

Understanding the Slope-Intercept Form of Linear Equations

Tutoring Centre Ferndale



In linear algebra, the slope-intercept form of a linear equation is a straightforward and intuitive way to represent a line.

Slope-Intercept Form: $y = mx + b$

In the slope-intercept form,

- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

Converting Standard Form to Slope-Intercept Form

To convert a linear equation from standard form $Ax + By = C$ to slope-intercept form $y = mx + b$:

$$Ax + By = C$$

$$By = -Ax + C$$

$$y = -\frac{A}{B}x + \frac{C}{B}$$

Here, the slope m is $-\frac{A}{B}$, and the y-intercept b is $\frac{C}{B}$.

Calculating the Slope m

The slope m is calculated using the change in x and the change in y between two points on the line.

The mathematical symbol for change is the Greek letter Δ delta, equivalent to the English letter D, standing for "difference."

$$\text{Thus, } m = \frac{\Delta x}{\Delta y}$$

These are also known as the rise and the run.

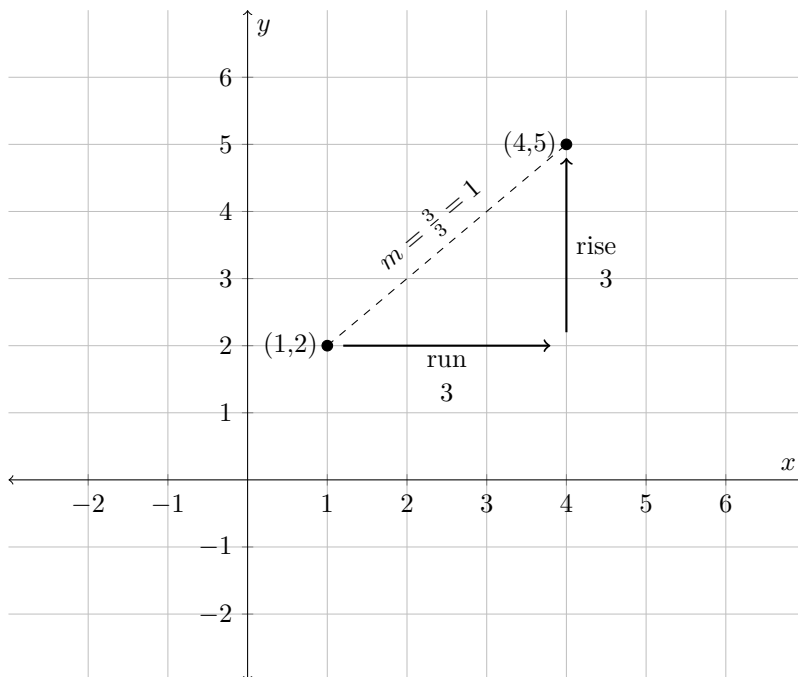
- The rise is the change in the y -coordinate.
- The run is the change in the x -coordinate.

$$m = \frac{\Delta x}{\Delta y} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

Consider two points $(1, 2)$ and $(4, 5)$. The slope m is calculated as follows:

$$m = \frac{5 - 2}{4 - 1} = \frac{3}{3} = 1$$



Effect of Changing the Slope m

- m determines the steepness and direction of the line.
- A positive m results in an upward-sloping line, while a negative m results in a downward-sloping line.
- The larger the absolute value of m , the steeper the line.

Absolute Value: The absolute value of a number is its distance from zero on the number line, regardless of direction. It is always a non-negative number. Any negative sign is stripped away, making the result positive or zero. The absolute value of a number x is denoted by $|x|$.

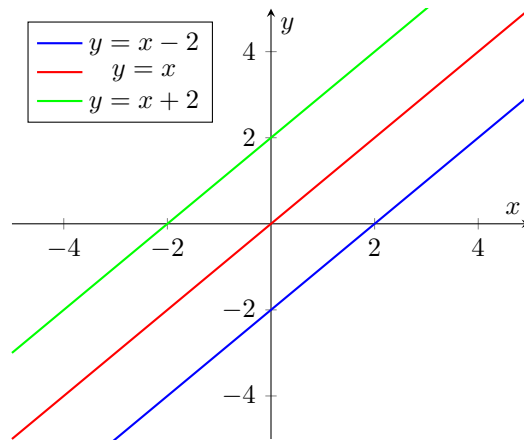
Example

The graph shows three lines with different slopes: $y = x + 1$, $y = 2x + 1$, and $y = -x + 1$. The slope m affects the steepness and direction of the lines.

Effect of Changing the y-intercept b

- b determines the point where the line crosses the y-axis.
- Changing b shifts the line up or down without altering its slope.

Example



The graph shows three lines with different y-intercepts: $y = x - 2$, $y = x$, and $y = x + 2$. The y-intercept b affects the vertical position of the lines.

Plotting the Graph Using Slope and y -intercept

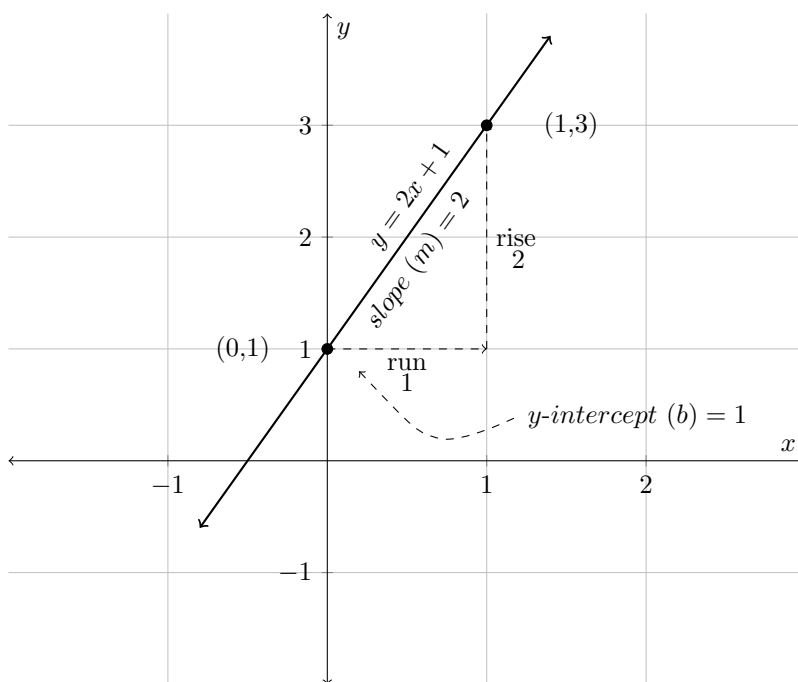
To plot a line using the slope-intercept form:

- Start at the y -intercept $(0, b)$.
- Use the slope m to determine the rise and run from the y -intercept.
- Plot additional points using the rise and run and draw the line through these points.

Example

For the equation $y = 2x + 1$:

- Start at the y -intercept $(0, 1)$.
- Use the slope $m = 2$ to plot the next point: rise = 2, run = 1, leading to point $(1, 3)$.
- Draw the line through these points.



Real-Life Examples

Example 1: Budgeting

Suppose you earn a fixed amount of money each week, and you have some initial savings. The total amount of money you have after x weeks can be modeled by a linear equation.

$$y = mx + b$$

Where:

- m is the weekly earnings.
- b is the initial savings.

Example 2: Distance Over Time

Suppose you are traveling at a constant speed. The distance traveled after x hours can be modeled by a linear equation.

$$y = mx + b$$

Where:

- m is the speed.
- b is the initial distance (if any).

Practice Questions

1. Find the slope and y-intercept of the line given by the equation $y = 3x - 4$.
2. Convert the standard form equation $2x + 3y = 6$ to slope-intercept form and identify the slope and y-intercept.
3. Write the equation of a line with a slope of 2 and a y-intercept of -3.
4. A person saves \$50 each week starting with \$200. Write the equation representing the total savings after x weeks.
5. For the equation $y = -\frac{1}{2}x + 4$, plot the graph and determine the coordinates of two points on the line.

Answers

1. Slope $m = 3$, y-intercept $b = -4$.
2. $y = -\frac{2}{3}x + 2$. Slope $m = -\frac{2}{3}$, y-intercept $b = 2$.
3. $y = 2x - 3$.
4. $y = 50x + 200$.
5. Two points on the line: $(0, 4)$ and $(2, 3)$.

