Powers and Roots Course

Tutoring Centre Ferndale

Powers and Roots

Power   Power means size and strength.

1. What does power mean, in your own words?

2. use power in a sentence.

   Powers of Numbers   When a number is multiplied by itself over and over, you raise it to a higher power. It gets bigger and it is called a power of that number.

   An example is the powers of 3:

   The first power of 3 is just 3.

   The second power of 3 is $3 \times 3 = 9$.

   The third power of 3 is $3 \times 3 \times 3 = 27$.

   And so on. You get the idea?

3. What does the powers of a number mean, in your own words?

4. Write 3 examples of the powers of a number..

   Root   Root means the base of something or where things grow from, like the root of a tree.

5. What does root mean, in your own words?

6. Use root in a sentence that shows its meaning.

   Roots of Numbers   The number being multiplied is called the root, and the number of times it is multiplied by itself is called the power.

$$3 \times 3 \times 3$$
$$3 \times 3$$
$$3$$

7. What is meant by the root of a number, in your own words?

8. Write 3 examples of the root of a number.

Writing Powers

You write the power as a small raised number to the right of the root.

$$\text{root} \rightarrow 3\,{}^{5} \leftarrow \text{power}$$

It's written that way because it's shorter, rather than having to write out in full that number of multiplications, so 3^5 means $3 \times 3 \times 3 \times 3 \times 3$.

9. Write 5 examples of a power of a number being written this way

10. Label them with arrows pointing to the root and to the power.

Reading Powers

When you are reading a number that is written with a power, such as 3^5, there are a few different ways of saying it.

You can call it the fifth power of 3.

You can say 3 to the power of 5.

You can say 3 raised to the fifth power.

You can say 3 to the fifth power.

Or you can just say 3 to the fifth.

11. Read these powers out loud: $3^5, 2^2, 6^4, 8^5, 7^8$.

Index

Index means a pointer, like the index of a book points to where you can find things in it.

12. What is an index, in your own words?

13. Use index in a sentence.

Index of a Number   The power of a number can also be called the index because it points to what power the number is being raised to.

For example, the powers of 3 are $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81$ and $3^5 = 273$ so you could say that the 4th power of 3 is 81, with 4 being the pointer to that particular power.

$$\text{root} \rightarrow 3\,5 \;\leftarrow \text{index}$$

3

14. What is the index of a number, in your own words?

15. Use index in a sentence that show this meaning.

   Indices   If you are talking about more than one index you don't say indexes. The 2 and the 3 in $5^2 + 6^3$ are called indices, and you say it as *indisees.* Index is a Latin word so it follows the Latin language's rules for making plurals.

16. What are indices, in your own words?

17. Use indices in a sentence.

   Exponent   Exponent means a person or thing which explains or makes clear.

   It is from Latin ex, meaning out of, and ponere, meaning place, with the idea of laying something out to view it's parts.

   For example, "The speaker is an exponent of the need for clean energy."

18. What does exponent mean, in your own words?

19. Use exponent in a sentence.

   Exponent of a Number   In maths, most of the world calls it the index or the power, but exponent is the preferred word in the US.

$$\text{root} \rightarrow 3\,5 \ \leftarrow \text{exponent}$$

20. What is the exponent of a number, in your own words?

21. Use exponent in a sentence, showing this meaning.

The ^ Caret Sometimes when writing indices on a computer, because we are using a keyboard which only allows the use of certain characters, instead of the small raised number we use the ^ 'caret' symbol. 3^5 would be written as 3^5. The caret symbol is also called a circumflex or a hat.

22. Write 5 examples of a power of a number using a caret.

23. Label your examples with arrows pointing to the root and to the power.

24. When might you have to write a power of a number with a caret?

Squares

A square has sides of equal length.

25. What is a square, in your own words?

26. Use square in a sentence.

**Area** Area means the size of a flat surface.

27. What is area, in your own words?

28. Use area in a sentence.
To find the area of a square you multiply the length of the two sides.

29. What is the area of a square that has sides of 5 cm?
That's why multiplying a number by itself is called squaring the number.

You wouldn't call it 2 to the second power or anything. You would say the square of 2 is 4 or that 2 squared is 4.

30. How do you read "6^2"?

$$\overset{\displaystyle 2}{\boxed{4}}$$

2
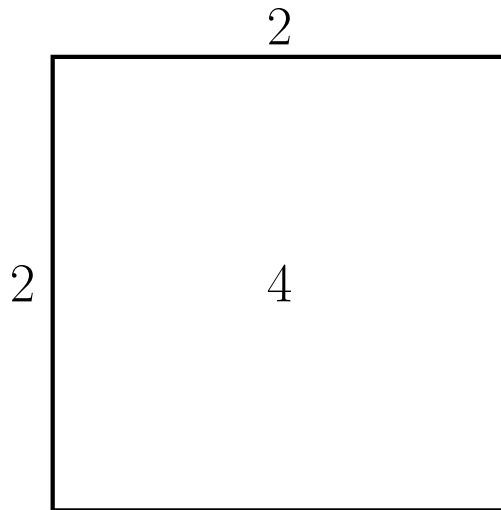
Area $= 2 \times 2 = 2^2 = 4$

Cubes

Volume   Volume means the size of something and how much space it takes up, not just a flat area.

31. What does volume mean, in your own words?

32. Use volume in a sentence.

    A cube has 3 sides all of the same length.

33. What is a cube, in your own words?

34. Use cube in a sentence.

    To find the volume of a cube, how big it is, you multiply the length of its side by itself 3 times.

35. What is the volume of a cube that has 5cm sides?

    That's why a number that has been multiplied by itself 3 times is called the cube of that number. Rather than saying "2 to the power of 3 is 8," you would say the cube of 2 is 8, or 2 cubed is 8.

36. How do you read "6^3"?
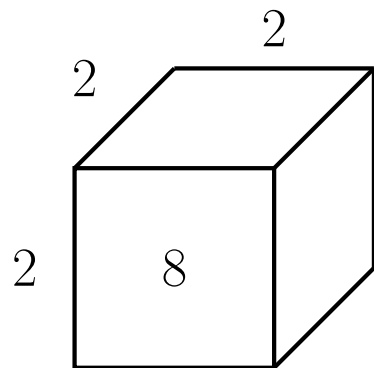
Volume $= 2 \times 2 \times 2 = 2^3 = 8$

Roots

Sometimes you only have a power of a number and you want to find out the root that it started with. Just like you can find the power of a number, you can also find the root.

This is called 'taking the root.'

If you start with a root of 3 and multiply it by itself 3 times, you get $3 \times 3 \times 3 = 27$, the third power of 3. The number that you have to multiply by itself 3 times to get 27, is 3, so 3 is the third root of 27.

Say you are asked to find the fourth root 256. That means you need to find what number, when multiplied by itself 4 times, gives 256.

$$4^1 = 4$$
$$4^2 = 4 \times 4 = 16$$
$$4^3 = 4 \times 4 \times 4 = 64$$
$$4^4 = 4 \times 4 \times 4 \times 4 = 256$$

As you can see, 256 is the fourth power of 4, so 4 is the fourth root of 256.

Here is a list of the first powers of 3 :

$$3^1 = 3$$

The first power of 3 is just 3, so the first root of 3 is also just 3.

$$3^2 = 3 \times 3 = 9$$

3 raised to the second power is 9, so the second root of 9 is 3.

$$3^3 = 3 \times 3 \times 3 = 27$$

3 raised to the third power is 27, so third root of 27 is 3.

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

3 raised to the fourth power is 81, so the fourth root of 81 is 3.

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

3 raised to the fifth power is 243, so the fifth root of 243 is 3.

37. Write out the first 6 powers of 2.

38. What is the 6th root of 64?

39. What is the 4th root of 16?

40. What is the 5th root of 32?

Radicals

There is a special symbol $\sqrt{}$ used for working with the roots of numbers. The Latin word for root is 'radical' and it is where the radical or root symbol $\sqrt{}$ comes from, being a sort of stretched out letter r.

41. What does radical mean, in your own words?

42. Use radical in a sentence.

**Radicand**   The value inside the radical symbol, the number for which the root is being taken, is the radicand.

43. What does radicand mean, in your own words?

44. Use radicand in a sentence.

You write the radical $\sqrt{}$ symbol and the radicand with a line along the top of it. The number to the left of the radical symbol that indicates which root is to be taken is called the index or the order or the degree of the root.

$$\text{index} \longrightarrow \sqrt[5]{243} = 3 \longleftarrow \text{root}$$

$\sqrt[5]{243} = 3$ means that 3 is the number which multiplied by itself 5 times is 243. In other words, the 5$^{\text{th}}$ root of 243 is 3.

45. what does the $\sqrt{}$ root symbol mean?

46. What does $\sqrt[4]{2401} = 7$ mean?

47. Using the root symbol, write that the 4$^{\text{th}}$ root of 256 is 4.

$$\overset{2}{\boxed{\qquad \sqrt{4} = 2 \qquad}}$$
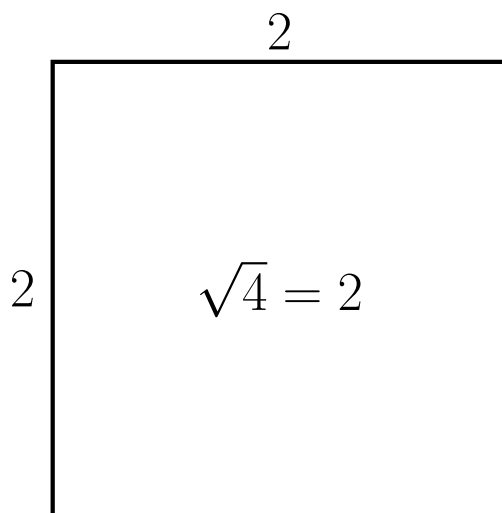
$\text{Area} = 2 \times 2 = 2^2 = 4$

Square Roots

The square of a number is the second power of the number, so the second root of a number is the square root.

For a square with an area of 4, the length of it's sides must be 2. The square root of 4 is 2.

If you see the $\sqrt{}$ symbol written without an index then its a square root. $\sqrt[2]{16} = 4$ is just written as $\sqrt{16} = 4$. It's the most commonly used root so the 2 isn't written unless there is some special reason.

48. In your own words, what is a square root?

49. Why is the second root called the square root?

50. Why isn't the 2 written next to the root symbol of a square root?

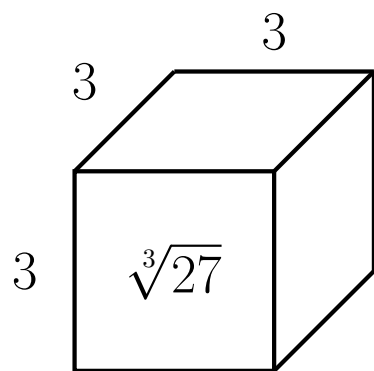51. What does $\sqrt{4} = 2$ mean?

Cube Roots

$$27 = 3^3$$
$$9 = 3^2$$
$$3 = 3^1$$

The cube of a number is its third power, and its third root is called the cube root.

A cube with a volume of 27 must have sides with lengths of 3, so the cube root of 27 is 3.

Volume $= 3 \times 3 \times 3 = 3^3 = 27$

You would write $\sqrt[3]{27} = 3$.

52. What is a cube root?

53. Why is the third root called a cube root?

54. Write that the cube root of 27 is 3.

Practice

1. What is 2 to the power of 4?

2. What is the square root of 64?

3. What is the cube root of 27?

4. What is 5 to the power of 3?

5. What is the fourth power of 2?

6. What is 3 raised to the fourth power?

7. What is the fourth root of 81?

8. What is 4 cubed?

Answers

1. $2^4 = 2 \times 2 \times 2 \times 2 = 16$

2. $\sqrt{64} = 8$

3. $\sqrt[3]{27} = 3$

4. $5^3 = 5 \times 5 \times 5 = 125$

5. $2^4 = 2 \times 2 \times 2 \times 2 = 16$

6. $3^4 = 3 \times 3 \times 3 \times 3 = 81$

7. $3^4 = 81$ so $\sqrt[4]{81} = 3$

8. $4^3 = 4 \times 4 \times 4 = 64$

More Practice

Powers of Numbers

(a) Define what a power is in mathematics.

(b) Explain why we use the notation 3^5 instead of writing out $3 \times 3 \times 3 \times 3 \times 3$.

(c) Calculate the value of 2^3.

(d) Express 5^2 in words.

(e) If x is raised to the power of 4, how would you write it?

(f) What is the result of 10^3?

(g) Express the fourth power of 6 in mathematical notation.

(h) What is the fifth power of 2.

(i) Can you find the eighth power of 10?

(j) Read the following powers out loud: 4^3, 6^2, 9^4, 2^5, 10^8.

(k) Explain the different ways you can verbally express a number with an index (e.g., "3 to the power of 5").

(l) Calculate the value of 7^2 and read it aloud using one of the verbal expressions.

Roots of Numbers

(a) What is the relationship between the root and the power of a number.

(b) Use the word "root" in a sentence related to mathematics.

(c) Calculate the roots of the following numbers: $\sqrt{16}$, $\sqrt{25}$, $\sqrt[3]{64}$.

(d) Write the fourth root of 81.

(e) Find the third root of 125.

Index

(a) Define what an index is in mathematics.

(b) Use the word "index" in a sentence.

(c) Explain how the index relates to the power of a number.

(d) Calculate the fourth power of 3. What is the index?

(e) What is the index in 2^{12}?

(f) Describe what "indices" are in mathematics.

(g) Use "indices" in a sentence.

(h) Explain why we use "indices" and not "indexes."

(i) Find the indices in the expression $2^7 - 3^5$.

Exponent

(a) What is an "exponent" in mathematics.

(b) Use "exponent" in a sentence.

(c) What is the origin of the word "exponent."

(d) How is an "exponent" related to "power" and "index"?

(e) What is the exponent in 10^6.

The Caret Symbol

(a) Explain when and why we might use the caret symbol (ˆ) in mathematical expressions.

(b) Write five examples of powers of numbers using the caret symbol.

Squares

(a) Define what a square is.

(b) Define "area."

(c) Calculate the area of a square with sides of length 6 cm.

(d) Explain why multiplying a number by itself is "squaring."

(e) Give an example of a real-life situation where understanding squares would be important.

Cubes

(a) Define what a cube is.

(b) Define "volume."

(c) Calculate the volume of a cube with side length 4 inches.

(d) Explain why raising a number to the third power is called "cubing."

(e) Provide an example of a real-life object that can be represented as a cube.

Roots

(a) How might you go about finding the root of a number?

(b) Find the third root of 8.

(c) How do roots and powers relate to each other?

Radical

(a) Define "radical."

(b) Explain the origin of the word "radical" and its connection to roots.

(c) Using the radical symbol, write the cube root of 64.

Radicand

(a) What is a "radicand."

(b) Write the the cube root of 27 and label the radicand.

Square Roots

(a) Explain the concept of a square root.

(b) Why is the second root called the "square root"?

(c) What is the square root of 81.

Cube Roots

(a) Describe what a cube root is.

(b) Why is the third root referred to as the "cube root"?

(c) What is the cube root of 64.

(d) What could be a real-world example where understanding cube roots is valuable.