Logical Problem Solving and Arguments

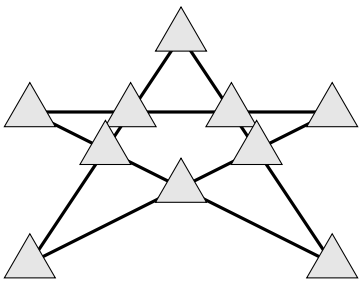
Being able to write proofs opens up your mind to being able to systematically solve general problems. Not only will you be able to reason your way through and solve the given problem, you will be able to explain how you *know* you have a correct solution. The following problems are not *math* problems; they are *logic* problems. You need a clear head and careful thought to solve them, not knowledge from a prior math class.

Goals:

- Analyze simple problems not based on given examples
- Solve problems with no algorithm provided
- Argue that solutions are valid

1. (Needy Children) Mrs. Jones has twin girls. When out shopping the family comes across a gum ball machine which offers red gum balls and purple gum balls. The savvy Mrs. Jones knows that she has to give the same color gum ball to each twin to ensure that no one throws a fit. What is the maximum number of gum balls that Mrs. Jones must purchase to ensure that each twin has the same color gum ball? After working out the maximum number, provide a carefully developed argument explaining how you know this is the correct answer.

2. (The King's Fortress) A king wants to design fortress with 10 towers and walls running between the towers. The king wants 4 towers on each wall and wants as many towers surrounded by walls as possible. One of the king's servants suggests the following arrangement.



   This meets the requirement for the walls, but none of the towers are contained entirely within walls. Give a better arrangement.

3. (Jelly Bean Mix-Up) You are presented with three jelly bean jars. One jar contains cherry flavored beans, another contains orange flavored beans, and the third contains a mix of cherry and orange. Each jar is labeled, but the labels are *all wrong*. What is the minimum number of jelly beans you must eat to properly correct the labels? After working out the minimum number, provide a carefully developed argument explaining how you know this to be the correct solution.

4. (Lost in the Woods) You are driving to a very important meeting and you realize that you're going to be late. Your GPS tells you that you can take a short cut through the woods that

will get you there on time. You quickly check the route and notice that it consists of a single road through the woods, so you decide to take it since you can't get lost. An hour into the woods you come to a fork in the road with no signs. You've lost your GPS signal, so you realize that you'll need to ask for directions. Fortunately, there is an inn at the fork in the road. As you walk up to the inn, you notice a sign on the front door that says the following.

> Two identical twin brothers live here and they can give you directions. However, you may only ask them one question per day and it will only be answered by the twin to which you address the question. One of the twins always lies, the other always tells the truth, and you can't tell them apart.

There is exactly one question that you can ask the twins that will help you. What is that question and how do you use the answer to get to the meeting on time?

5. (A Burning Problem) You are given two ropes that take exactly 60 minutes each to burn. The lengths of the ropes burn inconsistently, meaning that it's possible for half the rope to burn in a few seconds and the other half to take nearly an hour. Find a way to use the two ropes to measure 45 minutes. Provide a carefully written explanation of how you know your solution is correct.

6. (Find the Fake). You are given 7 gold coins that appear to be identical, but one is a fake and weighs slightly less than the other 6. You are also given a balancing scale. Find a way to determine which coin is that fake by using the scale *only twice*. Give a carefully written explanation of your solution.