Quantifiers

Most mathematical statements of any significance contain quantifiers. Sometimes they're implied, sometimes their explicit, but either way they're important.

Goals:

- Translate statements with quantifiers into logical symbols
- Determine the truth value of quantified statements

1. Translate the following logical statement into English sentences.

   (a) $\forall\, n \in \mathbb{Z},\ n = 2k$ for $k \in \mathbb{Z}$

   (b) $\forall\, x \in \mathbb{R},\ x^2 > 0$

   (c) $\exists\, n \in \mathbb{Z},\ 2^n < 0$

(d) $\forall\, n \in \mathbb{Z},\ \exists m \in \mathbb{Z},\ n + m = 2$

(e) $\exists\, k \in \mathbb{Z},\ \forall\, n \in \mathbb{Z},\ kn = 0$

(f) $\exists\, n \in \mathbb{Z},\ \forall\, m \in \mathbb{Z},\ n + m = 0$

(g) $\exists\, A \subseteq \mathbb{R},\ |A| < \infty$

(h) $\forall\ n \in \mathbb{Z},\ \exists\ A \in \mathscr{P}(\mathbb{N}), |A| < n$

2. Determine the truth value of every statement in Exercise 1.

3. Translate the following mathematical statements into symbolic form using the symbols $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$, $\exists$, and $\forall$.

   (a) If f is a continuous function on the interval $[a, b]$ and N is a number between $f(a)$ and $f(b)$ with $f(a) \neq f(b)$ then there exists $c \in (a, b)$ such that $f(c) = N$.

   (b) A function f is continuous on $[a, b]$ if and only if $\lim\limits_{x \to c} = f(c)$ for all $c \in [a, b]$.

(c) The limit of the sequence a_n equals L if and only if for all $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that if $n \geq N$ then $|a_n - L| < \epsilon$.

(d) The limit of a function f at $x = a$ equals L if and only if for all $\epsilon > 0$ there exists $\delta > 0$ such that $|f(x) - L| < \epsilon$ if $|x - a| < \delta$.