Proof by Contrapositive

Proof by contrapositive is a very powerful proof technique. Technically speaking, any statement that can be proven with a direct proof can be proven with a contrapositive proof and vice versa. However, it is almost always the case that one proof is significantly easier to write.

Goals:
• Write proofs using contrapositive • Determine when it is appropriate to use proof by contrapositive

1. Prove the following claims using a proof by contrapositive.

   Claim 1. *Suppose $n \in \mathbb{Z}$. If n^2 is even, then n is even.*

   Claim 2. *Suppose $x \in \mathbb{R}$. If $x^2 + 5x < 0$ then $x < 0$.*

Claim 3. *Suppose a and b are integers. If both ab and $a+b$ are even, then both a and b are even.*

You should recall the following definition from your reading.

Definition 1. Given integers a and b and an $n \in \mathbb{N}$, we say that a and b are **congruent modulo n** if (and only if) $n|(a-b)$. We express this as $a \equiv b(\text{mod } n)$. If a and b are not congruent modulo n, we write this as $a \not\equiv b(\text{mod } n)$.

2. Prove the following claim using both direct proof and proof by contrapositive. Which proof is easier to write? Which is easier to understand? To prove the claim, you may apply the following variation of a result known as *Euclid's Lemma*.

   Lemma 1. *Let $a, b, p \in \mathbb{Z}$. If p is prime and $p|ab$, then $p|a$ or $p|b$.*

   Claim 4. *Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b (\text{mod } n)$ and $a \equiv c(\text{mod } n)$, then $c \equiv b(\text{mod } n)$.*

3. There are no distinct rules about when to use a direct proof versus when to use proof by contrapositive. However, you may have developed some intuition about where you may want to start. List some ideas you have of how to decide between direct proof or proof by contrapositive.