CSE 543T Algorithms for Nonlinear Optimization: Homework 2

Due: Nov 8, Thursday, 4pm

1. **Problem 3.1.1.a) and 3.1.1.b)** . For both problems, you need to use the Second Order Sufficiency Condition in Proposition 3.2.1 to verify that your solution is indeed a local minimum. (15%)

3.1.1

Use the Lagrange multiplier theorem to solve the following problems:

(a) $f(x) = ||x||^2$, $h(x) = \sum_{i=1}^{n} x_i - 1$.

(b) $f(x) = \sum_{i=1}^{n} x_i$, $h(x) = ||x||^2 - 1$.

Proposition 3.2.1: (Second Order Sufficiency Conditions) Assume that f and h are twice continuously differentiable, and let $x^* \in \Re^n$ and $\lambda^* \in \Re^m$ satisfy

$$\nabla_x L(x^*, \lambda^*) = 0, \qquad \nabla_\lambda L(x^*, \lambda^*) = 0, \tag{3.31}$$

$$y' \nabla^2_{xx} L(x^*, \lambda^*) y > 0, \qquad \text{for all } y \neq 0 \text{ with } \nabla h(x^*)' y = 0. \tag{3.32}$$

Then x^* is a strict local minimum of f subject to $h(x) = 0$. In fact, there exist scalars $\gamma > 0$ and $\epsilon > 0$ such that

$$f(x) \geq f(x^*) + \frac{\gamma}{2} ||x - x^*||^2, \qquad \forall\, x \text{ with } h(x) = 0 \text{ and } ||x - x^*|| < \epsilon.$$

2. **Industrial design.** A cylindrical can is to hold 4 cubic inches of orange juice. The cost per square inch of constructing the metal top and bottom is twice the cost per square inch of constructing the cardboard side. What are the dimensions of the least expensive can? (15%)

3. **Duality.** Read Section 3.4 and study Example 3.4.2. Prove that the following two linear programs are dual to each other

$$\text{Min } c'x, \text{ subject to } A'x \geqslant b$$

$$\text{Max } b'\mu, \text{ subject to } A\mu = c, \ \mu \geqslant 0 \qquad (15\%)$$

4. **Problem 4.2.1 (a) (b) and (d)** (15%)

Hint: The augmented Lagrangian function with quadratic penalty is described in pages 398-404.

5. **Mathematical modeling for data mining.** (40%)

Linear regression is one of the fundamental models for data mining. The model describes a linear relationship between a number of numerical attributes $\mathbf{x} = (x_1, x_2, ..., x_n)$ and a predicted variable y in the form of

$$y = \boldsymbol{a}'\mathbf{x} + b,$$

where $\mathbf{a} \in R^n$ and $b \in R$ are parameters to be determined by *training*. The training process takes a set of K training examples

$$(\mathbf{X}, Y) = \{(\mathbf{x}^1, y^1), (\mathbf{x}^2, y^2), ..., (\mathbf{x}^K, y^K)\},$$

where each $\mathbf{x}^i \in R^n$ is a vector of attributes. The parameters $\mathbf{a}$ and b are determined by minimizing the mean squared error (MSE):

$$\text{MSE} = \sum_{i=1}^{K} [y^i - (\mathbf{a}'\mathbf{x}^i + b)]^2$$

Build a linear regression for the following **program effort data.** Each training sample consists of an index of social setting, an index of family planning effort, and the percentage change in the crude birth rate (CBR) between 1965 and 1975, for 20 countries in Latin America. Here, we want to predict *change* (y) using *setting* (x_1) and *effort* (x_2). Therefore, we have that n = 2 and K =20.

	setting(x_1)	effort(x_2)	change(y)
Bolivia	46	0	1

Brazil	74	0	10
Chile	89	16	29
Colombia	77	16	25
CostaRica	84	21	29
Cuba	89	15	40
DominicanRep	68	14	21
Ecuador	70	6	0
ElSalvador	60	13	13
Guatemala	55	9	4
Haiti	35	3	0
Honduras	51	7	7
Jamaica	87	23	21
Mexico	83	4	9
Nicaragua	68	0	7
Panama	84	19	22
Paraguay	74	3	6
Peru	73	0	2
TrinidadTobago	84	15	29
Venezuela	91	7	11

Write an AMPL model for the optimization problem, and submit it to NEOS to obtain the optimal parameters **a** and b in the linear regression model. You need to choose a suitable solver in NEOS. You cannot use any other existing software for linear regression. Submit the following:

1) The AMPL model file (and data file, if any)
2) A print-out of the solution from your NEOS solver.
3) A table listing the model error $y^i - (\mathbf{a}'\mathbf{x}^i + b)$ for all the 20 countries.
4) Discuss the insights you gained from this analysis, such as: How does each attribute influence the change? Which attribute seems to have stronger correlation with the change? Does the linear regression model seem accurate to you?