Factorizing Polynomials

Tutoring Centre Ferndale

Factorizing polynomials simplifies expressions and solves equations.

Perfect Squares

- A trinomial of the form $a^2 + 2ab + b^2$ factors into $(a + b)^2$.
- A trinomial of the form $a^2 - 2ab + b^2$ factors into $(a - b)^2$.

Examples

$$\begin{aligned} x^2 + 6x + 9 &= (x + 3)^2 \\ 4x^2 - 12x + 9 &= (2x - 3)^2 \\ y^2 + 10y + 25 &= (y + 5)^2 \end{aligned}$$

Exercises

Factor the following trinomials:

1. $x^2 + 8x + 16$
2. $9x^2 - 24x + 16$
3. $4y^2 + 12y + 9$
4. $z^2 - 14z + 49$

Answers

1. $x^2 + 8x + 16 = (x + 4)^2$
2. $9x^2 - 24x + 16 = (3x - 4)^2$
3. $4y^2 + 12y + 9 = (2y + 3)^2$
4. $z^2 - 14z + 49 = (z - 7)^2$

Difference of Squares

A binomial of the form $a^2 - b^2$ factors into $(a + b)(a - b)$.

Examples

1. $x^2 - 9 = (x + 3)(x - 3)$
2. $4x^2 - 25 = (2x + 5)(2x - 5)$
3. $y^2 - 16 = (y + 4)(y - 4)$

Exercises

Factor the following expressions:

1. $x^2 - 36$
2. $49x^2 - 1$
3. $25y^2 - 64$
4. $z^2 - 81$

Answers

1. $x^2 - 36 = (x + 6)(x - 6)$
2. $49x^2 - 1 = (7x + 1)(7x - 1)$
3. $25y^2 - 64 = (5y + 8)(5y - 8)$
4. $z^2 - 81 = (z + 9)(z - 9)$

General Factorization Techniques

There are several general methods for factorizing polynomials. These include:

- Factoring out the Greatest Common Factor (GCF)
- Factoring by Grouping
- Factoring Trinomials
- Special Cases (Cubic Polynomials)

Factoring out the Greatest Common Factor (GCF)

The first step in many factoring problems is to factor out the GCF, which is the largest factor common to all terms in the polynomial.

Examples

1. $6x^2 + 9x = 3x(2x + 3)$
2. $8x^3 - 4x^2 + 2x = 2x(4x^2 - 2x + 1)$
3. $15x^4y - 25x^2y^2 = 5x^2y(3x^2 - 5y)$

Exercises

Factor out the GCF:

1. $12x^3 + 18x^2$
2. $10x^2y + 15xy^2$
3. $24x^4y^2 - 16x^3y$
4. $14x^3 - 21x^2 + 28x$

Answers

1. $12x^3 + 18x^2 = 6x^2(2x + 3)$
2. $10x^2y + 15xy^2 = 5xy(2x + 3y)$
3. $24x^4y^2 - 16x^3y = 8x^3y(3x - 2)$
4. $14x^3 - 21x^2 + 28x$
   $= 7x(2x^2 - 3x + 4)$

Factoring by Grouping

When a polynomial has four or more terms, grouping terms to factor out a common factor can be effective.

Examples

$$\begin{aligned} x^3+3x^2+x+3 &= (x^3+3x^2)+(x+3) \\ &= x^2(x+3)+1(x+3) \\ &= (x^2+1)(x+3) \end{aligned}$$

$$\begin{aligned} 2x^3+4x^2+3x+6 &= (2x^3+4x^2)+(3x+6) \\ &= 2x^2(x+2)+3(x+2) \\ &= (2x^2+3)(x+2) \end{aligned}$$

$$\begin{aligned} 3x^2y-6xy+2x-4 &= (3x^2y-6xy)+(2x-4) \\ &= 3xy(x-2)+2(x-2) \\ &= (3xy+2)(x-2) \end{aligned}$$

Exercises

Factor the following polynomials by grouping:

1. x^3-2x^2+x-2
2. $2x^2+6x+x+3$
3. $4xy+8y-x-2$
4. $3x^2+5x+6x+10$

Answers

$$\begin{aligned} 1.\ x^3-2x^2+x-2 &= (x^3-2x^2)+(x-2) \\ &= x^2(x-2)+1(x-2) \\ &= (x^2+1)(x-2) \end{aligned}$$

$$\begin{aligned} 2.\ 2x^2+6x+x+3 &= (2x^2+6x)+(x+3) \\ &= 2x(x+3)+1(x+3) \\ &= (2x+1)(x+3) \end{aligned}$$

$$
\begin{aligned}
3.\ 4xy + 8y - x - 2 &= (4xy + 8y) - (x + 2) \\
&= 4y(x + 2) - 1(x + 2) \\
&= (4y - 1)(x + 2)
\end{aligned}
$$

$$
\begin{aligned}
4.\ 3x^2 + 5x + 6x + 10 &= (3x^2 + 5x) + (6x + 10) \\
&= x(3x + 5) + 2(3x + 5) \\
&= (x + 2)(3x + 5)
\end{aligned}
$$

Factoring Trinomials

Trinomials of the form $ax^2 + bx + c$ can often be factored into the product of two binomials.

Examples

1. $x^2 + 5x + 6 = (x + 2)(x + 3)$
2. $2x^2 + 7x + 3 = (2x + 1)(x + 3)$
3. $3x^2 - x - 4 = (3x + 4)(x - 1)$

Exercises

Factor the following trinomials:

1. $x^2 + 7x + 12$
2. $2x^2 + 3x - 2$
3. $3x^2 + 8x + 4$
4. $4x^2 + 11x + 6$

Answers

1. $x^2 + 7x + 12 = (x + 3)(x + 4)$
2. $2x^2 + 3x - 2 = (2x - 1)(x + 2)$
3. $3x^2 + 8x + 4 = (3x + 2)(x + 2)$
4. $4x^2 + 11x + 6 = (4x + 3)(x + 2)$

Sum and Difference of Cubes

Sum of Cubes

The sum of cubes is a polynomial of the form $a^3 + b^3$. It factors into:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Examples

1. $x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$
2. $27x^3 + 1 = (3x)^3 + 1^3 = (3x + 1)(9x^2 - 3x + 1)$
3. $y^3 + 125 = y^3 + 5^3 = (y + 5)(y^2 - 5y + 25)$

Exercises

Factor the following sums of cubes:

1. $x^3 + 27$
2. $8x^3 + 1$
3. $y^3 + 64$
4. $125x^3 + 8$

Answers

1. $x^3 + 27 = (x + 3)(x^2 - 3x + 9)$
2. $8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$
3. $y^3 + 64 = (y + 4)(y^2 - 4y + 16)$
4. $125x^3 + 8 = (5x + 2)(25x^2 - 10x + 4)$

Difference of Cubes

The difference of cubes is a polynomial of the form $a^3 - b^3$. It factors into:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

The difference of cubes is similar to the sum of cubes, except that the sign between the two cubes is negative.

Examples

1. $x^3 - 8 = x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$
2. $27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)(9x^2 + 3x + 1)$
3. $y^3 - 125 = y^3 - 5^3 = (y - 5)(y^2 + 5y + 25)$

Exercises

Factor the following differences of cubes:

1. $x^3 - 27$
2. $8x^3 - 1$
3. $y^3 - 64$
4. $125x^3 - 8$

Answers

1. $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$
2. $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$
3. $y^3 - 64 = (y - 4)(y^2 + 4y + 16)$
4. $125x^3 - 8 = (5x - 2)(25x^2 + 10x + 4)$