Example first test
CS/Math 375

Name: ______________________________

1. (10 points) Write a short essay describing the binary format and storage of double precision floating–point numbers represented as $(-1)^{\mathbf{sign}} \times 2^{\mathbf{exponent}-(\mathbf{exponent\ bias})} \times 1.\mathbf{mantissa}$. How is such a number laid out in computer memory? Discuss in particular the possible ranges of the **exponent** and true exponent: **exponent** − **exponent bias**.

2. (10 points) Consider the function $f(x) = x(1 - \cos x)$.

(a) Define a *root of multiplicity* m, and find the value of m for the root $r = 0$ of $f(x)$.

(b) Find the forward and backward errors of the approximation $x_c = 0.01$ to r.

3. (10 points) Consider $f(x) = 2x^4 - 4x^3$, with simple root $r_1 = 2$ and multiple root $r_2 = 0$.

(a) Write down Newton's method for finding the roots of this function.

(b) For the root $r_1 = 2$, determine whether Newton's Method or the Bisection method will converge faster. *Do not carry out the methods*, and justify your answer.

(c) Same question as **(b)** for the root $r_2 = 0$.

4. (10 points)

(a) Construct by hand the LU–factorization of the matrix

$$T = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

(b) Assume that your computer can solve 500-by-500 linear system $A\mathbf{x} = \mathbf{b}$ by LU factorization in 10 seconds (with A full and not tridiagonal). How many floating point operations per second (`flops/sec`) does this performance amount to? Based on this `flops/sec` value, estimate how long it would take to solve a 500-by-500 tridiagonal system $T\mathbf{x} = \mathbf{b}$.