Expanding & Simplifying Polynomials

Tutoring Centre Ferndale

Polynomials are algebraic expressions consisting of variables and coefficients, constructed using only addition, subtraction, multiplication, and non-negative integer exponents of variables. Expanding and simplifying polynomials are fundamental skills in algebra that are essential for solving equations, analyzing functions, and more.

Definitions

Polynomials

A **polynomial** in one variable x is an expression of the form:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_0$ are coefficients and n is a non-negative integer.

Degree of a Polynomial

The **degree** of a polynomial is the highest power of the variable in the expression.

Leading Coefficient

The **leading coefficient** is the coefficient of the term with the highest power.

Methods for Expanding Polynomials

The Distributive Property

The distributive property states that:

$$a(b + c) = ab + ac$$

This property is fundamental in expanding polynomials.

Example

Expand $3(x + 4)$.

$$3(x + 4) = 3 \cdot x + 3 \cdot 4 = 3x + 12$$

Example

Expand $2(3x - 5)$.

$$2(3x - 5) = 2 \cdot 3x + 2 \cdot (-5) = 6x - 10$$

FOIL Method

The FOIL method is used to expand the product of two binomials:

$$(a + b)(c + d) = ac + ad + bc + bd$$

FOIL stands for First, Outer, Inner, Last, which is the order in which the terms are multiplied.

Example

Expand $(x + 2)(x + 3)$ using FOIL.

$$(x + 2)(x + 3) = x \cdot x + x \cdot 3 + 2 \cdot x + 2 \cdot 3 = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

Simplifying Polynomials

Simplifying polynomials involves combining like terms and performing arithmetic operations to reduce the expression to its simplest form.

Combining Like Terms

Like terms are terms that have the same variable raised to the same power. To combine like terms, add or subtract their coefficients.

Example

Simplify $3x^2 + 5x - 2x^2 + 7x + 4$.

$$3x^2 - 2x^2 + 5x + 7x + 4 = (3 - 2)x^2 + (5 + 7)x + 4 = x^2 + 12x + 4$$

Factoring Polynomials

Factoring is the reverse process of expanding. It involves writing the polynomial as a product of its factors.

Example

Factor $x^2 + 5x + 6$.

Find two numbers that multiply to 6 and add to 5. These numbers are 2 and 3.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Special Polynomials

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Example

Expand $x^2 - 9$.

$$x^2 - 9 = (x - 3)(x + 3)$$

Perfect Square Trinomial

$$a^2 + 2ab + b^2 = (a + b)^2 a^2 - 2ab + b^2 = (a - b)^2$$

Example

Expand $(x + 4)^2$.

$$(x + 4)^2 = x^2 + 2 \cdot x \cdot 4 + 4^2 = x^2 + 8x + 16$$

Examples

Example

Expand and simplify $2(x+3)(x-2)$.

First, expand $(x+3)(x-2)$ using FOIL:

$$(x+3)(x-2) = x^2 - 2x + 3x - 6 = x^2 + x - 6$$

Now, multiply by 2:

$$2(x^2 + x - 6) = 2x^2 + 2x - 12$$

Example

Simplify $4x(x-5) + 3(x^2 - x + 2)$.

First, distribute:

$$4x(x-5) = 4x^2 - 20x3(x^2 - x + 2) = 3x^2 - 3x + 6$$

Combine like terms:

$$4x^2 - 20x + 3x^2 - 3x + 6 = (4x^2 + 3x^2) + (-20x - 3x) + 6 = 7x^2 - 23x + 6$$

Practice Questions

Question 1

Expand $5(x-4)$.

Answer: $5(x-4) = 5x - 20$

Question 2

Expand $(2x+3)(x-5)$ using the FOIL method.

Answer:
$(2x+3)(x-5) = 2x \cdot x + 2x \cdot (-5) + 3 \cdot x + 3 \cdot (-5) = 2x^2 - 10x + 3x - 15 = 2x^2 - 7x - 15$

Question 3

Simplify $6x^2 - 2x + 4x^2 + x - 7$.

Answer: $6x^2 + 4x^2 - 2x + x - 7 = 10x^2 - x - 7$

Question 4

Factor $x^2 - 16$.

Answer: $x^2 - 16 = (x-4)(x+4)$

Answers to Practice Questions

1. **Question 1 Answer:** $5x - 20$
2. **Question 2 Answer:** $2x^2 - 7x - 15$
3. **Question 3 Answer:** $10x^2 - x - 7$
4. **Question 4 Answer:** $(x-4)(x+4)$