Power Laws Course

Applied Scholastics, Ferndale WA

Powers

Powers are numbers multiplied by themselves a given number of times. The number being multiplied is called the *base*, and the number of times that it is multiplied by itself is called the *power* (or the *index*, or the *exponent*.)

The power or index or exponent is written as a small raised number to the right of the base.

The word *power* is from Latin *potentias* which is a mistranslation of the Greek word *dynamis* meaning *amplification* used by the mathematician Euclid around 300BC for the square of a line.

The word *exponent* was coined in a book from 1544 called *Arithmetica Integra*. It is from Latin *expo*, out of, and *ponere*, place, with the idea of laying something out to view it's parts.

Exponent is the preferred term in the US but the rest of the world prefers the terms *index* or *power*.

Index is a Latin word meaning pointer, and the plural of *index* is *indices*. (Pronounced "indisees.") The use of the term index derived from 'pointing' to which of the powers of a number was meant. For example, the powers of 3 are $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81$ and $3^5 = 273$ so you could say that the 4^{th} power of 3 is 81, with 4 being the pointer to that particular power.

The word power is also used to mean the result of raising a number to a power, such as when we say that 8 is a power of 2 because $2^3 = 8$.

If we use a as the base and n as the exponent, a^n is read as the n^{th} *power of* a, or as a *to the power of* n, or as a *the the* n^{th} *power*.

a to the power of n is a times a, n times.

$a^n = a \cdot a \cdot a \cdot ... \cdot a$ {for n factors} e.g. $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

In writing computer programs, and in some maths programs, because they use only standard text characters and can't write the small raised n, the "caret" symbol,ˆ is used to write the power of a number.

e.g. 3ˆ5 $= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

1. What does power mean?
2. What does exponent mean?
3. What does index mean?
4. What does 2^5 mean?
5. What does 3ˆ6 mean?

Roots

The *base* can also be called the *root*, particularly when given some number and you are looking for the number which when multiplied by itself a given number of times will result in that number.

The Latin word for *root* is *radical* and that word is also sometimes used to mean the root. It is also thought to be where the radical symbol $\surd$ comes from, being a sort of stretched out letter *r*.

The number to the left of the radical symbol that indicates which root is to be taken and it is called the *index* or the *order* or the *degree* of the root.

The value inside the radical symbol is known as the *radicand*.

$\sqrt[n]{a}$ is read as 'the n^{th} root of a.'

e.g. $\sqrt[5]{243} = 3$ is read as 'the 5^{th} root of 243 equals 3.' (notice that $3^5 = 243$.)

6. What does base mean?
7. What does root mean?
8. What does radical mean?
9. What does radicand mean?
10. In $\sqrt[4]{81} = 3$, which number is the radicand?
11. In $\sqrt[4]{81} = 3$, which number is the index?
12. In $\sqrt[4]{81} = 3$, which number is the root?

The second root of a number is also called its 'square' root, because the area of a square is given by the product of the lengths of its sides. The number is usually not written. If a radical symbol has no number given, then it is implied to be a square root that is meant.

e.g. $\sqrt{a}$ is read as 'the square root of a.'

The third root of a number is also called its 'cube' root, because the volume of a cube is given by the product of the lengths of its three dimensions.

e.g. $\sqrt[3]{a}$ is read as 'the cube root of a.'

13. What is a square root?

14. What is a cube root?

Power Laws

The powers of numbers follow some useful laws:

Unit Power Law

$$a^1 = a$$

$$\text{e.g. } 3^1 = 1$$

15. What happens when any number is raised to the power of 1?

16. Does the unit power law apply to negative numbers as well?

17. Why do you think it is that any number to the first power is equal to itself?

18. What is 27^1?

19. What is 0.2^1?

20. What is $(-5)^1$?

21. What is $(\frac{3}{4})^1$?

Product Law

$$a^m \times a^n = a^{m+n}$$

$$\begin{aligned} \text{e.g. } 3^2 \times 3^3 = 3^{2+3} &= 3^5 \\ &= 3 \cdot 3 \times 3 \cdot 3 \cdot 3 \\ &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5 \end{aligned}$$

22. How does the product law work when multiplying two numbers with the same base raised to different powers?

23. Why do you think the product law works this way?

24. What is $2^3 \times 2^4$?

25. What is $4^6 \times 4^7$?

26. What is $x^2 \times x^5$?

27. Express 3^{2+5} as a product of powers of 3.

Quotient Law

$$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$$

$$\text{e.g. } \frac{3^5}{3^3} = 3^{5-3} = 3^2$$
$$= (3 \times 3 \times 3 \times 3 \times 3) \div (3 \times 3 \times 3)$$
$$= \frac{3 \cdot 3 \cdot \cancel{3 \cdot 3 \cdot 3}}{\cancel{3 \cdot 3 \cdot 3}} = 3 \cdot 3 = 3^2$$

28. How does the quotient law work when dividing two numbers with the same base raised to different exponents?

29. Why do you think the quotient law works this way?

30. Why do you think the base can't equal 0 with this law?

31. What is $4^6 \div 4^3$?

32. What is $\frac{3^9}{3^6}$?

33. What is $36^5 \div 36^4$?

34. Express 2^4 as a quotient of powers of 2.

Zero Power Law

$$a^0 = 1 \quad (a \neq 0)$$

$$\begin{aligned}(\frac{n^x}{n^x} &= \frac{\cancel{n^x}}{\cancel{n^x}} = 1 \\ &= n^{x-x} = n^0)\end{aligned}$$

$$\text{e.g. } 3^0 = 1$$

35. What is the zero power law?
36. Does the zero power law apply to a base of 0?
37. Why do you think it could be that a number to the power of 0 is equal to 1?
38. Does the zero power law apply to negative numbers?
39. What is $(2x)^0$?
40. What is $(-5)^0$?
41. What is $(\frac{22}{7})^0$?

Power of a Power Law

$$(a^m)^n = a^{m \times n}$$

$$\begin{aligned} \text{e.g. } (3^2)^3 &= 3^{2\times 3} = 3^6 \\ &= (3\cdot 3)\times(3\cdot 3)\times(3\cdot 3) \\ &= 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3 = 3^6 \end{aligned}$$

42. In words, what is the power of a power law?
43. Why do you think the power of a power law works this way?
44. What is $(2^3)^4$?
45. What is $(x^4)^2$?
46. What is $((7x)^2)^3$?

Power of a Product Law

$$(a \times b)^m = a^m \times b^m$$

$$\text{e.g. } (3\cdot 3)^2 = 3^2\cdot 3^2 = 9\times 9 = 81$$

47. How does the power of a product law work when raising a product to a power?
48. What is $(7\times 5)^2$?
49. What is $(3x)^2$?
50. What is $(3xy)^3$

Power of a Quotient Law

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$$

$$\text{e.g. } \left(\tfrac{3}{9}\right)^2 = \tfrac{3^2}{9^2} = \tfrac{9}{81} = \tfrac{1}{9}$$

51. What does the power of a quotient law state?

52. What is $(\frac{5}{6})^2$

53. What is $(\frac{2}{x})^5$

54. What is $(\frac{3x}{2y})^3$

Negative Power Laws

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^{-n}} = a^n$$

$$(a^{-n} = a^{0-n} = \frac{a^0}{a^n} = \frac{1}{a^n})$$

$$\text{e.g. } 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\text{e.g. } \frac{1}{5^{-3}} = 5^3 = 75$$

Use of brackets in writing negative powers

Brackets must be used properly when writing negative powers because the meaning is completely different depending on their placement.

e.g. $-3^2 = -(3 \times 3) = -9$,

but $(-3)^2 = -3 \times -3 = 9$.

55. In words, what are the two negative power laws?
56. Why are brackets important in writing powers of a negative number?
57. Give an example of a negative number raised to a power, and how its meaning is changed by placement of brackets.
58. What is 2^{-4}?
59. What is $2x^{-2}$?
60. What is $(2x)^{-2}$?
61. What is $(3^0)^{-2}$?
62. What is $\frac{1}{5^{-2}}$?
63. What is $\frac{1}{2^{-5}}$?
64. What is 3^{-1}?

Multiplication of Negative Powers

same bases:

$$a^{-m} \times a^{-n} = a^{-(m+n)} = \frac{1}{a^{m+n}}$$

e.g. $2^{-2} \times 2^{-3} = \frac{1}{2^{2+3}} = \frac{1}{2^5} = \frac{1}{32}$

65. How do you multiply powers with negative indices and the same base?

66. What is $3^{-2} \times 4^{-3}$?

different bases:

$$a^{-n} \times b^{-n} = (a \times b)^{-n}$$

e.g. $2^{-3} \times 3^{-3} = (2 \times 3)^{-3} = 6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

67. How do you multiply powers with the same negative indices but different bases?

68. What is $5^{-3} \times 3^{-3}$?

different bases and powers:

$$a^{-m} \times b^{-n} \text{ (calculate separately)}$$

$$\text{e.g.}\quad 2^{-3} \times 3^{-2} = \frac{1}{2^3} \times \frac{1}{3^2} = \frac{1}{8} \times \frac{1}{9} = \frac{1}{72}$$

69. How do you multiply powers with the different negative indices and different bases?

70. What is $5^{-3} \times 3^{-2}$?

Division of Negative Powers

same bases:

$$a^{-m} \div a^{-n} = a^{-m-(-n)} = a^{-m+n}$$

$$\text{e.g.}\quad 3^{-3} \div 3^{-2} = 3^{-3+2} = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

71. How do you divide powers with negative indices and the same base?

72. What is $3^{-5} \div 3^{-3}$?

different bases:

$$a^{-m} \div b^{-n} = \frac{b^n}{a^m}$$

$$\text{e.g.}\ 2^{-3} \div 3^{-2} = \frac{3^2}{2^3} = \frac{9}{8}$$

73. How do you divide powers with negative indices and different bases?

74. What is $5^{-2} \div 4^{-3}$?

75. What is $\frac{5^{-4}}{6^{-3}}$?

Fractions with Negative Powers

$$(\frac{a}{b})^{-n} = \frac{b^n}{a^n}$$

$$((\frac{a}{b})^{-n} = \frac{1}{(\frac{a}{b})^n} = \frac{1}{\frac{a^n}{b^n}} = \frac{b^n}{a^n})$$

$$\text{e.g.} \quad (\tfrac{2}{3})^{-2} = \tfrac{3^2}{2^2} = \tfrac{9}{4}$$

76. What is $\left(\frac{3}{4}\right)^{-2}$?

77. What is $\left(\frac{1}{2}\right)^{-3}$?

78. What is $\left(\frac{2}{3}\right)^{-1}$?

79. What is $\left(\frac{4}{5}\right)^{-3}$?

80. What is $\left(\frac{1}{8}\right)^{-4}$?

Powers of a Negative Number

A negative number multiplied by a negative number results in a positive product. When that positive product is multiplied by a negative number, the result is a negative product. The sign of the powers of a negative number alternates depending on whether the power is an odd or an even number.

A negative number taken to an even power gives a positive result.

e.g. $(-4)^4 = -4 \times -4 \times -4 \times -4 = 256$

A negative number taken to an odd power gives a negative result.

e.g. $(-4)^5 = -4 \times -4 \times -4 \times -4 \times -4 = 1024$

81. What are the rules about the powers of a negative number?
82. What is -2^2?
83. What is -5^3?

Roots of an Even Power of a Negative Number

There are no roots of an even power of a negative number.

e.g. No real number can be multiplied by itself to find $\sqrt{-16}$.

Roots of an Odd Power of a Negative Number

You can, however, find the roots of odd powers.

e.g. $-3 \times -3 \times -3 = -27$, so $\sqrt[3]{-27} = -3$.

84. What are the rules about finding the roots of negative numbers?

85. What is $\sqrt[3]{-125}$?

86. Does $\sqrt[6]{64}$ have a real answewr?

Reciprocal Power law

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\text{e.g. } 32^{\frac{1}{5}} = \sqrt[5]{32} = 2$$

87. What is a reciprocal?

88. Why is $\frac{1}{n}$ the reciprocal of n?

89. In words, what is the reciprocal power law?

90. What is $16^{\frac{1}{4}}$?

91. What is $75^{\frac{1}{3}}$?

92. Express $\sqrt[3]{81}$ as a power.

Fractional Power Law

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$(a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m)$$

$$(a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m})$$

e.g. $3^{\frac{2}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9} = (\sqrt[3]{3})^2$

e.g. $-4^{\frac{2}{4}} = \sqrt[4]{-4^2} = 2$

93. In words, what is the fractional power law?
94. What is $8^{\frac{3}{2}}$?
95. What is $32^{\frac{2}{5}}$?
96. Express $\sqrt[8]{256} = 2$ as a base with a fractional power.
97. Express $\sqrt[3]{8^2} = 4$ as a base with a fractional power.

Negative Fractional Powers

$$a^{\frac{-m}{n}} = 1/a^{\frac{m}{n}} = \frac{a}{(\sqrt[m]{a})^n}$$

e.g. $3^{\frac{-2}{3}} = 1/3^{\frac{2}{3}} = \frac{1}{(\sqrt[2]{3})^3} \approx \frac{1}{1.73^3} \approx \frac{1}{5.20} \approx 0.19$

98. What is $2^{\frac{-1}{4}}$?

99. What is $4^{\frac{-2}{7}}$?

100. What is $3^{\frac{-3}{8}}$?

101. What is $5^{\frac{-2}{3}}$?

Final Practice

Here is a set of final questions to test your knowledge of the various power laws:

1. What is the value of 3^0?
2. Simplify 2^3.
3. What is the result of 5^{-2}?
4. Evaluate 6^1.
5. Simplify $7^{\frac{1}{2}}$.
6. Calculate 8^{-1}.
7. Evaluate $3^2 \times 3^4$.
8. Simplify $(2^5)^2$.
9. Compute $5^3 \div 5^2$.
10. What is the result of $6^3 \times 6^{-2}$?

11. Evaluate $\frac{4^3}{2^3}$.

12. Calculate $(3^2)^3$.

13. What is the value of $(5^4)^{\frac{1}{2}}$?

14. Compute $7^2 \times 7^{-2}$.

15. Evaluate $2^{-3} \div 2^{-5}$.

16. Simplify $10^{\frac{3}{2}}$.

17. Calculate $\frac{4^2}{2^4}$.

18. What is the result of $8^{\frac{1}{3}}$?

19. Evaluate $(9^2)^{\frac{1}{2}}$.

20. Simplify $(2^3)^{-2}$.

21. Compute $11^2 \div 11^1$.

22. Calculate $(2^2)^2 \times 2^3$.

23. Evaluate $5^{\frac{3}{2}} \times 5^{\frac{1}{2}}$.

24. Simplify: $2^{-3} \times 2^{-4}$.

25. Calculate: $5^{-2}/5^{-3}$.

26. Determine the value of $3^{-2} \cdot 3^4$.

27. Evaluate: $(1/2)^{-3}$.

28. Simplify: $2^{-2}/3^{-2}$.

29. Calculate: $5^{-1} \cdot 10^{-1}$.

30. Find the value of $4^{-1/2}$.

31. Simplify: $8^{-3/2}$.

32. Calculate: $9^{-1/2} \cdot 3^{-1/2}$.

33. Compute -2^3.

34. Calculate -5^4.

35. Determine the value of -1^{100}.

36. Find -4^2.

37. Express the cube root of 8^2 as a base with fractional indices.

38. Calculate the fourth root of -16^4.

39. What is $2^{-3} \cdot 2^{-4}$?

40. What is $5^{-2}/\ 5^{-3}$?

41. Evaluate: $3^{-2} \cdot 3^4$.

42. Calculate: $\frac{1}{2}^{-3}$.

43. What is $2^{-2} \div 3^{-2}$

44. What is $5^{-1} \cdot 10^{-1}$?

45. What is $4^{-\frac{1}{2}}$?

46. What is $8^{-\frac{3}{2}}$?

47. What is $9^{-\frac{1}{2}} \cdot 3^{-\frac{1}{2}}$?

48. Evaluate: $(3^2)^1$.

49. What is $4^{\frac{1}{2}} \cdot 4^{\frac{1}{3}}$?

50. What is -5^4?

Answers:

1. $3^0 = 1$
2. $2^3 = 8$
3. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
4. $6^1 = 6$
5. $7^{1/2} = \sqrt{7}$
6. $8^{-1} = \frac{1}{8}$
7. $3^2 \times 3^4 = 3^{2+4} = 3^6$
8. $(2^5)^2 = 2^{5\cdot 2} = 2^{10} = 1024$
9. $\frac{5^3}{5^2} = 5^{3-2} = 5^1 = 5$
10. $6^3 \times 6^{-2} = 6^{3-2} = 6^1 = 6$
11. $\frac{4^3}{2^3} = \frac{(2^2)^3}{2^3} = \frac{2^6}{2^3} = 2^{6-3} = 2^3 = 8$
12. $(3^2)^3 = 3^{2\cdot 3} = 3^6$

13. $(5^4)^{1/2} = 5^{4(1/2)} = 5^2 = 25$

14. $7^2 \times 7^{-2} = 7^{2-2} = 7^0 = 1$

15. $2^{-3} \div 2^{-5} = 2^{-3+5)} = 2^2 = 4$

16. $10^{3/2} = \sqrt{10^3} = \sqrt{1000}$

17. $\frac{4^2}{2^4} = \frac{(2^2)^2}{2^4} = \frac{2^4}{2^4} = 1$

18. $8^{1/3} = \sqrt[3]{8} = 2$

19. $(9^2)^{1/2} = 9^{2\times\frac{1}{2}} = 9^1 = 9$

20. $(2^3)^{-2} = 2^{3(-2)} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

21. $11^2 \div 11^1 = 11^{2-1} = 11^1 = 11$

22. $(2^2)^2 \times 2^3 = 2^{2\times2+3} = 2^{4+3} = 2^7 = 128$

23. $5^{3/2} \times 5^{1/2} = 5^{(3/2+1/2)} = 5^2 = 25$

24. $2^{-3} \times 2^{-4} = 2^{(-3-4)} = 2^{-7} = \frac{1}{128}$

25. $5^{-2}/5^{-3} = 5^{-2+3} = 5^1 = 5$

26. $3^{-2} \cdot 3^4 = 3^{(-2+4)} = 3^2 = 9$

27. $(1/2)^{-3} = 2^3 = 8$

28. $\frac{2^{-2}}{3^{-2}} = \frac{\frac{1}{2^2}}{\frac{1}{3^2}} = \frac{\frac{1}{4}}{\frac{1}{9}} = \frac{1}{4} \times \frac{9}{1} = \frac{9}{4}$

29. $5^{-1} \cdot 10^{-1} = \frac{1}{5} \times \frac{1}{10} = \frac{1}{50}$

30. $4^{-\frac{1}{2}} = 1/4^{\frac{1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

31. $8^{-3/2} = 1/8^{\frac{3}{2}} = \frac{1}{\sqrt{8^3}} = \frac{1}{\sqrt{512}}$

32. $9^{-1/2} \times 3^{-1/2} = (9 \times 3)^{-\frac{1}{2}} = 27^{-\frac{1}{2}} = \frac{1}{\sqrt{27}}$

33. $-2^3 = -8$

34. $-5^4 = 625$

35. $-1^{100} = 1$

36. $-4^2 = 16$

37. $\sqrt[3]{8^2} = 8^{\frac{2}{3}}$

38. $\sqrt[4]{-16^4} = (-16^4)^{\frac{1}{4}} = -16^{4\times\frac{1}{4}} = -16$

39. $2^{-3} \cdot 2^{-4} = \frac{1}{2^{3+4}} = \frac{1}{2^7} = \frac{1}{128}$

40. $\frac{5^{-2}}{5^{-3}} = 5^{-2+3} = 5^1 = 5.$

41. $3^{-2} \cdot 3^4 = 3^{(-2+4)} = 3^2 = 9.$

42. $(\frac{1}{2})^{-3} = \frac{1}{(\frac{1}{2})^3} = \frac{1}{\frac{1^3}{2^3}} = \frac{1}{\frac{1}{8}} = 8.$

43. $\frac{2^{-2}}{3^{-2}} = \frac{\frac{1}{2^2}}{\frac{1}{3^2}} = \frac{\frac{1}{4}}{\frac{1}{9}} = \frac{1}{4} \times \frac{9}{1} = \frac{9}{4}$

44. $5^{-1} \cdot 10^{-1} = \frac{1}{5} \cdot \frac{1}{10} = \frac{1}{50}$

45. $4^{-1/2} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

46 $8^{-\frac{3}{2}} = \frac{8}{(\sqrt[3]{8})^2} = \frac{8}{2^2} = \frac{8}{4} = 2$

47. $9^{-\frac{1}{2}} \cdot 3^{-\frac{1}{2}} = (9 \cdot 3)^{-\frac{1}{2}} = 27^{-\frac{1}{2}} = \frac{1}{\sqrt{27}}$

48. $(3^2)^{-1} = \frac{1}{3^2} = \frac{1}{9}$

49. $4^{1/2} \cdot 4^{1/3} = \sqrt{4} \cdot \sqrt[3]{4} = 2\sqrt[3]{4}$

50. $-5^4 = 625$