P.4 Basic Set Theory

Sets are arguable the most basic building blocks of mathematics. It is not possible to discuss writing proofs nor to write proofs without know some basic set theory and notation.

Goals:

- Practice with set-builder notation.
- Practice finding cardinalities.

1. Write each of the following sets by listing their elements between braces.

   (a) $\{3x + 2 : x \in \mathbb{Z}\}$

   (b) $\{x \in \mathbb{Z} : -2 \leq x < 7\}$

   (c) $\{x \in \mathbb{R} : x^2 + 5x = -6\}$

   (d) $\{5a + 2b : a, b \in \mathbb{Z}\}$

2. Write each of the following sets in set-builder notation.

   (a) $\{3, 4, 5, 6, 7, 8\}$

   (b) $\{\ldots, -6, -3, 0, 3, 6, 9, 12, 15, \ldots\}$

3. Describe (in your own words) what the cardinality of set is.

4. Determine the following cardinalities.

   (a) $|\{2, 3, 4, 5\}|$

   (b) $|\{2, 3, \{4, 5\}\}|$

   (c) $|\varnothing|$

   (d) $|\{\varnothing\}|$

   (e) $|\{x \in \mathbb{Z} : x^2 - 3 \leq 0\}|$

5. (a) Suppose that A, B, and C. What does the notation $A \subseteq B$ and $A \not\subseteq C$ mean? Be explicit.

(b) List all of the subsets of $A = \{2, 5, 7\}$.

The set of all of the subsets of A is called the **power set** of A and is denoted $\mathscr{P}(A)$. Symbolically, $\mathscr{P}(A) = \{X : X \subseteq A\}$.

6. Let $B = \{0, \{1\}\}$. Determine $\mathscr{P}(B)$.

We'll need to use set notation and the concepts listed here and in sections 1.1 and 1.3 of the text throughout the semester. If you ever find yourself unsure about set notation, don't hesitate to review these concepts and to ask questions.