Solutions for Calculus Vol 1: One variable calculus, with an introduction to Linear Algebra (2nd Edition) by Tom M. Apostol

Michael Rocke

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0.1 Introduction

0.1.1 1.4 Exercises

1

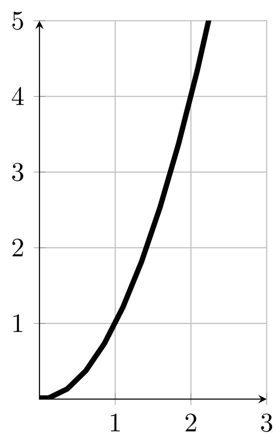
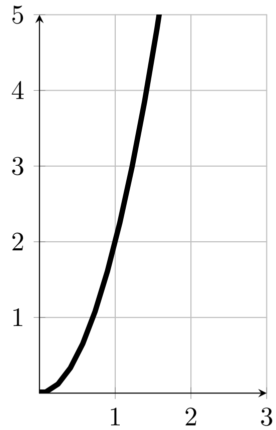


Figure 1.3: $y = x^2$

a) Modify the region in Figure 1.3 by assuming that the ordinate at each x is $2x^2$ instead of x^2.

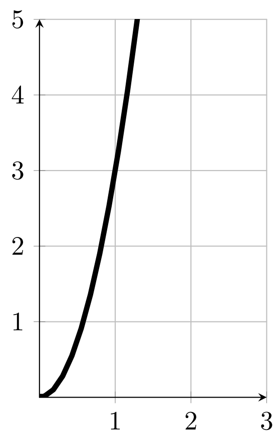
Draw the new figure.



$y = 2x^2$

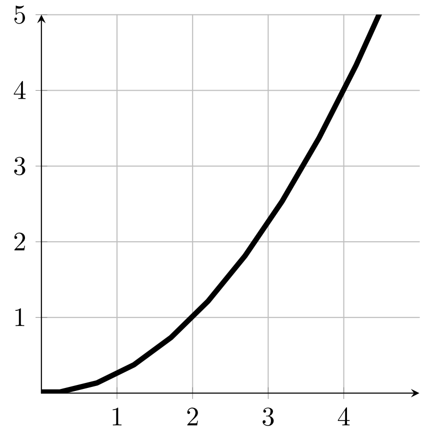
Check through the principal steps in the forgoing section and find what effect this has on the calculation of the area. Do the same if the ordfinate at each x

b) $3x^2$



$y = 3x^2$

c) $\frac{1}{4}x^2$



$y = \frac{1}{4}x^2$

d) $2x^2 + 1$
e) $ax^2 + c$

2

Modify the region in Figure 1.3 by assuming that the ordinate at each x is x^3 instead of x^2.

Draw the new figure.

a) Use a construction similar to that illustrated in Figure 1.5 and show that the outer and inner sums S_n and s_n are given by

$$S_n = \frac{b^4}{n^4}(1^3 + 2^3 + ... + n^3),\ s_n = \frac{b^4}{n^4}[1^3 + 2^3 + ... + (n-1)^3]$$

b)

0.2 The concepts of integral calculus

0.2.1 1.5 Exercises

1

Let $f(x) = x + 1$ for al real x. Compute the following

$f(2) = 3$
$f(-2) = -1$
$-f(2) = -3$
$f(\frac{1}{2}) = \frac{3}{2}$
$1/f(2) = \frac{1}{3}$
$f(a+b) = a + b + 1$
$f(a) + f(b) = a + b + 2$
$f(a)f(b) = (a+1)(b+1) = ab + a + b + 1$

2

Let $f(x) = 1 + x$ and let $g(x) = 1 - x$ for all real x. Compute the following:

$f(2) + g(2) = 3 + (-1) = 2$
$f(2) - g(2) = 3 - (-1) = 4$
$f(2)g(2) = -3$
$f(2)/g(2) = -3$
$f[g(2)] = f(-1) = 0$
$g[f(2)] = g[3] = -2$
$f(a) + g(-a) = 1 + a + (1 - (-a)) = 2 + 2a$
$f(t)g(-t) = (1 + t)(1 - (-t)) = 1 + 2t + t^2$

3

Let $\psi(x) = |x - 3| + |x - 1|$ for all real x. Compute the following:

$\psi(0) = |0 - 3| + |0 - 1| = 3 + 1 = 4$
$\psi(1) = |1 - 3| + |1 - 1| = 2$
$\psi(2) = |2 - 3| + |2 - 1| = 1 + 1 = 2$
$\psi(3) = |3 - 3| + |3 - 1| = 2$
$\psi(-1) = |-1 - 3| + |-1 - 1| = 6$
$\psi(-2) = |-2 - 3| + |-2 - 1| = 5 + 3 = 8$

Find all t for which $\psi(t + 2) = \psi(t)$ $\psi(t) = |t - 3| + |t - 1|$ $\psi(t + 2) = |t + 2 - 3| + |t + 2 - 1| = |t - 1| + |t + 1|$

Given $|t - 3| + \cancel{|t - 1|} = \cancel{|t - 1|} + |t + 1|$
thus $|t - 3| = |t + 1|$
Only value that will satisfy is when $t = 1$

4

Let $f(x) = x^2$ for all real x. Verify each of the following formulas. In each case describe the set of real x, y, t, etc., for which the given formula is valid

(a) $f(-x) = f(x)$, So $f(-x) = (-x) \cdot (-x) = (x) \cdot (x) = f(x)$ for all x

(b) $f(y) - f(x) = (y - x)(y + x)$, So $(y - x)(y + x) = y^2 - x^2 = f(y) - f(x)$ for all x and y

(c) $f(x + h) - f(x) = 2xh + h^2$, So, $f(x + h) - f(x) = (x + h)^2 - x^2 = \cancel{x^2} + 2xh + h^2 - \cancel{x^2}$ for all x and h

(d) $f(2y) = 4f(y)$, So $f(2y) = (2y)^2 = 4y^2 = 4f(y)$ for all y

(e) $f(t^2) = f(t)^2$, So $f(t^2) = (t^2)^2 = f(t)^2$ for all t

(f) $\sqrt{f(a)} = |a|$, So $\sqrt{f(a)} = \sqrt{a^2}$ which when taking the positive root, is $|a|$ for all a

$\int_a^b x^2 dx$