Error formula for Composite NC Quadrature

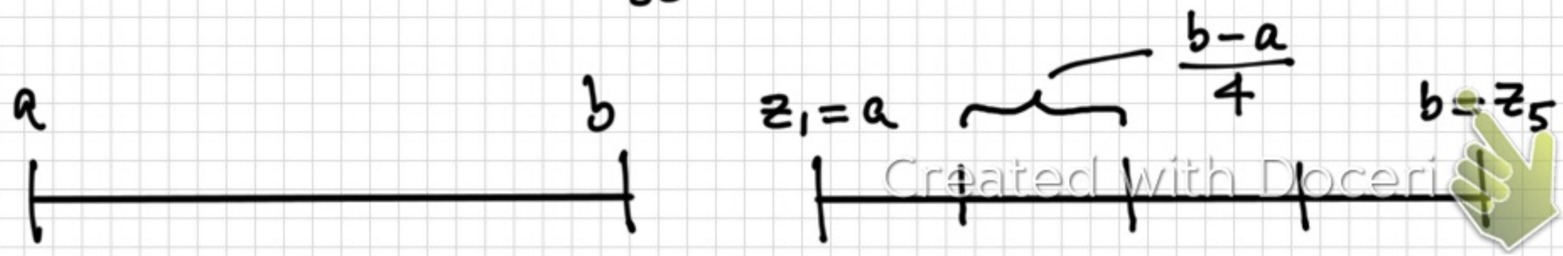
Recall $$\left| \int_a^b f(x)\,dx - Q_{NC(m)} \right| \leq |C_m|\, M_{d+1} \left(\frac{b-a}{m-1}\right)^{d+2}$$

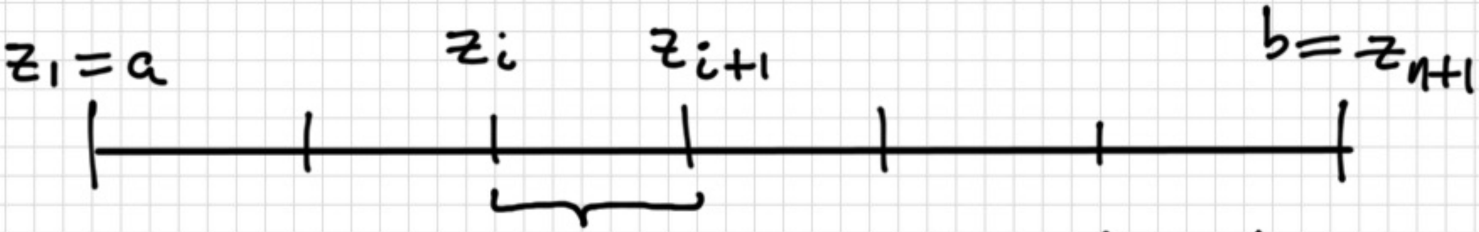
$\boxed{\text{quad3}}$

$\frac{1}{12} M_2 (b-a)^3$ for $m=2$

$\frac{1}{90} M_4 \left(\frac{b-a}{2}\right)^5$ for $m=3$

$\frac{3}{80} M_4 \left(\frac{b-a}{3}\right)^5$ for $m=4$





subinterval of width $\frac{b-a}{n}$

If we apply $\Phi_{NC(m)}$ over $[z_i, z_{i+1}]$ we get

$$\left| \int_{z_i}^{z_{i+1}} f(x)\,dx - \Phi_{NC(m)} \right| \leq |C_m| \underbrace{M_{d+1}^{(i)}}_{\text{bound on } |f^{(d+1)}(x)| \text{ over } [z_i, z_{i+1}]} \left(\frac{b-a}{n(m-1)} \right)^{d+2}$$

$$\therefore \left| \int_a^b f(x)\,dx - \Phi_{NC(m)}^{(n)} \right| \leq |C_m| M_{d+1} \left(\frac{b-a}{m-1} \right)^{d+2} \frac{n}{n^{d+2}}$$

Error formula

$$\left| \int_a^b f(x)\,dx - Q^{(n)}_{NC(m)} \right| \le |C_m|\, M_{d+1} \left(\frac{b-a}{m-1}\right)^{d+2} \frac{1}{n^{d+1}}$$

Error for Simpson rule:

$$\left| \int_a^b f(x)\,dx - Q_{NC(3)} \right| \le \frac{1}{90} M_4 \left(\frac{b-a}{2}\right)^5$$

Error for Simpson $\frac{3}{8}$ rule:

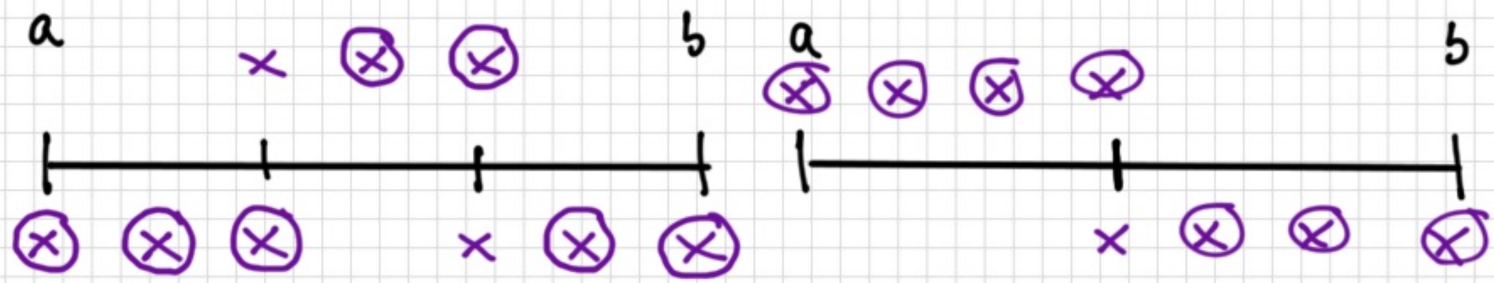
$$\frac{1}{2^5 90} = \frac{1}{2880}$$

$$\frac{1}{3^4 80} = \frac{1}{6480}$$

$$\left| \int_a^b f(x)\,dx - Q_{NC(4)} \right| \le \frac{3}{80} M_4 \left(\frac{b-a}{3}\right)^5$$

So $\varphi_{NC(4)}$ more accurate than $\varphi_{NC(3)}$.

But proper comparison is $\varphi^{(3)}_{NC(3)}$ and $\varphi^{(2)}_{NC(4)}$



Both require 7 function evaluations

Errors: for $\varphi^{(3)}_{NC(3)} \propto \frac{1}{2880}\,\frac{1}{3^4} = \frac{1}{233280}$

for $\varphi^{(2)}_{NC(4)} \propto \frac{1}{6480}\,\frac{1}{2^4} = \frac{1}{103680}$