Exponential Equations

Tutoring Centre Ferndale

An exponential equation is an equation in which a constant base is raised to a variable exponent. The general form of an exponential equation is:

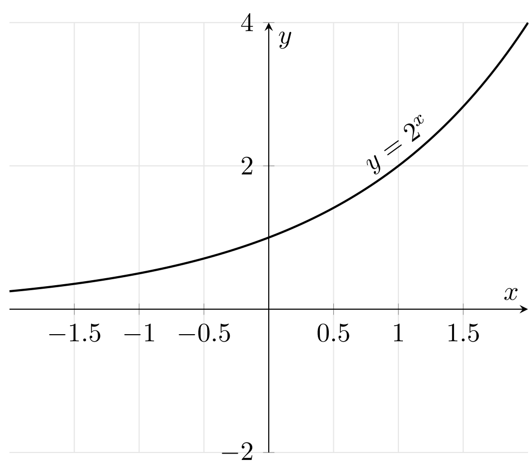
$$y = a \cdot b^x$$

where a is a constant, b is the base, and x is the exponent.

- If $b > 1$, the function is increasing.
- If $0 < b < 1$, the function is decreasing.
- The function never touches the x-axis: it approaches but never reaches zero.
- The y-intercept is at $y = a$ when $x = 0$.

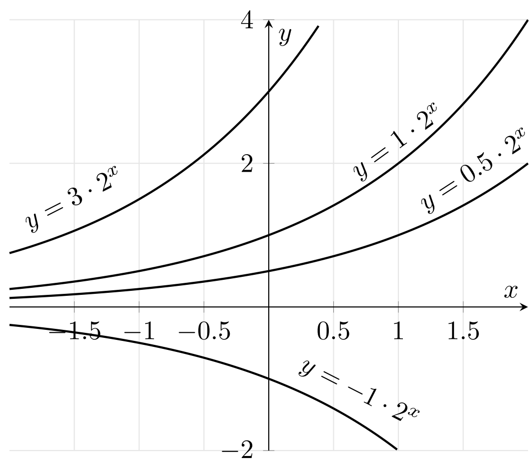
Graphing Exponential Equations

Let's plot the graph of $y = 2^x$:



Effects of Different Values of a

Here are the graphs of $y = a \cdot 2^x$ for different values of a.

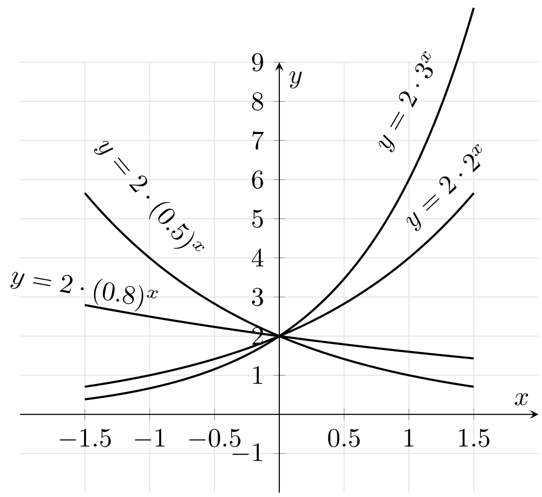


The value of a affects the vertical stretch or compression and the vertical translation of the graph:

- If $a > 1$, the graph stretches vertically.
- If $0 < a < 1$, the graph compresses vertically.
- If $a < 0$, the graph reflects across the x-axis and then stretches or compresses based on the magnitude of a.
- The y-intercept of the graph is at $y = a$ when $x = 0$.

Effects of Different Values of b

Here are the graphs of $y = 2 \cdot b^x$ for different values of b.



The value of b affects the rate of growth or decay of the graph:

- If $b > 1$, the function is an increasing exponential function, showing exponential growth.
- If $0 < b < 1$, the function is a decreasing exponential function, showing exponential decay.
- The base b determines how rapidly the function increases or decreases:
  - Larger values of $b > 1$ result in steeper growth.
  - Smaller values of $0 < b < 1$ result in slower decay.

Practical Uses

Exponential equations have practical applications, including:

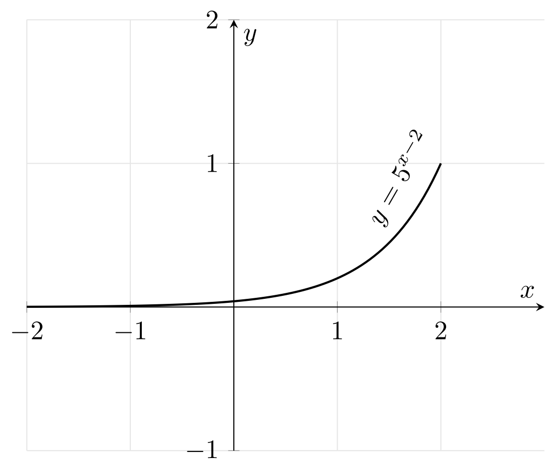
- **Population Growth**: The population of a species or community can be modeled using exponential functions.
- **Radioactive Decay**: The decay of radioactive substances follows an exponential pattern.
- **Interest Calculations**: Compound interest in finance is calculated using exponential functions.

Exercises

1. Solve for x: $2^x = 16$.
2. Graph the function $y = 5^{x-2}$.
3. If a population doubles every 3 years, express the population P as a function of time t (in years), given the initial population is P_0.
4. Solve for x: $3^{x+1} = 27$.

Answers

1. $2^x = 16$
   $16 = 2^4$
   Therefore, $x = 4$.
2. The graph of $y = 5^{x-2}$:



3. The population P as a function of time t can be expressed as:

$$P(t) = P_0 \cdot 2^{t/3}$$

4. $3^{x+1} = 27$
   $27 = 3^3$
   Therefore, $x + 1 = 3$
   $x = 2$.