Simultaneous Equations

Tutoring Centre Ferndale

Simultaneous equations are a set of equations that are solved together. The solution is the set of values that satisfy all equations simultaneously.

When two equations that represent two lines are graphed, for example, the solution to the simultaneous equations is the point where these two lines intersect. This point gives the values of the variables that make both equations true at the same time.

This same principle can be used with sets of any sorts of equations where the curves of their lines intersect. Sets of equations with more than two variables can be graphed as plane and their solution will be the line of intersection of the planes.

Adding or Subtracting Equations

When solving simultaneous equations, you can add or subtract one equation from another to eliminate one of the variables. This works because of a basic property of equality: if two things are equal, then adding or subtracting the same amount from both will keep them equal.

For example, if you have:

$$\begin{cases} x + y = 7 \\ x - y = 3 \end{cases}$$

You can add the two equations together to eliminate y:

$$(x + y) + (x - y) = 7 + 3$$

This simplifies to:

$$2x = 10 \implies x = 5$$

Now that you know x, you can substitute it back into one of the original equations to find y.

This method works because adding or subtracting the same values (the equations) maintains the balance, allowing us to isolate and solve for one variable at a time.

Multiplying or Dividing Equations

Sometimes, the coefficients of the variables in the equations are not suitable for direct addition or subtraction. In such cases, you can multiply or divide one or both equations by a number to make the coefficients of one of the variables match.

For example, if you have:

$$\begin{cases} 2x + 3y = 8 \\ 3x + 2y = 7 \end{cases}$$

You can multiply the first equation by 3 and the second equation by 2 to match the coefficients of x:

$$\begin{cases} 3(2x + 3y) = 3(8) \\ 2(3x + 2y) = 2(7) \end{cases}$$

This simplifies to:

$$\begin{cases} 6x + 9y = 24 \\ 6x + 4y = 14 \end{cases}$$

Now, you can subtract the second equation from the first to eliminate x:

$$(6x + 9y) - (6x + 4y) = 24 - 14$$

This simplifies to:

$$5y = 10 \implies y = 2$$

Now that you know y, you can substitute it back into one of the original equations to find x:

$$2x + 3(2) = 8 \implies 2x + 6 = 8 \implies 2x = 2 \implies x = 1$$

This method works because multiplying both sides of an equation by the same number maintains the equality, allowing us to manipulate the equations to make solving for the variables easier.

Methods for Solving Simultaneous Equations

There are several methods to solve simultaneous equations:

Substitution Method

1. Solve one equation for one variable.
2. Substitute this expression into the other equation.
3. Solve the resulting single-variable equation.
4. Substitute back to find the other variable.

Example

Solve the following system:

$$\begin{cases} x + y = 7 \\ 2x - y = 3 \end{cases}$$

- Solve the first equation for y: $y = 7 - x$.
- Substitute into the second equation: $2x - (7 - x) = 3$.
- Simplify and solve: $3x - 7 = 3 \Rightarrow 3x = 10 \Rightarrow x = \frac{10}{3}$.
- Substitute x back into $y = 7 - x$: $y = 7 - \frac{10}{3} = \frac{11}{3}$.
- Solution: $\left(\frac{10}{3}, \frac{11}{3}\right)$.

Elimination Method

1. Multiply equations to align coefficients of one variable.
2. Add or subtract equations to eliminate that variable.
3. Solve the resulting single-variable equation.
4. Substitute back to find the other variable.

Example

Solve the following system:

$$\begin{cases} 3x + 2y = 16 \\ 4x - 3y = 2 \end{cases}$$

- Multiply the first equation by 3 and the second by 2:

$$\begin{cases} 9x + 6y = 48 \\ 8x - 6y = 4 \end{cases}$$

- Add the equations: $17x = 52 \Rightarrow x = \frac{52}{17} = 3\frac{1}{17}$.
- Substitute x back into $3x + 2y = 16$:

$$3(\frac{52}{17}) + 2y = 16 \Rightarrow \frac{156}{17} + 2y = 16 \Rightarrow 2y = 16 - \frac{156}{17} = \frac{116}{17} \Rightarrow y = \frac{(\frac{116}{17})}{2} = \frac{232}{17} = 13\frac{11}{17}.$$

- Solution: $(3\frac{1}{17}, 13\frac{11}{17})$.

Practical Examples

Simultaneous equations are used in various fields such as economics, engineering, and science to model and solve real-world problems.

Example: Supply and Demand

Consider the following supply and demand equations:

$$\begin{cases} S = 3P + 10 \\ D = 50 - 2P \end{cases}$$

where S is the supply, D is the demand, and P is the price.

Find the equilibrium price and quantity (the price and quantity at which supply equals demand):

- At equilibrium, supply equals demand: $3P + 10 = 50 - 2P$.
- Solve for P: $5P = 40 \Rightarrow P = 8$.
- Substitute P back into S: $S = 3(8) + 10 = 34$.
- Solution: The equilibrium price is 8, and the equilibrium quantity is 34.

Exercises

Solve the following systems of equations:

Exercise 1

$$\begin{cases} 2x + y = 5 \\ 3x - y = 4 \end{cases}$$

Exercise 2

$$\begin{cases} x - 2y = 1 \\ 2x + 3y = 12 \end{cases}$$

Answers

Exercise 1

- Add the equations: $(2x + y) + (3x - y) = 5 + 4 \Rightarrow 5x = 9 \Rightarrow x = \frac{9}{5} = 1.8$.
- Substitute x back into $2x + y = 5$: $2(1.8) + y = 5 \Rightarrow 3.6 + y = 5 \Rightarrow y = 1.4$.
- Solution: $(1.8, 1.4)$.

Exercise 2

- Solve the first equation for x: $x = 1 + 2y$.
- Substitute into the second equation: $2(1 + 2y) + 3y = 12 \Rightarrow 2 + 4y + 3y = 12 \Rightarrow 7y = 10 \Rightarrow y = \frac{10}{7}$.
- Substitute y back into $x = 1 + 2y$: $x = 1 + 2\left(\frac{10}{7}\right) = 1 + \frac{20}{7} = \frac{27}{7}$.
- Solution: $\left(\frac{27}{7}, \frac{10}{7}\right)$.