Midterm Review

CAS CS 132: Geometric Algorithms

1 Solving Systems of Linear Equations

Find a solution the following system of linear equations.

$$\begin{aligned} x_1 + x_2 - x_3 &= -9 \\ x_2 - 2x_3 &= -1 \\ x_1 + x_2 &= -10 \end{aligned}$$

Solution.

2 LAA 1.2.18

Determine all values of h such that the following matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix}$$

Solution.

3 General Form Solutions

Consider the following system of linear equations.

$$
\begin{aligned}
x_1 - 5x_2 - x_3 - 2x_4 &= 3 \\
x_3 - 2x_4 &= 11 \\
(-2)x_3 + 5x_4 &= -24
\end{aligned}
$$

A. Write down a general form solution which describes the solution set of the following system of linear equations.

B. Write down a different general form solution which describes the same solution set (i.e., one in which a different variable is free).

Solution.

4 LAA 1.3.13

Consider the following matrix A and vector $\mathbf{b}$.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ h \end{bmatrix}$$

A. Determine if $\mathbf{b}$ can be written as a linear combination of the columns of A if $h = -3$.

B. For what values of h can $\mathbf{b}$ be written as a linear combination of the columns of A.

Solution.

5 Intersections of Spans

Consider the following vectors.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

Find a nonzero vector which lies in both $\mathsf{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ and $\mathsf{span}\{\mathbf{v}_3, \mathbf{v}_4\}$.

Solution.

6 LAA 1.7.31

Find a nontrivial solution to matrix equation $A\mathbf{x} = \mathbf{b}$ without performing any row reductions. (Hint. What is the relationship between the first two columns and the last column of A?)

$$A = \begin{bmatrix} 2 & 3 & 5 \\ -5 & 1 & -4 \\ -3 & -1 & -4 \\ 1 & 0 & 1 \end{bmatrix}$$

Solution.

7 Linearly Independent Vectors

Consider three arbitrary vectors

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

and suppose that they are linearly independent.

A. What is the maximum number of entries of these vectors which can be 0? Note that the solution will be a number between 0 and 9.

B. What is the minimum number?

In each case provide an example.

Solution.

8 Drawing Linear Transformations

Draw the unit square after being transformed by the matrix transformation implemented by

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Solution.

9 Matrices of Linear Transformations

Find the matrix which implemented the following transformation.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \mapsto \begin{bmatrix} v_2 \\ v_1 \\ v_3 + v_4 \end{bmatrix}$$

Solution.

10 3D Linear Transformations

Considering the transformation T implemented by the following matrix.

$$\begin{bmatrix} \cos 2 & 0 & -\sin 2 \\ 0 & 1 & 0 \\ \sin 2 & 0 & \cos 2 \end{bmatrix}$$

Describe geometrically what T does. Then find a vector $\mathbf{v}$ whose span is not changed by this transformation (i.e., $\mathsf{span}\{\mathbf{v}\} = \mathbf{span}\{T(\mathbf{v})\}$).

Solution.