Introductory Mathematics: Algebra and Analysis Solutions

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0.1 Chapter 1

0.1.1 Exercises

Notes

$\mathbb{N}$ = Set of Natural numbers, $\{1, 2, 3, ..\}$

$\mathbb{Z}$ = Set of Integers, $\{..., -2, -1, 0, 1, 2, ..\}$

$\mathbb{Q}$ = Set of Rational Numbers, $Q = \{\frac{a}{b} | a, b \in \mathbb{Z}, b \neq 0\}$

$\mathbb{R}$ = Set of Real numbers

1.1

$A = \{1, 2, 3\}, B = \{1, 2\}, C = \{1, 3\}, D = \{2, 3\}, E = \{1\}, F = \{2\}, G = \{3\}, H = \emptyset$

a) $A \cap B = B$

b) $A \cup C = A$

c) $A \cap (B \cap C) = E$

d) $(C \cup A) \cap B = B$

e) $A \setminus B = G$

f) $C \setminus A = H$

g) $(D \setminus F) \cup (F \setminus D) = G$

h) $G \setminus A = H$

j) $A \cup ((B \setminus C) \setminus F) = A$

k) $H \cup H = H$

l) $A \cap A = A$

m) $((B \cup C) \cap C) \cup H = C$

1.2

a) i and ii are the same, iii is different
b) i and ii are the same, iii is different
c) $i = \{1, 2, 3, 4, 5, 6, 7\}, ii = \{1, 2, 3, 4, 5, 6, 7, -1, -2, -3, -4, -5, -6, -7\}, iii = \{1, 2, 3, 4, 5, 6, 7\}$, so i and iii are the same, ii is different
d) $i = \{0, 1, 2, 3, ...\}, ii = \{1, 2, 3, ...\}, iii = \{1, 2, 3, ...\}$, ii and iii are the same, i is different
e) i and iii are the same, ii is different

f) ii and iii are same, i is different

g) ii and iii are same, i is different
h) i and iii are same, ii is different

j) $i = \emptyset, ii = \emptyset, iii = \{\emptyset\}$ i and ii are same, iii are different
k) ii and iii are the same, i is different
l) ii and iii are the same, i is different
m) $i = \{\emptyset, \{\emptyset\}, 0\}, ii = \{\emptyset, \{\emptyset\}, 0\}, iii = \{\emptyset, 0\}$ i and ii are same, iii different

1.3

a)

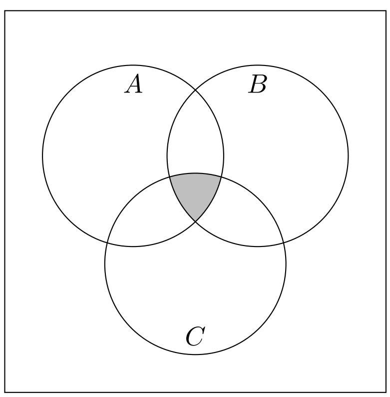


Figure 1: $A \cap B \cap C$

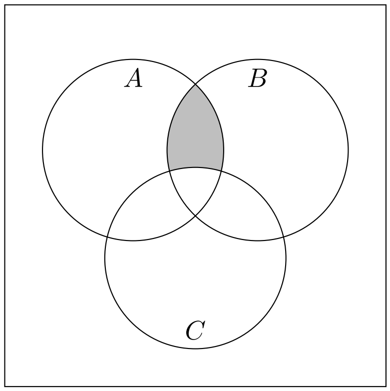


Figure 2: $A \cap B \cap C'$

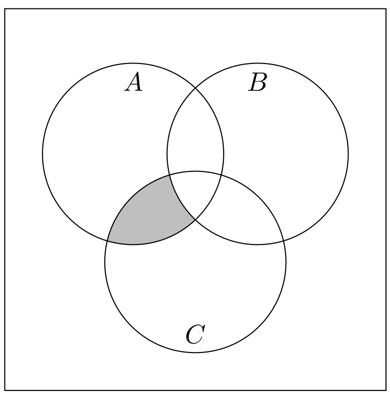


Figure 3: $A \cap B' \cap C$

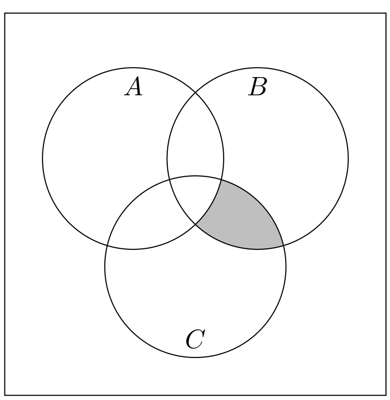


Figure 4: $A' \cap B \cap C$

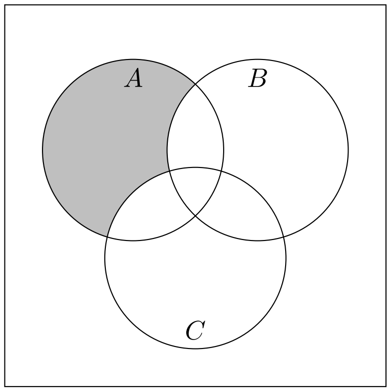


Figure 5: $A \cap B' \cap C'$

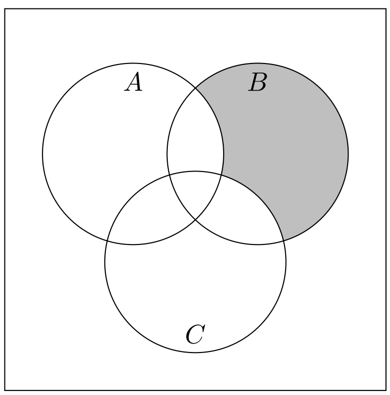


Figure 6: $A' \cap B \cap C'$

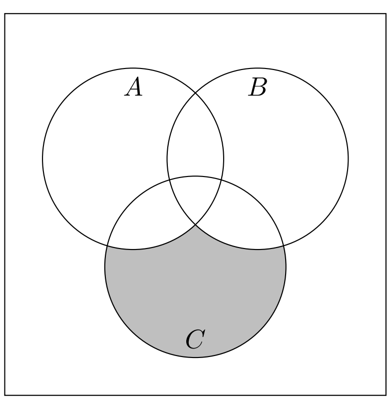


Figure 7: $A' \cap B' \cap C$

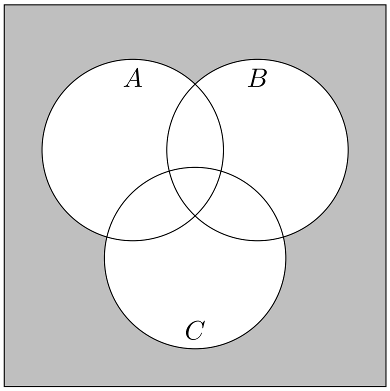


Figure 8: $A' \cap B' \cap C'$

b)

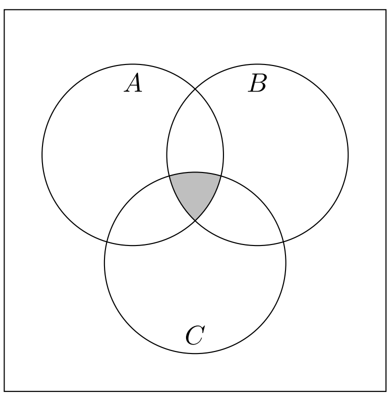


Figure 9:

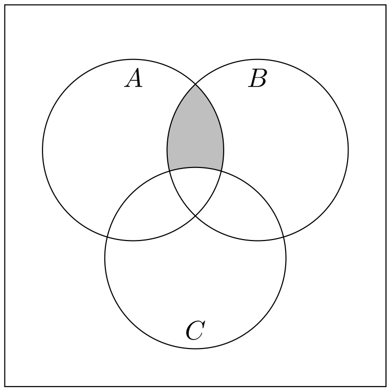


Figure 10:

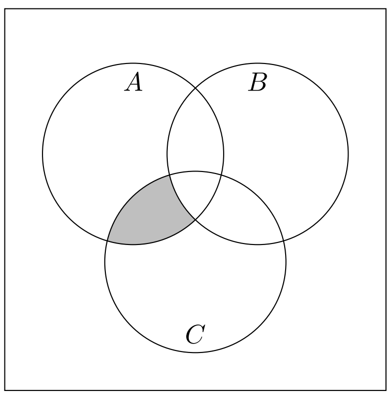


Figure 11:

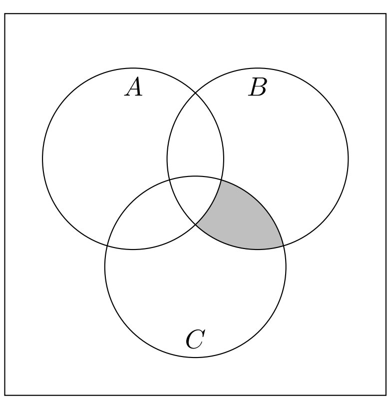


Figure 12:

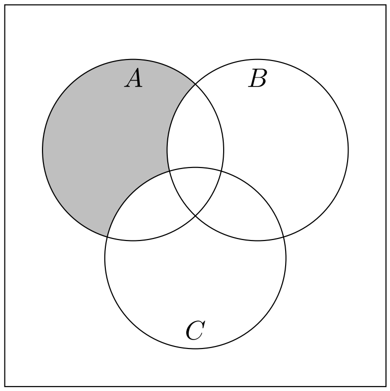


Figure 13: $(A' \cup B \cup C)'$

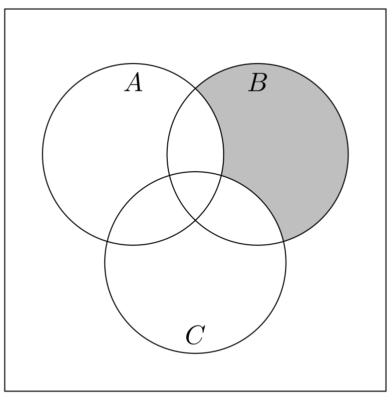


Figure 14: $(A \cup B' \cup C)'$

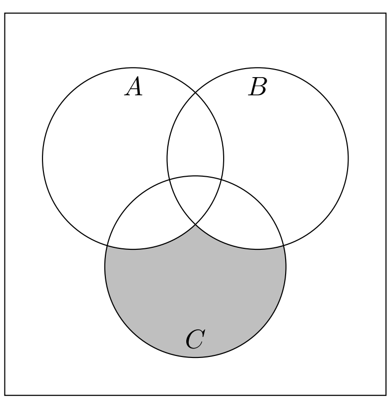


Figure 15: $(A \cup B \cup C')'$

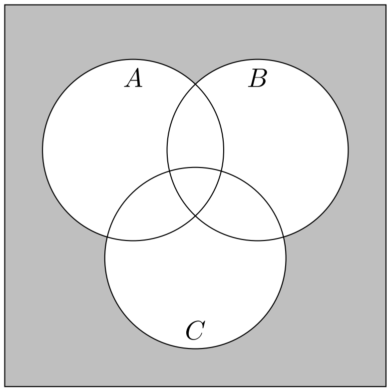


Figure 16: $(A \cup B \cup C)'$

1.4

a) Prove that $1, 2, 3 = 1, 1, 2, 3$

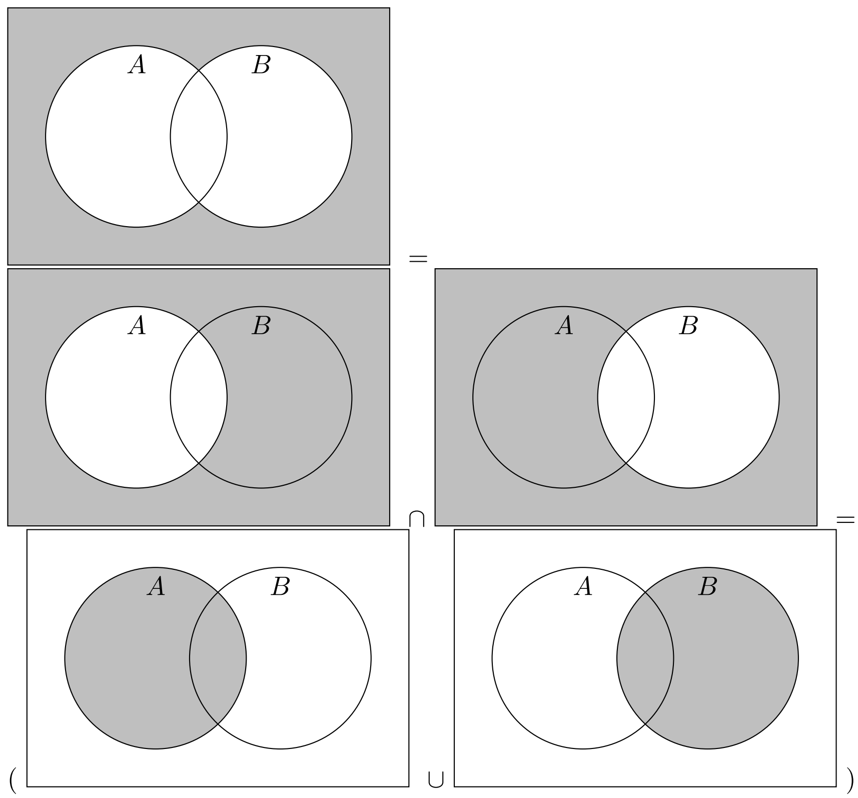
Proof. Lets define the following sets $A = \{1, 2, 3\}$ and $B = \{1, 1, 2, 3\}$. For two sets to be equal, every element in one set must belong to the other, meeting the property $A = A \cup B = B$. Just looking empirically, A contains the elements $1, 2, 3$ and B contains the same elements. We can also prove this by defining both sets as follows $A = \{a | a \in \mathbb{N}, 1 \leq a \leq 3\}$ and $B = \{b | b \in \mathbb{N}, 1 \leq b \leq 3\}$ which is the same definition $\square$

b) Using $P(A)$ to represent the Power Set of A. How many elements are there in the set $P(\{1, 2, 3\})$? $2^3 = 8$. These are: $\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$
c) Deterime the cardinality of $P(\emptyset)$? $2^0 = 1$ resulting in $\{\emptyset\}$
d) $P(P(\emptyset))$? $2^1 = 2$ $\{\emptyset, \{\emptyset\}\}$
e) $P(P(P(P(P(P(\emptyset))))))$? Using substitution, we can see $P(P(P(P(\{\emptyset, \{\emptyset\}\}))))$, and the cardinality of a powerset is defined 2^n where n is the cardinality of the argument set. This becomes the following

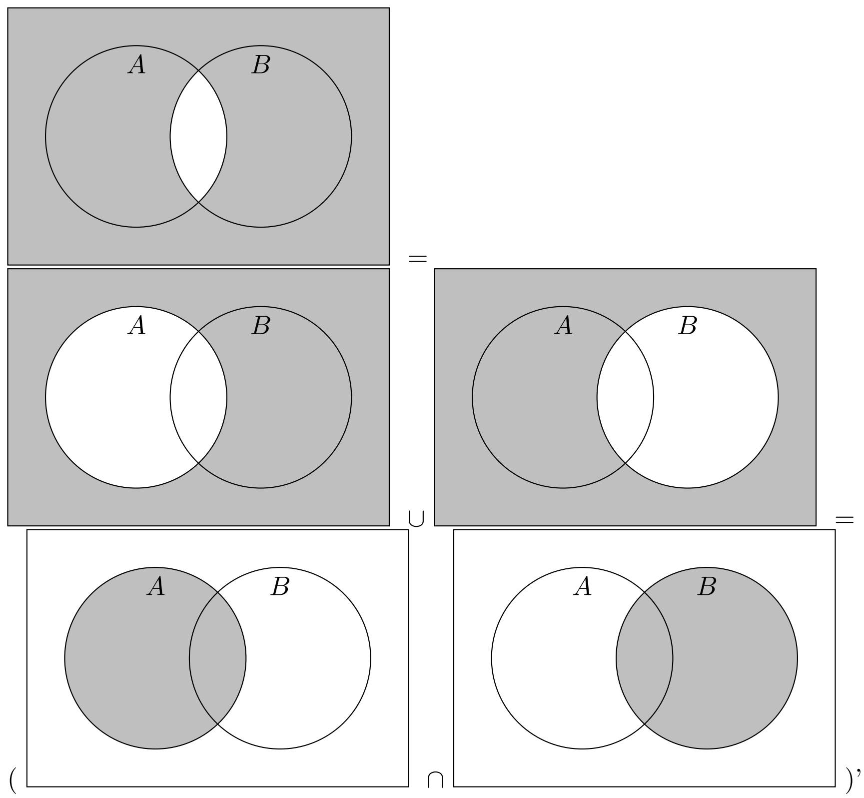
$$
\begin{aligned}
A_1 = P(\emptyset), |A_1| = 2^0 = 1 \\
A_2 = P(A_1), |A_2| = 2^1 = 2 \\
A_4 = P(A_2), |A_4| = 2^2 = 4 \\
A_16 = P(A_4), |A_16| = 2^4 = 16 \\
A_65536 = P(A_16), |A_65536| = 2^16 = 65536
\end{aligned}
$$

1.5

a) Draw Venn diagrams illustrating the truth to De Morgan's laws
$(A \cup B)' = A' \cap B'$



$(A \cap B)' = A' \cup B'$



b) Prove De Morgan's laws.

Proof. First we prove $(A \cup B)' = A' \cap B'$. $\square$