Logarithms Course

Applied Scholastics, Ferndale

Logarithms

History & Purpose

A logarithm is the power to which a base number must be raised to equal a given number.

The base is written as a subscript next to the word 'log.'

$$number = base^{power}$$
$$log_{base} number = power$$

(Given $base > 0, base \neq 1$, and $number > 0$.)

For example, for a base of 10, since $100 = 10^2$, the logarithm of 100 is 2.

$$100 = 10^2 \text{ so } \log_{10} 100 = 2$$

1. What is a logarithm?

2. For a base of 10, what is the logarithm of 1000?
3. How do you write this?
4. For a base of 2, what is the logarithm of 16?
5. How do you write this?

   Logarithms were developed in the 1600s by mathematicians John Napier and John Briggs. The word was coined by Napier from the Latin words logos and arithmos and means 'ratio-number.'

   With scientific advances in fields such as astronomy, navigation, time keeping and map making, it had become necessary to perform more and more long and tedious calculations with large numbers by hand. Logarithms were invented to make working with large numbers easier and faster.

6. Why were logarithms invented?

   Napier used a base of e for his logarithms. This is 'Euler's number,' roughly equal to 2.7183. Euler was a famous mathematician and e was named after him. It is a constant that often turns up in studies of the natural world and it is used in many branches of mathematics. It was first discovered in solving a problem to do with compound interest. Logarithms with a base of e are called natural logarithms and are written as $\ln n$ or $\log_e n$.

   Briggs later introduced the base of 10 as being easier to work with decimal numbers. Logarithms with a base of 10 are called common logarithms and are written simply as $\log n$ without indicating the base. You can assume that the base of a logarithm is 10 unless it is indicated otherwise.

$$\text{e.g.}\quad \begin{array}{ll} \log 25 = \log_{10} 25 \approx 1.3979 & 10^{1.3979} \approx 25 \\ \ln 25 = \log_e 25 \approx 3.2189 & e^{3.2189} \approx 25 \end{array}$$

$$\text{e.g.}\quad \begin{array}{ll} 16 = 4^2 & \log_4 16 = 2 \\ 4 = 8^{\frac{2}{3}} & \log_8 4 = \frac{2}{3} \\ 1000 = 10^3 & \log 1000 = 3 \end{array}$$

7. What is Euler's number?
8. What are logarithms with a base of e called?
9. How are natural logarithms written?
10. What are common logarithms?
11. Why was a base of 10 preferred?
12. What base is assumed in $\log 25$?

Tables were calculated where the logarithm of any number could be easily looked up. The product of two numbers could be found simply by looking up their logarithms, adding these logarithms, and then looking in the table for the antilogarithm, which meant the number with that logarithm.

Similarly, any two numbers could be quickly divided by taking the difference of their logarithms and looking up that antilogarithm as their quotient.

Lengthy problems in multiplication and division were changed by use of logarithms to simple problems of addition and subtraction.

I. FOUR-PLACE LOGARITHMS.

1	0	1	2	3	4	5	6	7	8	9
0	0000	0000	3010	4771	6021	6990	7782	8451	9031	9542
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388

Part of a table from a book of logarithms

For example, 123 and 234 can be multiplied by looking up their logarithms (2.09 and 2.37), adding them together ($2.09 + 2.37 = 4.46$), and looking through the values of the table to find the number that has that logarithm, the antilogarithm.

$$\begin{aligned}
\log_{10} 123 &= 2.09 \\
\log_{10} 234 &= 2.37 \\
2.09 + 2.37 &= 4.46 \\
\log 28,782 &= 4.46 \\
\text{so, } 123 \times 234 &= 28,782
\end{aligned}$$

13. How is multiplication done with logarithms?

14. Given that $\log 456 = 2.5690$ and $\log 567 = 2.7536$, and that the antilog of $2.5690 + 2.7536 = 5.4125$ is 258524, what is 456×567?

That's the basic idea, but the real power of logarithms comes from their use with larger numbers.

To demonstrate the amount of work that is saved by using logarithms, here is an example of multiplying two long numbers by hand, calculating partial products for each digit of the multiplier multiplied by each digit of the multiplicand, and then adding all of the partial products. Multiplying like this can made shorter by "carrying" digits and so on, but even with that it is still long, tedious and error-prone.

$$\begin{array}{r}
1\,2\,3\,4\,5\,6 \\
\times \quad 2\,3\,4\,5\,6\,7 \\
\hline
4\,2 \\
3\,5 \\
2\,8 \\
2\,1 \\
1\,4 \\
7
\end{array}$$

36
30
24
18
12
6
30
25
20
15
10
5
24
20
16
12
8
4
18
15
12
9
6
3
12
10
8
6
4
2

2
15
14
22
28
34
15

5

$$
\begin{array}{l}
\qquad 3\;4 \\
\quad\;\; 1\;6 \\
\quad\;\; 7 \\
+2 \\
\hline
\quad 28{,}9\,5\,8{,}7\,0\,3{,}5\,5\,2 \\
\hline\hline
\end{array}
$$

You could go through all that to get an exact answer, but on the other hand, you can multiply 123,456 by 234,567 much more easily using logarithms.

$$
\begin{aligned}
\log_{10} 123,456 &= 5.0915 \\
\log_{10} 234,567 &= 5.3703 \\
5.0912 + 5.3714 &= 10.4618 \\
\log_{10} 28,960,096,189 &= 10.4618 \\
123 \times 234 &\approx 28,960,096,189
\end{aligned}
$$

This product is only approximately correct. The exact answer differs from this approximate answer by $1,392,637$, but that is $\frac{1{,}392{,}637}{28{,}958{,}703{,}552}$ which is only an error of 0.004809%.

Logarithm tables are only calculated to a certain number of decimal places, usually 4, but this level of precision is good enough for practical purposes and answers can be arrived at quickly.

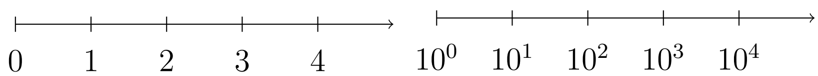
In the same way, division problems became simpler subtraction problems, and the calculation of powers and roots were also simplified. These were important problems to solve, and particularly helped in the field of navigation as sailors began to venture further around the world.

15. Are products calculated by logarithms exact?

16. Why was it still better to use logarithms when dealing with large numbers rather than calculating exact numbers?

    Logarithms were still commonly used for calculations up until the 1980s and the advent of electronic calculators.

    Logarithmic scales, where points go up by powers rather than going up linearly as on a ruler, are still used in measurement of acidity, sound, earthquakes, and in various other fields.

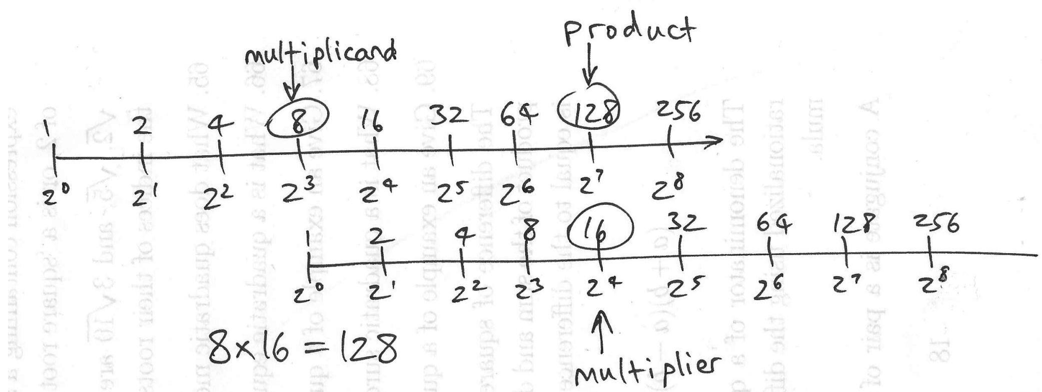


Linear Scale   Logarithmic Scale

17. What is a logarithmic scale?

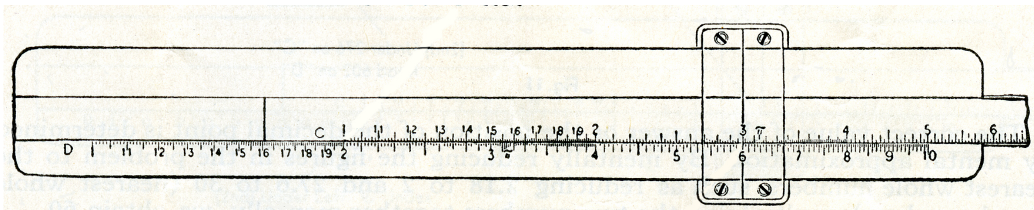
18. What is the difference between a linear scale and a logarithmic scale?

    Slide rules were often used in the place of pocket calculators and without having to carry a table of logarithms.



Multiplication by Slide Rule (using powers of 2)

A slide rule consists of two logarithmic scales that can be slid against each other. Multiplication is done by aligning the start of the bottom scale to the multiplicand on the top scale, so the product becomes the number above the multiplier.



Slide Rule

Prior to the 1980s, slide rules were indispensable tools for scientists, mathematicians, and engineers. These essential instruments played a pivotal role in the calculations behind every major engineering project and significant scientific advancement. Even as NASA made its initial foray into early computing, slide rules remained a constant companion for their engineers. In fact, it's remarkable to note that the remarkable achievement of sending humans to the moon was accomplished with slide rules, rather than the modern calculators of today.

19. What is a slide rule?

20. How were slide rules better than a table of logarithms?

21. How do you multiply using a slide rule?

22. How would you do division with a slide rule?

Antilogarithms

As you have seen in the final step of the use of logarithms to simplify operations with large numbers, an antilogarithm reverses a logarithm. It is the number that was turned into a logarithm in the first place. Given the logarithm of an unknown number, how do you find the number? Usually finding the antilogarithm gives the solution to the problem.

Antilogarithm is usually abbreviated as antilog.

Say $\log x \approx 0.4771$. The antilog is the number that 0.4771 is the log of.

The definition of a log is that if $n = b^e$, $\log_b n = e$, so $\log_{10} x \approx 0.4771$ can be rearranged to become x$\approx 10^{0.4771} \approx 3$. In other words, the antilog of 0.4771 is 3.

It is the same for a base other than 10, such as $\log_5 x \approx 0.6065$. This rearranges to become x (the antilog) $\approx 5^{0.6065} \approx 2.6541$.

The antilogarithm can be calculated as a power, or looked up in reverse from a table of logarithms.

23. What is an antilogarithm?
24. How do you find the antilogarithm of a number?
25. If $\log_{10} x = 2.1673$, then what is the antilog?
26. If $\log_5 x = 13.1326$, then what is the antilog?

Properties of Logarithms

$$\log_a 1 = 0$$

(because $n^0 = 0$

also: $\ln 1 = 0$
and: $\log 1 = 0$

$$\log_a a = 1$$

(because $n^1 = n$

$$\text{also: } \ln e = 1$$
$$\text{and: } \log 10 = 1$$

$$log_a a^n = n$$

$$\text{also: } \ln e^n = n$$
$$\text{and: } \log 10^n = n$$

$$a^{log_b n} = n$$

$$\text{also: } e^{\ln x} = x$$
$$\text{and: } 10^{\log x} = x$$

27. What is $\log 1$?
28. What is $\log_5 1$?
29. What is $\log_2 1$?
30. What is $\ln 1$?
31. What is $\log 10$?
32. What is $\ln e$?
33. What is $\log_5 5$?
34. What is $\log_2 2$?
35. What is $\log_5 5^2$?

36. What is $\log_3 3^5$?

37. What is $\log 10^3$?

38. What is $\ln e^6$?

39. What is $3^{\log_5 4}$?

40. What is $8^{\log_5 1}$?

41. What is $e^{\ln 7}$?

42. What is $10^{\log 9}$?

Log Rules

Product Rule

$$\log_b A + \log_b B = \log_b AB$$

proof: let $log_b A = x$ $\quad A = b^x$
let $\log_b B = y$ $\quad B = b^y$

so $AB = b^x b^y = b^{x+y}$
$log_b AB = x + y = log_b A + log_b B$

e.g. $\log_2 5 + log_2 4 = \log_2 (5 \cdot 4) = \log_2 20$

e.g. given $\log 2 = 0.3010$
and $\log 3 = 0.4771$
and $\log 5 = 0.6990$
then $\log 30 = \log (2 \cdot 3 \cdot 5)$
$= \log 2 + \log 3 + \log 5 = 1.4771$

43. What is $\log_2 8 + \log_2 2$?

44. What is $\log_3 9 + \log_3 27$?

45. What is $\ln e + \ln 7$?

46. If $\log_a 2 = 0.3010$ and $\log_a 3 = 0.4771$, find $\log_a (2 \cdot 3)$.

47. If $\log_b 4 = 1.3863$ and $\log_b 5 = 0.6989$, find $\log_b (4 \cdot 5)$.

48. If $\log_c 10 = 1$ and $\log_c 7 = 0.8451$, find $\log_c (10 \cdot 7)$.

Quotient Rule

$$\log_b A - \log_b B = \log_b \frac{A}{B}$$

proof: let $\log_b A = x$ $\qquad A = b^x$
let $\log_b B = y$ $\qquad B = b^y$

so $\frac{A}{B} = \frac{b^x}{b^y} = b^{x-y}$
$\log_b \frac{A}{B} = x - y = \log_b A - \log_b B$

e.g. $\log_2 20 + log_2 4 = \log_2 (\frac{20}{4}) = \log_2 5$

e.g. given $\log 2 = 0.3010$

$$\begin{aligned} \text{then } \log 50 &= \log(\frac{100}{2}) \\ &= \log 100 - \log 2 \ (\log_{10} 100 = 2) \\ &= 2 - 0.3010 = 1.699 \end{aligned}$$

49. What is $\log_3 27 - \log_3 9$

50. What is $\log_4 64 - \log_4 8$

51. What is $\log_2 16 - \log_2 4$

52. What is $\log_5 125 - \log_5 5$

53. If $\log_a 8 = 0.9031$ and $\log_a 2 = 0.3010$, find $\log_a (8/2)$.

54. If $\log_b 100 = 2$ and $\log_b 4 = 0.6021$, find $\log_b (100/4)$.

Power Rule

$$log_a x^n = n \log_a x$$

proof: let $m = \log_a x = x$ $\qquad x = a^m$
so $x^n = (a^m)^n = a^{mn}$
$\log x^n = mn = nm = n \log_a x$

e.g. $\log x^4 = 4 \log x$

e.g. $\log_3 \frac{1}{3} = log_3 1 - log_3 3$
$= 0 - 1 = -1$
$(\log_a 1 = 0 \text{ and } \log_a a = 1)$

e.g. $\log \sqrt{10} = \log 10^{\frac{1}{2}}$
$= \frac{1}{2} \log 10$
$= \frac{1}{2} \cdot 1 = \frac{1}{2}$

55. What is $\log_2 (2^4)$?
56. What is $\log_5 (5^2)$?
57. What is $\log_3 (3^2) + \log_3 (3^4)$?
58. What is $\log (10^3) - \log (10^2)$?
59. What is $\ln (e^5) + \ln (e^3)$?
60. What is $\log_2 (2^7) - \log_2 (2^5)$?
61. What is $2 \log_2 3$ as a single logarithm?
62. What is $4 \log_{10} 7$ as a single logarithm?

63. What is $-2\ln(e^2)$ as a single logarithm?

64. What is $\log_4(4^{3/2})$?

65. What is $\log_2(2^{7/2}) - \log_2(2^{3/2})$?

66. What is $\ln(e^{1/3}) + \ln(e^{5/3})$?

Root Rule

$$\log_a(\sqrt[n]{x}) = \frac{\log_a x}{n}$$

$$\text{e.g.} \log_2 \sqrt{8} = \frac{\log_2 8}{2} = \frac{3}{2}$$

$$\text{e.g.} \log_3 \sqrt{9} = \frac{\log_3 9}{2} = \frac{2}{2} = 1$$

67. What is $\log_2(\sqrt[3]{64})$?

68. What is $\log(\sqrt[4]{10,000})$?

69. What is $\ln(\sqrt[5]{e^5})$?

70. What is $\log_3(\sqrt[2]{81})$?

71. What is $\log_9(\sqrt[2]{9^{10}})$?

72. What is $\ln(\sqrt[4]{e^8})$?

73. What is $\frac{1}{3}\log_2 8$?

74. What is $\frac{1}{4}\log_{10} 10,000$?

75. What is $\frac{1}{5}\ln(e^{10})$?

76. What is $\log_{11}(\sqrt[3]{11^9})$?

77. What is $\log_2(\sqrt[5]{2^{15}})$?

78. What is $\ln(\sqrt[7]{e^{21}})$?

Change of Base Law

It is unusual to have to find a log for a number with a base other than 10 or e, but there is a formula that can be used:

$$\log_a n = \frac{\log_b n}{\log_b a}$$

for base 10 $\log_a n = \frac{\log_{10} n}{\log_{10} a} = \frac{\log n}{\log a}$

for base e $\log_a n = \frac{\log_e n}{\log_e a} = \frac{\log n}{\log a}$

$$\begin{array}{llll}
\text{proof:} & \log_a n & = \mathrm{mn} = \mathrm{a}^m & \text{(by definition)} \\
 & \log_b n & = \log_b a^m & \text{(taking log of both sides)} \\
 & \log_b n & = \mathrm{m} \log_b a & \text{(using power rule)} \\
 & \frac{\log_b n}{\log_b a} & = \mathrm{m} = \log_a n & \text{(rearranging)}
\end{array}$$

$$\text{e.g. } \log_4 40 = \frac{\log 40}{\log 4} = \frac{16.0206}{0.60206} = 2.661$$

$$\text{e.g. } \log_2 5 + log_3 7 = (\frac{\log 5}{\log 2}) + (\frac{\log 7}{\log 3}) = 4.0932$$

$$\text{e.g. } \log x + 3\log y - 4\log z = \log x + \log y^3 - \log z^4 = \log \frac{xy^3}{z^4}$$

$$\text{e.g. } 2 + 4\log_3 x = \log_3 9 + \log_3 x^4 = \log_3 9x^4$$

(express 2 as a log with base 3:
$2 = 2\log_3 3 = \log_3 3^2 = \log_3 9$)

79. What is $\log_6 36$ with a base of 2?

80. What is $\log 1000$ with a base of 5?

81. What is $\ln 100$ with a base of 4?

82. What is $\log_7 49$ with a base of 3?

83. What is $\log_3 7 + \log_2 9$?

84. What is $\log_4 16 - \log_2 64$?

85. What is $\log 100 + \log_5 25$?

86. What is $\ln e^3 - \ln e^2$?

87. What is $\log_2 16 + \log_3 9$ with a base of 4?

88. What is $\log_5 125 - \log 100$ with a base of 2?

89. What is $\log_5 25 + \log_5 5$ with a base of 10?

90. What is $\ln e^5 + \log_2 2^3$ with a base of 10?

Logarithmic Equations

Using the Definition of a Logarithm

e.g.	$\log_1 0x = 2$	$\longrightarrow x = 10^2$	$\longrightarrow x = 100$
e.g.	$\log_x 25 = 2$	$\longrightarrow 25 = x^2$	$\longrightarrow x = \pm 5 = 5$
e.g.	$\log_x 22 = 2.1$	$\longrightarrow 22 = x^{2.1}$	$\longrightarrow x = \sqrt[2.1]{22} = 4.358$
e.g.	$\log_4 5 = x$	$\longrightarrow \frac{\log 5}{\log 4} = 1.161$	

91. Solve for x: $\log_3 x = 4$

92. Solve for x: $\log x = 1$

93. Solve for x: $\log_2 x = 3$

94. Solve for x: $\ln x = 0$

95. Solve for x: $\log_4 x = 2$

96. Solve for x: $\log_7 x = 1.5$

97. Solve for x: $\ln ex = -2$

98. Solve for x: $\log_3 x = 2.5$

99. Solve for x: $\log x = 2.5$

100. Solve for x: $\log_2 x = 2.5$

101. Solve for x: $\ln x = 2.5$

102. Solve for x: $\ln x = 0.5$

Using Equivalence

(equivalence: if $log_b x = log_b y$ then $x = y$.)

$$\begin{aligned} \text{e.g.}\quad \log x + \log 5 &= \log 20 \\ \log 5x &= \log 20 \text{ (product rule)} \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

103. Solve for x: $\log x + \log 3 = \log 9$.

104. Solve for x: $\log 2x = \log 16$.

105. Solve for x: $\log 4x + \log 2 = \log 32$.

106. Solve for x: $\log x - \log 4 = \log 2$.

107. Solve for x: $\log x + \log 5 = \log 125$.

108. Solve for x: $\log 3x = \log 27$.

109. Solve for x: $\log x + \log 7 = \log 49$.

110. Solve for x: $\log 8x + \log 2 = \log 128$.

111. Solve for x: $\log 3x - \log 9 = \log 1$.

112. Solve for x: $\log x + \log 11 = \log 121$.

113. Solve for x: $\log 12x + \log 2 = \log 288$.

114. Solve for x: $\log 2x - \log 16 = \log 8$.

Grouping Log Terms to one side

(Rearranging to get log terms on on side
and numbers on the other)

$$
\begin{aligned}
\text{e.g. } \log_4(2x+4) - 2 &= \log_4 3 \\
\log_4(2x+4) - \log_4 3 &= 2 \\
\log_4 \frac{2x+4}{3} &= 2 \\
\frac{2x+4}{3} &= 4^2 \\
\frac{2x+4}{3} &= 16 \\
2x+4 &= 48 \\
2x &= 44 \\
x &= 22
\end{aligned}
$$

115. Solve for x: $\log_3(3x+6) - 2 = \log_3 9$.

116. Solve for x: $\log_2(4x+8) - \log_2 2 = 3$.

117. Solve for x: $\log_5(5x+10) - \log_5 5 = 1$.

118. Solve for x: $\log(10x+20) - \log 2 = 4$.

119. Solve for x: $\ln(ex+2e) - \ln e = 2$.

Replacing a Number with the Equivalent Log

$$
\begin{aligned}
\text{e.g. } \log_4(2x+4) - 2 &= \log_4 3 \\
\log_4(2x+4) - \log_4 16 &= \log_4 3 \\
\log_4(2x+4) - \log_4 4^2 &= \log_4 3 \\
\log_4 \frac{2x+4}{16} &= log_4 3 \\
\frac{2x+4}{16} &= 3 \\
2x+4 &= 48 \\
2x &= 44 \\
x &= 22
\end{aligned}
$$

120. Solve for x: $\log_2(x+4) = 3$.

121. Solve for x: $\log(2x+6) = \log 100$.

122. Solve for x: $\log_5(3x+9) = \log_5 125$.

123. Solve for x: $\ln(4x+8) = \ln e^3$.

124. Solve for x: $\log_3(5x+10) = \log_3 3^2$.

Exponential Equations

Exponential equations deal with rates of growth and decay. They take their name from the fact that they contain terms with exponents. Exponential equations were first used in calculating compound interest and in population growth. They have many uses. Logarithms provide a method to solve exponential equations that would be difficult or impractical to solve algebraically.

Here are some examples from the field of finance:

How long will it take investor who deposits \$20,000 compounding at 5 % p.a. to increase this amount to \$30,000?

Compound Interest Formula: $A = P(1+i)^n$

- A is the final amount (in this case, $\$30,000$).

- P is the principal amount (initial investment, in this case, $\$20,000$).

- i is the annual interest rate (in this case, 5% or 0.05 as a decimal).

- n is the number of years (which we need to find).

$$\begin{aligned}
30,000 &= 20,000(1.05)^n \\
\frac{30,000}{20,000} &= 1.05^n \\
1.5 &= 1.05^n \\
\log 1.5 &= \log 1.05^n \\
\log 1.5 &= n \log 1.05 \\
\frac{\log 1.5}{\log 1.05} &= n \\
\frac{0.1761}{0.2119} &\approx 8.3 \text{ years} \\
[1.05^{8.31} &\approx 1.5 \checkmark]
\end{aligned}$$

125. How long will it take for an investment of $\$10,000$ to grow to $\$15,000$ when compounded annually at a rate of 4%?

$$\begin{gathered}
\$15,000 = \$10,000(1+0.04)^n \\
1.5 = (1.04)^n \\
\ln(1.5) = \ln(1.04^n) \\
\ln(1.5) = n \cdot \ln(1.04) \\
n = \frac{\ln(1.5)}{\ln(1.04)} \\
n \approx \frac{0.4055}{0.0392} \approx 10.34 \text{ years}]
\end{gathered}$$

126. An individual invests $\$25,000$ in a savings account, and the account grows to $\$30,000$ after a certain number of years with an annual interest rate of 3%. Find the number of years it took to reach this amount.

127. If $\$5,000$ is invested at a 6% annual interest rate, how many years will it take for the investment to double in value?

128. A deposit of $\$8,000$ earns 7% annual interest. Determine the number of years it takes for the deposit to reach $\$10,000$.

129. If an investment of $\$50,000$ grows to $\$60,000$ with a 5% annual interest rate, how many years did it take to achieve this growth?

Here is an example from the field of physics involving radioactive decay:

The decay of a radioactive substance over time can be modeled using the exponential decay equation:

$$N(t) = N_0 \times e^{-\lambda t}$$

Where:

$$\begin{aligned} N(t) &: \text{Remaining quantity of radioactive substance at time } t \\ N_0 &: \text{Initial quantity of radioactive substance} \\ \lambda &: \text{Decay constant} \\ t &: \text{Time elapsed} \end{aligned}$$

Suppose we have an initial quantity of a radioactive substance, and we want to find the time it takes for the substance to decay to a certain fraction of its initial amount. Let's say we want to find the time when the remaining quantity is $\frac{N_0}{2}$.

Using the exponential decay equation, we have:

$$\frac{N_0}{2} = N_0 \times e^{-\lambda t}$$

Dividing both sides by N_0 and taking the natural logarithm (base e) of both sides:

$$
\begin{aligned}
\ln\left(\frac{N_0}{2}\right) &= \ln\left(N_0 \times e^{-\lambda t}\right) \\
\ln\left(\frac{1}{2}\right) &= \ln\left(e^{-\lambda t}\right) \\
\ln\left(\frac{1}{2}\right) &= -\lambda t \ln e \\
\ln\left(\frac{1}{2}\right) &= -\lambda t \\
t &= -\frac{\ln\left(\frac{1}{2}\right)}{\lambda}
\end{aligned}
$$

This formula allows us to calculate the time it takes for any radioactive substance to decay to half of its initial amount, given its decay constant.

Here is another worked example:

Suppose a sample of a radioactive substance has an initial quantity of $N_0 =$ 1000 grams and a decay constant $\lambda = 0.05$ per year. We want to find the time it takes for the substance to decay to 250 grams.

Using the exponential decay equation:

$$N(t) = N_0 \times e^{-\lambda t}$$

Substituting the known values:

$$
\begin{gathered}
250 = 1000 \times e^{-0.05t} \\
\frac{250}{1000} = \frac{1}{4} = e^{-0.05t} \\
\ln\left(\frac{1}{4}\right) = \ln\left(e^{-0.05t}\right) \\
\ln\left(\frac{1}{4}\right) = -0.05t \ln e \\
(lne = 1) \\
\ln\left(\frac{1}{4}\right) = -0.05t
\end{gathered}
$$

$$t = \frac{\ln\left(\frac{1}{4}\right)}{-0.05} \approx 27.73 \text{ years}$$

1. A radioactive substance initially has a quantity of $N_0 = 200$ grams and a decay constant $\lambda = 0.02$ per year. Calculate the time it takes for the substance to decay to 50 grams.

2. In a laboratory experiment, a radioactive material with an initial quantity of $N_0 = 5000$ grams is observed to decay to 2500 grams in a certain time. Find the decay constant λ for this substance.

3. A sample of a radioactive element with $N_0 = 300$ grams and $\lambda = 0.03$ decays over time. Calculate the time it takes for the substance to decay to 75 grams.

4. The half-life of a radioactive substance is 10 years. Find the decay constant λ for this substance.