The Division Algorithm and Proof Writing

We need to revisit the exercises from the Week 4 homework and hopefully gain a better knowledge of proof writing.

Goals:

- Revisit the definitions of even and odd and the division algorithm.
- Formulate a formal proof that an odd integer has the form $2k + 1$.
- Apply the division algorithm in another setting.

Definition 1 (Even)**.** An integer n is *even* if $n = 2k$ for some $k \in \mathbb{Z}$.

Definition 2 (Odd)**.** An integer n is *odd* if it is not even.

Claim 1: If an integer n is odd, then $n = 2k + 1$ for some $k \in \mathbb{Z}$.

1. Write a formal proof of Claim 1.

Claim 2: For any integer n, either n, $n+1$, or $n+2$ is divisible by 3.

2. Write a formal proof of Claim 2.