Loss of significance

loss

$x_A \equiv$ approximate value

$x_T \equiv$ true value

The # of significant digits (significant figures) in x_A relative to x_T is roughly the # of leading digits in x_A which match those in x_T

$$x_T = \pi = 3.14159265\cdots$$

$$x_A = \frac{22}{7} = 3.14285714\cdots$$

3 sig figs

More examples

$$x_T = \frac{7}{9} = 0.777777\ldots$$

$$x_A = 0.77770$$

Counting from left, difference of $7 > 5$ in 5^{TH} digit. 3 not 4 sig figs

$$x_T = \frac{7}{9} = 0.777777\ldots$$

$$x_A = 0.77773$$

4 sig figs

Often we just have a number x "by itself," but want to view it as an x_A even though we don't know x_T.

Guidelines for assigning # of significant digits in x

1004.2

143.2

- all non-zero digits are significant
- zeros which lie between non-zeros are significant
- zeros before first nonzero are <u>not</u> significant

001.2

- zeros after last nonzero <u>maybe</u> significant

1000 45.032300

Loss of significance

$\boxed{\text{Subtraction of nearly equal \#s is prone to loss of significance.}}$

Ex

$x = 123.01 \qquad x_T = 123.01289764\ldots$

$y = 123.02 \qquad y_T = 123.02175347\ldots$

True difference $y_T - x_T = 0.00885582\ldots$

many sig figs

Approx difference $y - x = 0.01$

only 1 sig fig

One should avoid such bad subtractions where possible in computations.

Ex find roots of $2.1x^2 - 4.5x + 10^{-11} = 0$

Using MATLAB or OCTAVE "roots" command

$$r_1^{\text{octave}} = 2.142857142854921e{+}00$$

$$r_2^{\text{octave}} = 2.222222222224527e{-}12$$

These values are good

Try computing roots w/ niave quadratic formula

$a = 2.1, \; b = -4.5, \; c = 10^{-11}$

use Octave

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = 2.14285714285492\text{l}e{+}00$$

use Octave

$$r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = 2.22213781766879\text{o}e{-}12$$

$$r_2^{\text{Octave}} = 2.22222222222224527e{-}12$$

$$\left| \frac{r_2 - r_2^{\text{octave}}}{r_2^{\text{octave}}} \right| \simeq 3.7982 \times 10^{-5}$$

$\boxed{r_2 \text{ is bad} \quad \text{WHY?}}$

$$\boxed{r_2 = \frac{1}{2a}\left(-b - \sqrt{b^2-4ac}\right)}$$

$$\boxed{\begin{aligned} a &= 2.1 \\ b &= -4.5 \\ c &= 10^{-11} \end{aligned}}$$

$$-b = 4.5$$

$$\sqrt{b^2-4ac} = \sqrt{(4.5)^2 - 8.4\times 10^{-11}}$$

$$= 4.5\,\underbrace{\sqrt{1 - \frac{8.4}{(4.5)^2}\times 10^{-11}}}$$

binomial expansion

$$\sqrt{1+\delta} \simeq 1 + \tfrac{1}{2}\delta$$

$$= 1 - \frac{4.2}{(4.5)^2}\times 10^{-11}$$

$$\simeq 4.5 - \frac{42}{45}\times 10^{-11}$$

We can remedy the trouble

$$r_2 = \frac{1}{2a}\left(-b-\sqrt{b^2-4ac}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)$$

$$= \frac{1}{2a}\,\frac{4ac}{-b+\sqrt{b^2-4ac}}$$

$$= \frac{2c}{-b+\sqrt{b^2-4ac}}$$

addition of nearly equal #s. NO TROUBLE

$$= 2.222222222224527e-12$$

octave

now good, same as r_2