## **Proof by Contrapositive**

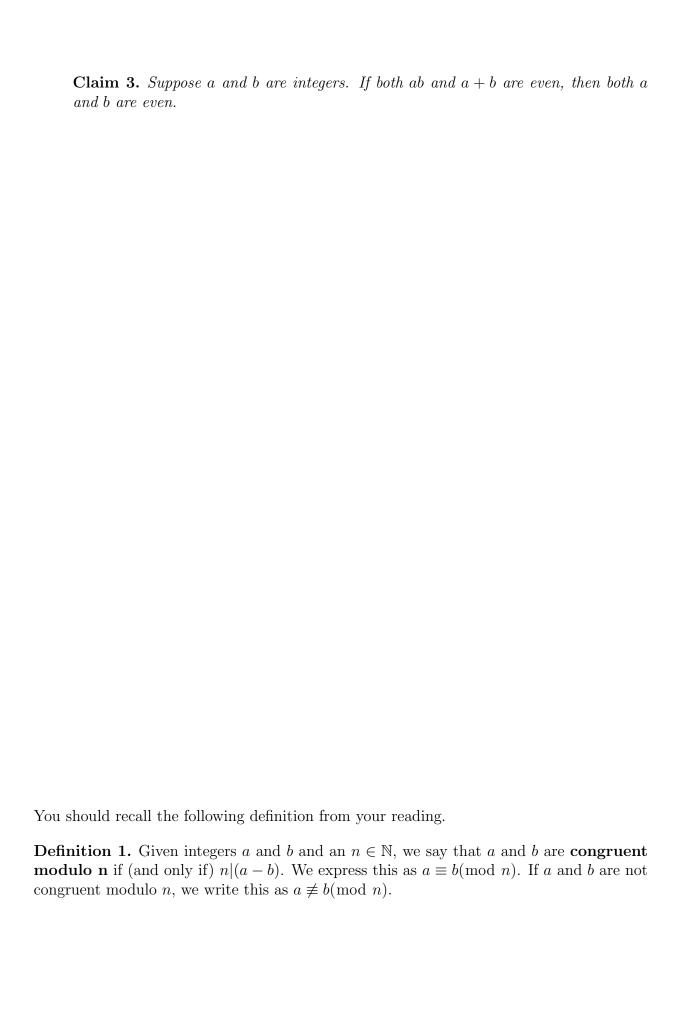
Proof by contrapositive is a very powerful proof technique. Technically speaking, any statement that can be proven with a direct proof can be proven with a contrapositive proof and vice versa. However, it is almost always the case that one proof is significantly easier to write.

## Goals:

- Write proofs using contrapositive
- Determine when it is appropriate to use proof by contrapositive
- 1. Prove the following claims using a proof by contrapositive.

Claim 1. Suppose  $n \in \mathbb{Z}$ . If  $n^2$  is even, then n is even.

Claim 2. Suppose  $x \in \mathbb{R}$ . If  $x^2 + 5x < 0$  then x < 0.



2. Prove the following claim using both direct proof and proof by contrapositive. Which proof is easier to write? Which is easier to understand? To prove the claim, you may apply the following variation of a result known as *Euclid's Lemma*.

**Lemma 1.** Let  $a, b, p \in \mathbb{Z}$ . If p is prime and p|ab, then p|a or p|b.

**Claim 4.** Let  $a, b \in \mathbb{Z}$  and  $n \in \mathbb{N}$ . If  $a \equiv b \pmod{n}$  and  $a \equiv c \pmod{n}$ , then  $c \equiv b \pmod{n}$ .

