Introduction to Linear Algebra

Tutoring Centre Ferndale

Linear Equations

A linear equation is an equation that describes a straight line when its solutions are plotted on a graph. There are three common forms of linear equations:

Standard Form

The standard form of a linear equation is given by:

$$Ax + By = C$$

where:

- A, B, and C are constants.
- x and y are variables.

Slope-Intercept Form

The slope-intercept form of a linear equation is given by:
$y = mx + b$
where:

- y is the dependent variable (the output value).
- x is the independent variable (the input value).

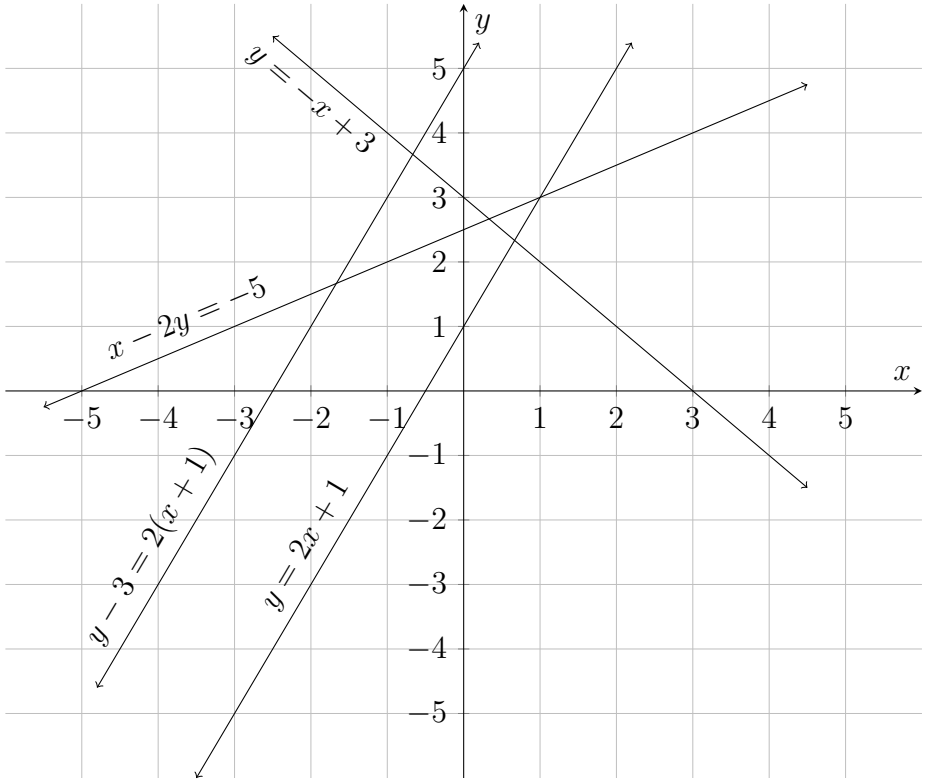
- m represents the slope of the line.
- b is the y-intercept, which is the point where the line crosses the y-axis.

Point-Slope Form

In the point-slope form, $y - y_1 = m(x - x_1)$:

- (x_1, y_1) is a point on the line.
- m is the slope of the line.

Graphs of Linear Equations



Can you identify the forms of the linear equations that are plotted here?

Effects of Changing Values

Changing the slope m:

- A positive slope means the line rises from left to right.
- A negative slope means the line falls from left to right.
- A larger absolute value of the slope means a steeper line.

Changing the y-intercept b:

- Increases in b shift the line up.
- Decreases in b shift the line down.

Practice Problems

Solve the following real-world problems using linear equations:

Problem 1

A taxi company charges a base fare of \$3 and an additional \$2 per mile driven. Write the linear equation representing the total fare y in terms of the number of miles x driven. Calculate the total fare for a 5-mile trip.

Solution:

$$y = 2x + 3$$

For $x = 5$:

$$y = 2(5) + 3 = 10 + 3 = 13$$

Total fare: \$13

Problem 2

A plant grows at a constant rate. After 2 weeks, the plant is 10 cm tall, and after 5 weeks, it is 25 cm tall. Write the linear equation representing the height y of the plant in terms of the number of weeks x. Determine the height of the plant after 8 weeks.

Solution:
First, find the slope m:

$$m = \frac{\Delta y}{\Delta x} = \frac{25 - 10}{5 - 2} = \frac{15}{3} = 5$$

Using the point-slope form $y - y_1 = m(x - x_1)$ and point (2, 10):

$$y - 10 = 5(x - 2)$$

Simplify to slope-intercept form:

$$y - 10 = 5x - 10$$

$$y = 5x$$

For $x = 8$:

$$y = 5(8) = 40$$

Height after 8 weeks: 40 cm

Conclusion

Linear algebra provides the tools to describe and analyze linear relationships. Understanding the forms of linear equations, their graphs, and the effects of changing values helps solve real-world problems. Practice with these problems to strengthen your understanding of linear equations and their applications.