The Equation of a Circle

Tutoring Centre Ferndale

What is the Equation of a Circle?

The equation of a circle is a mathematical expression that describes all the points that are a fixed distance (the radius) from a central point (the center). The standard form of the equation of a circle is derived from the Pythagorean theorem.

Key Terms

- **Circle:** A set of all points in a plane that are a fixed distance (the radius) from a central point (the center).
- **Center:** The fixed point from which all points on the circle are equidistant, denoted as (h, k).
- **Radius:** The fixed distance from the center to any point on the circle, denoted as r.

Equation of a Circle

The standard form of the equation of a circle with center at (h, k) and radius r is:

$$\boldsymbol{(x-h)^2+(y-k)^2=r^2}$$

The Pythagorean Theorem

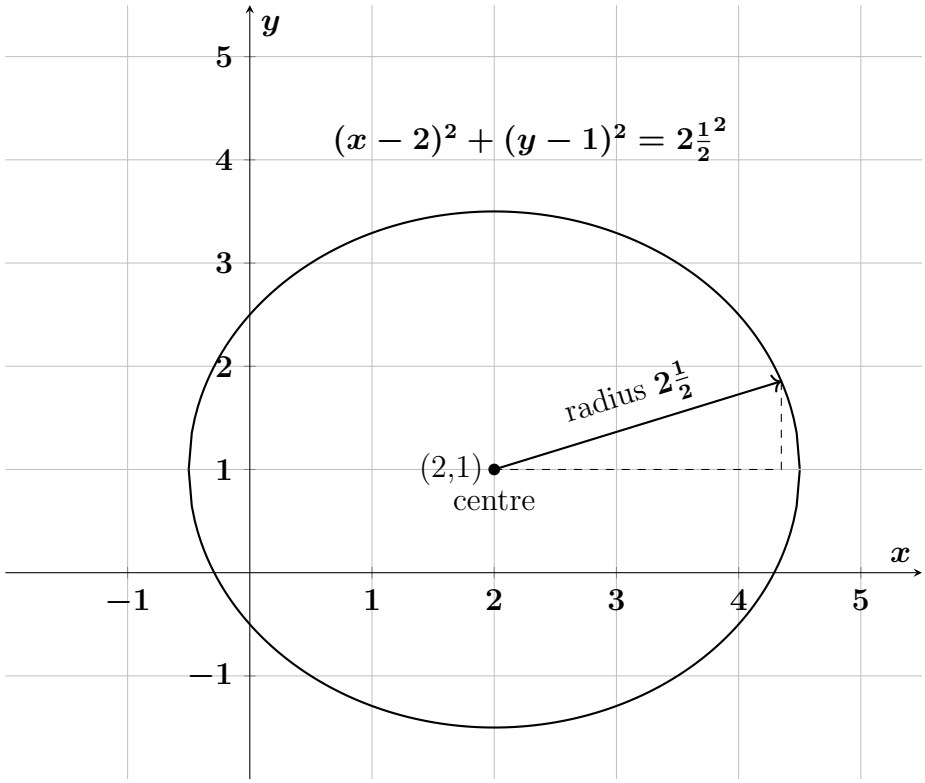
The equation of a circle is derived from the Pythagorean theorem. In a right triangle with legs parallel to the coordinate axes, the hypotenuse is the distance between a point (x, y) on the circle and the center (h, k). Thus, the distance formula:

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Squaring both sides gives the standard form of the equation of a circle.

Effects of Each Variable

- **Center (h, k):** Changing h or k moves the circle horizontally or vertically, respectively.
- **Radius r:** Changing r changes the size of the circle. A larger r makes the circle larger, and a smaller r makes the circle smaller.



Examples

Example 1

Find the equation of a circle with center at $(3, -2)$ and radius 5.

Solution:
Using the standard form:

$$(x-h)^2 + (y-k)^2 = r^2$$

Substitute $h = 3$, $k = -2$, and $r = 5$:

$$(x-3)^2 + (y+2)^2 = 5^2$$

Simplify:

$$(x-3)^2 + (y+2)^2 = 25$$

Equation of the circle: $(x-3)^2 + (y+2)^2 = 25$

Example 2

Find the center and radius of the circle given by the equation $(x+1)^2 + (y-4)^2 = 16$.

Solution:
Compare with the standard form:

$$(x-h)^2 + (y-k)^2 = r^2$$

We have:

$$(x-(-1))^2 + (y-4)^2 = 4^2$$

Thus, the center is $(-1, 4)$ and the radius is 4.

Center: $(-1, 4)$
Radius: 4

Practice Problems

Solve the following problems related to the equation of a circle:

Problem 1

Find the equation of a circle with center at $(2, 3)$ and radius 7.

Solution:
Using the standard form:

$$(x - 2)^2 + (y - 3)^2 = 7^2$$
$$(x - 2)^2 + (y - 3)^2 = 49$$

Equation of the circle: $(x - 2)^2 + (y - 3)^2 = 49$

Problem 2

Find the center and radius of the circle given by the equation $x^2 + y^2 = 9$.

Solution:
Compare with the standard form:

$$(x - 0)^2 + (y - 0)^2 = 3^2$$

Thus, the center is $(0, 0)$ and the radius is 3.

Center: $(0, 0)$
Radius: 3

Conclusion

The equation of a circle is a crucial concept in geometry that helps us understand the properties and relationships of circles. By learning how to derive and manipulate this equation, students can solve a variety of geometric problems and deepen their understanding of mathematical relationships. Practice with these problems to strengthen your understanding of the equation of a circle and its applications!