Introduction to Combinations, Permutations & Probability Course

Tutoring Centre Ferndale

Permutations

A permutation is an arrangement of objects in a specific order. The number of permutations of n distinct objects is given by $n!$ (n factorial), which is the product of all positive integers up to n.

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

Example

How many ways can you arrange the letters A, B, and C?

Solution:
The number of permutations is:
$3! = 3 \times 2 \times 1 = 6$
The arrangements are: $ABC, ACB, BAC, BCA, CAB, CBA$.

Permutations of a Subset

The number of ways to arrange r objects from n distinct objects is given by:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Example

How many ways can you arrange 2 letters out of A, B, C?

Solution:

The number of permutations is:

$$P(3,2) = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{6}{1} = 6$$

The arrangements are: AB, AC, BA, BC, CA, CB.

Combinations

A combination is a selection of objects without regard to the order. The number of combinations of r objects from n distinct objects is given by:

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

($\binom{n}{r}$ is read as "n choose r.")

Example

How many ways can you choose 2 letters from A, B, C?

Solution:

The number of combinations is:

$$C(3,2) = \binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3!}{2! \cdot 1!} = \frac{6}{2 \cdot 1} = 3$$

The selections are: AB, AC, BC.

Probability

Probability is a measure of the likelihood of an event occurring. It is defined as the ratio of the number of favorable outcomes to the total number of possible outcomes.

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Example

What is the probability of rolling a 3 on a fair six-sided die?

Solution:

The number of favorable outcomes is 1 (rolling a 3), and the total number of possible outcomes is 6 (rolling any number from 1 to 6).

$$P(\text{rolling a 3}) = \frac{1}{6}$$

Probability of Multiple Events

For independent events, the probability of both events occurring is the product of their individual probabilities.

$$P(A \cap B) = P(A) \times P(B)$$

(The $\cap$ synbol used here means intersection, meaning where two different sets have elements in common.)

Example

What is the probability of rolling a 3 on a fair six-sided die and then flipping a heads on a fair coin?

Solution:

Probability of rolling a 3 is $\frac{1}{6}$ and probability of flipping heads is $\frac{1}{2}$.

$$P(\text{rolling a 3 and flipping heads}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

Examples

Example 1: Permutations

How many ways can 4 people be seated in a row?

Solution:

The number of permutations is:

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Example 2: Combinations

How many ways can you choose 3 out of 5 books to take on a trip?

Solution:

The number of combinations is:

$$C(5,3) = \binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3! \cdot 2!} = \frac{120}{6 \cdot 2} = 10$$

Example 3: Probability

What is the probability of drawing an ace from a standard deck of 52 cards?

Solution:

The number of favorable outcomes is 4 (the 4 aces), and the total number of possible outcomes is 52.

$$P(\text{drawing an ace}) = \frac{4}{52} = \frac{1}{13}$$

Practice

Problem 1: Permutations

How many ways can you arrange the letters in the word "MATH"?

Solution:

The number of permutations is:

$$4! = 24$$

Problem 2: Combinations

How many ways can you choose 2 cards from a standard deck of 52 cards?

Solution:

The number of combinations is:

$$C(52,2) = \binom{52}{2} = \frac{52!}{2!(52-2)!} = \frac{52!}{2! \cdot 50!} = \frac{52 \times 51}{2 \times 1} = 1326$$

Problem 3: Probability

What is the probability of drawing a red card from a standard deck of 52 cards?

Solution:

The number of favorable outcomes is 26 (the 13 hearts and 13 diamonds), and the total number of possible outcomes is 52.

$$P(\text{drawing a red card}) = \frac{26}{52} = \frac{1}{2}$$