmatrix}
\times_{1} \\
\times_{2} \\
\times_{3}
\end{pmatrix} = \begin{pmatrix}
b_{1} \\
b_{2} \\
b_{3} \\
b_{4}
\end{pmatrix}$$

$$R = \vec{b}$$

$$|| R \times - W || = || \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{21} & r_{23} \\ 0 & 0 & r_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} |$$

$$|| \begin{pmatrix} r_{11} \times_1 + r_{12} \times_2 + r_{13} \times_3 - w_1 \\ r_{21} \times_1 + r_{23} \times_3 - w_2 \\ r_{33} \times_3 - w_3 \\ - w_4 \end{pmatrix} |$$

$$+ || \begin{cases} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{23} \\ r_{22} & r_{23} \\ x_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \text{ at each of the last of the properties of the last of the$$



$$\begin{vmatrix}
2 & 1 & 3 \\
0 & \sqrt{5} & \sqrt{5} \\
0 & 0 & 2
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