Understanding the Point-Slope Form of Linear Equations

Tutoring Centre Ferndale

In linear algebra, the point-slope form $y - y_1 = m(x - x_1)$ is useful for writing the equation of a line when the slope m and a point (x_1, y_1) on the line are known. Understanding how to use and manipulate this form can simplify the process of graphing linear equations and solving related problems.

Point-Slope Form: $y - y_1 = m(x - x_1)$

In the point-slope form, $y - y_1 = m(x - x_1)$:

- (x_1, y_1) is a point on the line.
- m is the slope of the line.

Questions

1. What is the general form of the point-slope equation?
2. Identify the slope and a point in the equation $y - 3 = 2(x + 1)$.
3. How do you derive the point-slope form from a given point and slope?

Graphing Using Point-Slope Form

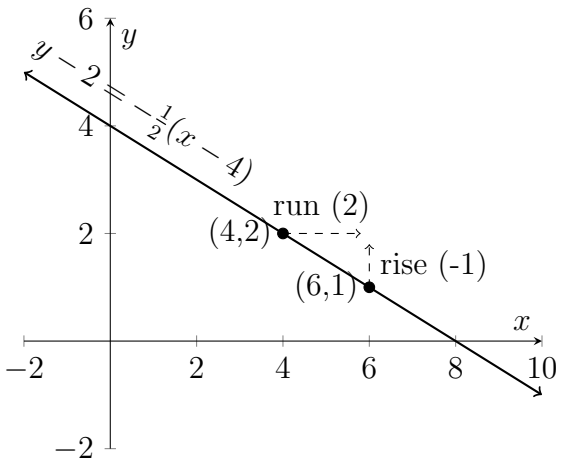
To graph a line using the point-slope form:

- Plot the point (x_1, y_1) on the coordinate plane.
- Use the slope m to determine the rise and run from the point (x_1, y_1).
- Draw the line through the point with the given slope.

Example

Given the equation $y - 2 = -\frac{1}{2}(x - 4)$:

- The point $(4, 2)$ is on the line.
- The slope is $-\frac{1}{2}$.



Questions

1. Plot the point $(4, 2)$ and draw the line with slope $-\frac{1}{2}$. 2. What is the rise and run from the point $(4, 2)$ if the slope is $-\frac{1}{2}$? 3. How does the graph change if the slope is positive instead of negative?

Converting Point-Slope to Slope-Intercept Form

The point-slope form can be converted to the slope-intercept form $y = mx+b$:

$$y - y_1 = m(x - x_1)$$

$$y = mx - mx_1 + y_1$$

$$y = mx + (y_1 - mx_1)$$

Here, the slope m remains the same, and the y-intercept b is given by $y_1 - mx_1$.

Example

Convert $y - 3 = 2(x + 1)$ to slope-intercept form:

$$y - 3 = 2(x + 1)$$

$$y - 3 = 2x + 2$$

$$y = 2x + 5$$

Questions

1. Convert $y - 1 = 3(x - 2)$ to slope-intercept form.
2. Identify the slope and y-intercept in the slope-intercept form of $y - 4 = -\frac{2}{3}(x + 3)$.
3. Explain why the slope remains unchanged when converting from point-slope to slope-intercept form.

Summary

- The point-slope form $y - y_1 = m(x - x_1)$ is useful for writing the equation of a line given a point and the slope.
- To graph a line using the point-slope form, plot the point and use the slope to draw the line.
- The point-slope form can be converted to slope-intercept form for easier interpretation of the y-intercept.

Understanding the point-slope form of linear equations can simplify the process of writing, graphing, and analyzing lines in various mathematical contexts.

Answers

Point-Slope Form: $y - y_1 = m(x - x_1)$

1. The general form is $y - y_1 = m(x - x_1)$.
2. The slope is 2 and the point is $(-1, 3)$.
3. Given a point (x_1, y_1) and a slope m, the point-slope form is $y - y_1 = m(x - x_1)$.

Graphing Using Point-Slope Form

1. The point $(4, 2)$ is plotted, and the line with slope $-\frac{1}{2}$ is drawn.
2. The rise is -1 and the run is 2 from the point $(4, 2)$.
3. If the slope is positive, the line would slant upwards to the right instead of downwards.

Converting Point-Slope to Slope-Intercept Form

1. $y - 1 = 3(x - 2)$:
$$y - 1 = 3x - 6$$
$$y = 3x - 5$$
2. The slope is $-\frac{2}{3}$ and the y-intercept is $4 - \left(-\frac{2}{3} \cdot (-3)\right) = 4 + 2 = 6$.
3. The slope remains unchanged because it directly represents the rate of change in both forms.