Linear Equations:
Lengths, Midpoints, Perpendicular Lines, & Intersection Points

Tutoring Centre Ferndale

Finding the Length of a Line Segment

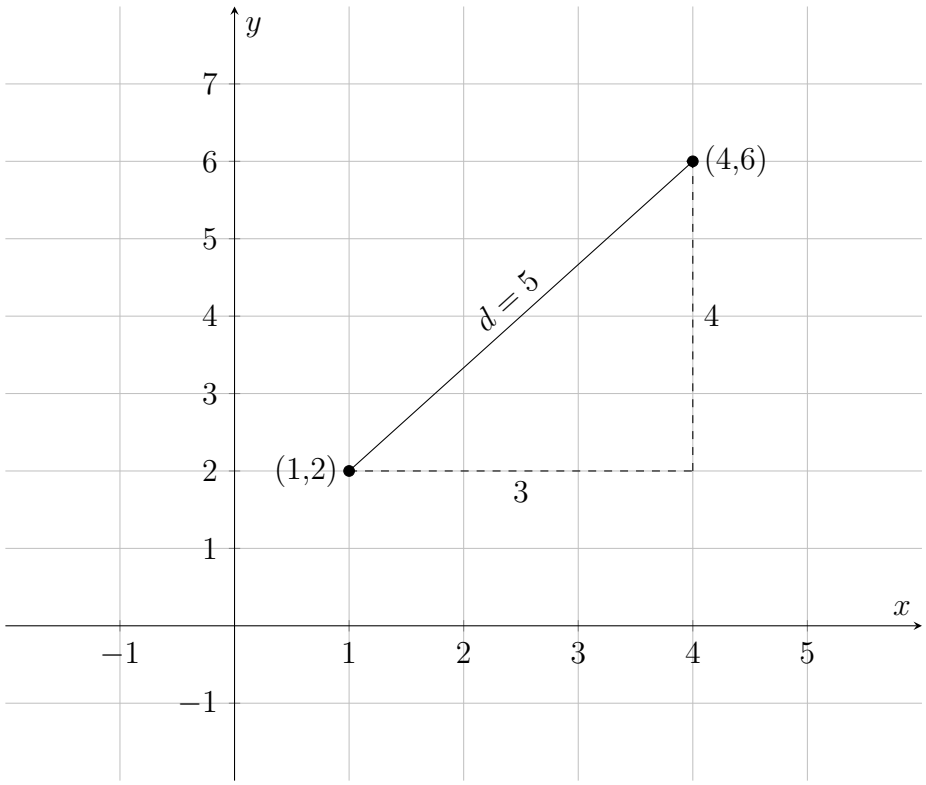
Using Pythagoras' formula for the lengths of the hypotenuse of a right triangle, the length of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

Find the length of the line segment with endpoints $(1, 2)$ and $(4, 6)$.

$$d = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$



Finding the Midpoint of a Line Segment

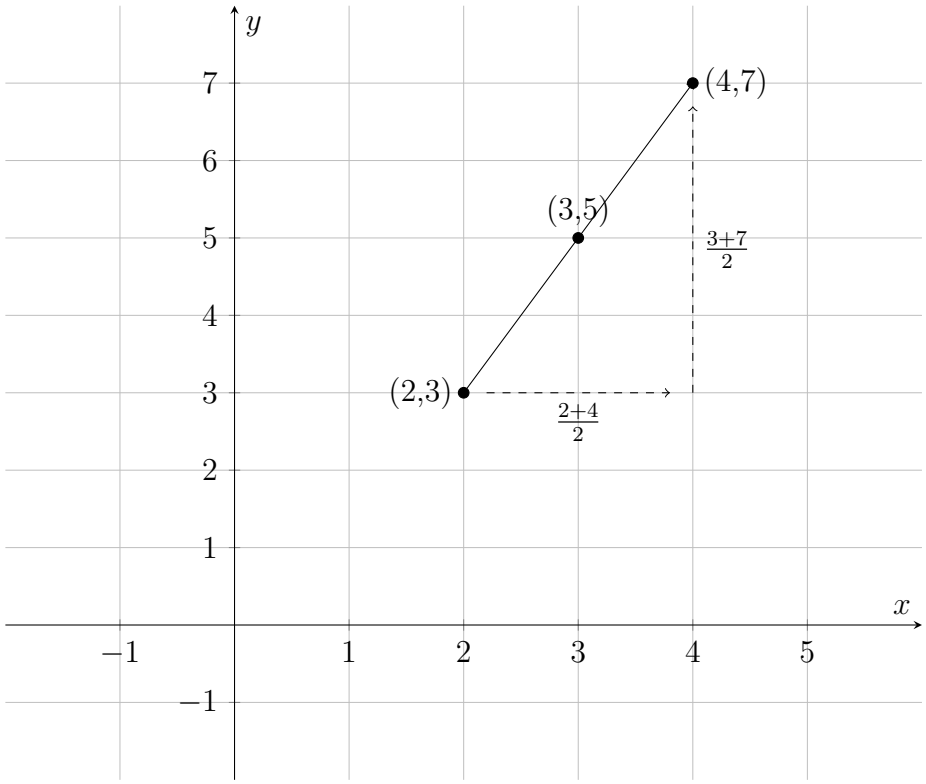
The midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example

Find the midpoint of the line segment with endpoints $(2, 3)$ and $(4, 7)$.

$$\left(\frac{2+4}{2}, \frac{3+7}{2}\right) = (3, 5)$$



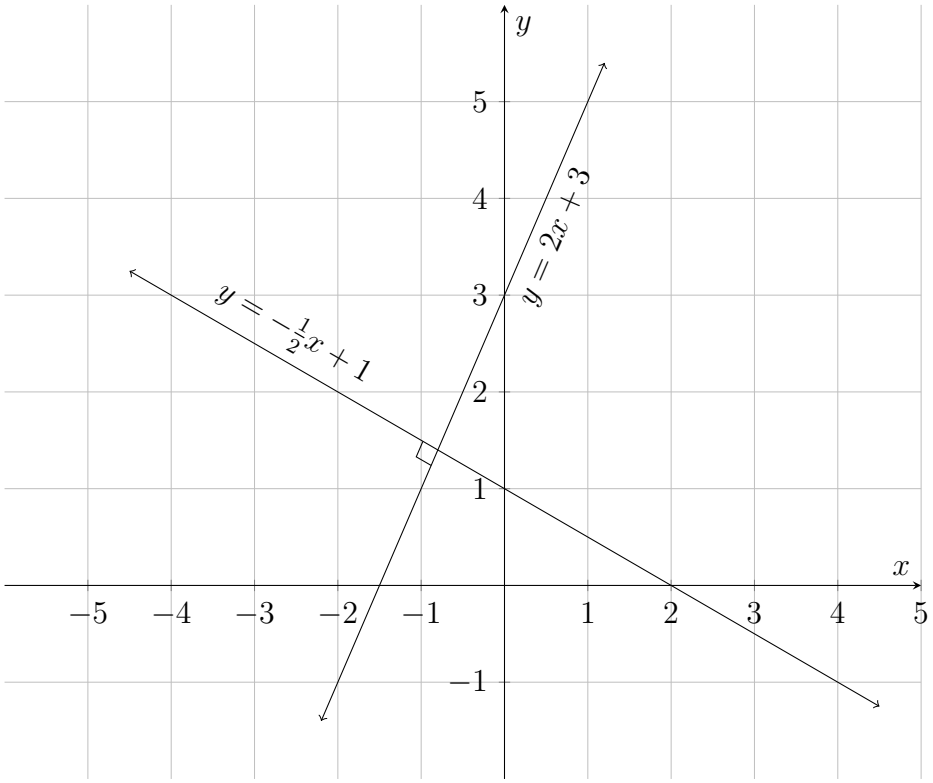
Equations of Perpendicular Lines

Perpendicular means at a right angles with the horizon, or more generally it means any line that forms a right angle with another line. (In diagrams, this is denoted by a small square in the right angle.)

In algebra, two lines are perpendicular if the product of their slopes is -1. If you have the equation of a line in slope-intercept form $y = mx + b$, the slope of a line perpendicular to it will be $-\frac{1}{m}$.

Example

Given the line $y = 2x + 3$, the slope is 2. Therefore, the slope of a line perpendicular to it is $-\frac{1}{2}$.



Finding the Intersection Points of Two Lines

To find the intersection point of two lines given by their equations, set the equations equal to each other and solve for x and y.

Example

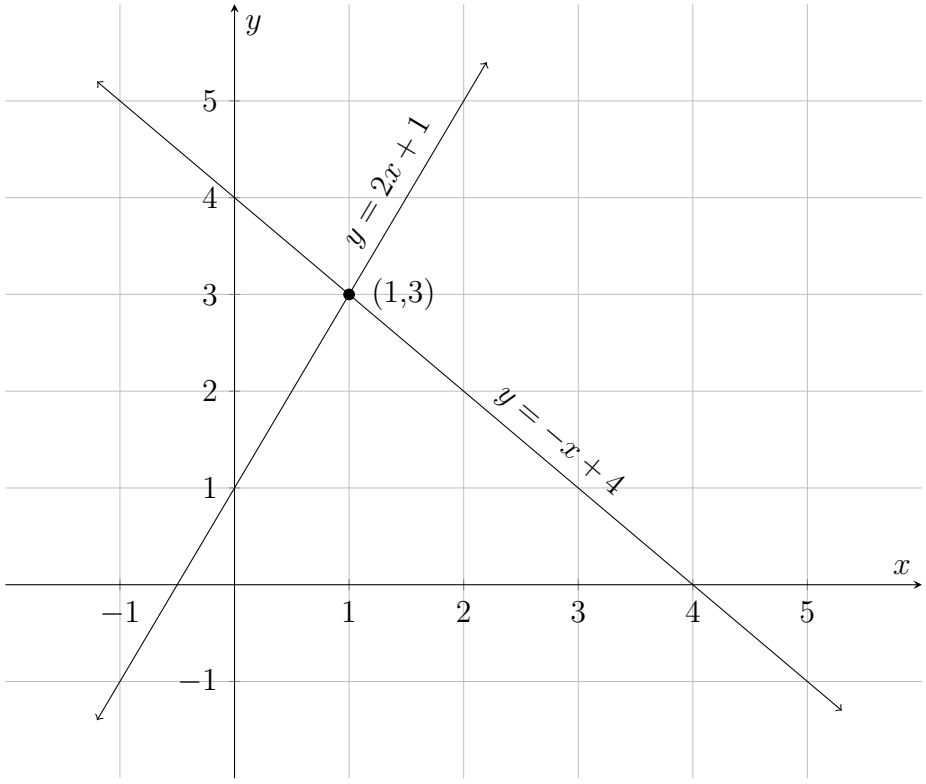
Find the intersection of the lines $y = 2x + 1$ and $y = -x + 4$.

$$2x + 1 = -x + 4$$

$$3x = 3 \implies x = 1$$

$$y = 2 \cdot 1 + 1 = 3$$

The intersection point is $(1, 3)$.



Exercises

Finding the Length of a Line Segment

1. Find the distance between the points $(1, 2)$ and $(4, 6)$.
2. Calculate the distance between the coordinates $(-3, -4)$ and $(3, 2)$.
3. Determine the distance between the points $(0, 0)$ and $(5, 12)$.

Finding the Midpoint of a Line Segment

1. Find the midpoint of the line segment with endpoints $(1, 2)$ and $(3, 6)$.
2. Now plot those points on a set of axes, draw a line between them, meaure it, and mark the midpoint.

   Do these coordinates agree with the coordinates that you found algebraically?
3. Find the midpoint of the line segment with endpoints $(0, 0)$ and $(4, 8)$.
4. Plot these three points and check that the midpoint does in fact look like the midpoint.

Equations of Perpendicular Lines

1. Find the equation of a line perpendicular to $y = 3x - 2$.
2. Plot these two lines to check that they are in fact perpendicular.
3. What is the slope of a line perpendicular to the line $y = -\frac{1}{4}x + 5$?

Finding the Intersection Points of Two Lines

1. Find the intersection point of the lines $y = x + 2$ and $y = -x + 4$.
2. Plot these two lines and mark their point of intersection. Does this agree with your alebraic solution?
3. Find the intersection of the lines $y = 2x - 1$ and $y = -\frac{1}{2}x + 3$.
4. Plot these two lines and mark their point of intersection. Does this agree with your alebraic solution?

Answers

Finding the Length of a Line Segment

1. The distance between the points $(1, 2)$ and $(4, 6)$ is:

$$d = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

2. The distance between the points $(-3, -4)$ and $(3, 2)$ is:

$$d = \sqrt{(3-(-3))^2 + (2-(-4))^2} = \sqrt{6^2 + 6^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$$

3. The distance between the points $(0, 0)$ and $(5, 12)$ is:

$$d = \sqrt{(5-0)^2 + (12-0)^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

Finding the Midpoint of a Line Segment

1. $(2, 4)$
2. $(2, 4)$

Equations of Perpendicular Lines

1. $y = -\frac{1}{3}x + c$
2. The slope is 4.

Finding the Intersection Points of Two Lines

1. $(1, 3)$
2. $(1.6, 2.2)$