Numbers and Counting Course

Applied Scholastics, Ferndale

Mathematics and Arithmetic

Mathematics   Mathematics is all the ways of handling any sort of numbers or shapes or patterns, and how to solve problems with them. It is a Greek word that originally meant all learning. Maths helps us to understand the world so that we can solve problems in the best possible way. It is a big subject that covers lots of different topics.

Usually mathematics is just called 'maths' for short (or 'math' in the US.)

Arithmetic   Arithmetic is all the ways of handling things that can be counted or numbered. It is a Greek word and it means methods of counting. Arithmetic is one part of the overall subject of Maths. Learning arithmetic well opens the gates to understanding all of mathematics.

1. Write, in your own words, what is mathematics.

2. Write a sentence that use the word 'maths.'

3. Write, in your own words, what is arithmetic.

4. Write a sentence that use the word 'arithmetic.'

5. What is the difference between maths and arithmetic?

6. Why might it be useful to learn arithmetic?

   Symbol A symbol is a thing that has been given a meaning. A symbol stands for or represents something.

7. Write, in your own words, what is a symbol.

8. Write 5 symbols and their meanings.

   Number A number is the idea of an amount or quantity of something. It can also mean the symbol or symbols used to represent an amount or quantity.

9. Write, in your own words, what is a number.

10. Write a sentence using the word 'number.'

    Numeral A numeral is a word or words, or a symbol or group of symbols, used to represent a number. "Four" and "4", "Twenty seven" and "27", are numerals.

11. Write, in your own words, what is a numeral.

12. Write a sentence that use the word 'numeral.'

13. What is the difference between a number and a numeral?

    Digit A digit is a finger. Because people count on their fingers, the numerals in a number (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) are also called digits. A digit is any single symbol that is a numeral in itself or that can stand with other digits as part of a larger numeral. "2" is a digit, and "23" is a numeral made up of two digits.

14. Write, in your own words, what is a digit.

15. Write a sentence using the word 'digit.'

16. What is the difference between a digit and a number?

Figure   'Figure' is another word for 'digit.' That's because a figure is a shape or outline of something, and digits (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) are all different shapes. A person's outline is known as their figure, for example, or diagrams in books are sometimes called figures. 'Figures' usually means numbers and numerals, not just single digits. Someone might want to look at "the figures," meaning the numbers involved, or that they will "figure it out," meaning they will solve some problem by working with the numbers involved, or that they will just generally think about how to solve some problem.

17. Write, in your own words, what are figures.

18. Write a sentence using the word 'figure.'

Count   Count means to say the names of the numbers one after the other. It is also to note each thing in a group of things while doing this to find out how many there are.

19. Write, in your own words, what is counting.

20. Write a sentence using the word 'count.'

21. What are 5 things that could be counted?

Numbering   To number things means to put a number on each thing as it is counted. The chapters of a book are numbered chapter 1, chapter 2, chapter 3, and so on. To number things can also mean to give a group of things the result of a count. You could say that the people of a town number 1000 or that the number of students in the class is 20.

22. Write, in your own words, what it means to number something.

23. Write 5 places where things have been numbered.

Natural Numbers Natural means things that are in the natural world, which is to say the real or physical world. Natural numbers are the numbers that are used for counting things, as in "I have six ducks," or for ordering things, as in "the third duck quacked."

Natural numbers do not include zero.

24. What does 'natural' mean?

25. In your own words, what is a natural number?

26. Write 5 examples of natural numbers.

Whole Numbers Whole means the entire thing and not just a part. Whole numbers count whole things.

They are natural numbers, but whole numbers do include zero.

27. What does 'whole' mean?

28. In your own words, what are whole numbers?

29. What is the difference between whole numbers and natural numbers?

Cardinal (Counting) Numbers Cardinal means main or most important. The four main directions (north, south, east and west) are called the cardinal directions, for example.

30. In your own words, what does 'cardinal' mean? Use it in a sentence.

Natural numbers or whole numbers, when they are used for counting things, are called cardinal numbers, or just counting numbers.

31. In your own words, what are cardinal numbers?

32. Why do you think they are called cardinal numbers?

33. In your own words, what are counting numbers?

34. Write 5 examples of cardinal or counting numbers.

Ordinal Numbers   Ordinal means having to do with what order or sequence that things are in.

35. What does it mean to arrange things in order?

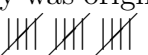36. In your own words, what does 'ordinal' mean?

    Natural numbers or whole numbers, when they give the order of things, are called ordinal numbers.

    Counting things as first, second, third, fourth, fifth, and so on, is ordinal numbering.

    Pointing to the "3rd" duck in a line of ducks uses an ordinal number.

37. In your own words, what are ordinal numbers?

38. Write 5 examples of ordinal numbers?

Tallies   Tallies are the simplest sort of numeral. For most of human history there were no other numbers. There were just a mark made for each things that was being counted. Tally is an old word for a stick, and a tally was originally a stick with notches cut into it. A tally looks like this: ||||| ||||| ||||| |||. Every fifth mark is drawn across the last four marks to make the number easier to see.

39. Write, in your own words, what is a tally.

40. Write 5 sentences using the word 'tally.'

Scores   A score is a scratch mark, and a score is also 20 of something. That comes from making a mark for every 20 of something being counted. Game 'scores' come from keeping a count this way. A famous American speech starts with "Four score and seven years ago..." meaning 87 years ago.

41. Write, in your own words, what is a score.

42. Write a sentence using the word 'score.'

Roman Numerals The Roman Empire ruled much of Europe, and large parts of Africa and Asia, for many centuries. They spread their language and their counting system with them. Roman numerals are still used today.

Roman numerals use I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, and M for 1000. The V is said to be a shorthand for a spread hand with 5 fingers and the X is said to be two Vs on top of each other.

XVIII in Roman numerals is shorter and easier to read and write than ‖‖ ‖‖ ‖‖ ||| as a tally.

A smaller amount is written either before or after the larger one, indicating that much more or less. That makes them even shorter to write. For example, XI means 11 because 11 is 1 more than 10, and IX means 9 because 9 is 1 less than 10.

Counting in Roman numerals is I II III IV V VI VII VIII IX X XI XII XIII XIV XV… and so on.

$$I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1000$$

43. What does 'Roman' mean?

44. Find Rome on a map.

45. What was the Roman Empire?

46. What are Roman numerals?

47. Are Roman numerals better than tallies? Why?

48. Have you seen Roman numerals before? Where?

49. Write the number of things on your table right now in Roman numerals.

50. Write from 1 to 20 using Roman numerals.

51. What is MMXXIV ?

Here are the Roman Numerals counting from 1 to 100:

1	I	2	II	3	III	4	IV	5	V
6	VI	7	VII	8	VIII	9	IX	10	X
11	XI	12	XII	13	XIII	14	XIV	15	XV
16	XVI	17	XVII	18	XVIII	19	XIX	20	XX
21	XXI	22	XXII	23	XXIII	24	XXIV	25	XXV
26	XXVI	27	XXVII	28	XXVIII	29	XXIX	30	XXX
31	XXXI	32	XXXII	33	XXXIII	34	XXXIV	35	XXXV
36	XXXVI	37	XXXVII	38	XXXVIII	39	XXXIX	40	XL
41	XLI	42	XLII	43	XLIII	44	XLIV	45	XLV
46	XLVI	47	XLVII	48	XLVIII	49	XLIX	50	L
51	LI	52	LII	53	LIII	54	LIV	55	LV
56	LVI	57	LVII	58	LVIII	59	LIX	60	LX
61	LXI	62	LXII	63	LXIII	64	LXIV	65	LXV
66	LXVI	67	LXVII	68	LXVIII	69	LXIX	70	LXX
71	LXXI	72	LXXII	73	LXXIII	74	LXXIV	75	LXXV
76	LXXVI	77	LXXVII	78	LXXVIII	79	LXXIX	80	LXXX
81	LXXXI	82	LXXXII	83	LXXXIII	84	LXXXIV	85	LXXXV
86	LXXXVI	87	LXXXVII	88	LXXXVIII	89	LXXXIX	90	XC
91	XCI	92	XCII	93	XCIII	94	XCIV	95	XCV
96	XCVI	97	XCVII	98	XCVIII	99	XCIX	100	C

Arabic Numerals The numerals that we use today, 0123456789, are very old. They are called Arabic numerals. They came to Europe though the Arab world and they are originally from India. They first appeared in Europe in the 10th century, over 1000 years ago, but they weren't in common use until around the 15th century.

0 1 2 3 4 5 6 7 8 9

52. In your own words, what is 'Arabic'?

53. Make a sentence using the word 'Arabic.'

54. In your own words, what is an Arabic numeral?

55. Write out the Arabic numerals.

The Place-Value System   Until Arabic numerals came to Europe, Roman numerals and systems of tallies and counting stones and so on were used.

The power of these new numbers was in the different way in which they were used.

A Roman number only ever stood for that one amount. A Roman V only ever meant five of something, even when it was part of a larger number containing other Roman numerals.

The amount that one of these Indian numerals stood for, though, when it was part of a larger number with more than one numeral, changed depending on it's position in that number. A '5' could mean five of something, or fifty, or five hundred, depending on it's position.

This is called the place-value system. The value of any digit in a number depends on it's place in that number. The idea is that a digit is read not just as itself but the value of each digit is some multiple more than the digit to it's right.
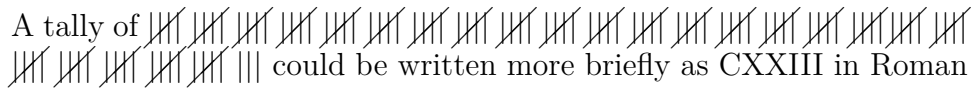
The Base   The number that is chosen to multiply each position by is known as as the base. Usually the base is 10, but any number can be used as a base. The base, if it isn't 10, is written as a small number under and to the right of the number.

123 with a base of 8, so each digit is 8 times the value of the digit to its right, is written as 123_8. It would mean $1 \times 8 \times 8 + 2 \times 8 + 3$. That is 83, which is $(8 \times 10) + 3$ when the same amount is written with a base of 10.

Decimal Base   We usually use the number 10 as the base, which is why most numbers that you see are called decimal numbers. Decimal means having to do with 10. In decimal numbers, the digit at the right of a number is just itself, but the digit to its left represents 10 times its value, and the next digit to the left is 100 times the value of that

digit, and so on.

123 is short for $(1 \times 10 \times 10) + (2 \times 10) + (3 \times 1)$.

A tally of ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||| could be written more briefly as CXXIII in Roman numerals, or simply as 123 in Hindu-Arabic numerals.

56. What does 'place-value' mean?

57. What does each of the digits in the number '234' mean?

58. What does 'base' mean in the place-value system?

59. What does 'decimal' mean?

60. Why are the numbers that we mostly use called decimal numbers?

61. How do you show what base you are using when writing a number that isn't a decimal?

Names of Large Numbers   Large numbers have some special names.

- A hundred is ten tens, written as 100.
- A thousand is ten hundreds, 1000.
- A million is a thousand thousand, 1000,000.
- A billion is a thousand million, 1000,000,000.
  (The 'bi-', meaning two, in 'billion' is because a billion once meant a million million.)
- A trillion is a thousand billion, 1000,000,000,000.
  (The 'tri-', meaning three, in 'trillion' is because a trillion once meant a million million million.)
- A googol is 1 followed 100 zeroes, and is where the internet company Google got its name.

Commas   To make decimal numbers easier to read, every third digit is separated by a comma. Thousands, Millions, Billions, and so on can be more easily read off when dealing with larger numbers that way.

Is the first digit of 1234567 millions? tens of millions?

Written as 1,234,567 it is easier to see that it is millions.

Large numbers are read from the left, in groups of three digits.

123,456,987,654,321 is read as 123 trillion, 456 billion, 987 million, 654 thousand, 3 hundred and 21.

62. How are commas used in writing large decimal number?

63. Write a decimal number with 8 digits, and then put commas every 3 places to make it easier to read.

64. Now read off your large number.

Fractions   A fraction is a part of a whole number that has been broken into equal-sized parts.

They are written as the number of parts and the number of parts that the whole number was broken into.

$\frac{3}{4}$ is a fraction where a whole number has been broken into 4 parts and we have 3 of those parts.

65. In your own words, what is a fraction?

66. Write 5 examples of fractions.

Decimal Fractions   Fractions can also be written as decimal numbers using the place-value system. Just as each digit is ten times the value of the digit to its right, each digit is also a tenth of the value of the digit to its left.

The Decimal Point   To write a decimal fraction, the decimal point is a full stop that marks where the whole number part of a number ends and where the fractional part starts.

Fractions written as decimal numbers are called decimal fractions. They are usually the answers given by calculators, and they are the answers that you get when doing division by hand.

Decimal numbers are sometimes just called decimals, and decimal fractions are also often just called decimals.

$$12.34 = 10 + 2 + \frac{3}{10} + \frac{4}{100}$$

For a number less than 1, the zero in the units position is still written to make it clear that the number is a fraction. You write 0.25, not just .25.

67. What is a decimal point?

68. In your own words, what are decimal fractions?

69. With a calculator, find $3 \div 4$ and write down what the digits of that answer mean.
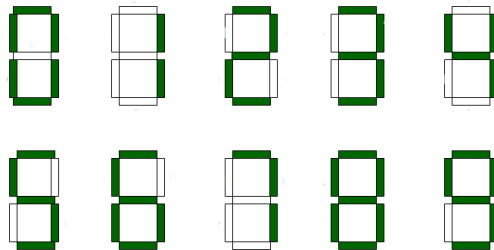
Binary Numbers   Computers use rows of switches that are either off or on to represent the ones and zeroes of numbers with a base of two. This is called binary, where the value of each digit is two times the vale of the digit to it's right.

1101 in binary is $(1 \times 2 \times 2 \times 2) + (1 \times 2 \times 2) + (0 \times 2) + (1 \times 1) = 8 + 4 + 0 + 1 = 13$ in decimal.

Doing arithmetic with binary numbers was worked out by mathematician George Boole in the 1800s. Binary numbers are still sometimes called Boolean numbers. The methods that Boole worked out for binary numbers are wired into all modern computers. The computer's work is all done in binary and then converted into decimal for human readers.

70. What does binary mean?

71. What does each position in a binary number mean?

72. What is the binary number 1101_2 when written as a decimal number?

Octal   Octal numbers, base 8, are also sometimes used with computers because each octal digit is equivalent to a group of three binary digits, which makes the electronics required simpler in running LED displays that are made up of 7 LEDs.

1101 binary (13 decimal) (or, placed into groups of 3 binary digits, 001 101) is 15 in octal (1 x 8 + 5 x 1.) Notice that 001 binary is 1 octal and 101 binary is 5 octal - the binary values that are seen in the octal digits.

73. What does octal mean?

74. What does each position in an octal number mean?

75. What is the octal number 123_8 when written as a decimal number?

Hexadecimal   You can also come across a base of sixteen used with computers, called hexadecimal, which uses the numbers 0 to 9 and then the letters A to F for 16 digits.

AF23 in hexadecimal means (A x 4096) + (F x 256) + (2 x 16) + 3 = (10 x 4096) + (16 x 256) + (2 x 16) + 3 = 40960 + 4096 + 32 + 3 = 45091 in decimal.

Hexadecimal is used for work with computers because each digit of a hexadecimal number is a four digit binary number.

For example, 1011 binary (11 decimal) is simply B in hexadecimal. Binary numbers in computers are typically 64 digits long so writing them in hexadecimal for human readers makes them much shorter and easier to deal with.

76. What does hexadecimal mean?

77. What does each position in a hexadecimal number mean?

78. What is the hexadecimal number $1C2_16$ when written as a decimal number?

Vigesimal You may remember that a score is a scratch mark, and that a score is also 20 of something, and that that comes from making a mark for every 20 of something being counted. That is known as the vigesimal system. Vigesimal means based on twenties. It can still be found, for example, in the way that the French count things. 20 is used as a base so that 70 is said as sixty-ten (soixante-dix, in French), and 80 is said as four twenties (quatre-vingts.)

79. What does vigesimal mean?

80. Write a sentence using the word 'vigesimal.'

Sexagesimal   The other number system that is commonly used around the world uses 60 as a base and is called the sexagesimal system. Sexigesimal means based on 60s. 60 is a very handy number to use because it divides evenly into so many other numbers – 30, 15, 20, 10, 5, 4, 3, and 2. It was first used by astronomers about three thousand years ago and it is why, among other things, there are 24 hours in a day, 60 minutes to an hour, sixty seconds to a minute, and 360 degrees in a circle.

81. What does sexagesimal mean?

82. What are some things that are based on the sexagesimal system?