Hintikka's World: scalable higher-order knowledge

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Abstract

Hintikka's World is a tool that shows how artificial agents can reason about higher-order knowledge (agent a knows that agent b knows that...). In this demonstration paper, we present symbolic models that enables to implement in *Hintikka's World* large examples such as real card games.

1 Introduction

The current trend is to construct programs that play games with imperfect information, for instance Hanabi [Bard *et al.*, 2019], but also video games such as Starcraft 2 [Hu *et al.*, 2018]. An important ingredient is to reason about higher-order knowledge (an agent knows that another agents knows that...). In these systems, epistemic logic and its dynamic extension, Dynamic epistemic logic ([Baltag *et al.*, 1998], [van Ditmarsch *et al.*, 2008]) may offer formal tools for providing explanations in such AI programs. needs to be understood is relevant in AI, especially in strategic reasoning [Aumann, 1999].

The only pedagogical tool we are aware of that explains such models is *Hintikka's world* and was presented at ECAI-IJCAI 2018 [Schwarzentruber, 2018]. *Hintikka's world* is a proof of concept of a graphical user interface that represent Kripke models by comic strips, as shown in Figure 1. It enables to explore mental states of agents. The tool is available at the following address: http://hintikkasworld.irisa.fr/.

Kripke models are graphs, represented explicitly in memory in the first version of the tool. Explicit models are useful to learn how dynamic epistemic logic works by means of toy examples: muddy children, Sally and Anne [Wimmer and Perner, 1983], etc. However, in real card games, such as Hanabi, there are an exponential number of possible configurations of cards. For instance Hanabi has 50 cards total and each player has 4 cards and the order of the cards is important. Therefore with 4 players, the initial Kripke model features $50 \times 49 \times 48 \times \cdots \times 38$ configurations, that is 2.2×10^{21}. Thus, it is impossible to represent explicitly the full graph in memory: the first version of Hintikka's world does not *scale*.

That is why, in this demonstration, we propose to represent Kripke models symbolically by using the approach in [Charrier and Schwarzentruber, 2017] and [Charrier and Schwarzentruber, 2018]. The implementation relies on Binary Decision Diagrams (BDDs) [Bryant, 1986] as it was done in the tool DEMO[1] [van Benthem *et al.*, 2015].

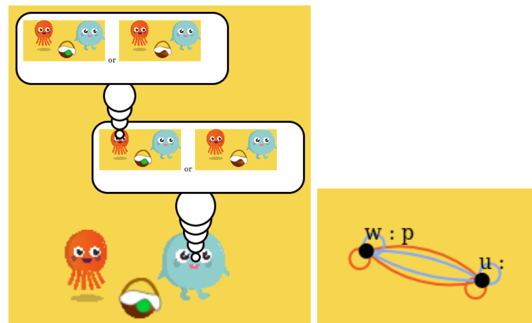

Figure 1: Graphical user interface of *Hintikka's world*.

Figure 2: Screenshot of Hanabi in *Hintikka's World*.

2 Demonstration Outline

In the demonstration we show a play of the game Hanabi. In this game, each agent has cards with a color and a number, but cannot see his own hand. In each turn, the user can play the role of one of the agent: he/she can either give the information to some other agent about a number or a color, or play a card. The goal is to play the cards in increasing order for each color. During the process, the system keeps track on the knowledge of the agents. More precisely, the system shows the real world (the real distribution of the cards). When the user clicks on an agent, it displays *some* randomly generated possible worlds for that agent (some possible distributions for him/her), as shown in Figure 2. Note that in this demonstration, we will continue to show examples that were already presented in 2018 [Schwarzentruber, 2018]: Sally and Anne, muddy children, consecutive numbers, etc.

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[1]The current implementation does not rely on DEMO since their work is not well-suited for a web application.

3 Symbolic models

In our tool, we definitely emphasize on the use of model checking over theorem proving, as advocated in [Halpern and Vardi, 1991]. More precisely, we use the same ideas than in symbolic model checking, as defined for temporal logics [Burch *et al.*, 1990], adapted to DEL, as explained in [Charrier and Schwarzentruber, 2017] and [Charrier and Schwarzentruber, 2018]. Our model checking procedure relies now on symbolic Kripke models, aimed at representing succinctly so-called pointed Kripke models. A pointed Kripke model is a graph whose nodes are *possible worlds*, edges are labeled by agents and an edge $w \rightarrow^a u$ means that agent a considers world u as possible in world w. Each world w is equipped with a valuation telling which atomic propositions is true in w. A special world is called the pointed world and represents the true situation, while the other possible worlds are worlds imagined by the agents. The tool shows that graph in the right-part of the screen (in the example of Figure 1, the Kripke model has two possible worlds, w and u, p is true in w but not in u, and $\rightarrow_a$ is given in red and $\rightarrow_b$ in blue).

A symbolic model gives a Boolean formula $\chi(\vec{x})$ that succinctly describes the set of possible worlds: a world is a valuation over Boolean variables $\vec{x}$ satisfing $\chi(\vec{x})$. It also gives, for each agent a, a Boolean formula $\pi_a(\vec{x}, \vec{x}')$ that tells whether there is an edge labeled by agent a from a world described by a valuation over $\vec{x}$ and a world described by a valuation over $\vec{x}'$. All these Boolean formula are then classically converted in BDDs.

Typically, for Hanabi, $\chi(\vec{x})$ tells that $\vec{x}$ describes an initial possible configuration. Formula $\pi_a(\vec{x}, \vec{x}')$ tells that the agents diferent from a have the same card in $\vec{x}$ and $\vec{x}'$ (it models the fact that agent a sees the cards of the other players).

Dynamic epistemic logic also provides so-called *event models* for describing actions (public announcements, public actions, private announcements/actions, etc.). The reader may refer to the textbook on DEL [van Ditmarsch *et al.*, 2008] and to [Charrier and Schwarzentruber, 2017] for symbolic event models, that we do not detail here.

4 System Description

Whereas the first version was written in Javascript, the new version is written in TypeScript and Angular 7.

4.1 Binary decision diagrams

The symbolic model checking of DEL is PSPACE-complete, thus is critical. We manipulate set of worlds, and relations by means of Binary decision diagrams. That is why, for manipulating the binary decision diagrams, we wrote a wrapup in C of the library CUDD [], that produces a Web Assembly library.

In order to show possible worlds for a given agent a in some world w, we first construct the BDD of $\pi_a(descr(w), \vec{x}')$ where $descr(w)$ are the Boolean values of $\vec{x}$ corresponding to world w. We then count the number of possible valuations $\vec{x}'$ than makes for randomly values for $\vec{x}'$ that makes $\pi_a(descr(w), \vec{x}')$ true (BDDs are convenient for counting, see TODO). If the number of such valuations is small, we show all the possible worlds, otherwise we randomly generate valuations for $\vec{x}'$ that lead to a true-leaf in the BDD of $\pi_a(descr(w), \vec{x}')$ (BDDs are also convenient for generating random valuations).

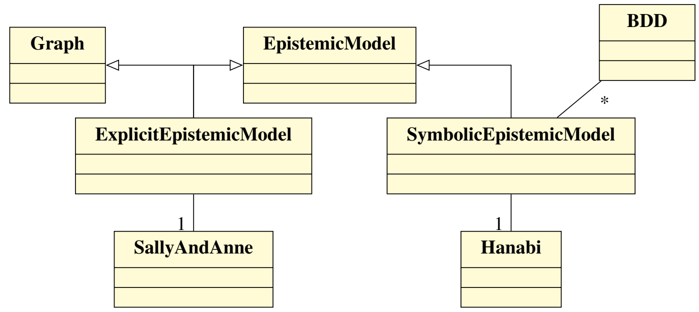

Figure 3: New architecture of *Hintikka's world*.

4.2 Class Architecture

Figure 3 shows the new architecture of *Hintikka's world*. `EpistemicModel` is an abstract class used by the graphical user interface (GUI), that is independent from the current runnin example (muddy children, Sally and Anne, Hanabi, etc.) but more interestingly independent from the representation of the epistemic model itself. In particular, an epistemic model can be an `ExplicitEpistemicModel` (a graph) or a `SymbolicEpistemicModel` that relies on BDDs, depending on the examples.

4.3 Adding new examples

The system is easy to use to provide new examples. Explicit epistemic models are directly described (set of nodes and of edges). Symbolic epistemic models are described by a Boolean formula χ, or Boolean formulas for π_a. The system provides a way to easily describe how worlds are displayed in the comic strips.

5 Future Work

In the future, we plan to implement many examples. We plan to extend our implementation to Algebraic decision diagrams (ADD) [Bahar *et al.*, 1997] in order to tackle examples with numerical values, and not only Boolean values.

We aim at studying the use of counting and sampling specific techniques (see for instance [Meel *et al.*, 2016]) for generating possible worlds.

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