Homework 2

CAS CS 132: Geometric Algorithms

Due: **September 21 at 2PM EST**

Submission Instructions

- Make the answer in your solution to each problem abundantly clear (e.g., put a box around your answer or used a colored font if there is a lot of text which is not part of the answer).

- Choose the correct pages corresponding to each problem in Gradescope. Note that Gradescope registers your submission as soon as you submit it, so you don't need to rush to choose corresponding pages.

Graders have license to dock points if either of the above instructions are not properly followed.

Practice Problems

The following list of problems comes from *Linear Algebra and its Application 5th Ed* by David C. Lay, Steven R. Lay, and Judi J. McDonald. They may be useful for solidifying your understanding of the material and for studying in general. **They are optional, so please don't submit anything for them**.

- (page 21) 1.2.1

- (page 22) 1.2.3, 1.2.15, 1.2.16, 1.2.17

- (page 32) 1.3.1, 1.3.6, 1.3.7, 1.3.12

- (page 33) 1.3.18, 1.3.19

1 (2×2) Echelon Forms

(10 pts) Write down all the 2×2 matrices in echelon form whose entries are either 0 or 1. Mark which ones are in reduced echelon form.

Solution.

2   Converting to Reduced Echelon Form

(10 pts) Convert the following matrix into reduced echelon form. Write down which columns are pivot columns.

$$\begin{bmatrix} 5 & 2 & 9 \\ 7 & 3 & 12 \\ 2 & 1 & 3 \end{bmatrix}$$

Solution.

3 Switching Free and Bound Variables

Consider the following augmented matrix in reduced echelon form.

$$\begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

A. (5 pts) Write down the solution to this system in general form.

B. (5 pts) Rewrite the solution in general form so that x_1 and x_3 are free (and x_2 and x_4 are written in terms of x_1 and x_3).

C. (5 pts) Write down the solution from part B as an augmented matrix (that is, your general form solution from part B should have two linear equations, you should rearrange these equations and write them as an augmented matrix). Write down the row operation of the form $R_i \leftarrow R_i + cR_j$ which converts the matrix from this part to the one above.

Solution.

4 Linear Combinations

(10 pts) Is the vector $\begin{bmatrix} 0 \\ -3 \\ -20 \end{bmatrix}$ in the span of $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$? Justify your answer.

Solution.

5 Creating New Spans

(15 pts) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

Write vectors $\mathbf{v}_3$ and $\mathbf{v}_4$ such that $\mathbf{v}_3$ is not in $\mathsf{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ and $\mathbf{v}_4$ is not in $\mathsf{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

Solution.

6 (Programming) Gaussian Elimination

(20 pt) As you might have guessed, this week you will be implementing Gaussian elimination. The point of this exercise is to wrestle with the pseudocode and turn it into working Python code. By the end, you should have a python program which you can use to solve the systems of linear equations we see in this course.

You are given starter code in the file `hw02prog.py`. Please do not change the name of this file when you submit. Also do not change any of the names of the functions included in the starter code. You may add your own functions, but you are not expected to. All you are required to do is **fill in the TODO items in the starter code**.

You will not be expected to handle floating point error perfectly. This is the purview of numerical analysis, and although it is extremely important in real world applications of linear algebra, it is quite difficult, and not as important as understanding the algorithm.

Some guidelines for this assignment (and others as well):

- Use the provided functions when possible. In particular, `zero_in_pivot_column` covers a bit of floating-point error.

- Take a look at the functions in the numpy library, it's good practice to build of your knowledge of a library as you use it. In particular, look at `np.nonzero` (e.g., look at the online documentation), it will be useful.

- Work incrementally. Don't try to implement the entire program in one go and then debug.

There are two test cases in the starter code. These are, of course, not exhaustive. Also note that if you want to do more tests, you can load text files of numpy matrices using `np.loadtxt` (see the documentation).

You will upload a single python file to Gradescope with your implementations of the required functions. We will be running autograding tests on your submission to determine its correctness. **You will not have access to the autograder tests.**