

Understanding the Standard Form of Linear Equations

Tutoring Centre Ferndale

In linear algebra, the slope-intercept form $y = mx + b$ is often preferred for its intuitive representation of the slope (m) and the y-intercept (b). However, the standard form of a linear equation, $Ax + By = C$, provides a different perspective on linear relationships. Understanding how changes in A , B , and C affect the graph can offer valuable insights.

Standard Form: $Ax + By = C$

In the standard form, $Ax + By = C$:

- A , B , and C are constants.
- A and B determine the slope and orientation of the line.
- C affects the position of the line without changing its slope.

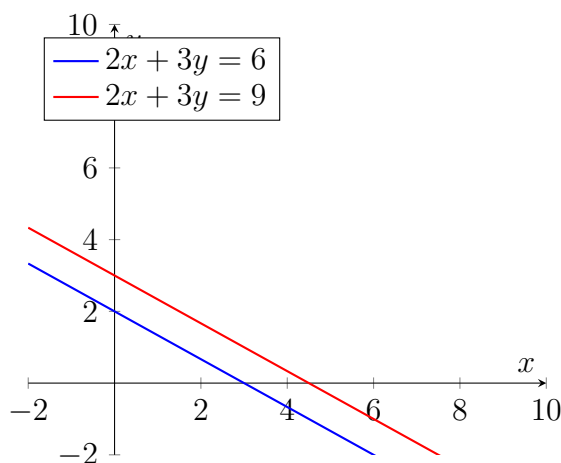
Questions

1. What are the constants in the standard form $Ax + By = C$?
2. How do A and B influence the line in standard form?
3. What is the role of C in the standard form?

Effect of Changing C

- C affects the line's position but not its slope.
- Increasing C shifts the line upwards or to the right, while decreasing C shifts it downwards or to the left.
- The line remains parallel to its original position because the slope, given by $-A/B$, remains unchanged.

Example



+The new line $2x + 3y = 9$ is parallel to $2x + 3y = 6$ but shifted upwards or to the right.

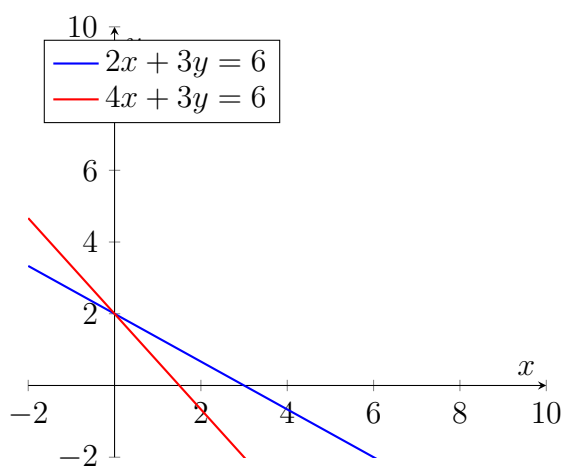
Questions

1. How does increasing C affect the line's position?
2. How does decreasing C affect the line's position?
3. Does the slope of the line change when C changes?

Effect of Changing A

- A influences the slope and x-intercept.
- Increasing A makes the line steeper in the x-direction, while decreasing A makes it less steep.

Example



The new line $4x + 3y = 6$ is steeper in the x-direction compared to $2x + 3y = 6$.

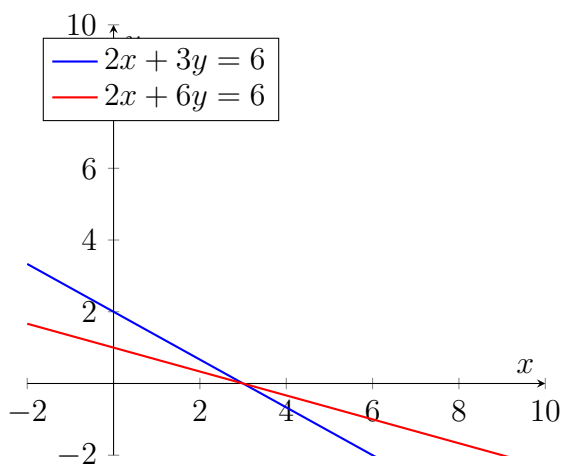
Questions

1. How does increasing A affect the line's steepness?
2. How does decreasing A affect the line's steepness?
3. What happens to the x-intercept when A changes?

Effect of Changing B

- B influences the slope and y-intercept.
- Increasing B makes the line less steep in the y-direction, while decreasing B makes it steeper.

Example



The new line $2x + 6y = 6$ is less steep in the y-direction compared to $2x + 3y = 6$.

Questions

1. How does increasing B affect the line's steepness?
2. How does decreasing B affect the line's steepness?
3. What happens to the y-intercept when B changes?

Converting Between Forms

To fully appreciate the effects, it can be helpful to convert the standard form $Ax + By = C$ to the slope-intercept form $y = mx + b$:

$$Ax + By = C$$

$$By = -Ax + C$$

$$y = -\frac{A}{B}x + \frac{C}{B}$$

Here, the slope m is $-\frac{A}{B}$, and the y-intercept b is $\frac{C}{B}$.

Questions

1. What is the slope in the slope-intercept form derived from $Ax + By = C$?
2. What is the y-intercept in the slope-intercept form derived from $Ax + By = C$?
3. Convert the standard form $4x + 5y = 20$ to the slope-intercept form.

Summary

- **Changing** C shifts the line without altering its slope.
- **Changing** A adjusts the line's steepness in the x-direction.
- **Changing** B adjusts the line's steepness in the y-direction.

Understanding these effects can help in analyzing and graphing linear equations in standard form. Each parameter plays a unique role in defining the line's properties and behavior on the Cartesian plane.

Answers

Standard Form: $Ax + By = C$

- (a) The constants are A , B , and C .
- (b) A and B influence the slope and orientation of the line.
- (c) C affects the position of the line without changing its slope.

Effect of Changing C

- (a) Increasing C shifts the line upwards or to the right.
- (b) Decreasing C shifts the line downwards or to the left.
- (c) The slope of the line does not change when C changes.

Effect of Changing A

- (a) Increasing A makes the line steeper in the x-direction.
- (b) Decreasing A makes the line less steep in the x-direction.
- (c) The x-intercept changes as A changes.

Effect of Changing B

- (a) Increasing B makes the line less steep in the y-direction.
- (b) Decreasing B makes the line steeper in the y-direction.
- (c) The y-intercept changes as B changes.

Converting Between Forms

- (a) The slope is $-\frac{A}{B}$.
- (b) The y-intercept is $\frac{C}{B}$.
- (c) Converting $4x + 5y = 20$ to slope-intercept form:

$$5y = -4x + 20$$

$$y = -\frac{4}{5}x + 4$$