Functions in Algebra

Tutoring Centre Ferndale

Equations

Equations describe the relationship between dependent variables and independent variables. Usually the equation is arranged so that y is the dependant variable, its value changing depending on the chosen value of x.

- It is not explicitly stated which is the independent variable and which is the dependent variable.
- For some equations there can be more than one possible value for the dependent variable.

Functions

A function is a rule that assigns exactly one output to each input.

- Functions leave no question about which are the dependent and independent variables, and avoid the confusion of more than one possible value for the dependent variable.
- If an equation gives more than one possible value of y for a given x then it is not considered to be a function.

Rules: The rule can be any mathematical statement or procedure.

A function is like a machine. You put a number in (the input), the machine does something to it (the rule), and a new number comes out (the output).

Notation:

The equation $y = 3x$ as a function would be $f(x) = 3x$.

- The notation $f(x)$ is read as "function of x" or "f of x" and denotes the value of the function f when given the input x. Here, y is the output and x is the input.
- The wording "is a function of" is sometimes used to mean that some quantity is determined by the value of some other variable, such as "distance travelled is a function of speed and time."
- Letters other than f can be used in defining more than one function.

Defining and Using Functions:

Once we define a function, we can use it over and over again.

For example, if $f(x) = x^2$, we can easily find $f(1)$, $f(2)$, $f(3)$, etc. This is widely used in computer programming.

Exercises

1. If $f(x) = x^2$, what is $f(3)$?
2. If $f(x) = x^2$, what is $f(-2)$?
3. If $f(x) = x^2$, what is $f(0)$?

Answers

1. $f(3) = 3^2 = 9$
2. $f(-2) = (-2)^2 = 4$
3. $f(0) = 0^2 = 0$

Combining Functions

More complex functions can be created by combining simpler ones.

For example:

- **Define the first function**: $f(x) = x^2$
  - This means $f(x)$ takes a number x and squares it.
- **Define the second function**: $g(x) = \sin(x)$
  - This means $g(x)$ takes a number x and finds its sine (a trigonometric function).
- **Combine the functions**: $h(x) = f(g(x))$
  - This means $h(x)$ takes a number x, applies $g(x)$ to it, and then applies f to the result.
  - So, first, we find $g(x) = \sin(x)$.
  - Then, we find $f(g(x))$ which is $f(\sin(x))$. Since $f(x) = x^2$, this becomes $(\sin(x))^2$.

Putting it all together, $h(x) = (\sin(x))^2$.

Domain and Range

Domain: The domain of a function $f(x)$ is the set of all possible input values (or x-values) that the function can accept. For example, if $f(x) = \sqrt{x}$, the domain is all non-negative real numbers because you can't take the square root of a negative number.

Range: The range of a function $f(x)$ is the set of all possible output values (or y-values) that the function can produce. For $f(x) = \sqrt{x}$, the range is all non-negative real numbers because the square root of a non-negative number is always non-negative.

Exercises

1. What is the domain of $f(x) = 3x + 1$?
2. What is the range of $f(x) = x^2$?

Answers

1. The domain of $f(x) = 3x + 1$ is all real numbers.
2. The range of $f(x) = x^2$ is all non-negative real numbers.

Examples of Non-Algebraic Functions

Trigonometric Functions

Trigonometric functions like sine (sin), cosine (cos), and tangent (tan) relate angles to ratios of side lengths in right-angled triangles.

- $f(\theta) = \sin(\theta)$
- $f(\theta) = \cos(\theta)$
- $f(\theta) = \tan(\theta)$

Exponential and Logarithmic Functions

These functions are widely used in various fields.

- Exponential function: $f(x) = e^x$
- Logarithmic function: $f(x) = \log(x)$

Piecewise Functions

These functions are defined by different expressions for different parts of their domain.

- $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

Discrete Functions

Functions that are defined for specific, separate values.

- A function that assigns grades based on scores: $f(\text{score}) = \text{grade}$

Real-World Applications

Functions can describe various real-world relationships.

- Distance traveled over time: $f(t) = \text{distance}$
- Population growth: $f(t) = \text{population}$
- Temperature changes: $f(t) = \text{temperature}$