Introductory Sets Exercises

Tutoring Centre Ferndale

Questions

1. Define the following terms:

   (a) Set

   (b) Element

   (c) Subset

   (d) Universal set

   (e) Empty set

2. Use set notation to write the set of all even numbers less than 20.

3. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. List the elements of $A \cup B$.

4. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. List the elements of $A \cap B$.

5. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. List the elements of $A - B$.

6. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. List the elements of $B - A$.

7. Let $A = \{x \mid x$ is a prime number less than $10\}$. List the elements of A.

8. Let $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 2, 3\}$. Find A' (the complement of A).

9. Is $\{1, 2\} \subseteq \{1, 2, 3, 4\}$? Justify your answer.

10. Is $\{1, 2, 5\} \subseteq \{1, 2, 3, 4\}$? Justify your answer.

11. Draw a Venn diagram to represent the sets A and B where $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$.

12. Using the Venn diagram from the previous question, shade the region representing $A \cup B$.

13. Using the Venn diagram from question 11, shade the region representing $A \cap B$.

14. Using the Venn diagram from question 11, shade the region representing $A - B$.

15. Using the Venn diagram from question 11, shade the region representing $B - A$.

16. Explain the difference between a finite set and an infinite set, providing examples of each.

17. Prove that the empty set is a subset of every set.

18. If A and B are sets, prove that $A \cap B \subseteq A$.

19. If A and B are sets, prove that $A \cap B \subseteq B$.

20. Explain the significance of the universal set in set theory.

21. Given the sets $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4, 5\}$, find $A \cup B$.

22. Given the sets $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4, 5\}$, find $A \cap B$.

23. Given the sets $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4, 5\}$, find $A - B$.

24. Given the sets $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4, 5\}$, find $B - A$.

25. Let $A = \{x \mid x$ is a vowel in the English alphabet$\}$. List the elements of A.

26. Let $B = \{x \mid x$ is a consonant in the English alphabet$\}$. List the elements of B.

27. Determine if the following statement is true or false: $\{x \mid x$ is a letter in the word 'book'$\} = \{b, o, k\}$.

28. Determine if the following statement is true or false: $\{x \mid x$ is a digit in the number $2024\} = \{2, 0, 4\}$.

29. Let $A = \{x \mid x$ is a natural number less than $5\}$. List the elements of A.

30. Let $B = \{x \mid x$ is a natural number greater than 5 and less than $10\}$. List the elements of B.

31. If $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$, find $A \cup B$.

32. If $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$, find $A \cap B$.

33. If $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$, find $A - B$.

34. If $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$, find $B - A$.

35. Let $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$. Draw a Venn diagram and shade the region representing $A \cup B$.

36. Let $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$. Draw a Venn diagram and shade the region representing $A \cap B$.

37. If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$, and $B = \{1, 2, 3, 4\}$, find $(A \cup B)'$.

38. If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$, and $B = \{1, 2, 3, 4\}$, find $(A \cap B)'$.

Answers

1. Definitions:

    (a) Set: A collection of distinct objects, considered as an object in its own right.

    (b) Element: An object that is a member of a set.

    (c) Subset: A set whose elements are all contained within another set.

    (d) Universal set: The set that contains all the objects under consideration, usually denoted by U.

    (e) Empty set: A set with no elements, denoted by $\emptyset$ or $\{\}$.

2. $\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

3. $A \cup B = \{1, 2, 3, 4, 5, 6\}$

4. $A \cap B = \{3, 4\}$
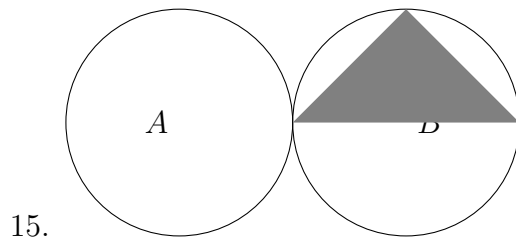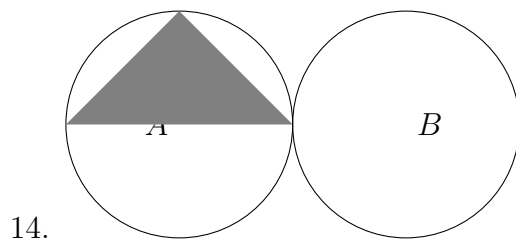
5. $A - B = \{1, 2\}$
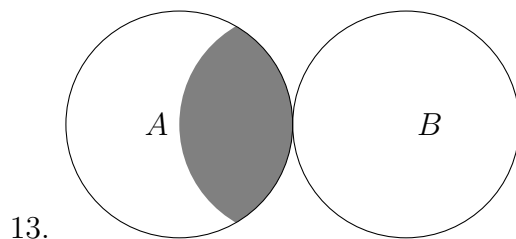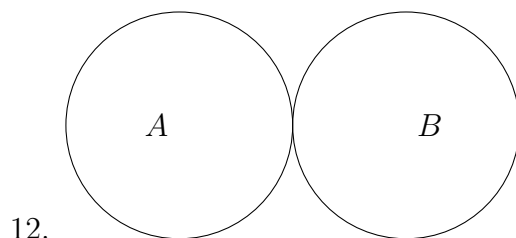
6. $B - A = \{5, 6\}$

7. $A = \{2, 3, 5, 7\}$

8. $A' = \{4, 5, 6\}$

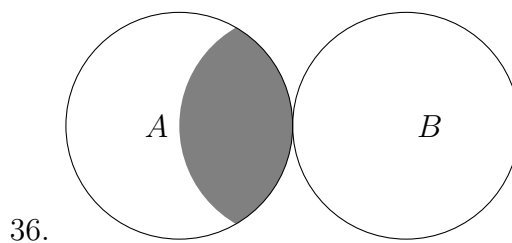9. Yes, because every element of $\{1, 2\}$ is also in $\{1, 2, 3, 4\}$.

10. No, because 5 is not in $\{1, 2, 3, 4\}$.
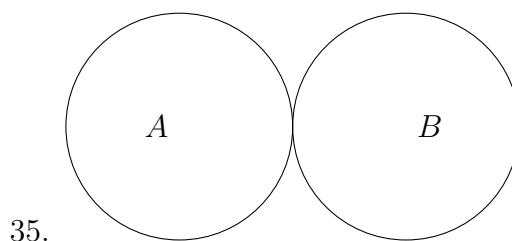
11.

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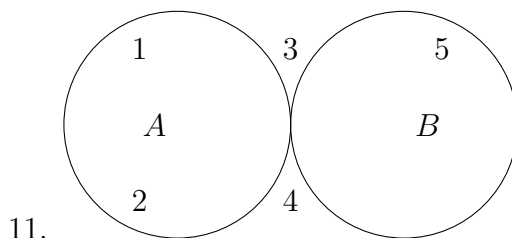
16. A finite set has a limited number of elements, e.g., $\{1, 2, 3\}$. An infinite set has unlimited elements, e.g., $\{1, 2, 3, \ldots\}$.

17. Proof: Let A be any set. The empty set $\emptyset$ has no elements. For any element $x \in \emptyset$, $x \in A$ is vacuously true. Hence, $\emptyset \subseteq A$.

18. Proof: If $x \in A \cap B$, then $x \in A$ and $x \in B$. Therefore, $x \in A$, so $A \cap B \subseteq A$.

19. Proof: If $x \in A \cap B$, then $x \in A$ and $x \in B$. Therefore, $x \in B$, so $A \cap B \subseteq B$.

20. The universal set U encompasses all objects under consideration, providing a context for defining complements and other set operations.

21. $A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

22. $A \cap B = \{2, 4\}$

23. $A - B = \{6, 8\}$

24. $B - A = \{1, 3, 5\}$

25. $A = \{a, e, i, o, u\}$

26. $B = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

27. True, because the set of letters in 'book' is $\{b, o, k\}$.

28. True, because the set of digits in 2024 is $\{2, 0, 4\}$.

29. $A = \{1, 2, 3, 4\}$

30. $B = \{6, 7, 8, 9\}$

31. $A \cup B = \{1, 2, 3, 5\}$

32. $A \cap B = \emptyset$

33. $A - B = \{1, 3, 5\}$

34. $B - A = \{2, 4, 6\}$

35.

36.

37. $(A \cup B)' = \{7, 8\}$

38. $(A \cap B)' = \{1, 3, 5, 7, 8\}$