Week 11 Discussion

CAS CS 132: Geometric Algorithms

November 13, 2023

During discussion sections, we will go over three problems.

- The first will be a warm-up question, to help you verify your understanding of the material.

- The second will be a solution to a problem on the assignment of the previous week.

- The third will be a problem similar to one on the assignment of the following week.

The remainder of the time will be dedicated to open Q&A.

1 Eigenvalues, Eigenvectors, Eigenspaces

A. Find an invertible 2×2 matrix with no eigenvalues.

B. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation which projects points onto the $x_1 x_2$-plane. Find the eigenvalues and bases for the corresponding eigenspaces of the matrix implementing this transformation *without doing any calculations.* Then write down the matrix implementing this transformation and find its characteristic polynomial. Check that the eigenvalues you get from the characteristic polynomial are the same.

C. Find the eigenvalues and bases for the corresponding eigenspace of

$$\begin{bmatrix} 1 & -4 \\ -3 & 5 \end{bmatrix}$$

Solution.

2 Complement of the Column Space

Let A be a $5 \times n$ matrix such that rank $A = 4$, which has an LU decomposition where

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 0 & 3 & -3 & 0 & 1 \end{bmatrix}$$

Determine if $\mathbf{v}$ in Col A, where

$$\mathbf{v} = \begin{bmatrix} 2 \\ -5 \\ -11 \\ 5 \\ -12 \end{bmatrix}$$

Solution.

3 Characteristic Polynomials

Find the characteristic polynomial for the matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 2 & -4 \\ 0 & 1 & 5 \end{bmatrix}$$

Use this to determine the eigenvalues of A.

Solution.