Fractions Course

Tuoring Centre Ferndale

1 What is a fraction

In English, a fraction means a small bit of something. You could say "I took a fraction of your food."

In maths, a fraction means equal parts of a whole.

1. What is a fraction in its usual English meaning?
2. Use fraction in a sentence that shows this meaning.

3. What is a fraction in maths?
4. Use fraction in a sentence that shows this meaning.

Writing Fractions

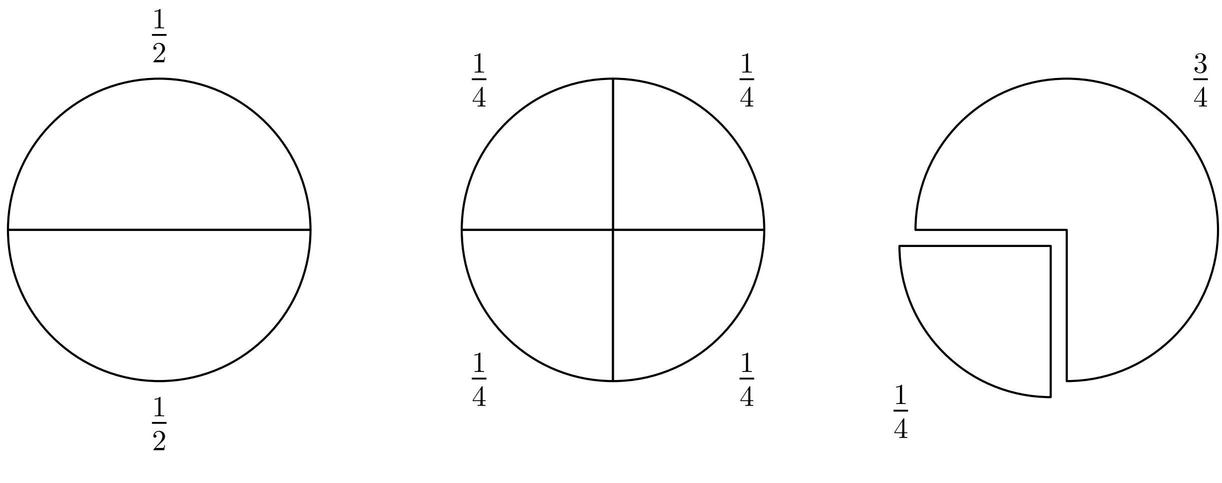
A fraction is written as two numbers on top of each other with a line between them, called the fraction bar.

The number on top is how many of those parts make up this fraction.

The number on the bottom is how many parts the whole thing has been divided into.

$$\frac{\text{how many parts make up the fraction}}{\text{how many parts the whole thing has been divided into}}$$

5. Write what fraction would you have if you had one part of something that was cut equally into two pieces.
6. What does the top number of this fraction mean?
7. What does the bottom number of this fraction mean?



Names of Fractions

There are some special words used to name some fractions:

- A half means a piece of something that has been cut into 2 pieces.
- A third means a piece of something that has been cut into 3 pieces.
- A quarter, or a fourth, means a piece of something that has been cut into four pieces.
- For the fractions of things that have been divided into 4 or more pieces, the word ending -th or -eth can be added to the denominator to name that fraction.

  That's the same way that numbers are made to show the order of things, as in first (1^{st}), second (2^{nd}), third (3^{rd}), fourth (4^{th}), and so on, but here it is used to show fractions.

  A piece of something that was cut into 20 pieces, $\frac{1}{20}$, would be called a twentieth, and it could be written as a 20^{th} for short.
- Fractions are also sometimes named by their numerator over their denominator. $\frac{1}{20}$ can be read as "1 over 20."

8. What is a half?

9. What is a third?

10. What is a quarter?

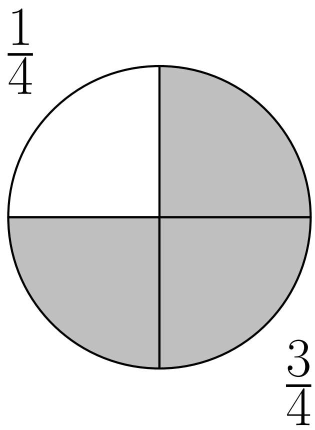
11. What does "th" mean when written after a number?

12. How would you write that you had three pieces of something that had been cut into ten pieces?

Numerator

The number above the fraction bar is called the numerator. It means the number of parts that are being talked about.

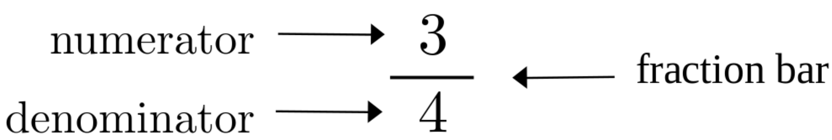
13. What is the numerator of a fraction, in your own words?



Denominator

Denominator In English, the denominator is the thing that gives something a name or a category. Doing a lot of sports is the denominator of being athletic, for example.

In maths, the denominator names what kind of fraction it is. It is the number at the bottom of a fraction that gives the total number of parts that the whole has been broken into.



$\frac{3}{4}$ means an amount that is three parts of something that has been broken into four parts.

14. What is the denominator of a fraction, in your own words?

2 Equivalent fractions

Express Express means to communicate an idea by putting it into a physical form. Ideas can be expressed by means of art, such as music or dance or pictures and so on, and by language with words and symbols.

Expressions The things that express the idea are called expressions. Any sentence is an expression, but expression is usually used to mean commonly used sentences.

15. What does express mean?

16. What is an expression?

Maths Expressions In maths, express means to put a maths idea into maths symbols. $3 \times 4 + 2$ is an idea expressed in maths. It's like using words in a sentence but you are using the numbers and symbols of maths. These maths sentences are also called expressions.

17. What does express mean in maths?

18. What is a maths expression?

Equals Equals means having the same value, especially when working out a final answer, such as $2+2+2=6$.

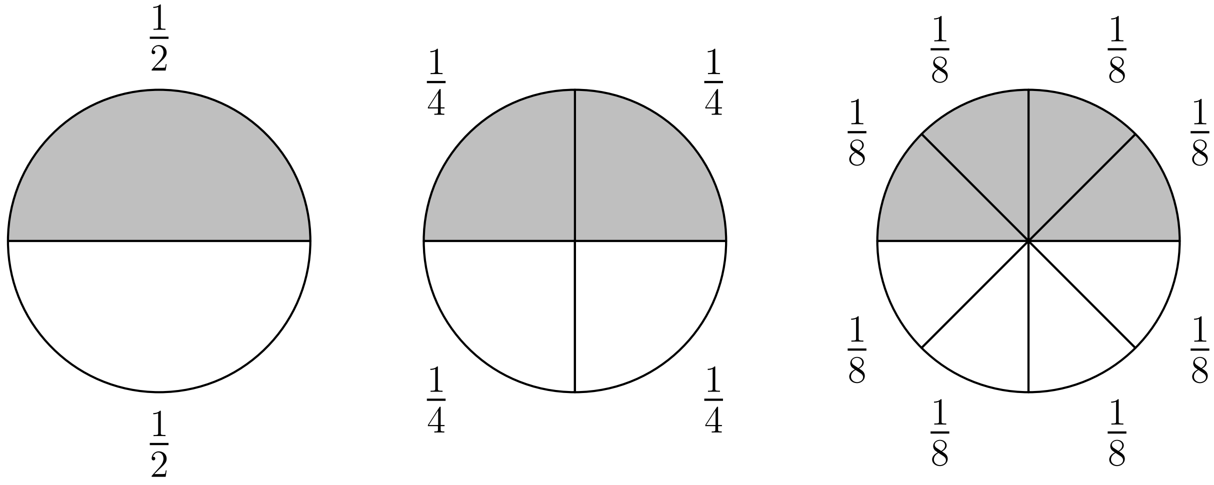
Equivalent Equivalent means the same as or just as good as. Coke and Pepsi are equivalent drinks. They taste the nearly same so it doesn't matter which one you get.

In maths, equivalent expressions are different ways of expressing the same amount. $2+2+2$ equals 3×2 and the expressions are equivalent.

19. What does equivalent mean?

20. What is the difference between equals and equivalent?

Equivalent Fractions Equivalent fractions are where you express the same fraction in different ways. $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ all express the same amount even though they have different numerators and denominators.



21. What are equivalent fractions, in your own words?

Making equivalent fractions

Fractions and Dividing Fractions and dividing are similar things. Just as you could write that $4 \div 2 = 2$, you could express the same idea as $\frac{4}{2} = 2$.

(As you do more maths, you will mostly stop seeing the division sign and divisions will be expressed as fractions instead.)

Multiplying and Dividing by 1 Dividing an amount by itself is equal to 1, and a fraction with the same numerator and denominator is also equal to 1.

$$2 \div 2 = 1 \qquad \frac{2}{2} = 1$$

Multiplying or dividing an amount by 1 doesn't change that amount.

$$2 \times 1 = 2 \qquad 2 \div 1 = 2$$

Multiplying or dividing an amount by 1 expressed as a fraction will leave the amount unchanged but it will change how it is expressed.

$$2 \times \frac{2}{2} = 2 \qquad 2 \div \frac{2}{2} = 2$$

To make equivalent fractions you multiply or divide both the numerator and denominator by the same number.

$$\frac{2}{4} = \frac{2}{4} \times \frac{2}{2} = \frac{4}{8} \qquad \frac{2}{4} = \frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$$

You can make any number of equivalent fractions this way.

22. What are 3 fractions that are equivalent to $\frac{2}{3}$?

3 Simplifying fractions

Simplify

Simplify means to make something simple so that it's easier.

23. What does simplify mean?

24. Use simplify in a sentence.

A fraction is easier to work with when it has been simplified. You do that by changing it into an equivalent fraction that has smaller numbers in it. When you simplify $\frac{4}{8}$ it becomes $\frac{1}{2}$.

25. What does it mean to simplify a fraction?

26. Use simplify in a sentence that show this meaning.

Greatest

In English, greatest means the best, as in a circus where they say its the Greatest Show on Earth. Greatest also meant biggest, like when you say things like my greatest fear is spiders.

27. What does greatest mean?

28. Use greatest in a sentence that shows its meaning as the biggest or the best.

In maths, greatest means the biggest number, like if you compare between 9 and 7, and say that 9 is the greatest.

29. What does greatest mean when it's a number?

30. Use greatest in a sentence that shows this meaning.

Common

Common means shared by two or more people or things, like when people share something in common.

31. What does common mean?

32. Use common in a sentence that shows its meaning as something shared.

In maths common means that two things have the same number in them somewhere. If you had two groups of numbers you would say that the common numbers were the ones that were in both groups. Say 2, 3, 4, 5 and 4, 5, 6, 7. The common numbers are 4 and 5.

33. What does common mean in maths?

34. What are the common numbers in 12,13,14,15 and 10, 11, 12, 13?

Factor

In English, a factor means a part of the cause for something. People might talk about the factors of some situation, meaning the different parts of it, time or money or the people involved.

35. What is a factor?

36. Use factor in a sentence that shows its meaning as a part of something.

In maths, factors are the numbers which will multiply to give some other number. For example, the factors of 8 are 1, 2, 4 and 8, because 1×8, 2×4, 4×2, and 8×1 all equal 8.

37. What is a factor, in maths?

38. What are some of the factors of 12?

Finding factors

You find the factors of a number by trying to divide it by each counting number, in order, looking for divisions that leave no remainder. These divisors and the quotients are pairs of factors.

For example, to find the factors of 15:

$15 \div 1 = 15 \ \longleftarrow$
$15 \div 2 = 7$ remainder 2
$15 \div 3 = 5 \ \longleftarrow$
$15 \div 4 = 3$ r 4 (quotient smaller than divisor)
$15 \div 5 = 3$ (same factors in reverse order)

The pairs of factors of 15 here are 1 & 15, and 3 & 5, so the factors of 15 are 1, 3, 5 and 15.

As you can see, continue only until the quotient is smaller than the divisor. Going past that point just results in the same factors but in reverse order.

39. What are the factors of 8?

40. What are the factors of 24?

Greatest Common Factor

The largest number that has both the numerator and denominator of a fraction as a multiple is called the greatest common factor.

It is sometimes written just as GCF.

To simplify a fraction you divide the numerator and the denominator by their greatest common factor.

41. In your own words, what does greatest common factor mean?
42. What is the purpose of finding the greatest common factor?
43. How is the greatest common factor of the numerator and denominator of a fraction used to simplify the fraction?

Simplified fractions are easier to work with. Always simplify a fraction if you can, both before you try to add, subtract, multiply or divide them, and in writing your final answer.

44. Why is it a good idea to simplify fractions?
45. Can $\frac{3}{6}$ be simplified?
46. What should you do if you worked out an answer that was $\frac{2}{4}$?

Cancelling

To cancel something means to cross it out.

47. What does cancel mean?
48. Use cancel in a sentence.

To cancel a fraction means to divide the numerator and the denominator by their greatest common factor and cross them out and replace them. That is the easiest and usual way to write it when you simplify a fraction.

For $\frac{4}{8}$, the largest number that divides evenly into both the numerator and denominator is 4, but you don't bother to write that. You just cross out the 4 and write a 1, and cross out the 8 and write a 2. That gives you the equivalent simplified fraction of 1/2.

So instead of writing $\dfrac{4}{8} = \dfrac{4 \div 4}{8 \div 4} = \dfrac{1}{2}$, you just write $\dfrac{4}{8} = \dfrac{{}^{1}\cancel{4}}{\cancel{8}_{2}}$.

49. What does cancelling a fraction mean?
50. Why do we cancel a fraction this way instead of writing what we are doing in full?
51. Simplify $\frac{6}{12}$ by cancelling.

How to Find the Greatest Common Factor

Knowing the times table

You can often just simplify fractions in your head, if you know the times table well. With practice you can get good enough that you will know the greatest common factor right away without having to think much about it.

For example, maybe you can see that the greatest common factor needed to simplify $\frac{3}{9}$ is 3.

$$\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3} \text{ or by cancelling you write } \frac{{}^{1}\cancel{3}}{\cancel{9}_{3}} = \frac{1}{3}$$

52. Using your knowledge of the times table, simplify $\frac{7}{21}$.

Simplifying in stages

If you can't spot the greatest common factor right away, it will still work if you find just any common factor and use that to start with.

If the numerator and denominator are both even then you can at least divide them both by 2.

You will get a simpler fraction and then you can look at that one for a new common factor. Keep going until you are sure there are no more common factors.

Say you have to simplify $\frac{63}{84}$, and you spot that 3 divides evenly into both 63 and 84. You cancel both numbers and replace them.

$$\frac{{}^{21}\cancel{63}}{\cancel{84}_{28}}$$

Now you have $\frac{21}{28}$ and you see that 21 and 28 are multiples of 7, so you cancel both numbers again and replace them.

$$\frac{63}{84} = \frac{{}^{21}\cancel{63}}{\cancel{84}_{28}} = \frac{{}^{3}\cancel{21}}{\cancel{28}_{4}} = \frac{3}{4}$$

53. In stages, simplify $\frac{66}{198}$.

Listing out all the factors

You can also work out the factors and write them down to pick the greatest common factor. You'll definitely get a right answer that way.

For example, to simplify $\frac{63}{105}$,

factors of 63: 1, 3, 7, 9, 21, 63

factors of 105: 1, 3, 5, 7, 15, 21, 35, 105

The greatest common factor of 63 and 105 is 21.

$$63 = \div 21 = 3 \text{ and } 105 \div 21 = 5, \text{ so } \frac{63}{105} = \frac{{}^{3}\cancel{63}}{\cancel{105}_{5}} = \frac{3}{5}$$

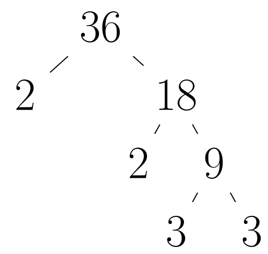
54. List out the factors and simplify these fractions: $\frac{14}{126}$, $\frac{8}{144}$, $\frac{21}{105}$.

Using Prime Factors to Simplify Fractions

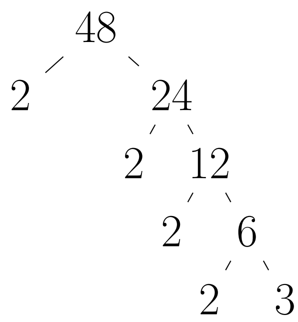
A foolproof way to simplify fractions is to find the prime factors of the numerator and denominator and rewrite the fraction as their product. Doing that can be easier than finding the greatest common factor when the numbers in the fraction get too big.

Remember prime factor trees? Any number that isn't a prime number can be expressed as a product of its prime factors.

To simplify $\frac{36}{48}$,



$36 = 2 \times 2 \times 3 \times 3$



$48 = 2 \times 2 \times 2 \times 2 \times 3$

$$\frac{36}{48} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times \times 3}$$

Now it's easy to see what to cancel.

$$\frac{36}{48} = \frac{{}^1\cancel{2 \times 2} \times 3 \times {}^1\cancel{3}}{{}^1\cancel{2 \times 2} \times 2 \times 2 \times \times \cancel{3}_1} = \frac{1 \times 3 \times 1}{1 \times 2 \times 2 \times 1} = \frac{3}{4}$$

55. Using prime factor trees and writing the denominator and numerator as products of prime numbers, simplify $\frac{84}{144}$, $\frac{48}{80}$, and $\frac{63}{127}$.

Euclid's Algorithm for Finding the Greatest Common Factor

Euclid, often referred to as the "Father of Geometry," was a Greek mathematician who lived around 300 BCE in Egypt. He is best known for his work 'Elements', a collection of 13 books that presented the knowledge of geometry, number theory, and mathematical logic of his time. Euclid's logical approach to mathematics set a standard for rigor in all branches of the subject.

An algorithm is a step-by-step procedure or set of rules designed to perform a specific task or solve a problem, often by breaking down complex problems into smaller, manageable steps.

Euclid's algorithm is an ancient and efficient method for finding the greatest common factor of two integers, which is the largest integer that divides both numbers without leaving a remainder.

The modulo operator, from modular arithmetic and used here, gives the remainder of a division.

Euclid's algorithm is based on the following principle:

> If a and b are two integers where $a > b$, the GCD of a and b is the same as the GCD of b and $a \mod b$.

This means that instead of working with a and b, we can replace a with the remainder of $a \div b$, and repeat the process until the remainder becomes 0. The last nonzero remainder is the GCD.

Steps of Euclid's Algorithm

(a) Divide a by b and find the remainder r (i.e., $a = bq + r$).

(b) Replace a with b and b with r.

(c) Repeat the process until $r = 0$.

(d) The last nonzero b is the GCD.

Example

Find the GCD of 252 and 105 using Euclid's algorithm.

Step 1: Initial Division

$$252 \div 105 = 2 \text{ remainder } 42 \quad \Rightarrow \quad 252 = 105 \times 2 + 42$$

Step 2: Replace and Divide Again

$$105 \div 42 = 2 \text{ remainder } 21 \quad \Rightarrow \quad 105 = 42 \times 2 + 21$$

Step 3: Repeat

$$42 \div 21 = 2 \text{ remainder } 0 \quad \Rightarrow \quad 42 = 21 \times 2 + 0$$

Since the remainder is now 0, the process stops. The last nonzero remainder is 21, so:

$$\text{GCD}(252, 105) = 21$$

Verification: The factors of 252 are

$$1, 2, 3, 4, 6, 7, 9, 12, 14, 18, 21, 28, 36, 42, 63, 84, 126, 252,$$

and the factors of 105 are $1, 3, 5, 7, 15, 21, 35, 105$. The largest common factor is indeed 21.

4 Proper fractions and Improper fractions

Proper

In English, proper mean that something is done right. Having proper manners means that you are doing the right thing.

56. What does proper mean, in your own words?

57. Use proper in a sentence.

Proper Fractions A fraction means some part of a whole. A fraction is proper when the numerator is smaller than the denominator so that it represents a value less than 1. $\frac{3}{4}$ is a proper fraction.

58. What is a proper fraction?

59. Why do you think they are called proper fractions?

60. Give an example of a proper fraction.

Improper

Improper means that something is not being done right. Improper behaviour means bad conduct or being rude.

61. What does improper mean, in your own words?

62. Use improper in a sentence.

Improper Fractions An improper fraction is one where the numerator is bigger than the denominator, so that the value of the fraction is greater than 1. It's not just an equal part of something so it's not properly a fraction. It's some value expressed as a fraction but it's not really a fraction, so it's called an improper fraction.

For example, $\frac{3}{2}$ means that you have 3 halves of something, an improper fraction, because their total is greater than 1.

63. What is an improper fraction?

64. Why are they called improper fractions?

65. Give an example of an improper fraction.

Changing a whole number into an improper fraction

When you have a problem that involves both whole numbers and fractions, the first step to solving it is to express the whole number as a fraction. That's done by writing the whole number as the numerator and using a denominator of 1.

That works because fractions and dividing are similar things. $\frac{1}{2}$ is just another way of expressing $1 \div 2$. Any number divided by 1 doesn't change the number, so a fraction with a denominator of 1 means the same as dividing the numerator by 1, so the fraction still has the same overall value as just its numerator.

For example, $236 = 236 \div 1 = \frac{236}{1}$.

You can change any whole number into an improper fraction just by writing it as a fraction with a denominator of 1.

66. Pick a whole number and change it into an improper fraction.

5 Mixed fractions

Mixed means that two or more things have been put together. You might hear of a dog being a mixed breed. A labradoodle is a mix of labrador and poodle.

67. What does mixed mean, in your own words?

68. Use mixed in a sentence.

A fraction that is a mix of a whole number plus a proper fraction is called a mixed fraction.

$1\frac{3}{4}$ is really saying $1 + \frac{3}{4}$, so it is a mixed fraction.

69. What is a mixed fraction, in your own words?

70. What is an example of a mixed fraction?

As well as simplifying a fraction if you can, you also change an improper fraction to a mixed fraction when you are writing your final answer.

Changing an improper fraction into a mixed fraction

To change an improper fraction to a mixed fraction you divide the numerator by the denominator. The result is the whole number part and the remainder is the numerator of the fraction part.

For example, $\frac{22}{7}$:

$22 \div 7 = 3$, with a remainder of 1.

So $\frac{22}{7} = 3\frac{1}{7}$.

71. What is the improper fraction $\frac{3}{2}$ as a mixed fraction?
72. What is the improper fraction $\frac{14}{5}$ as a mixed fraction?
73. What is the improper fraction $\frac{5}{3}$ as a mixed fraction?

Changing a mixed fraction into an improper fraction

Sometimes you have to change a mixed fraction back into an improper fraction. To do that you write the mixed fraction as the whole number part plus the fraction part. Then you change the whole number part into an improper fraction and add the two fractions.

To find the numerator of the whole number part, multiply the whole number and the denominator of the fraction part.

The denominator of the whole number part is the same denominator as the fraction part.

$$\text{mixed fraction} = \text{whole number} + \frac{\text{numerator}}{\text{denominator}}$$

$$= \frac{\text{whole number} \times \text{denominator}}{\text{denominator}} + \frac{\text{numerator}}{\text{denominator}}$$

$$\text{For example, } 2\frac{3}{5} = 2 + \frac{3}{5} = \frac{2 \times 5}{5} + \frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$$

74. What is the mixed fraction $1\frac{1}{3}$ as an improper fraction?

75. What is the mixed fraction $2\frac{1}{8}$ as an improper fraction?

76. What is the mixed fraction $1\frac{4}{7}$ as an improper fraction?

6 Multiplying fractions

Multiplying two fractions is easy. You just multiply the two numerators and the two denominators and that's your answer.

For example, $\frac{3}{7} \times \frac{5}{9} = \frac{15}{63}$.

And simplify your answer, of course: $\frac{{}^{5}\cancel{15}}{\cancel{63}_{21}} = \frac{5}{21}$.

77. What is $\frac{3}{4} \times \frac{2}{3}$?

78. What is $\frac{4}{5} \times \frac{7}{8}$?

79. What is $\frac{2}{7} \times \frac{1}{8}$?

of In word problems, the word "of" means "times." Things such as "seven eighths of fifteen" are solved by doing $\frac{7}{8} \times 15 = \frac{105}{8} = 13\frac{1}{8}$.

80. What is half of $\frac{3}{4}$?

81. What is $\frac{2}{3}$ of 11?

82. What is $\frac{7}{8}$ of $\frac{5}{6}$?

Simplifying before Multiplying

It is easier to simplify fractions before multiplying because the numbers will be smaller, and the answer will already be simplified.

$$\frac{9}{15} \times \frac{3}{9} = \frac{{}^{3}\cancel{9}}{\cancel{15}_{5}} \times \frac{{}^{1}\cancel{3}}{\cancel{9}_{3}} = \frac{3 \times 1}{5 \times 3} = \frac{1}{15}$$

83. Simplify and multiply: $\frac{6}{8} \times \frac{3}{9}$.

84. Simplify and multiply: $\frac{12}{15} \times \frac{8}{18}$.

85. Simplify and multiply: $\frac{4}{12} \times \frac{12}{15}$.

Cross-Cancelling

Because the order doesn't matter when you multiply, you can cancel either denominator with either numerator.

$$\frac{3}{10} \times \frac{2}{5} = \frac{3 \times 2}{10 \times 5} = \frac{2 \times 3}{10 \times 5} = \frac{2}{10} \times \frac{3}{5}$$

That's why you can do $\frac{3}{\cancel{10}_5} \times \frac{{}^1\cancel{2}}{5} = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$.

You can also do this with any number of fractions.

$$\frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{{}^1\cancel{4}}{5} \times \frac{{}^1\cancel{3}}{\cancel{4}_1} \times \frac{2}{\cancel{3}_1} = \frac{1 \times 1 \times 2}{5 \times 1 \times 1} = \frac{2}{5}$$

86. Cross-cancel and multiply: $\frac{9}{10} \times \frac{2}{3}$
87. Cross-cancel and multiply: $\frac{16}{15} \times \frac{21}{24}$
88. Cross-cancel and multiply: $\frac{2}{7} \times \frac{35}{6}$

Multiplication by Switching Numerators

Instead of cross cancelling it is also possible to reverse the order of multiplication in the numerators resulting in more easily simplified fractions.

$$\frac{21}{5} \times \frac{15}{14} = \frac{21 \times 15}{5 \times 14} = \frac{15 \times 21}{5 \times 14} = \frac{\cancel{15}^3}{\cancel{5}_1} \times \frac{\cancel{21}^3}{\cancel{14}_2} = \frac{9}{2}$$

89. Switch the numerators, simplify and multiply: $\frac{14}{6} \times \frac{3}{21}$.
90. Switch the numerators, simplify and multiply: $\frac{4}{10} \times \frac{5}{6}$.
91. Switch the numerators, simplify and multiply: $\frac{4}{10} \times \frac{5}{8}$.

Multiplying Mixed Fractions

There are a few different ways that you can go about multiplying mixed fractions.

Converting to Improper Fractions

To multiply mixed fractions, convert any mixed fraction to an improper fraction before you do the multiplication.

For example, $1\frac{1}{3} \times 2\frac{1}{4}$:

$$1\frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$$
$$2\frac{1}{4} = \frac{8}{4} + \frac{1}{4} = \frac{9}{4}$$
$$\frac{4}{3} \times \frac{9}{4} = \frac{{}^{3}\cancel{36}}{\cancel{12}_{1}} = \frac{3}{1} = 3$$

92. Convert mixed fractions to improper fractions and multiply: $2\frac{2}{3} \times 1\frac{5}{7}$

Making Mixed Fractions Whole Numbers

A fraction multiplied by its denominator becomes a whole number, which is easier to multiply. If you then also divide by the same amount, the overall product is unchanged.

$$\begin{aligned} 36 \times 3\frac{1}{2} &= 36 \times 3\frac{1}{2} \times 2 \div 2 \\ &= 36 \div 2 \times 3\frac{1}{2} \times 2 \\ &= 18 \times 7 = 126 \end{aligned}$$

93. Convert fractions to whole numbers and multiply: $3\frac{1}{3} \times 4$.

Making Mixed Fractions Mixed Decimal Fractions

Another way is to convert mixed fractions to a decimal fractions.

$$36 \times 3\tfrac{1}{2} =$$

$$\begin{array}{r} 36.0 \\ \times\ {}_{1\,3}3.5 \\ \hline {}^{1}1800 \\ +10800 \\ \hline 126.00 \\ \hline\hline \end{array}$$

The decimal point is placed at the total number of fractional digits of the factors being mutliplied.

94. Convert $2\frac{1}{2}$ to a decimal fraction and multiply by 3.

7 Dividing fractions

Reciprocal Reciprocal means existing on both sides. For example, Australia has a reciprocal agreement with China to receive steel in exchange for iron ore.

95. What does reciprocal mean?

96. What is an example of something else that is reciprocal?

Reciprocal of a fraction The flipped version of a fraction is called its reciprocal. $\frac{2}{1}$ is the reciprocal of $\frac{1}{2}$

97. What does the reciprocal of a fraction mean?

98. What is the reciprocal of $\frac{3}{4}$?

How to divide fractions Dividing fractions is easy. You flip the second fraction first, to get its reciprocal, and then just multiply them.

> (Be careful, though: If you flip the first fraction instead of the second one you will get a wrong answer.)

For example,

$$\frac{3}{7} \div \frac{5}{9} = \frac{3}{7} \times \frac{9}{5} = \frac{27}{35}$$

Another example:

$$\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5} = \frac{{}^{4}\cancel{12}}{\cancel{15}_{5}} = \frac{4}{5}$$

99. What is $\frac{3}{4} \div \frac{2}{3}$?

100. What is $\frac{3}{8} \div \frac{5}{9}$?

101. What is $\frac{2}{7} \div \frac{1}{6}$?

Dividing mixed fractions

Convert mixed fractions to improper fractions first, do the division, simplify, and then convert the answer back to a mixed fraction.

For example, divide $8\frac{1}{3}$ by 3 :

First convert both numbers to improper fractions:

$$8\frac{1}{3} \div 3 = \frac{24}{3} + \frac{1}{3} \div \frac{3}{1} = \frac{25}{3} \div \frac{3}{1}$$

Flip one of the fractions, and multiply:

$$\frac{25}{3} \div \frac{3}{1} = \frac{25}{3} \times \frac{1}{3} = \frac{25}{9}$$

Then convert the answer back to a proper fraction:

$$\frac{25}{9} = 2\frac{7}{9}$$

102. What is $1\frac{1}{2} \div 4$?

103. What is $3 \div \frac{2}{3}$?

104. What is $3\frac{1}{4} \div 2\frac{2}{5}$?

8 Comparing fractions

Obviously, comparing fractions that have the same denominator is easy because the parts that you are comparing are all the same size and you just have to compare the numerators.

$$\frac{4}{7} < \frac{5}{7}$$

Common Denominator Comparing two fractions with different denominators isn't so easy.

Which is bigger: $\frac{2}{3}$ or $\frac{3}{5}$? You can only tell once you have converted them both into equivalent fractions with the same denominators.

Once you work out that $\frac{2}{3} = \frac{10}{15}$ and $\frac{3}{5} = \frac{9}{15}$, then you can easily see that the answer is $\frac{2}{3} > \frac{3}{5}$.

Common means shared by both people or things, like when two people have something in common, so finding the denominator that both fractions can be changed into is called finding the common denominator.

105. Compare $\frac{1}{2}$ and $\frac{5}{8}$. Which is bigger?

Cross-multiplying A faster way of comparing fractions is cross-multiplying. That means multiplying the numerator of one fraction by the denominator of the other fraction.

$$\text{For example, comparing } \frac{2}{3} \text{ and } \frac{3}{5}:$$

$$2 \times 5 = 10 \text{ and } 3 \times 3 = 9, \text{ so } \frac{2}{3} > \frac{3}{5}.$$

106. Cross-multiply and compare $\frac{5}{7}$ and $\frac{6}{11}$. Which is bigger?

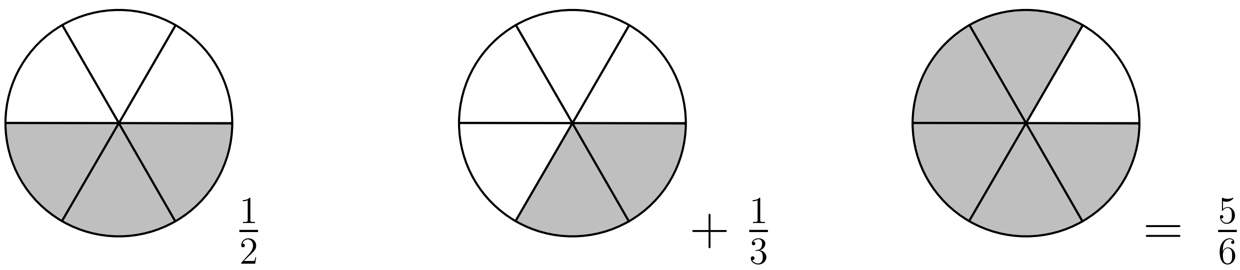
9 Adding and Subtracting Fractions

Adding and Subtracting fractions that have the same denominator is easy. Just add or subtract their numerators.

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

Adding or subtracting two fractions that have different denominators isn't so easy. You can't just add the numerators or you'll get a wrong answer. The parts that you are adding or subtracting have to be of the same size or your answer won't be right.

Common Denominator To add or subtract fractions with different denominators, you have to change both fractions into equivalent fractions with a common denominator. Then you can add or subtract the numerators and you'll get a right answer.



$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Halves and thirds have to be cut into sixths before you can add them.

Finding the common denominator

You need to find a number to multiply the top and bottom of each fraction by that will give both fractions the same denominator.

Multiply the denominators

The easiest way to find a common denominator is to just multiply the two denominators.

For example, $\frac{1}{3} + \frac{1}{4}$:

Common denominator: $3 \times 4 = 12$.

Multiply both fractions to convert them to equivalent fractions with this common denominator:

$$\frac{1}{3} + \frac{1}{4} = (\frac{1}{3} \times \frac{4}{4}) + (\frac{1}{4} \times \frac{3}{3}) = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Lowest Common Denominator Finding the lowest common denominator will make sure that you are multiplying by the smallest possible numbers. Also, the total of the fractions will be already expressed in it's simplest form.

The lowest common denominator is also called the lowest common multiple. These are sometimes written just as LCD or LCM.

Listing the multiples of the denominators

A thorough way to find common denominators is to list out the multiples of each denominator and pick the lowest number that is common to both lists.

107. Add $\frac{3}{7}$ and $\frac{2}{5}$ by listing the multiples of their denominators to find the lowest common denominator.

So if we were adding $\frac{1}{4} + \frac{1}{5}$:

Multiples of 4: 4, 8, 12, 16, 20

Multiples of 5: 5, 10, 15, 20, 25

The lowest common multiple is 12, so:

$$\frac{1}{4} + \frac{1}{5} = (\frac{1}{4} \times \frac{5}{5}) + (\frac{1}{5} \times \frac{4}{4}) = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}.$$

Dividing the GCF of the denominators by the product of the denominators

Another way to find the lowest common denominator is to multiply the two denominators and divide their product by the greatest common factor of the denominators.

$$\text{Lowest Common Denominator} = \frac{\text{Greatest Common Factor of the Denominators}}{\text{Product of the Denominators}}$$

For example, $\frac{3}{8} + \frac{5}{12}$:

factors of 8: 1, 2, 4, 8

factors of 12: 1, 2, 3, 4, 6, 12

Greatest common factor of the denominators is 4.

Product of the denominators is $8 \times 12 = 96$.

LCD for $\frac{3}{8} + \frac{5}{12}$ is $\frac{8 \times 12}{4} = \frac{{}^{24}\cancel{96}}{\cancel{4}_1} = 24$

So, $\frac{3}{8} + \frac{5}{12} = (\frac{3}{8} \times \frac{3}{3}) + (\frac{5}{12} \times \frac{2}{2}) = \frac{9}{24} + \frac{10}{24} = \frac{19}{24}$

108. Using this method, what is $\frac{4}{9} + \frac{7}{8}$?

Finding which multiplier to use

To find just what multiplier is needed, divide the lowest common denominator by the denominator.

Say you have $\frac{1}{3}$ and $\frac{1}{4}$.
The lowest common denominator of 3 and 4 is 12.
The denominator of $\frac{1}{3}$ is 3.
$12 \div 3 = 4$, so multiply $\frac{1}{3}$ by $\frac{4}{4}$ to get $\frac{4}{12}$.
In the same way, $12 \div 4 = 3$, so multiply $\frac{1}{4}$ by $\frac{3}{3}$ to get $\frac{3}{12}$.

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}.$$

Here is another example with subtraction:

$$\frac{3}{5} - \frac{1}{4}$$

Common denominator: $5 \times 4 = 20$.

$$\frac{20}{5} = 4, \text{ so multiply } \frac{3}{5} \text{ by } \frac{4}{4}$$

$$\frac{20}{4} = 5, \text{ so multiply } \frac{1}{4} \text{ by } \frac{5}{5}:$$

$$\frac{3}{5} - \frac{1}{4} = \left(\frac{3}{5} \times \frac{4}{4}\right) - \left(\frac{1}{4} \times \frac{5}{5}\right) = \frac{12}{20} - \frac{5}{20} = \frac{7}{20}$$

109. What is $\frac{1}{2} + \frac{1}{7}$?

110. What is $\frac{2}{5} + \frac{3}{8}$?

111. What is $\frac{4}{9} + \frac{2}{5}$?

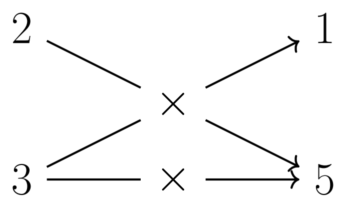
112. What is $\frac{3}{4} + \frac{4}{9}$?

113. What is $\frac{9}{11} + \frac{2}{7}$?

Cross-Multiplying

A simple method to add and subtract fractions is cross-multiplying:

$$\frac{2}{3} + \frac{1}{5} =$$



$$= \frac{(2 \times 5) + (3 \times 1)}{(3 \times 5)}$$

$$= \frac{10 + 3}{15} = \frac{13}{15}$$

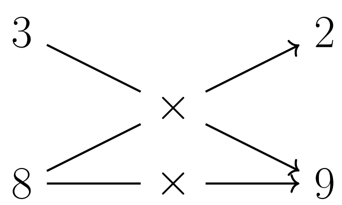
114. Using cross-multiplying, what is $\frac{2}{3} + \frac{2}{5}$?

115. Using cross-multiplying, what is $\frac{3}{4} + \frac{4}{7}$?

116. Using cross-multiplying, what is $\frac{4}{9} + \frac{1}{12}$?

Here is another example of cross multiplying to add fractions:

$$\frac{3}{8} + \frac{2}{9} =$$



$$= \frac{(3 \times 9) + (8 \times 2)}{(8 \times 9)}$$

$$= \frac{27 + 16}{72} = \frac{43}{72}.$$

Congratulations!

You have mastered fractions.

Now review any parts of this course that you are unsure of, and then work your way through these final questions:

Questions:

1. Compare $\frac{1}{3}$ and $\frac{2}{5}$.
2. Add $\frac{3}{4}$ and $\frac{1}{8}$.
3. Subtract $\frac{5}{6}$ from $\frac{2}{3}$.
4. Multiply $\frac{2}{3}$ and $\frac{4}{5}$.
5. Divide $\frac{3}{4}$ by $\frac{2}{3}$.
6. Find an equivalent fraction for $\frac{1}{4}$ with a denominator of 12.
7. Simplify $\frac{6}{12}$.
8. Calculate $\frac{9}{15}$ in its simplest form.
9. Find the greatest common factor (GCF) of 24 and 36.
10. Determine the lowest common denominator for $\frac{1}{3}$ and $\frac{1}{4}$.
11. Add $\frac{2}{7}$ and $\frac{3}{7}$.
12. Subtract $\frac{5}{8}$ from $\frac{3}{8}$.
13. Multiply $\frac{2}{9}$ and $\frac{3}{4}$.
14. Divide $\frac{5}{6}$ by $\frac{1}{2}$.
15. Find an equivalent fraction for $\frac{3}{5}$ with a denominator of 20.
16. Simplify $\frac{12}{24}$.
17. Calculate $\frac{8}{10}$ in its simplest form.
18. Find the GCF of 18 and 27.
19. Determine the LCD for $\frac{1}{5}$ and $\frac{1}{6}$.
20. Add $\frac{1}{8}$ and $\frac{1}{16}$.

21. Subtract $\frac{5}{9}$ from $\frac{4}{9}$.

22. Multiply $\frac{2}{5}$ and $\frac{5}{7}$.

23. Divide $\frac{3}{4}$ by $\frac{2}{5}$.

24. Find an equivalent fraction for $\frac{2}{3}$ with a denominator of 9.

25. Simplify $\frac{15}{30}$.

26. Calculate $\frac{10}{20}$ in its simplest form.

27. Find the GCF of 36 and 48.

28. Determine the LCD for $\frac{2}{9}$ and $\frac{1}{6}$.

29. Add $\frac{1}{2}$ and $\frac{3}{4}$.

30. Subtract $\frac{5}{6}$ from $\frac{7}{8}$.

31. Multiply $\frac{1}{3}$ and $\frac{4}{9}$.

32. Divide $\frac{2}{5}$ by $\frac{3}{4}$.

33. Find an equivalent fraction for $\frac{5}{7}$ with a denominator of 35.

34. Simplify $\frac{20}{40}$.

35. Calculate $\frac{14}{28}$ in its simplest form.

36. Find the GCF of 42 and 56.

37. Determine the LCD for $\frac{3}{5}$ and $\frac{2}{3}$.

38. Add $\frac{4}{7}$ and $\frac{5}{7}$.

39. Subtract $\frac{9}{10}$ from $\frac{3}{10}$.

40. Multiply $\frac{3}{5}$ and $\frac{4}{7}$.

41. Divide $\frac{2}{3}$ by $\frac{1}{4}$.

42. Find an equivalent fraction for $\frac{4}{6}$ with a denominator of 12.

43. Simplify $\frac{16}{32}$.

44. Calculate $\frac{9}{18}$ in its simplest form.

45. Find the GCF of 30 and 45.

46. Determine the LCD for $\frac{2}{8}$ and $\frac{3}{12}$.

47. Add $\frac{1}{3}$ and $\frac{1}{6}$.

48. Subtract $\frac{7}{8}$ from $\frac{1}{8}$.

49. Multiply $\frac{3}{4}$ and $\frac{2}{3}$.

50. Divide $\frac{5}{6}$ by $\frac{2}{5}$.

51. Convert the mixed fraction $1\frac{3}{4}$ into an improper fraction.

52. Add $2\frac{2}{3}$ and $3\frac{1}{6}$.

53. Subtract $4\frac{3}{5}$ from $5\frac{4}{7}$.

54. Multiply $2\frac{1}{2}$ and $3\frac{2}{3}$.

55. Divide $6\frac{1}{4}$ by $2\frac{1}{2}$.

56. Convert the improper fraction $\frac{11}{4}$ into a mixed fraction.

57. Add $\frac{9}{2}$ and $3\frac{1}{3}$.

58. Subtract $\frac{7}{5}$ from $4\frac{3}{10}$.

59. Multiply $\frac{5}{8}$ and $2\frac{3}{4}$.

60. Divide $7\frac{2}{9}$ by $\frac{3}{4}$.

Answers

1. $\frac{1}{3} < \frac{2}{5}$
2. $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$
3. $\frac{2}{3} - \frac{5}{6} = \frac{1}{6}$
4. $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$
5. $\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$
6. $\frac{1}{4}$ with a denominator of 12 is $\frac{3}{12}$
7. $\frac{6}{12} = \frac{1}{2}$
8. $\frac{9}{15} = \frac{3}{5}$
9. GCF of 24 and 36 is 12.
10. LCD for $\frac{1}{3}$ and $\frac{1}{4}$ is 12.
11. $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$
12. $\frac{3}{8} - \frac{5}{8} = -\frac{1}{8}$
13. $\frac{2}{9} \cdot \frac{3}{4} = \frac{1}{6}$
14. $\frac{5}{6} \div \frac{1}{2} = \frac{5}{3}$
15. $\frac{3}{5}$ with a denominator of 20 is $12_{\overline{20}}$
16. $\frac{12}{24} = \frac{1}{2}$
17. $\frac{8}{10} = \frac{4}{5}$
18. GCF of 18 and 27 is 9.
19. LCD for $\frac{1}{5}$ and $\frac{1}{6}$ is 30.
20. $\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$
21. $\frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$

22. $\frac{2}{5} \cdot \frac{5}{7} = \frac{2}{7}$

23. $\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$

24. $\frac{2}{3}$ with a denominator of 9 is $\frac{6}{9}$

25. $\frac{15}{30} = \frac{1}{2}$

26. $\frac{10}{20} = \frac{1}{2}$

27. GCF of 36 and 48 is 12.

28. LCD for $\frac{2}{9}$ and $\frac{1}{6}$ is 18.

29. $\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$

30. $\frac{7}{8} - \frac{5}{6} = \frac{1}{24}$

31. $\frac{1}{3} \cdot \frac{4}{9} = \frac{4}{27}$

32. $\frac{2}{5} \div \frac{3}{4} = \frac{8}{15}$

33. $\frac{5}{7}$ with a denominator of 35 is

34.

35. $\frac{25}{35}$

36. $\frac{20}{40} = \frac{1}{2}$

37. $\frac{14}{28} = \frac{1}{2}$

38. GCF of 42 and 56 is 14.

39. LCD for $\frac{3}{5}$ and $\frac{2}{3}$ is 15.

40. $\frac{4}{7} + \frac{5}{7} = \frac{9}{7}$

41. $\frac{3}{10} - \frac{9}{10} = -\frac{6}{10}$

42. $\frac{3}{5} \cdot \frac{4}{7} = \frac{12}{35}$

43. $\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$

44. $\frac{4}{6}$ with a denominator of 12 is $\frac{8}{12}$

45. $\frac{16}{32} = \frac{1}{2}$

46. $\frac{9}{18} = \frac{1}{2}$

47. GCF of 30 and 45 is 15.

48. LCD for $\frac{2}{8}$ and $\frac{3}{12}$ is 12.

49. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

50. $\frac{1}{8} - \frac{7}{8} = -\frac{3}{4}$

51. $\frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$

52. $\frac{5}{6} \div \frac{2}{5} = \frac{25}{12}$

53. $1\frac{3}{4}$ as an improper fraction is $\frac{7}{4}$.

54. $2\frac{2}{3} + 3\frac{1}{6} = 6\frac{1}{2}$.

55. $5\frac{4}{7} - 4\frac{3}{5} = 1\frac{25}{35}$ (simplified to $2\frac{5}{7}$).

56. $2\frac{1}{2} \cdot 3\frac{2}{3} = 7\frac{1}{6}$.

57. $6\frac{1}{4} \div 2\frac{1}{2} = 2\frac{1}{8}$.

58. $\frac{11}{4}$ as a mixed fraction is $2\frac{3}{4}$.

59. $\frac{9}{2} + 3\frac{1}{3} = 5\frac{2}{3}$.

60. $4\frac{3}{10} - \frac{7}{5} = 3\frac{1}{10}$.

61. $\frac{5}{8} \cdot 2\frac{3}{4} = 1\frac{11}{16}$.

62. $7\frac{2}{9} \div \frac{3}{4} = 9\frac{1}{3}$.